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A New Group Acceptance Sampling Plans Based on Percentiles for the Weibull Fréchet Model

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Abstract When the life test is truncated at a pre-determined duration, group acceptance sampling plans for the Weibull Fréchet distribution percentiles are introduced in this article. Under a given group size, acceptance limit, and customer risk, the minimum number of groups needed to guarantee the specified life percentile is calculated. The operating characteristic values are discovered, additionally the producer's risk. To illustrate the process mentioned here, two experimental are given. Also, real data set is used to demonstrate the flexibility of the Weibull Fréchet model.

Keywords Life test, Weibull Fréchet distribution, Minimum number of group sampling strategy, Consumers risk, Operating characteristic function

AMS 2010 subject classifications 62D05, 62D99

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1. Introduction

A process for ensuring that output product abides by a given collection of quality standards or meets the needs of customers is known as quality control. Acceptance sampling is one of the most critical elements of statistical quality control. It is used by the customer to determine whether or not to approve or reject a large number of goods that have been delivered from the manufacturer. When product inspection is expensive or destructive, acceptance sampling is commonly used; Duncan [9] provides more details. The main problem with most sampling plans for a truncated existence test is estimating the sample size from a lot under investigation. In the traditional sampling plan, it is tacitly presumed that only one item is placed in a tester. In practice, however, fitters that can test a large number of items at once are used because testing time and expense can be minimized by testing many items at once. This form of the tester is often used in sudden death tests.

Implementations of sudden death research are discussed in Pascual and Meeker [19], Vlcek et al.[25], and Jun et al. [14]. The number of items that must be included in this type of tester is determined by the specification. A group sampling strategy would be used when these types of testers are used. When creating a group sampling strategy, the number of groups must be determined since the group size is already known. For a pre-determined time, the items in a group are tested independently, identically and, simultaneously on the different testers. If more than the allowable number of failures occurred in any group during the experiment period, the experiment is truncated.
The aim of this article is to design a group acceptance sampling plan when the lifetime of the items follows

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the Weibull Fréchet model with known shape parameters. The main problem deemed is to decide the minimum number of groups needed to assure a certain 10^{th} percentile life when the life test is truncated at a pre-assigned period t and the observed number of nonconforming items does not exceed a specified acceptance limit. Gupta and Groll [13] said the decision operation is to accept a lot only if the specified quantile lifetime can be instituted with a predetermined probability $\mathbb{C}^*$ which contributes the consumer's safeguard. It is worth mentioning that the decision to accept can only be made at the terminus of period t and if the number of nonconforming items does not transcend the stated acceptance limit c. The life test experiment can be stopped when the $(c+1)^{th}$ failure is spotted or when time t reaches, whichever comes first. The only decision that can be made in the first case is to reject the lot.

Such a group acceptance sampling plan based on truncated life test for various lifetime distributions are recently used by many researchers Aslam and Jun [2], [3], Rao [20], [21], [22], Aslam et. al. [5], Aslam et al. [6] and Aslam et. al. [4]. The popular acceptance sampling plans according to lifetime distributions are used by some researchers. It was carried out by Epstein [10] under the assumption that a truncated existence test follows an exponential distribution. Good and Kao [11] performed a follow-up analysis, stating that the sampling plan for reliability and life testing is based on the premise that an item's life distribution follows the Weibull distribution. Gupta and Groll [13], Gupta [12], Kantam and Rosaiah [15], Kantam et al. [16], Rosaiah and Kantam [23], Balakrishnan et al. [7] and Mervat, et al. [17] are only a few of the several studies that are introduced.

The remainder of the article is laid out as follows: Section 2 contains a description of the Weibull Fréchet model. Section 3 contains the design of the proposed sampling plan. Section 4 introduces the operating characteristics function. Section 5 contains producer risk. Section 6 introduces tables and instance explanations. Section 7 performs the application on a real data set.

2. Definition of Weibull Fréchet model

The Weibull Fréchet model as a four-parameter family of models has the density function of T is given by (for $t > 0$)

$$g\underline{\psi}(t) = ab\theta\lambda^{\theta}t^{-\theta-1}\exp\left[-b\left(\frac{\lambda}{t}\right)^{\theta}\right]\left\{1-\exp\left[-\left(\frac{\lambda}{t}\right)^{\theta}\right]\right\}^{-b-1}\exp\left(-a\left\{\exp\left[\frac{\lambda}{t}\right]^{\theta}-1\right\}^{-b}\right), \tag{1}$$

where $\underline{\psi} = (\lambda, a, \theta, b)$ is the parameters vector, $\lambda > 0$ is the scale parameter and $a > 0$, $\theta > 0$ and $b > 0$ are shape parameters. We denote a random variable having PDF in Equation 1 by $T \sim WF(\underline{\psi})$. The WF distribution is a very elastic model that approximates diverse distributions when its parameters are assorted. As $b = 1$ and $\theta = 1$, the WF model converges to an exponential inverse exponential model. For $b = 1$ and $\theta = 2$, the WF model converges to the exponential inverse Rayleigh model. As $\theta = 2$, the WF model refers to Weibull inverse Rayleigh model. The WF model is reduced to the exponential Fréchet distribution when $b = 1$. When $\theta = 1$, the WF model reduces to Weibull inverse exponential model. The CDF corresponding to Equation 1 is given by

$$G\underline{\psi}(t) = 1-\exp\left(-a\left\{\exp\left[\left(\frac{\lambda}{t}\right)\right]^{\theta}-1\right\}^{-b}\right). \tag{2}$$

