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Reliability Analysis of Exponentiated Exponential Distribution for Neoteric and Ranked Sampling Designs with Applications

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Abstract The neoteric ranked set sampling (NRSS) scheme is an effective design compared to the usually ranked set sampling (RSS) scheme. Herein, we regard reliability estimation of the stress-strength (SS) model using the maximum likelihood procedure via NRSS and RSS designs. Assume that stress Y and strength X are exponentiated exponential random variables with the same scale parameter. Various sample strategies are used to evaluate the reliability estimator. We acquire an estimate of R when the samples of stress and strength random variables are chosen from the same sampling methods, such as RSS or NRSS. Furthermore, we derive R estimator when X and Y are chosen from RSS and NRSS, respectively, and vice versa. A simulation investigation is formed to assay and compare the accuracy of estimates for all proposed schemes. We conclude based on study outcomes that the reliability estimates of the stress-strength model via NRSS are more efficient than the others via RSS. Analysis of real data is displayed to investigate the usefulness of the proposed estimators.

Keywords Exponentiated Exponential distribution, Neoteric ranked set sampling, Maximum likelihood method, Relative Efficiency

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1. Introduction

[26] proposed the RSS technique for finding a more informative sample to estimate the population mean than the commonly used simple random sample (SRS). In this case, visual examination of the study variable or the use of auxiliary variables can be used to rank a small group of elected units. RSS is employed when precise sample measurements, such as for environmental management, ecology, sociology, and agriculture, for a given unit is either problematic or expensive and time-consuming. The following is the technique for enrolling a sample under RSS:

Allocate n^2 randomly selected units from the population into n sets, each of size n. Without yet knowing any values for the variable of study, we sort the units within each set according to the interesting variable based on personal professional experience or on a concomitant variable associated with the variable of interest. Determine a sample for actual quantification by including the smallest ordered unit in the first set, the second smallest ordered unit in set two, and continue the operation in this path until the biggest ordered unit is chosen from the ultimate set. Repeating this process h times to acquire a sample of size $N = nh$ for the actual measurement.

The NRSS scheme was prepared in [36] and it differs from the original RSS scheme. This design presents more efficient estimators for the population mean and variance than SRS and RSS schemes. The NRSS method is characterized as follows: Allocate n^2 randomly selected units from the desired population and ranked the sample items based on the pre-determined sorting procedure.

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For odd n, choose the $[((n+1)/2)+(\varsigma-1)n]^{th}$ ranked value for $\varsigma=1,\ldots,n$. For n is even select the$[J+(\varsigma-1)n]^{th}$ ranked unit, where$J=(n/2)$ if ς is an even and $J=((n+2)/2)$ if ς is an odd for $\varsigma=1,\ldots,n$. Repeating this process h times to gain a sample with size $N=nh$. For more detailed information about RSS; [34] mentioned that the mean of the RSS is an unbiased estimator of the population mean as well as it is more efficient than SRS. [6] discussed maximum likelihood (ML) and a modified ML estimator of exponential distribution using moving extremes RSS. [17] provided tests for exponentiated Pareto distribution using an extreme RSS scheme. [18] discussed ML and Bayesian estimators for exponentiated exponential distribution (EED) using the RSS method. L ranked set sampling was discussed as a method for estimating the distribution function in [3]. [36] presented a generalization of the RSS for estimating the population mean and variance. [16] presented an unbiased sampling approach called paired double RSS for estimating the population mean. Some entropy estimators of a continuous random variable using different sampling schemes were proposed in [4]. [31] used the NRSS scheme to estimate the inverse Weibull distribution parameters. [9] handled the data with Zubair Lomax parameter estimators under RSS. [10] introduced and discussed parameter estimators of half logistic inverted Topp-Leone distribution from RSS scheme. [12] examined the ML estimator for the scale family of distributions viz some RSS schemes.

The SS model, in its straightforward form, assigns the reliability of a unit as the probability that the strength of the unit (X) is greater than the stress (Y) forced on it. The SS model is represented as $R=P(Y<X)$, where R is the reliability parameter that has many applications in physics, civil, mechanical, and aerospace engineering. The SS model was first proposed in [11]. Many authors have worked on estimating the SS reliability (SSR) model when both X and Y follow certain distributions based on SRS. The reader can refer to [8, 25, 30, 23]. An excellent review of the SS model can be found in [24].

Recently, the inference of the SS model has been evaluated by several researchers based on RSS and its modifications. [27] discussed the estimation of the SSR when strength X and stress Y are two independent exponential distributions under RSS. [22] argued for the estimation of the SSR from generalized inverted exponential distributions under RSS. [20, 21] provided an SSR estimator for independent Burr XII distributions under several RSS schemes. [1] gave the SSR estimator for independent Weibull distributions under RSS. The SSR estimator for independent Lindley populations was developed in [2]. [33] obtained the SSR when Y and X are independent exponential distributions under RSS. [5] discussed the SSR when X and Y are exponentiated Pareto distributions. [13] considered the SSR when both X and Y are generalized exponential distributions under RSS. [19] discussed the SSR when X and Y are generalized inverted exponential distributions. [7] investigated the SSR when both X and Y are Topp-Leone distributions under RSS and MRSS. [32] investigated SSR for proportional reversed hazard rate model via lower record RSS.

[14] presented a generalization of the exponential distribution with an extra parameter, named as the EED. The EED is popularly used in analyzing lifetime or survival data. The cumulative distribution function (cdf) and the probability density function (pdf) of the EED with scale parameter θ and shape parameter ω are given, respectively, by

$$G(x;\omega,\theta)=(1-e^{-\theta x})^{\omega}; \qquad x,\omega,\theta>0, \tag{1}$$

and,

$$g(x;\omega,\theta)=\theta\omega e^{-\theta x}(1-e^{-\theta x})^{\omega-1}; \qquad x,\omega,\theta>0. \tag{2}$$

Many researchers have looked into the properties and applications of the EED, including [29, 15, 28].
Let X have the EED with parameters (ω,θ) and Y be another EED with parameters(δ,θ), where X and Y are independent. The SSR is discussed under the claim that both X and Y have common scale parameter and dissimilar shape parameters,*i.e.*; $X\sim EED(\omega,\theta)$ and $Y\sim EED(\delta,\theta)$. Hence, the SSR is given by

$$\begin{aligned} R=P\left[Y<X\right]&=\int_0^\infty G_y(x)g(x)dx=\int_{x=0}^\infty\int_{y=0}^x\delta\omega\theta^2e^{-\theta(y+x)}\left(1-e^{-\theta x}\right)^{\omega-1}\left(1-e^{-\theta y}\right)^{\delta-1}dydx\\ &=\int_0^x\omega\theta e^{-\theta x}\left(1-e^{-\theta x}\right)^{\omega+\delta-1}dx=\frac{\omega}{\delta+\omega}. \end{aligned} \tag{3}$$

There have been no previous studies that used the NRSS design to estimate SSR. The key contribution here is investigated reliability estimator of $R=P\left[Y<X\right]$, where the strength $X\sim EED(\omega,\theta)$ and the stress $Y\sim EED(\delta,\theta)$ are independent variables via RSS and NRSS. As a consequence, we examine the reliability estimator using four sampling designs:$R=P\left[Y_{RSS}<X_{RSS}\right]$,$R=P\left[Y_{NRSS}<X_{NRSS}\right]$,$R=P\left[Y_{RSS}<X_{NRSS}\right]$ and

$R = P\left[Y_{NRSS} < X_{RSS}\right]$. To examine the efficiency of the SSR and to analyze how these estimates behave for various sample sizes, statistical analysis via simulation research is undertaken comparing these schemes. The accuracy of these estimates is assessed in this context using some measures of accuracy.