The k^{th} moment of WF model is stated as follows:

$$\mu'_{k,T} \mid k < \theta = \sum_{j,r=0}^{\infty} v_{j,r}\lambda^{k}[(r+1)b+j]^{\frac{k}{\theta}}\Gamma(1-\frac{k}{\theta}), \tag{3}$$

Putting $k = 1$ in Equation 3, we have the mean of WF model. The quantile function of WF($\underline{\psi}$), say $Q_{\nabla} = G^{-1}_{WF}(\nabla;\underline{\psi})$, is given by

$$t_\nabla \mid 0 < \nabla < 1 = \lambda\Big[log\Big\{1 + [(-a^{-1})log(1-\nabla)^{-\frac{1}{b}}\Big\}\Big]^{-\frac{1}{\theta}}, \tag{4}$$

By putting $\nabla = 0.5$, in Equation 4 gives the median of WF model. For more details about WF distribution see Afify et al. [1]. The quantile t_∇ is a function of the scale parameter λ and at a pre-fixed value of t_∇ say t^0_∇, we may obtain the value of λ, say λ_0, as

$$\lambda_0 = t^0_\nabla\Big[log\Big\{1 + [-\frac{1}{a}log(1-\nabla)]^{-\frac{1}{b}}\Big\}\Big]^{\frac{1}{\theta}}. \tag{5}$$

Clearly for a given t_∇, t^0_∇, λ and λ_0, we have

$$t_\nabla \geq t^0_\nabla \Leftrightarrow \lambda \geq \lambda_0, \tag{6}$$

and we may also note λ_0 depend on θ_0, a_0 and b_0. To build up group sampling plans for WF model it is an ascertained that t_∇ exceeds t^0_∇ equivalently λ exceeds λ_0.

3. Design of the group sampling plan

When the life test is terminated at a pre-determined time t^0_∇ and the confirmed number of failures doesn't exceed a specified acceptance limit c, the essential problem is to determine the minimum number of groups needed to ensure a percentile life. Many products are considered conforming when the actual percentile life t_∇ is more than the specified life t^0_∇. The lot is accepted if $t_\nabla \geq t^0_\nabla$ at a specific level of consumers risk. Other than this the lot is rejected. The following is the group acceptance sampling strategy according to the truncated life test:

I . Determine the number of groups $g[\mathbb{N}]$ and assign k items to each group, resulting in $n = k.g[\mathbb{N}]$.
II . For each group, the acceptance limit c is chosen, and the termination time t_0 is fixed.
III . The experiment for each group is carried out simultaneously, and the number of failures for each group is reported. The lot is accepted if there are at least c failures in each all group. Otherwise, the lot is rejected, and the experiment is terminated.

When $k = 1$, the group sampling plans are reduced to a single sampling plan. Our aim is to determine the number of groups $g[\mathbb{N}]$ needed for WF model and diverse values of acceptance limit c, while the group size k and termination time t_0 are defined. Because it is suitable to set the termination time as a redoubled of the specified life t^0_∇, $t_0 = u_\nabla t^0_\nabla$ for a specified constant u_∇ (termination ratio). The consumer's risk $\beta(\mathbb{C}^*)$ is the probability of accepting a non-conforming lot. The consumer's risk is often represented by the consumer's confidence level. If the consumer's confidence level is $\mathbb{C}^*$, then the consumer's risk is $\beta(\mathbb{C}^*) = 1 - \mathbb{C}^*$.
The number of the group in the suggested sampling plan is decided so that the consumer's risk doesn't override $\beta(\mathbb{C}^*)$. When the lot size is very large so that the lot lies in two categories (accepted or rejected), a binomial distribution is used to develop group sampling plans Stephens [24]. Based on the group sampling plan, the lot is accepted if there were maximum c failures in each of $g[\mathbb{N}]$ groups. So, the lot acceptance probability of the proposed plan is given by

$$Pr(p) = \Big[\sum_{j=0}^{c}\binom{k}{j}p^j(1-p)^{k-j}\Big]^{g[\mathbb{N}]}, \tag{7}$$

where p is the probability that an item in any group fails before the termination time t_0. It would be appropriate to determine the termination time t_0 as a multiple of the specified life percentile t^0_∇. That is, $t_0 = u_\nabla t^0_\nabla$ for a constant u_∇. As a result, the distribution function can be expressed as follows :

$$p = 1 - \exp\Bigg(-a\Big\{\exp\Big[S(a,b,\nabla)\Big(\frac{t_\nabla}{u_\nabla t^0_\nabla}\Big)^{\theta}\Big] - 1\Big\}^{-b}\Bigg), \tag{8}$$

where

$$S(a, b, \nabla) = log\Big\{1 + [(-a^{-1})log(1-\nabla)]^{-\frac{1}{b}}\Big\}, \tag{9}$$

when the shape parameters, multiplier u_∇, and ratio $\frac{t_\nabla}{t^0_\nabla}$ are fixed, the proportion p can be evaluated. The minimum number of groups required can be determined by considering the consumer's risk when the actual percentile life equals the specified percentile life $t_\nabla = t^0_\nabla$ through the following inequality:

$$1 \leq \Big[\sum_{j=0}^{c}\binom{k}{j}p_0^j(1-p_0)^{k-j}\Big]^{g[\mathbb{N}]} \leq \beta(\mathbb{C}^*), \tag{10}$$

where p_0 is the probability of failure that an item in each group before the termination time at $t_\nabla = t^0_\nabla$, and it is given by:

$$p_0 = 1 - \exp\left(-a\Big\{\exp\Big[S(a,b,\nabla)\Big(\frac{1}{u_\nabla}\Big)^\theta\Big] - 1\Big\}^{-b}\right) \tag{11}$$