The remainder of this essay will be presented in the following manner. The estimator of SSR is supplied using RSS in Section 2, and the subsequent sections are planned as follows. The SSR estimator is derived in Section 3 when the needed samples of *X* are in the form of RSS and the samples of *Y* are drawn from NRSS, and vice versa. Section 4 contains an NRSS-based SSR estimator. Sections 5, and 6 deal with the simulation technique and real-data analysis, respectively. Section 7 brings the paper to a close.

2. Estimator of R Using RSS

Herein, the SSR estimator of the system is obtained using the ML method. The SSR estimator is argued assuming that the selected samples of stress and strength based on the same RSS design.

Suppose that $X = \{x_{\varsigma(\varsigma)t}, \varsigma = 1, 2, \ldots n, t = 1, 2, \ldots h_x\}$ be a chosen RSS from EED$(\omega, \theta)$with sample size *N* =nh_x, *n* is the set size and $h_x$is the number of cycles. Let $Y = \{y_{\tau(\tau)a}, \tau = 1, 2, \ldots m, a = 1, 2, \ldots h_y\}$ be a chosen RSS from EED(δ, θ), with sample size *M*=mh_y, such that *m* is the set size and $h_y$is the number of cycles. The likelihood function of stress and strength under RSS is

$$\begin{aligned} L_1 = C_1 C_2 \prod_{t=1}^{h_x} \prod_{\varsigma=1}^{n} g(x_{\varsigma(\varsigma)t}) \left[G(x_{\varsigma(\varsigma)t})\right]^{\varsigma-1} \left[1 - G(x_{\varsigma(\varsigma)t})\right]^{n-\varsigma} \\ \prod_{a=1}^{h_y} \prod_{\tau=1}^{m} g(y_{\tau(\tau)a}) \left[G y_{\tau(\tau)a})\right]^{\tau-1} \left[1 - G(y_{\tau(\tau)a})\right]^{m-\tau} \end{aligned},$$

where, $C_1 = \prod_{t=1}^{h_x} \prod_{\varsigma=1}^{n} \frac{n!}{(\varsigma-1)!(n-\varsigma)}$, and $C_2 = \prod_{a=1}^{h_y} \prod_{\tau=1}^{m} \frac{m!}{(\tau-1)!(m-\tau)}$.

Then, the log-likelihood function

$$\begin{aligned} \ln L_1 =& N \ln C_1 + M \ln C_2 + (N+M) \ln \theta + N \ln \omega + M \ln \delta + \sum_{t=1}^{h_x} \sum_{\varsigma=1}^{n} (\omega\varsigma - 1) \ln(1 - e^{-\theta x_{\varsigma(\varsigma)t}}) \\ &+ \sum_{a=1}^{h_y} \sum_{\tau=1}^{m} (\delta\tau - 1) \ln(1 - e^{-\theta y_{\tau(\tau)a}}) + \sum_{t=1}^{h_x} \sum_{\varsigma=1}^{n} (n - \varsigma) \ln(1 - (1 - e^{-\theta x_{\varsigma(\varsigma)t}})^{\omega}) \\ &- \sum_{t=1}^{h_x} \sum_{\varsigma=1}^{n} \theta x_{\varsigma(\varsigma)t} - \sum_{a=1}^{h_y} \sum_{\tau=1}^{m} \theta y_{\tau(\tau)a} + \sum_{a=1}^{h_y} \sum_{\tau=1}^{m} (m - \tau) \ln(1 - (1 - e^{-\theta y_{\tau(\tau)a}})^{\delta}). \end{aligned}$$

The first derivative of L_1 with respect to θ, ω and $\delta$are given by:

$$\begin{aligned} \frac{\partial \ln L_1}{\partial \theta} =& \frac{N+M}{\theta} + \sum_{t=1}^{h_x} \sum_{\varsigma=1}^{n} \frac{(\omega\varsigma - 1) x_{\varsigma(\varsigma)t}}{e^{\theta x_{\varsigma(\varsigma)t}} - 1} + \sum_{a=1}^{h_y} \sum_{\tau=1}^{m} \frac{(\delta\tau - 1) y_{\tau(\tau)a}}{e^{\theta y_{\tau(\tau)a}} - 1} - \sum_{t=1}^{h_x} \sum_{\varsigma=1}^{n} x_{\varsigma(\varsigma)t} - \sum_{a=1}^{h_y} \sum_{\tau=1}^{m} y_{\tau(\tau)a} \\ &- \sum_{t=1}^{h_x} \sum_{\varsigma=1}^{n} \frac{\omega(n-\varsigma) e^{-\theta x_{\varsigma(\varsigma)t}} x_{\varsigma(\varsigma)t} (1 - e^{-\theta x_{\varsigma(\varsigma)t}})^{\omega-1}}{1 - (1 - e^{-\theta x_{\varsigma(\varsigma)t}})^{\omega}} - \sum_{a=1}^{h_y} \sum_{\tau=1}^{m} \frac{\delta(m-\tau) e^{-\theta y_{\tau(\tau)a}} y_{\tau(\tau)a} (1 - e^{-\theta y_{\tau(\tau)a}})^{\delta-1}}{1 - (1 - e^{-\theta y_{\tau(\tau)a}})^{\delta}}, \end{aligned} \quad (4)$$

$$\frac{\partial \ln L_1}{\partial \omega} = \frac{N}{\omega} + \sum_{t=1}^{h_x} \sum_{\varsigma=1}^{n} \left\{ \varsigma \ln(1 - e^{-\theta x_{\varsigma(\varsigma)t}}) - \frac{(n-\varsigma) \ln(1 - e^{-\theta x_{\varsigma(\varsigma)t}})}{(1 - e^{-\theta x_{\varsigma(\varsigma)t}})^{-\omega} - 1} \right\}, \quad (5)$$

and,

$$\frac{\partial \ln L_1}{\partial \delta} = \frac{M}{\delta} + \sum_{a=1}^{h_y}\sum_{\tau=1}^{m}\left[\tau\ \ln(1-e^{-\theta y_{\tau(\tau)a}}) - \frac{(m-\tau)\ln(1-e^{-\theta y_{\tau(\tau)a}})}{(1-e^{-\theta y_{\tau(\tau)a}})^{-\delta}-1}\right]. \tag{6}$$

We get the ML estimators of θ, ω and δ after setting Equations (4)-(6) with zero and solving numerically. Then the ML estimator of *R* is obtained, based on invariance property of ML method, by inserting the ML of population parameters in (3).

3. Estimator of R via NRSS with RSS

Here, the SSR estimator is obtained in two different cases. In the first state, the ML estimator of *R* is obtained when *X* and *Y* observations are selected from NRSS and RSS respectively. In the second case, we derive the SSR estimator when *X* and *Y* samples are chosen via RSS and NRSS respectively.

3.1. Likelihood Function of $R = P(X_{NRSS} < Y_{RSS})$

Let $X = \{x_{d(\varsigma)t}, \varsigma = 1, ..., n; t = 1, ..., h_x\}$ is a NRSS selected from EED(ω, θ), with sample size N=nh_x,where n is the set size and h_x is the number of cycles. The likelihood function of *X* based on NRSS (see [31]) is given by

$$L_2 = \prod_{t=1}^{h_x} C_3 \prod_{\varsigma=1}^{n} g(x_{d(\varsigma)t}) \prod_{\varsigma=1}^{n+1}[G(x_{d(\varsigma)t}) - G(x_{d(\varsigma-1)t})]^{d(\varsigma)-d(\varsigma-1)-1}],$$

where, $C_3 = \frac{n!}{\prod_{\varsigma=1}^{n+1}(d(\varsigma)-d(\varsigma-1)-1)!}$, $\quad d\,(0) \ = 0, d\,(n+1) = n+1, \quad x_{(d(0))} = -\infty \ \ and \ \ x_{(d(\varsigma+1))} = \infty.$
Consider $Y = \{y_{\tau(\tau)a}, \tau = 1, 2, ..., m; a = 1, 2, ..., h_y\}$ is the RSS taken from EED(δ, θ), with sample size $M = mh_y$, such that *m* and h_y are set size and number of cycles, respectively. The likelihood function of stress and strength is given by

$$L_2 = C_2 \prod_{t=1}^{h_x}\left[C_3 \prod_{\varsigma=1}^{n} g(x_{d(\varsigma)t}) \prod_{\varsigma=1}^{n+1}[G(x_{d(\varsigma)t}) - G(x_{d(\varsigma-1)t})]^{d(\varsigma)-d(\varsigma-1)-1}]\right]$$
$$\prod_{a=1}^{h_y}\prod_{\tau=1}^{m} g(y_{\tau(\tau)a}) \left[G(y_{\tau(\tau)a}\right]^{\tau-1}\left[1-G(y_{\tau(\tau)a}\right]^{m-\tau}.$$

Then, the log-likelihood function, say L_2, of observed samples is given by

$$\begin{aligned}\ln L_2 = {} & N\,(\ln C_3 + \ln\theta + \ln\omega) + M\,(\ln C_2 + \ln\theta + \ln\delta) + \sum_{t=1}^{h_x}\sum_{\varsigma=1}^{n}(\omega-1)\ln(1-e^{-\theta x_{d(\varsigma)t}}) \\ & - \sum_{t=1}^{h_x}\left[\sum_{\varsigma=1}^{n}\theta x_{d(\varsigma)t} - \sum_{\varsigma=1}^{n+1}[d(\varsigma)-d(\varsigma-1)-1]\ln[(1-e^{-\theta x_{d(\varsigma)t}})^{\omega} - (1-e^{-\theta x_{d(\varsigma-1)t}})^{\omega}]\right] \\ & + \sum_{a=1}^{h_y}\sum_{\tau=1}^{m}\left[(\delta\tau-1)\ln(1-e^{-\theta y_{\tau(\tau)a}}) - \theta y_{\tau(\tau)a} + (m-\tau)\ln(1-(1-e^{-\theta y_{\tau(\tau)a}})^{\delta})\right].\end{aligned}$$

The partial derivative of L_2 with respect to δ is derived in Equation (6). The partial derivatives with respect to θ and ω are as follows:

$$
\begin{aligned}
\frac{\partial \ln L_2}{\partial \theta} &= \frac{N+M}{\theta} + \sum_{t=1}^{h_x}\sum_{\varsigma=1}^{n} \frac{(\omega-1)x_{d(\varsigma)t}}{e^{\theta x_{d(\varsigma)t}}-1} + \sum_{a=1}^{h_y}\sum_{\tau=1}^{m} \frac{(\delta\tau-1)y_{\tau(\tau)a}}{e^{\theta y_{\tau(\tau)a}}-1} \\
&+ \sum_{t=1}^{h_x}\sum_{\varsigma=1}^{n+1} \frac{\omega[d(\varsigma)-d(\varsigma-1)-1]\left[(1-e^{-\theta x_{d(\varsigma)t}})^{\omega-1}x_{d(\varsigma)t}e^{-\theta x_{d(\varsigma)t}} - (1-e^{-\theta x_{d(\varsigma-1)t}})^{\omega-1}x_{d(\varsigma-1)t}e^{-\theta x_{d(\varsigma-1)t}}\right]}{(1-e^{-\theta x_{d(\varsigma)t}})^{\omega} - (1-e^{-\theta x_{d(\varsigma-1)t}})^{\omega}} \\
&- \sum_{a=1}^{h_y}\sum_{\tau=1}^{m} \frac{\delta(m-\tau)y_{\tau(\tau)a}e^{-\theta y_{\tau(\tau)a}}(1-e^{-\theta y_{\tau(\tau)a}})^{\delta-1}}{1-(1-e^{-\theta y_{\tau(\tau)a}})^{\delta}} - \sum_{t=1}^{h_x}\sum_{\varsigma=1}^{n} x_{d(\varsigma)t} - \sum_{a=1}^{h_y}\sum_{\tau=1}^{m} y_{\tau(\tau)a},
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
\frac{\partial \ln L_2}{\partial \omega} &= \frac{N}{\omega} + \sum_{t=1}^{h_x}\sum_{\varsigma=1}^{n} \ln(1-e^{-\theta x_{d(\varsigma)t}}) \\
&+ \sum_{t=1}^{h_x}\sum_{\varsigma=1}^{n+1} \frac{[d(\varsigma)-d(\varsigma-1)-1]\left[(1-e^{-\theta x_{d(\varsigma)t}})^{\omega}\ln(1-e^{-\theta x_{d(\varsigma)t}}) - (1-e^{-\theta x_{d(\varsigma-1)t}})^{\omega}\ln(1-e^{-\theta x_{d(\varsigma-1)t}})\right]}{(1-e^{-\theta x_{d(\varsigma)t}})^{\omega} - (1-e^{-\theta x_{d(\varsigma-1)t}})^{\omega}}.
\end{aligned}
\tag{8}
$$

The ML estimators of θ, ω and δ are obtained after putting (6), (7) and (8) equal zero and solving numerically via iterative technique. Consequently, the SSR estimator is obtained using (3).

3.2. Likelihood Function of R=P$(X_{RSS} < Y_{NRSS})$

Suppose that $X = \{x_{\varsigma(\varsigma)t}, \varsigma = 1, 2, ..., n; t = 1, 2, ..., h_x\}$is the allowed RSS from EED$(\omega, \theta)$with sample size *N*=nh_x. Let $Y = \{y_{d(\tau)a}, \tau = 1, 2, ..., m; a = 1, 2, ..., h_y\}$ is elected NRSS from EED(δ, θ) with sample size *M*=$mh_{y.}$. Therefore, the likelihood function of the observed samples is given by

$$
\begin{aligned}
L_3 = C_1 \prod_{t=1}^{h_x}\prod_{\varsigma=1}^{n} & g(x_{\varsigma(\varsigma)t})\left[G(x_{\varsigma(\varsigma)t}\right]^{\varsigma-1}\left[1-G(x_{\varsigma(\varsigma)t}\right]^{n-\varsigma} \\
& \prod_{a=1}^{h_y}\left[C_4 \prod_{\tau=1}^{m} g(y_{d(\tau)a}) \prod_{\tau=1}^{m+1}[G(y_{d(\tau)a}) - G(y_{d(\tau-1)a})]^{d(\tau)-d(\tau-1)-1}\right],
\end{aligned}
$$

where,

$$
C_4 = \frac{m!}{\prod_{\tau=1}^{m+1}(d(\tau)-d(\tau-1)-1)!},\ d\,(0)\ = 0, d\,(m+1) = m+1,\ y_{(d(0))} = -\infty\ and\ y_{(d(\tau+1))} = \infty.
$$