The minimum number of groups $g[\mathbb{N}]$ satisfying inequality 10 for the proposed plan for WF model with $a = 1.5$, $\theta = 2$ and $b = 0.2$ according to a diverse of consumer's risk values $0.25, 0.10, 0.05,$ and 0.01, the numbers of tester $k = 2(1)7$, acceptance limit $c = 0(1)5$ and termination time ratio $u_\nabla = 0.85, 1.0, 1.5, 2.5,$ and 3.5 at $\nabla = 0.10$ or 0.50 are present in Table 1.

This table mentions that the desired minimum number of the group decreases as the termination time ratio and quantile point increase. Moreover, the desired minimum number of the group increases as the group size increases.

4. Operating characteristics function

It will be simple to evaluate the probability of acceptance lot when the product quality is good by determining the minimum number of groups in the sampling plan at the consumer's risk does not exceed. As previously stated, if $t_\nabla > t^0_\nabla$ or $t_\nabla/t^0_\nabla > 1$, the product is regarded to be good. The probabilities of acceptance according to equality 7 for various values of the percentile ratios t_∇/t^0_∇, consumer's risk $\beta(\mathbb{C}^*)$, and time multiplier u_∇ at $\nabla = 0.10$ or 0.50 are given in Tables 2 and 3 respectively. Tables 2 and 3 indicate that OC values decrease as the quantile point increases. Moreover, the OC values increase as the quality ratio increases.

5. Producer's risk

When $t_\nabla \geq t^0_\nabla$, the producer's risk is the probability of rejecting a good lot. The minimum ratio of t_∇/t^0_∇ would ensure that the producer's risk is less than or equal to 0.05 for the group sampling plan under consideration and a given value of the producer's risk (say, 0.05) by satisfying the following inequality.

$$1 \geq \Big[\sum_{j=0}^{c}\binom{k}{j}p^j(1-p)^{k-j}\Big]^{g[\mathbb{N}]} \geq 0.95. \tag{12}$$

Where p is given inequality 8 and $g[\mathbb{N}]$ is chosen according to the consumer's risk $\beta(\mathbb{C}^*)$ when $t_\nabla/t^0_\nabla = 1$. The minimum ratio of t_∇/t^0_∇ at $\nabla = 0.10$ or 0.50 for WF model with parameters $a = 1.5, \theta = 2$ and $b = 0.2$ are given in Tables 4 and 5 respectively. These tables indicate that the required minimum ratio increases as the termination time ratio and consumer's risk increase. Furthermore, as the number of groups and the acceptance limit increase, the desired minimum ratio in a life test decreases.

Table 1. Minimum number of groups for the designed plan for WF model with $a = 1.5$, $b = 0.2$ and $\theta = 2$ using percentile $\nabla = 0.10$ or 0.50.

$\beta(\mathbb{C}^*)$	k	c	$u_{0.10}$					$u_{0.50}$				
			0.85	1.0	1.5	2.5	3.5	0.85	1.0	1.5	2.5	3.5
0.25	2	0	2	2	1	1	1	2	1	1	1	1
	3	1	9	6	4	2	2	4	3	2	2	1
	4	2	29	19	8	4	3	10	7	5	3	2
	5	3	99	74	19	9	6	23	19	9	5	4
	6	4	346	295	45	16	10	55	37	17	9	6
	7	5	1236	738	104	31	18	133	93	32	15	10
0.10	2	0	4	3	2	2	2	3	2	2	1	1
	3	1	14	11	6	4	3	7	6	4	3	2
	4	2	48	31	14	7	6	16	12	8	5	4
	5	3	165	148	32	14	10	38	37	15	9	7
	6	4	575	295	74	27	17	91	74	28	14	10
	7	5	2054	1475	173	51	30	220	185	53	24	17
0.05	2	0	5	5	3	2	2	3	3	2	2	2
	3	1	19	14	8	5	4	9	7	5	4	3
	4	2	63	41	18	10	7	21	16	10	7	5
	5	3	214	148	42	19	13	49	37	19	11	9
	6	4	749	590	97	35	22	118	148	36	19	14
	7	5	2672	1475	226	67	39	287	185	69	32	22
0.01	2	0	8	7	5	4	3	5	5	4	3	3
	3	1	29	21	12	8	6	13	11	8	6	5
	4	2	97	62	28	15	11	32	25	15	10	8
	5	3	329	295	64	28	20	75	74	29	17	13
	6	4	1151	590	148	54	34	181	148	56	29	21
	7	5	4107	2950	347	103	60	441	369	106	49	33