The log-likelihood function based on RSS and NRSS is formed as

$$
\begin{aligned}
\ln L_3 = N(\ln C_1 + \ln\theta + \ln\omega) &+ \sum_{t=1}^{h_x}\sum_{\varsigma=1}^{n}\left[(\omega\,\varsigma-1)\ln(1-e^{-\theta x_{\varsigma(\varsigma)\,t}}) - \theta x_{\varsigma(\varsigma)\,t}\right] + M\,(\ln C_4 + \ln\theta + \ln\delta) \\
&+ \sum_{t=1}^{h_x}\sum_{\varsigma=1}^{n}(n-\varsigma)\ln(1-(1-e^{-\theta x_{\varsigma(\varsigma)\,t}})^{\omega}) + \sum_{a=1}^{h_y}\sum_{\tau=1}^{m}(\delta-1)\ln(1-e^{-\theta y_{d(\tau)a}}) - \sum_{a=1}^{h_y}\sum_{\tau=1}^{m}\theta y_{d(\tau)a} \\
&+ \sum_{a=1}^{h_y}\sum_{\tau=1}^{m+1}[d(\tau)-d(\tau-1)-1]\ln[(1-e^{-\theta y_{d(\tau)a}})^{\delta} - (1-e^{-\theta y_{d(\tau-1)a}})^{\delta}].
\end{aligned}
$$

The partial derivative of ω is given in (5). The following equations provide partial derivatives of θ and δ as follows:

$$\frac{\partial \ln L_3}{\partial \theta} = \frac{N+M}{\theta} + \sum_{t=1}^{h_x}\sum_{\varsigma=1}^{n}\left[\frac{x_{\varsigma(\varsigma)\,t}(\omega\varsigma-1)}{e^{\theta x_{\varsigma(\varsigma)\,t}}-1} - \frac{\omega(n-\varsigma)e^{-\theta x_{\varsigma(\varsigma)\,t}}x_{\varsigma(\varsigma)\,t}(1-e^{-\theta x_{\varsigma(\varsigma)\,t}})^{\omega-1}}{1-(1-e^{-\theta x_{\varsigma(\varsigma)\,t}})^{\omega}} - x_{\varsigma(\varsigma)\,t}\right]$$
$$+\sum_{a=1}^{h_y}\sum_{\tau=1}^{m+1}\frac{\delta[d(\tau)-d(\tau-1)-1]\left[(1-e^{-\theta y_{d(\tau)a}})^{\delta-1}e^{-\theta y_{d(\tau)a}}y_{d(\tau)a}-(1-e^{-\theta y_{d(\tau-1)a}})^{\delta-1}e^{-\theta y_{d(\tau-1)a}}y_{d(\tau-1)a}\right]}{(1-e^{-\theta y_{d(\tau)a}})^{\delta}-(1-e^{-\theta y_{d(\tau-1)a}})^{\delta}}$$
$$+\sum_{a=1}^{h_y}\sum_{\tau=1}^{m}\left[\frac{(\delta-1)y_{d(\tau)a}}{e^{\theta y_{d(\tau)a}}-1}-y_{d(\tau)a}\right], \tag{9}$$

and,

$$\frac{\partial \ln L_3}{\partial \delta} = \frac{M}{\delta} + \sum_{a=1}^{h_y}\sum_{\tau=1}^{m}\ln(1-e^{-\theta y_{d(\tau)a}})$$
$$+\sum_{a=1}^{h_y}\sum_{\tau=1}^{m+1}\frac{[d(\tau)-d(\tau-1)-1]\left[(1-e^{-\theta y_{d(\tau)a}})^{\delta}\ln(1-e^{-\theta y_{d(\tau)a}})-(1-e^{-\theta y_{d(\tau-1)a}})^{\delta}\ln(1-e^{-\theta y_{d(\tau-1)a}})\right]}{(1-e^{-\theta y_{d(\tau)a}})^{\delta}-(1-e^{-\theta y_{d(\tau-1)a}})^{\delta}}. \tag{10}$$

Equations (5), (9) and (10) are very complicated to be solved, so we use numerical technique to gain the solution. Consequently, the SSR estimator is obtained using (3).

4. Reliability Estimator Based on NRSS

Consider $X = \{x_{d(\varsigma)t}, \varsigma = 1,2,...,n; t = 1,2,...,h_x\}$ is enrolled NRSS from EED $(\omega,\theta)$with sample size $N = nh_x$. Let $Y = \{y_{d(\tau)a}, \tau = 1,2,...,m; a = 1,2,...,h_y\}$ is a chosen NRSS from EED (δ,θ) with sample size $M = mh_y$. The likelihood function of the observed samples will be as follows:

$$L_4 = \prod_{t=1}^{h_x}\left[C_3\prod_{\varsigma=1}^{n}g(x_{d(\varsigma)t})\prod_{\varsigma=1}^{n+1}[G(x_{d(\varsigma)t})-G(x_{d(\varsigma-1)t})]^{d(\varsigma)-d(\varsigma-1)-1}\right]$$
$$\prod_{a=1}^{h_y}\left[C_4\prod_{\tau=1}^{m}g(y_{d(\tau)a})\prod_{\tau=1}^{m+1}[G(y_{d(\tau)a})-G(y_{d(\tau-1)a})]^{d(\tau)-d(\tau-1)-1}\right].$$

The log likelihood function, based on NRSS, is given by

$$\ln L_4 = N(\ln C_3+\ln\theta+\ln\omega)+\sum_{t=1}^{h_x}\sum_{\varsigma=1}^{n}\left[(\omega-1)\ln(1-e^{-\theta x_{d(\varsigma)t}})-\theta x_{d(\varsigma)t}\right]-\sum_{a=1}^{h_y}\sum_{\tau=1}^{m}\theta y_{d(\tau)a}$$
$$+\sum_{t=1}^{h_x}\sum_{\varsigma=1}^{n+1}[d(\varsigma)-d(\varsigma-1)-1]\ln[(1-e^{-\theta x_{d(\varsigma)t}})^{\omega}-(1-e^{-\theta x_{d(\varsigma-1)t}})^{\omega}]+\sum_{a=1}^{h_y}\sum_{\tau=1}^{m}(\delta-1)\ln(1-e^{-\theta y_{d(\tau)a}})$$
$$+M(\ln C_4+\ln\theta+\ln\delta)+\sum_{a=1}^{h_y}\sum_{\tau=1}^{m+1}[d(\tau)-d(\tau-1)-1]\ln[(1-e^{-\theta y_{d(\tau)a}})^{\delta}-(1-e^{-\theta y_{d(\tau-1)a}})^{\delta}].$$

Obtaining the partial derivatives with respect to δ and ω in (10) and (8) while the partial derivative of θ is obtained as follows:

$$\frac{\partial \ln L_4}{\partial \theta} = \frac{N+M}{\theta} + \sum_{t=1}^{h_x}\sum_{\varsigma=1}^{n}\left[\frac{(\omega-1)x_{d(\varsigma)t}}{e^{\theta x_{d(\varsigma)t}}-1} - x_{d(\varsigma)t}\right] + \sum_{a=1}^{h_y}\sum_{\tau=1}^{m}\left[\frac{(\delta-1)y_{d(\tau)a}}{e^{\theta y_{d(\tau)a}}-1} - y_{d(\tau)a}\right]$$
$$+\sum_{t=1}^{h_x}\sum_{\varsigma=1}^{n+1}\frac{\omega[d(\varsigma)-d(\varsigma-1)-1]\left[(1-e^{-\theta x_{d(\varsigma)t}})^{\omega-1}e^{-\theta x_{d(\varsigma)t}}x_{d(\varsigma)t} - (1-e^{-\theta x_{d(\varsigma-1)t}})^{\omega-1}e^{-\theta x_{d(\varsigma-1)t}}x_{d(\varsigma-1)t}\right]}{(1-e^{-\theta x_{d(\varsigma)t}})^{\omega} - (1-e^{-\theta x_{d(\varsigma-1)t}})^{\omega}}$$
$$+\sum_{a=1}^{h_y}\sum_{\tau=1}^{m+1}\frac{\delta[d(\tau)-d(\tau-1)-1]\left[(1-e^{-\theta y_{d(\tau)a}})^{\delta-1}e^{-\theta y_{d(\tau)a}}y_{d(\tau)a} - (1-e^{-\theta y_{d(\tau-1)a}})^{\delta-1}e^{-\theta y_{d(\tau-1)a}}y_{d(\tau-1)a}\right]}{(1-e^{-\theta y_{d(\tau)a}})^{\delta} - (1-e^{-\theta y_{d(\tau-1)a}})^{\delta}} \tag{11}$$