6. Tables and instances explanations

The numerical outcomes for $T \sim WF\ (t; \underline{\psi})$ when $a = 1.5$, $\theta = 2$ and $b = 0.2$, are given in tables 1:5. Table 1 displays the minimum number of group necessary to assert that the percentiles (0.10 and 0.50) life exceed a given value of consumer's risk $\beta(\mathbb{C}^*)$, with confidence level $\mathbb{C}^* = 0.99$ and the acceptance limit c, using binomial probabilities. This table indicates that the minimum number of the group decreases as the termination time ratio and quantile point increase. For instance, from table 1, if $k = 4$, $c = 2$, $\beta(\mathbb{C}^*) = 0.01$ at $\nabla = 0.10$ and the termination time ratio proceed from 0.85 to 1.0 the coveted values of the parameter of group acceptance sampling plan change from 97 to 62. Also, if $k = 4$, $c = 2$, $\beta(\mathbb{C}^*) = 0.01$ at $\nabla = 0.50$ and the termination ratio shift from 0.85 to 1.0 the desired values of the parameter of group acceptance sampling plan change from 32 to 25. This tendency, however, is not monotonic because it is influenced by the acceptance limit. Tables 2 and 3 show the probability of acceptance for the lot at the percentiles (0.10 and 0.50) ratio based on the producer's risk. These tables indicate that OC values increases as the quality ratio increases. Moreover, OC values decrease as the quantile point increases. Finally, the minimum ratios of actual percentiles (0.10 and 0.50) to the specified percentiles for the acceptance of a lot with producer's risk 0.05 for the given value of parameter are presented in tables 4 and 5. Now, two illustrative experimental are presented.

6.1. Experimental 1

Presume that the product lifetime follows the WF model and that the fitter is concerned to establish the actual unknown 10^{th} percentile lifetime for the pistons to be at least 10000 hours with confidence level $\mathbb{C}^* = 0.99$. It is

Table 2. The OC values of group sampling plan with k = 4 and c = 2 for a given $\beta(\mathbb{C}^*)$ for WF model with $a = 1.5, \theta = 2$ and $b = 0.2$.

$\beta(\mathbb{C}^*)$	$g(\mathbb{N})$	$u_{0.10}$	$t_{0.10}/t^0_{0.10}$							
			2	3	4	5	6	7	8	9
0.25	29	0.85	0.985	1	1	1	1	1	1	1
	19	1	0.974	0.999	1	1	1	1	1	1
	8	1.5	0.924	0.988	0.998	1	1	1	1	1
	4	2.5	0.841	0.941	0.980	0.994	0.998	0.999	1	1
	3	3.5	0.792	0.894	0.949	0.976	0.989	0.995	0.998	0.999
0.10	48	0.850	0.976	0.999	1	1	1	1	1	1
	31	1	0.956	0.998	1	1	1	1	1	1
	14	1.5	0.877	0.980	0.997	1	1	1	1	1
	7	2.5	0.750	0.905	0.967	0.989	0.997	0.999	1	1
	6	3.5	0.679	0.830	0.916	0.960	0.982	0.992	0.997	0.998
0.05	63	0.850	0.968	0.999	1	1	1	1	1	1
	41	1	0.944	0.997	1	1	1	1	1	1
	18	1.5	0.843	0.974	0.996	1	1	1	1	1
	10	2.5	0.688	0.878	0.957	0.986	0.996	0.999	1	1
	7	3.5	0.605	0.785	0.892	0.949	0.977	0.99	0.995	0.998
0.01	97	0.85	0.952	0.999	1	1	1	1	1	1
	62	1	0.915	0.995	1	1	1	1	1	1
	28	1.5	0.769	0.961	0.994	0.999	1	1	1	1
	15	2.5	0.563	0.818	0.935	0.979	0.993	0.998	0.999	1
	11	3.5	0.462	0.689	0.839	0.922	0.964	0.984	0.993	0.997

coveted to end the experiment at 10000 hours. Then, when $c = 2$, $k = 4$, $u_{0.10} = 1.0$ and $a = 1.5$, $\theta = 2$, $b = 0.2$, from Table 1, the required minimum number of group $g[\mathbb{N}]$ is 62. It ought to be noted that if a minimum sample size is wanted, it can be acquired by $n = k.g[\mathbb{N}]$. As a result, we'll draw a random sample of size $n = 248$ items and assign four items to each of the 62 groups to test for 10000 hours. If no more than two failures out of 62 units are observed over the course of 10000 hours. The fitter can then say with 99 % certainty that the 10^{th} percentile life is at least 10000 hours, and the fitter can accept the lot. The OC values for group acceptance sampling plan (k, $g[\mathbb{N}]$, c and $u_{0.10}$) = (4, 62, 2, and 1.0) and confidence level $\mathbb{C}^* = 0.99$ under WF model from table 2 is as follows:

$t_{0.10}/t^0_{0.10}$	2	3	4	5	6	7	8	9
OC	0.915	0.955	1	1	1	1	1	1

This stated that if the actual 10^{th} percentile is twice the specified 10^{th} percentile $t_{0.10}/t^0_{0.10} = 2$ the producer's risk is roughly 0.085. When the actual 10^{th} percentile is greater than or equal to 4 times the specified 10^{th} percentile, the producer's risk is nearly zero. From table 4, we get the value of the minimum ratio of the actual 10^{th} percentile to the specified 10^{th} percentile for various choices of k, $g[\mathbb{N}]$, c,$u_{0.10}$ in order that the producer's risk may not exceed 0.05. For instance, the value of $t_{0.10}/t^0_{0.10}$is 7.979 for $k = 4$, $u_{0.10} = 1.0$, $c = 2$ and $\mathbb{C}^* = 0.99$. This betokens that the product must have actual 10^{th} percentile life of 7.979 times the specified 10^{th} percentile life in order for the lot to be accepted with a probability of 0.95.

6.2. *Experimental 2*

Assume the product's lifetime follows the WF distribution, with $a = 1.5$, $\theta = 2$ and $b = 0.2$. The fitter needs to establish that the actual median life is at 10000 hours ($t^0_{0.50}$) with confidence level $\mathbb{C}^* = 0.99$ and testing time be 10000 hours. As a result, for the confidence level of 0.99 and an acceptance limit of $c = 2$, the demanded $g[\mathbb{N}]$ is found from table 1 to be 25; that is, in this state, the group acceptance sampling plan ($k = 4$, $g[\mathbb{N}] = 25$, $c = 2$,

Table 3. The OC values of group sampling plan with $k = 4$ and $c = 2$ for a given $\beta(\mathbb{C}^*)$ for WF model with $a = 1.5$, $\theta = 2$ and $b = 0.2$.

$\beta(\mathbb{C}^*)$	$g(\mathbb{N})$	$u_{0.50}$	$t_{0.50}/t^0_{0.50}$							
			2	3	4	5	6	7	8	9
0.25	10	0.850	0.935	0.991	0.999	1	1	1	1	1
	7	1	0.910	0.983	0.997	1	1	1	1	1
	5	1.5	0.844	0.944	0.981	0.994	0.998	0.999	1	1
	3	2.5	0.773	0.871	0.929	0.963	0.981	0.99	0.995	0.998
	2	3.5	0.740	0.827	0.885	0.925	0.952	0.97	0.981	0.988
0.10	16	0.850	0.895	0.986	0.998	1	1	1	1	1
	12	1	0.856	0.972	0.995	0.999	1	1	1	1
	8	1.5	0.754	0.908	0.969	0.99	0.997	0.999	1	1
	5	2.5	0.652	0.796	0.886	0.939	0.968	0.984	0.992	0.996
	4	3.5	0.606	0.729	0.817	0.879	0.921	0.95	0.969	0.981
0.05	21	0.85	0.865	0.981	0.998	1	1	1	1	1
	16	1	0.816	0.964	0.994	0.999	1	1	1	1
	10	1.5	0.693	0.882	0.96	0.987	0.996	0.999	1	1
	7	2.5	0.574	0.743	0.854	0.921	0.959	0.979	0.989	0.995
	5	3.5	0.521	0.663	0.769	0.845	0.899	0.935	0.959	0.975
0.01	32	0.850	0.800	0.972	0.997	1	1	1	1	1
	25	1	0.732	0.945	0.990	0.998	1	1	1	1
	15	1.5	0.569	0.825	0.939	0.981	0.994	0.998	0.999	1
	10	2.5	0.426	0.633	0.784	0.881	0.937	0.968	0.984	0.992
	8	3.5	0.367	0.532	0.667	0.772	0.849	0.902	0.938	0.961

$u_{0.50} = 1.0$). Therefore, a random sample of size $n = 100$ items and assign 4 items to each of the 25 groups to put on the test for 10000 hours is drawn. This elucidates that a combined of 100 products are needed and that 4 items are assigned to each of the 25 testers. The lot is accepted if at most 2 failures are found in 10000 hours in every group. Otherwise, the lot is rejected. For the group acceptance sampling plan ($k = 4$, $g[\mathbb{N}] = 25$, $c = 2$, $u_{0.50} = 1.0$), the OC values from table 3 are as follows:

$t_{0.50}/t^0_{0.50}$	2	3	4	5	6	7	8	9
OC	0.732	0.945	0.999	0.998	1	1	1	1

This denotes that the producer's risk is about 0.268 if the actual median life is twice the specified median life $t_{0.50}/t^0_{0.50} = 2$, and about 0.01 if the actual median life is 4 times the specified median life. Table 5 can be utilized to acquire the value of $t_{0.50}/t^0_{0.50}$for the various values of (k, $c = 2$,$u_{0.50}$) such that the producer's risk may not exceed 0.05. For instance, the value of $t_{0.50}/t^0_{0.50}$is 11.168 for ($k = 4$, $c = 2$,$u_{0.50} = 1.0$) and $\mathbb{C}^* = 0.99$. This signifies that the product should have a median life of 11.168 times the specified median life so as to for the lot to be approved with a probability of 0.95.

7. An application

In this Section, we use real data from Nichols and Padgett [18] to explain the proposed group acceptance sampling plan in this section. The data consists of 100 observations of breaking stress of carbon fibers (in Gba). First, exploring a real data set can be considered either numerically or graphically or with both techniques. In this Section, we will consider the graphical techniques such as the Cullen and Frey plot for exploring initial fit to the theoretical common distributions such as normal, uniform, exponential, lognormal, logistic, beta and Weibull

Table 4. Minimum ratio of actual $u_{0.10}$ for the acceptability of a lot for WF model and producer's risk of 0.05