Solving numerically Equations (8), (10) and (11) via iterative technique, we get the ML estimators of θ, ω and δ. Consequently, the SSR estimator is obtained using (3).

5. Numerical Clarification

We come up with a simulation illustration to explore and compare the behavior of the SSR estimates from different sampling schemes. Measures like, the absolute bias (AB), the mean squared error (MSE), and the relative efficiency (RE) criteria are utilized to check the validity of estimates. The set sizes of *X* and *Y* are taken as (*n, m*)=(2, 2), (2, 3), (3, 2), (3, 3), (3, 4), (4, 3), (4, 4), (5, 5), (6, 6), (7, 7) and number of cycles are chosen as $h_x = h_y = 5$. Consequently, the sizes of the samples are $(N = n\, h_x,\ M = m\, h_y)$= (10, 10), (10, 15), (15, 10), (15, 15), (15, 20), (20, 15), (20, 20), (25, 25), (30, 30), (35, 35). The values of parameters are selected as (ω, δ)= (0.5, 0.4), (4, 0.5), (5, 0.4) and θ=1, therefore, the exact value of *R* is equal to 0.5556, 0.8889 and 0.925. 1000 random samples from *X*∼EED $(\omega, \theta)$and *Y*∼ EED (δ, θ) are generated. The RE of the RSS with respect to NRSS is defined as:

$$RE_1 = MSE\left(\hat{R}(X_{RSS} < Y_{RSS})\right)\Big/MSE\left(\hat{R}(X_{NRSS} < Y_{NRSS})\right),$$

$$RE_2 = MSE\left(\hat{R}(X_{RSS} < Y_{RSS})\right)\Big/MSE\left(\hat{R}(X_{RSS} < Y_{NRSS})\right)$$

and

$$RE_3 = MSE\left(\hat{R}(X_{RSS} < Y_{RSS})\right)\Big/MSE\left(\hat{R}(X_{NRSS} < Y_{RSS})\right).$$

Values of ABs, MSEs, and REs of the SSR estimate are listed in Table 1 for specific values of (*n*, *m*) and distribution parameters.

Table 1. Some measures of SSR estimates based on RSS and NRSS

R = 0.5556												
(n, m)	$X_{RSS} < Y_{RSS}$		$X_{NRSS} < Y_{NRSS}$		RE_1	$X_{RSS} < Y_{NRSS}$		RE_3	$X_{NRSS} < Y_{RSS}$		RE_3
	AB	MSE	AB	MSE		AB	MSE		AB	MSE	
(2, 2)	0.1389	0.0193	0.0292	0.0096	2.01	0.0595	0.0117	1.65	0.0515	0.0123	1.57
(2, 3)	0.0687	0.0146	0.0423	0.0084	1.74	0.0525	0.0107	1.36	0.0514	0.0115	1.27
(3, 2)	0.3633	0.0146	0.0609	0.0078	1.87	0.0536	0.0091	1.6	0.066	0.0097	1.51
(3, 3)	0.0388	0.01	0.0509	0.0053	1.89	0.0624	0.0077	1.3	0.0627	0.008	1.25
(3, 4)	0.1205	0.0091	0.0536	0.0055	1.65	0.0431	0.0076	1.2	0.0686	0.0077	1.18
(4, 3)	0.071	0.0062	0.055	0.0048	1.29	0.0069	0.0051	1.22	0.0477	0.0052	1.19
(4, 4)	0.0158	0.0057	0.0471	0.0038	1.5	0.0006	0.0041	1.39	0.0462	0.0046	1.24
(5, 5)	0.0396	0.0051	0.05	0.0033	1.55	0.0025	0.0038	1.34	0.054	0.0042	1.21
(6, 6)	0.0061	0.0047	0.0485	0.003	1.57	0.0142	0.0032	1.47	0.0526	0.0037	1.27
(7, 7)	0.0072	0.0037	0.0487	0.0029	1.28	0.0514	0.0031	1.19	0.0505	0.0032	1.16
R = 0.8889											
(n, m)	$X_{RSS} < Y_{RSS}$		$X_{NRSS} < Y_{NRSS}$		RE_1	$X_{RSS} < Y_{NRSS}$		RE_3	$X_{NRSS} < Y_{RSS}$		RE_3
	AB	MSE	AB	MSE		AB	MSE		AB	MSE	
(2, 2)	0.062	0.0069	0.071	0.0054	1.28	0.0132	0.0062	1.11	0.0768	0.0063	1.1
(2, 3)	0.0679	0.0066	0.0718	0.0053	1.25	0.0761	0.0059	1.12	0.0764	0.0061	1.08
(3, 2)	0.0573	0.006	0.0669	0.0045	1.33	0.0715	0.0052	1.15	0.0744	0.0056	1.07
(3, 3)	0.057	0.0057	0.065	0.0043	1.33	0.0689	0.0048	1.19	0.0727	0.0053	1.08
(3, 4)	0.0525	0.0052	0.062	0.004	1.3	0.0641	0.0043	1.21	0.069	0.0048	1.08
(4, 3)	0.0453	0.0046	0.0603	0.0037	1.24	0.0633	0.0041	1.12	0.0667	0.0045	1.02
(4, 4)	0.0466	0.0043	0.0557	0.0032	1.34	0.0594	0.0036	1.19	0.0631	0.004	1.08
(5, 5)	0.0385	0.0042	0.0545	0.003	1.4	0.0577	0.0034	1.24	0.0618	0.0038	1.11
(6, 6)	0.0371	0.004	0.0516	0.0027	1.48	0.0546	0.0031	1.29	0.0597	0.0035	1.14
(7, 7)	0.0314	0.0033	0.0476	0.0023	1.43	0.0475	0.0026	1.27	0.0549	0.003	1.1
R = 0.925											
(n, m)	$X_{RSS} < Y_{RSS}$		$X_{NRSS} < Y_{NRSS}$		RE_1	$X_{RSS} < Y_{NRSS}$		RE_3	$X_{NRSS} < Y_{RSS}$		RE_3
	AB	MSE	AB	MSE		AB	MSE		AB	MSE	
(2, 2)	0.0516	0.0052	0.0005	0.0035	1.49	0.0007	0.0037	1.41	0.063	0.004	1.3
(2, 3)	0.0155	0.0042	0.0555	0.0031	1.35	0.0597	0.0035	1.2	0.0619	0.0038	1.11
(3, 2)	0.0542	0.0039	0.0554	0.003	1.3	0.0564	0.0032	1.22	0.0608	0.0037	1.05
(3, 3)	0.0464	0.0037	0.0523	0.0027	1.37	0.0546	0.003	1.23	0.0591	0.0035	1.06
(3, 4)	0.0523	0.0036	0.0479	0.0023	1.57	0.053	0.0028	1.29	0.0574	0.0033	1.09
(4, 3)	0.0479	0.0031	0.0445	0.002	1.55	0.0476	0.0023	1.35	0.051	0.0026	1.19
(4, 4)	0.037	0.0026	0.0405	0.0017	1.53	0.0463	0.0021	1.24	0.0476	0.0023	1.13
(5, 5)	0.0342	0.0024	0.0387	0.0015	1.6	0.0401	0.0017	1.41	0.0442	0.0019	1.26
(6, 6)	0.0333	0.0021	0.0336	0.0012	1.75	0.0365	0.0014	1.5	0.0412	0.0017	1.24
(7, 7)	0.0235	0.0017	0.0269	0.0009	1.89	0.0342	0.0012	1.42	0.0386	0.0015	1.13