$\beta(\mathbb{C}^*)$	k	c	$u_{0.10}$				
			0.85	1.0	1.5	2.5	3.5
0.25	2	0	3.743	4.226	5.831	9.001	12.274
	3	1	3.39	3.903	5.044	5.632	7.493
	4	2	3.20	3.629	4.616	5.065	5.646
	5	3	2.868	3.583	4.084	4.892	5.576
	6	4	2.61	3.097	3.73	4.651	4.838
	7	5	2.487	2.824	3.101	4.118	4.514
0.10	2	0	5.646	6.131	6.758	10.242	13.853
	3	1	4.996	5.776	6.109	9.528	11.327
	4	2	4.583	5.307	5.72	8.168	9.799
	5	3	4.108	4.781	4.759	7.562	9.261
	6	4	3.828	4.042	4.392	7.35	8.434
	7	5	3.323	3.937	4.306	6.166	8.082
0.05	2	0	7.492	7.949	8.062	10.887	14.755
	3	1	7.083	7.321	7.834	9.615	13.279
	4	2	6.246	6.62	7.477	8.483	11.389
	5	3	6.178	6.354	6.689	8.045	9.997
	6	4	5.306	5.708	6.161	7.676	9.038
	7	5	5.034	5.117	5.744	7.391	8.62
0.01	2	0	8.423	9.119	9.575	11.939	16.228
	3	1	7.738	8.158	8.88	9.78	14.585
	4	2	7.166	7.979	8.119	8.573	11.834
	5	3	6.983	7.517	7.607	7.819	9.18
	6	4	5.95	6.469	7.144	7.4	8.833
	7	5	5.268	6.121	6.959	7.253	7.692

models. Bootstrapping method is also applied and plotted in the same plot for the same purpose. Noting that, Cullen and Frey plot is just to compare models in the space of (squared skewness and kurtosis). Many other graphical plots are also used and presented such as the nonparametric Kernel density estimation (NKDE) plot, the quantile-quantile (Q-Q) plot, the total time in test (TTT) plot and the box plot. For exploring the autocorrelation between any two breaking stress value, the autocorrelation function (ACF) and the partial autocorrelation function (ACF) plots are presented along with useful comments. Moreover, the Scattergram plots are presented for more accuracy. The theoretical ACF provides some information about the distribution of hills and valleys across the surface (Lag = 1). The theoretical partial ACF (Lag= 1) is also presented. Figure 1 (top left) gives the box plot for the breaking stress of carbon fibers. Figure 1 (top right) gives the Q-Q plot for the breaking stress of carbon fibers. Figure 1 (bottom left) gives the TTT plot for the breaking stress of carbon fibers. Figure 1 (bottom right) gives the NKDE plot for the breaking stress of carbon fibers. Figure 2 (first, second and third plots) shows the scattergrams for breaking stress of carbon fibers (Lag = 1). Figure 2 (fourth plot) gives the Cullen and Frey plot for the breaking stress of carbon fibers. Based on Figure 2 (fourth panel), it is noted that we have a left skewed data and do not follow any of the above theoretical models. Figure 2 (fifth and sixth panels) gives the ACF (Lag = 1) and partial ACF (Lag = 1) respectively. Based on Figure 2 (top left and top right panels), we see that one extreme observation were spotted. Based on Figure 2 (bottom left panel), it is noted that the HRF of the breaking stress of carbon fibers is "monotonically increasing". Figure 2 (bottom right panel) the initial NKDE is asymmetric function with right tail. Based on Figure 2 (fifth and sixth panels), we note that that the first lag value (Lag = 1) is statistically significant, whereas the other partial autocorrelations for all other lags are not significant. Thus, the autoregressive model (1) (AR(1)) model is suggested for the breaking stress of carbon fibers distribution. The "R"

Table 5. Minimum ratio of actual $u_{0.50}$ for the acceptability of a lot for WF model and producer's risk of 0.05.

$\beta(\mathbb{C}^*)$	k	c	$u_{0.50}$				
			0.85	1.0	1.5	2.5	3.5
0.25	2	0	5.485	6.27	8.863	14.355	17.226
	3	1	4.889	5.017	5.55	8.58	10.063
	4	2	4.155	4.726	4.952	6.431	7.499
	5	3	3.913	4.269	4.398	5.951	6.051
	6	4	3.671	3.687	4.172	5.476	5.747
	7	5	3.356	3.381	3.638	5.163	5.243
0.10	2	0	8.716	9.261	10.083	15.974	21.75
	3	1	7.981	8.876	8.361	10.522	13.059
	4	2	6.941	7.82	7.874	8.907	11.072
	5	3	6.513	7.353	6.985	7.271	10.027
	6	4	5.82	6.804	6.379	6.752	8.887
	7	5	5.261	6.201	5.936	5.847	7.958
0.05	2	0	10.261	10.651	12.862	17.028	23.227
	3	1	7.685	9.19	9.72	15.202	20.112
	4	2	7.021	8.858	8.253	13.521	19.099
	5	3	5.931	7.353	8.063	10.762	17.11
	6	4	5.74	6.541	7.451	10.271	15.289
	7	5	6.404	6.597	7.138	9.965	13.392
0.01	2	0	13.79	14.785	15.095	18.751	25.639
	3	1	11.898	13.447	14.035	17.665	21.32
	4	2	10.51	11.168	13.088	16.198	18.609
	5	3	9.496	10.361	11.773	14.992	16.177
	6	4	8.37	9.075	10.16	12.899	15.87
	7	5	7.112	8.638	9.499	12.115	14.81

software is used for sketching all figures in this Section. However, the MATHCAD is used for all other numerical results.