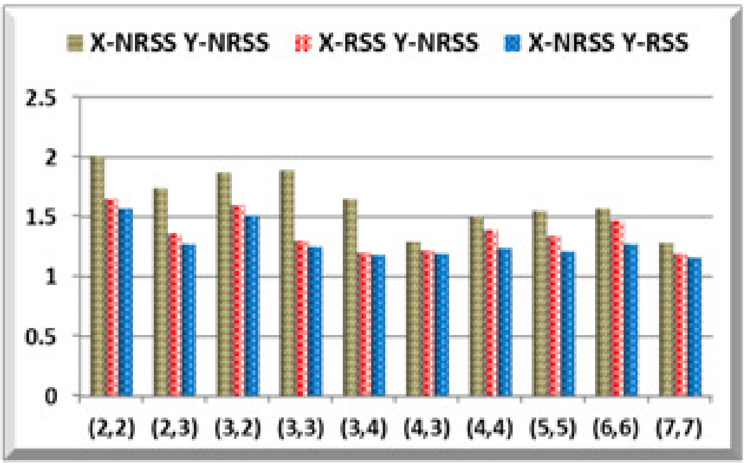

Figure 1. **R**E of SSR estimates at R=0.5556

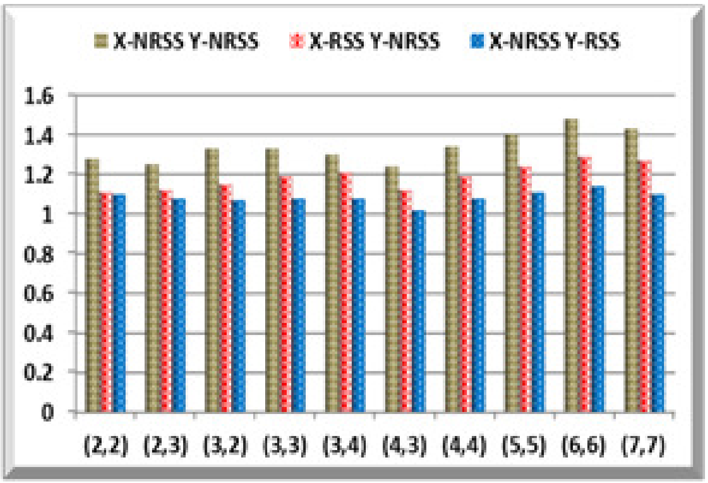

Figure 2. **R**E of SSR estimates at R=0.8889

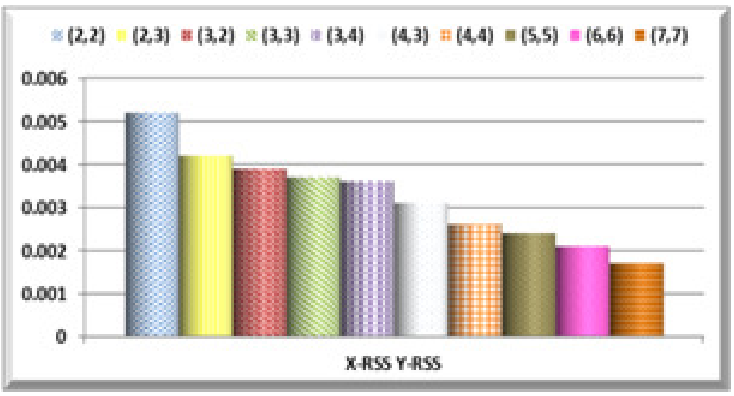

Figure 3. **M**SE of SSR estimate for R=0.5556

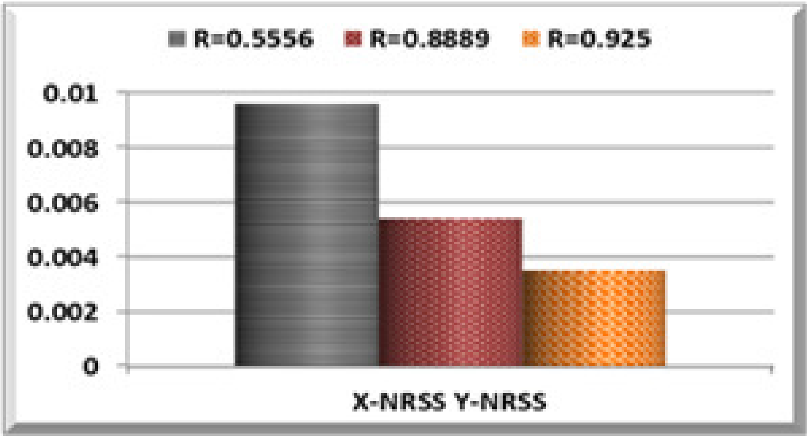

Figure 4. **M**SE of SSR estimate in case of $P(X_{NRSS} < Y_{NRSS})$ at (2,2)