We applied the following criteria to adapt the data for the WF model, which are the Akaike information (AIC), Bayesian information (BIC), consistent Akaike information (CAIC), Hannan-Quinn information (HQIC) and, Kolmogorov-Smirnov (K-S). Table 6 summarizes the results of these criteria, which show that the WF correctly match the data. The maximum likelihood estimators (MLE) of the WF parameters with standard deviation (Std. Dev.) are given in Table 7.

Table 6. Goodness of fit criteria for breaking stress of carbon fibers data

AIC	BIC	CAIC	HQIC	-2L	K-S	P-value
296.2	301.8	297.2	300.7	285.3	0.172	0.9578

Table 7. MlEs and standard deviation (in parentheses) for breaking stress of carbon fibers data

Estimates	MLEs	Srd. Dev.
$\widehat{\lambda}$	0.6855	0.363
$\widehat{\theta}$	0.6315	0.284
$\widehat{a}$	0.0981	0.456
$\widehat{b}$	1.3513	0.742

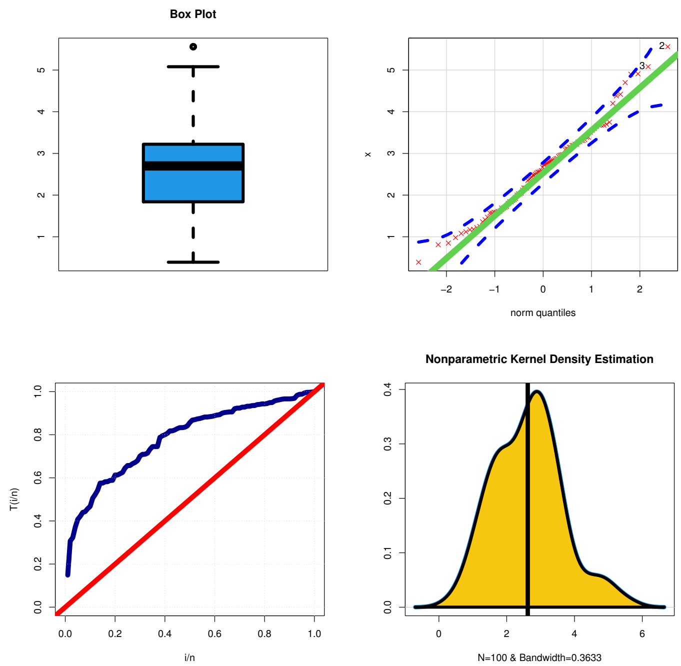

Figure 1. Box plot, Q-Q plot, TTT plot and NKDE plot, for raw breaking stress of carbon fibers data.

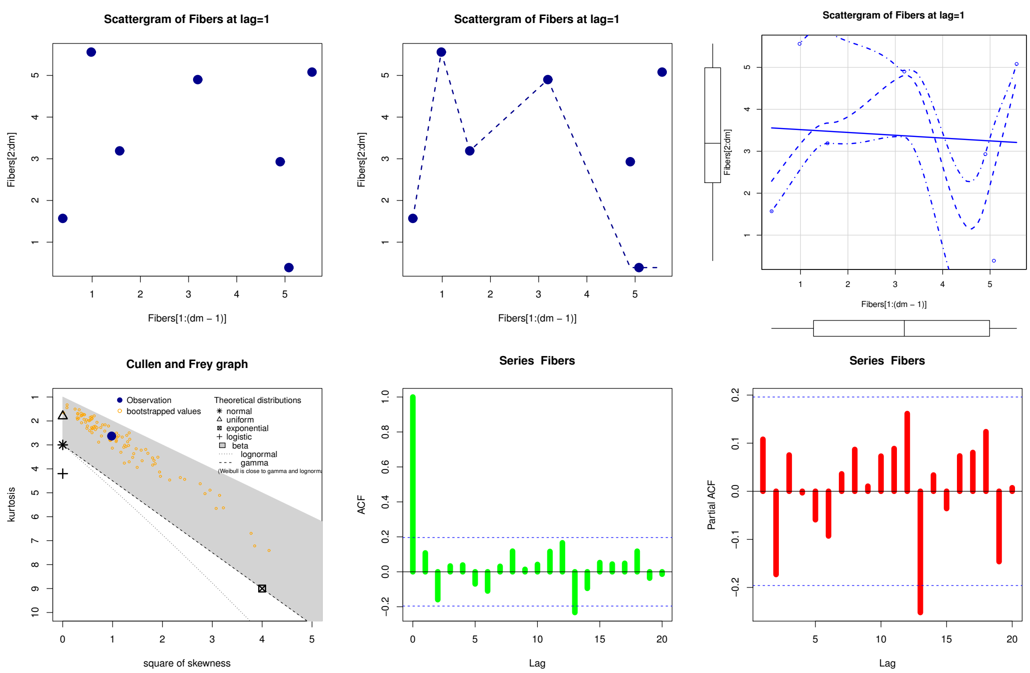

Figure 2. Scattergrams plots, Cullen-Frey plot, ACF and partial ACF for breaking stress of carbon fibers data