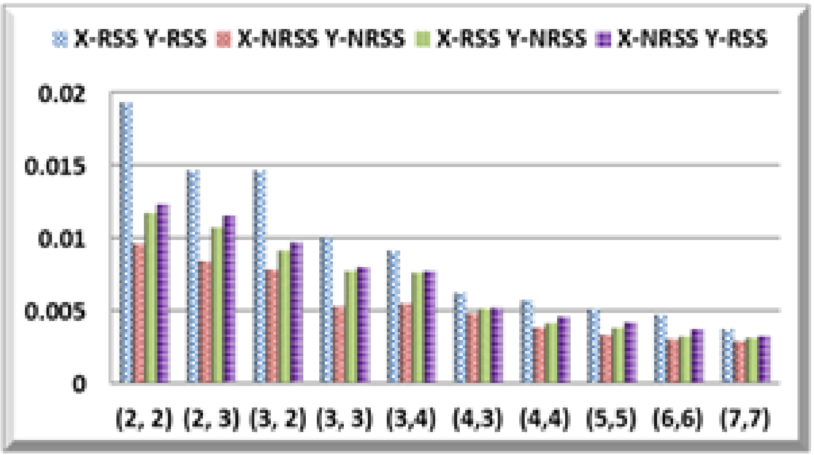

Figure 5. **M**SE of SSR estimate at R=0.5556

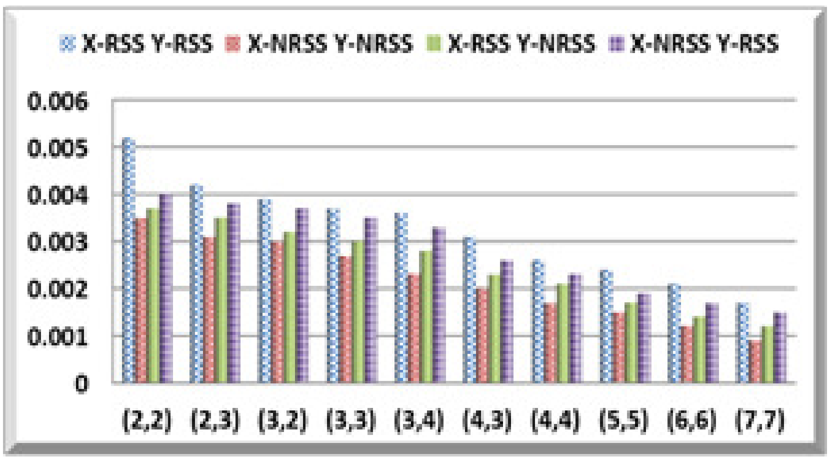

Figure 6. **M**SE of SSR estimate at R=0.925

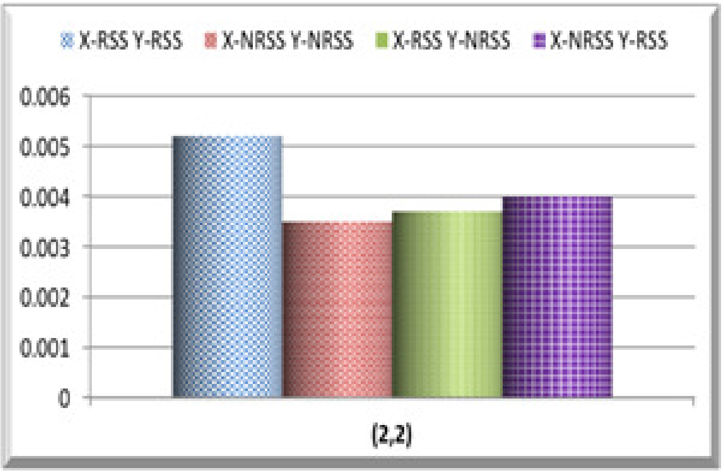

Figure 7. **T**he MSE of SSR estimates at *R*=0.5556 and (2, 2)

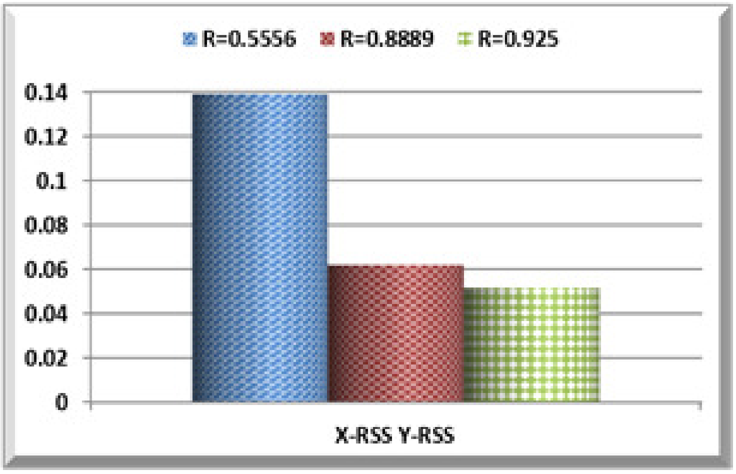

Figure 8. **T**he ABs of $X_{RSS} < Y_{RSS}$at (2, 2)

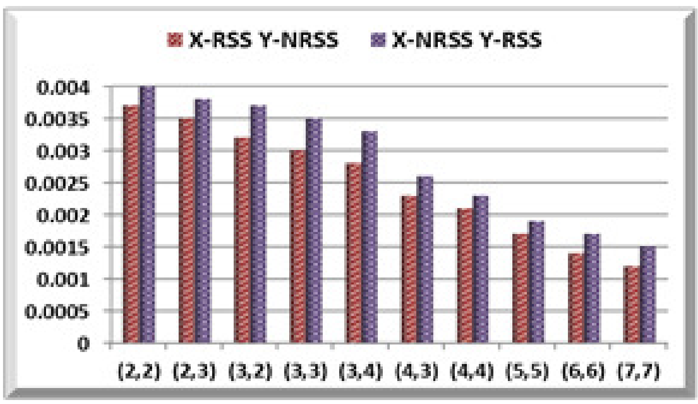

Figure 9. **M**SE of SSR estimate for $X_{RSS} < Y_{NRSS}$ and $X_{NRSS} < Y_{RSS}$ at R=0.925

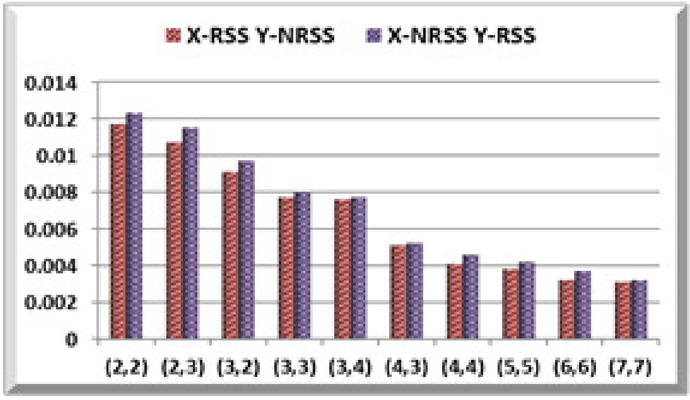

Figure 10. **M**SE of SSR estimate for $X_{RSS} < Y_{NRSS}$ and $X_{NRSS} < Y_{RSS}$ at R=0.556

Some conclusions, based on Table 1 and Figures 1 to 10, are summarized as follows:

- The SSR estimates when *X* and *Y* random variables are elected from NRSS are more efficient than the others under different ranking schemes, that is (X_{RSS}, Y_{NRSS}), (X_{NRSS}, Y_{RSS}) and (X_{RSS}, Y_{RSS}) for chosen values of *N*, *M* and true value of *R* (see for example Figures 1 and 2 and Table 1).
- The MSE of SSR estimates under different schemes for all set sizes decreases as *N* and *M* increase for all true values of *R* (see for example Figure 3 and Table 1).
- The MSE of SSR estimates decreases for different set sizes as the true value of *R* increases in all ranked schemes (see for example Figure 4 and Table 1).
- We can see that the SSR have the lowest MSE in the approximately most of situations via NRSS for all true values of *R* (see Figures 5, 6 and 7 and Table 1).
- The AB of SSR estimates decreases with increases value of *R* for different ranked schemes in almost of situations (see Figure 8 and Table 1).
- The estimates get increasingly accurate as the sample size grows higher, suggesting that they are asymptotically unbiased. Furthermore, in all cases, the MSE reduces as the sample size increases, illustrating that the various estimates are consistent.
- The MSE of $\hat{R} = P(X_{RSS} < Y_{NRSS})$ is smaller than the corresponding for $\hat{R} = P(X_{NRSS} < Y_{RSS})$ at all values of *R* (see Figures 9 and 10 and Table 1). In other words, we find that when the strength *X* is elected from RSS and the stress *Y* is chosen from NRSS, the SSR estimates perform better than when the strength *X* is selected from NRSS and the stress *Y* is chosen from RSS.

6. Data Analysis

We consider two data sets and depict all the details for illustrative purposes. The two data sets were originally reported in [35], which represent the fiber diameter, tensile behavior and coefficient of variations of fiber diameter (CVFD) of jute fibers at 5 mm, 10 mm, 15 mm, and 20 mm gauge lengths. We used the data sets about the breaking strengths of jute fiber at gauge lengths of 10 and 20mm with sizes of the samples N = M=30. The data sets are scheduled as:

Data set 1 (10 mm) MPa:
693.73 704.66 323.83 778.17 123.06 637.66 383.43 151.48 108.94 50.16 671.49 183.16 257.44 727.23 291.27 101.15 376.42 163.40 141.38 700.74 262.90 353.24 422.11 43.93 590.48 212.13 303.90 506.60 530.55 177.25.

Data set 2 (20 mm) MPa:
71.46 419.02 284.64 585.57 456.60 113.85 187.85 688.16 662.66 45.58 578.62 756.70 594.29 166.49 99.72 707.36 765.14 187.13 145.96 350.70 547.44 116.99 375.81 581.60 119.86 48.01 200.16 36.75 244.53 83.55.

We'll do some basic data analysis before going any further. We verify the effectiveness of the fitted model using Kolmogorov-Smirnov (KS) goodness of fit test and its P-value (PV). The KS distance between the empirical and the fitted distribution is 0.1012 with PV = 0.918 for the first dataset and 0.14978 with PV = 0.511 for the second dataset, indicating satisfactory fits. The estimated densities, estimated cdf, PP plots and empirical survival for the considered data are shown in Figures 11 and 12. Therefore, the EED can be used to examine the two sets of data.

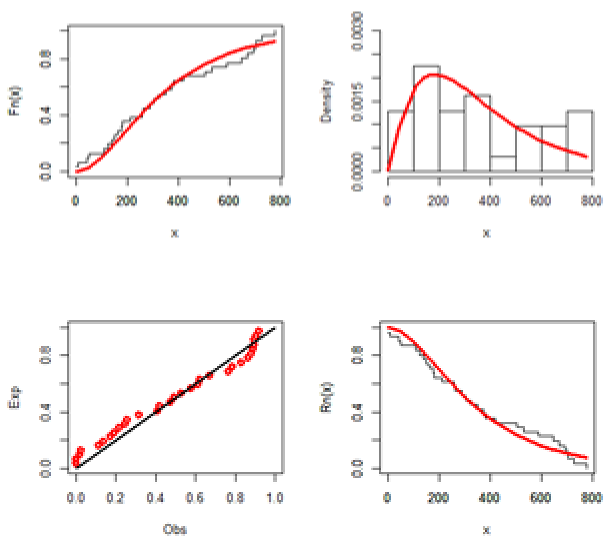

Figure 11. **P**lots of density, cdf and survival of the EED for data set 1

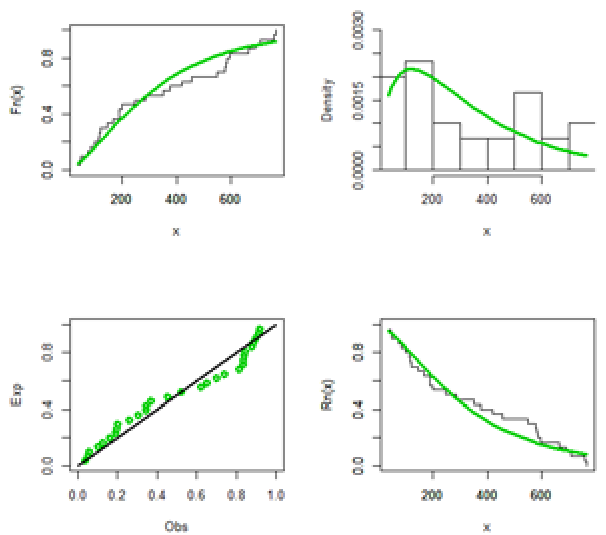

Figure 12. **P**lots of density, cdf and survival of the EED for data set 2

Table 2. SSR estimate of the fiber diameter, tensile behavior and CVFD of jute fibers

(n, m)	$X_{RSS} < Y_{RSS}$	$X_{NRSS} < Y_{NRSS}$	$X_{RSS} < Y_{NRSS}$	$X_{NRSS} < Y_{RSS}$
(2,2)	0.5758	0.757	0.6907	0.614
(3,2)	0.6206	0.7798	0.7411	0.6879
(2,3)	0.6718	0.8147	0.8035	0.7675
(3,3)	0.7356	0.8613	0.8439	0.7872
(3,4)	0.7672	0.9081	0.8619	0.8241
(4,3)	0.7924	0.9226	0.8847	0.8394
(4,4)	0.8158	0.9561	0.9267	0.8516
(5,5)	0.8242	0.9817	0.9613	0.886

First, it is assumed that $X \sim EED\,(\omega,\,\theta_1)$ and $Y \sim EED\,(\delta,\,\theta_2)$. The ML estimates of $\omega,\,\theta_1,\delta$ and θ_2 are as follows: $\hat{\omega} = 3.164$, $\hat{\theta}_1 = 0.9914$, $\hat{\delta} = 1.222$, $\hat{\theta}_2 =$15.968 and the log-likelihood value is $\ln L_1 = -406.628$. Second, suppose that $X \sim EED\,(\omega,\,\theta)$ and $Y \sim EED\,(\delta,\,\theta)$, the ML estimates of ω, δ and θ are as follows: $\hat{\omega} =$3.566, $\hat{\delta} =$1.392, $\hat{\theta} =$ 0.9918 and the associated log-likelihood value is $\ln L_2 = -406.690$. We perform the following testing of hypothesis;
$H_0\,:\,\theta_1 = \theta_2$vs $H_1\,:\,\theta_1 \neq \theta_2$
and in this case $-2\,(L_2 - L_1) =$0.124, hence, the null hypothesis cannot be rejected. Therefore, in this case the assumption of $\theta_1 = \theta_2$ is justified. Thus, both tests accept the null hypothesis that each data set is drawn from EED. Based on the ML estimate of $\hat{\omega}$,$\hat{\delta}$, and $\hat{\theta}$, the ML estimate of R via NRSS and RSS are obtained. Table 2 displays the SSR estimates obtained from the EED assuming recommended sampling strategies for a variety of *n* and *m* values over 5 repeated cycles. The MATHEMATICA program is used to generate NRSS and RSS algorithms from both data sets 1 and 2. We can see from these results that for large set size, the estimated value of $R = P(X_{NRSS} < Y_{NRSS})$ is approximately 0.9. Furthermore, we observe that, the estimated values of $R = P(X_{RSS} < Y_{NRSS})$ are greater than the corresponding at $R = P(X_{NRSS} < Y_{RSS})$, leading to the selection of NRSS samples from the stress random variable. Thus, the obtained results in this section confirm the results of the previous one.

7. Concluding Remarks

We address the estimation of the SSR, that is $R = P\,[Y\,<\,X]$, assuming the strength *X* and stress *Y* are independent random variables. The SSR estimators are considered in four cases under RSS and NRSS schemes. Two cases of them are considered when both the stress and the strength random variables have the same sample design, while the other two issues are considered when *X* and *Y* distributions have dissimilar sampling designs. We implement the simulation study to appreciate and compare different behavior estimates through some criteria. Outputs of the study showed that the MSE of SSR estimates via NRSS get the smallest values than the others based on RSS data in all issues. Generally speaking, the SSR estimates when data of both the stress and strength are drawn from NRSS are more efficient than others when strength and stress data are selected from RSS. Furthermore, the SSR estimates when *X* is RSS and *Y* is NRSS data are more efficient than the SSR estimates in the reversed case (when *X* is NRSS and *Y* is RSS data). Real data application illustrates these results.

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