The MLEs of the four parameters of WF for the above breaking stress of carbon fibers data are $\widehat{\lambda} = 0.6855$, $\widehat{\theta} = 0.6315$, $\widehat{a} = 0.0981$, and $\widehat{b} = 1.3513$ and hence the 10^{th} quantile of the WF model equal to 0.1735, and the

median is 0.338. The maximum distance between the data by Kolmogorov¨CSmirnov test is 0.172 and P-value is 0.9578. Hence, WF provides a good fit for the breaking stress of carbon fiber data.
Figure 3 presents the estimated PDF (E-PDF) plot, estimated CDF (E-CDF) plot, Kaplan-Meier survival plot and probability-probability (P-P) plot for breaking stress of carbon fiber data the four plots of Figure 3 are plotted based on the estimates in Table 7. Based on Figure 3 (top left panel), it is seen that the E-PDF of the WF fits the histogram of the breaking stress of carbon fiber. Based on Figure 3 (top right panel), it is seen that the theoretical CDF fits the E-CDF. Based on Figure 3 (bottom left panel), it is seen that the estimated survival function of the WF fits the Kaplan-Meier survival plot of the breaking stress of carbon fiber data. Based on Figure 3 (bottom right), it is seen that the WF model provided an adequate fit to the breaking stress of carbon fiber data via the P-P plot.

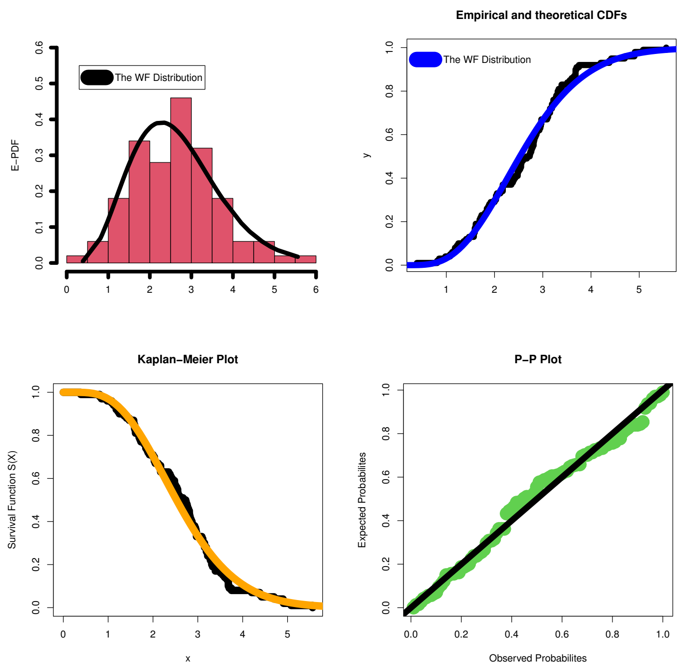

Figure 3. E-PDF, E-CDF, KM and P-P plots for breaking stress of carbon fibers data.

Second, presume that the products' lifetime follows the WF model. Consequently, let's pretend that $\widehat{\theta} = 0.63$, $\widehat{a} = 0.098$, $\widehat{b} = 1$ and the specified 10^{th} percentile $t^0_{0.10} = 0.17$ Gba and testing time $t_{0.10} = 0.1445$ Gba, with $\mathbb{C}^* = 0.99$, this leads to the experiment specified multiplier of $u_{0.10} = 0.85$ Gba. From Table 8, we can observe that the minimum number of groups available is 5. Therefore, a random sample of size $n = 20$ items and assign 4 items to each of the 5 groups to put on the test for 0.1445 Gba is drawn. Consequently, the lot is accepted if at most 2 failures are found in 0.85 Gba in every group. Otherwise, the lot is rejected.

Table 8. Minimum number of groups for the designed plan for WF model with $\widehat{a} = 0.098$, $\widehat{b} = 1$ and $\widehat{\theta} = 0.63$ using percentile $\nabla = 0.10$.

$\beta(\mathbb{C}^*)$	k	c	$u_{0.10}$				
			0.85	1.0	1.5	2.5	3.5
0.25	2	0	1	1	1	1	1
	3	1	3	1	1	1	1
	4	2	5	2	1	1	1
	5	3	10	5	1	1	1
	6	4	19	5	1	1	1
	7	5	38	12	1	1	1
0.10	2	0	2	1	1	1	1
	3	1	4	2	1	1	1
	4	2	8	3	1	1	1
	5	3	16	9	1	1	1
	6	4	32	10	1	1	1
	7	5	82	23	1	1	1
0.05	2	0	3	2	1	1	1
	3	1	5	3	1	1	1
	4	2	11	4	1	1	1
	5	3	21	9	1	1	1
	6	4	42	19	1	1	1
	7	5	82	23	1	1	1
0.01	2	0	4	2	1	1	1
	3	1	8	4	1	1	1
	4	2	17	7	1	1	1
	5	3	33	19	1	1	1
	6	4	64	20	2	1	1
	7	5	127	23	2	1	1

8. Concluding Remarks

In the case of the Weibull Fréchet distribution, this article proposes a group acceptance sampling plan based on a truncated life test. When the consumer's risk and the other plan parameters are fixed, the number of groups is determined. The minimum number of groups is observed to decrease as the termination time ratio and quantile point increase. Furthermore, as the group size increases, the optimal minimum number of groups increases. In addition, as quality improves, the operating characteristics feature increases disproportionately. When a large number of products are being checked at the same time, this group sampling plan may be used. Clearly, such a tester would save time and cost during the testing process.

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