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Detection Model With a Maximum Discounted Effort Reward Search to Maintenance a Best Decision Under the Quality Control Process

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Abstract This paper aims to get a needed service by making the best decision of choosing one suitable company (queue) from $K-$ independent Markovian queues (companies). The customers arrive at each queue according to a Poisson process. The service time of each customer has an exponential distribution. In a steady-state, the best decision depends on the minimum cost of detecting the suitable company which provides the best service with high speed (maximum service rate). To minimize the detection cost and maximize the probability of detection, we consider the search effort bounded by a Gaussian distribution as a function with a discounted parameter. The effectiveness of this model appears in a simulation study and the comparison with other models.

Keywords Quality control process, Optimization, Detection model, Steady-state probability.

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1. Introduction

The companies differ in many respects or distinguishing features such as their mission, goals, strategies, systems, or business philosophies to achieve their primary goals. There is one thing in common among these features, which is the recognition that the basis for achieving success, growth, and continuity in the market depends on the extent of the presence of a customer base and the quality of the relationship it has with these customers. Providing good customer service is no longer an optional matter but rather a reality imposed by the nature of conditions and changes in the contemporary business environment. The success formula is no longer based on innovative promotion, positive trends, distinctive products, or fair prices. But it is based primarily on providing its services according to its customers' personal needs and expectations. The companies are constantly competing to provide excellent service to their customers to ensure that the desired benefits are achieved. This can be done by providing perfect customer service by understanding customers and their types. On the other hand, customers seek to obtain the service at the lowest possible cost and quickly as possible. In addition, one of the most important factors to consider is the waiting time to get the service. Therefore, the customers aimed to detect the appropriate company to perform this service.
One of the most important applications of the search theory for the lost targets is to detect the suitable company among a group of independent companies to provide the service as desired by customers,

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with the maximum detection probability and the minimum cost of detection. Based on the customers' financial capabilities and previous assumptions, an appropriate decision is taken to select the appropriate company. [1] presented a distinguished model to detect a suitable company from K-independent companies with minimum cost and maximum probability. In [1] model, the customers arrive to each company ($M/M/C_i/N_i, i = 1, 2, ..., K$ queue) according to a Poisson process with exponential service time. He used an interesting exponential detection function, which considered in [2, 3] and [4, 5, 6]. [1] solved a difficult optimization problem under the steady-state situation to get a suitable company with the highest service rate. The search models for the lost targets vary according to the circumstances in which the searching process takes place, for example, [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32]. The main purpose of these models is to find these targets with minimum cost and maximum probability.

Improving the level of the service provided increases the turnout of customers to the company. This feature facilitates the task of making the appropriate decision. One of the most important ways to improve the service is to use the concept of cooperation between servers, as in [38], where this model contributes to reducing the rate of congestion. In this regard, some different models were discussed in [39], [40, 41] and [42]. These models depend on the optimal use of the available resources and how to manage them to reduce congestion and improve the level of service.

This paper presents a new detection model that can link probability theory, stochastic processes, queuing theory, searching problems for the lost targets, and operations research. This model helps the customer make the right and appropriate decision to find the right company in terms of the costs and the waiting service time. This decision allows the customer to choose the appropriate company among the $K-$ independent companies. Each company contains a set of independent servers (C servers). To detect a suitable company with maximum probability and minimum cost, we include the effort bounded by a Gaussian distribution as a function with a discount effort parameter. We consider the discount reward search function at time step t is $u(t) = \varpi^t$, where $0 < \varpi^t < 1$ is a known discount parameter, see [5]. More than allowing the system to be more controlled (preserving the company's profits) to facilitate the choosing process for the customers, we maximize the discount effort-reward, which minimizes the detection cost and maximizes the detection probability.

Before end of this section, the nomenclature used in this model is listed. The rest of this paper is organized as follows: Some fundamental principles used and described in this model are presented in Section 2. The formulation of the model is presented in Section 3. After solving a complex optimization problem in Section 4, we use a random generation method to obtain the optimal service rate and cost values in Section 5; we present a numerical example to get the model's effectiveness measures. The results and the conclusion of our work are discussed in the last section.

Symbol	Description
Q_i	The company number i, $i = 1, 2, \dots, K$.
N_i	The capacity of Q_i.
n_{ij}	The number of the customers in the Q_i at a time step j,where $0 \leq n_{ij} < N_i$, $j = 1, 2, \dots, T$.
μ_{ij}	The Q_i servicing rate at a time step j.
λ_{ij}	The Q_i arrival rate at time step j.
$P_{n_{ij}}$	Steady-state probability when there are n_{ij} in the Q_i at a time step j.
$P_{0_{ij}}$	Steady-state probability when there are no customers in Q_i at a time step j.
L_{ij}	The customers expected number in the Q_i at time step j.
$L_{q_{ij}}$	The waiting customers expected number to be served in Q_i at a time step j. i.e., the service will be presented if it found at least one customer in Q_i at a time step j.
$L_{S_{ij}}$	$= L_{ij} - L_{q_{ij}}$. The average number of occupied (customers in) service in Q_i at a time step j.
W^*_{ij}	The smallest expected waiting time in the system at a time step j.
$W^*_{q_{ij}}$	The smallest expected waiting time in Q_i at a time step j.
$W^*_{S_{ij}}$	The smallest expected time of the service at a time step j.
$Z_{n_{ij}}$	The effort random variable to detect Q_i at a time step j.

2. Fundamental Principles

he company (organization) seeks to provide a good service by performing the service under the promises made by the organization to customers. The customer obtains the same level of service no matter how circumstances change. In addition, the company should provide accuracy, independence, and familiarity with the work and perform the service in the right way from the first time. The willingness and constant desire to provide good service are related to the element of time. Therefore, we found the rate and time of service changes from one-time interval to another. Accordingly, we take into account the following fundamental principles to describe Q_i, $i = 1, 2, \dots, K$ at any time interval j, $j = 1, 2, , \dots, T$.

- The arrival process of the consumers depends on the Poisson process with rate $\lambda_{ij} > 0$, and mean inter-arrival time $1/\lambda_{ij}$.
- At the time step j, the clients' service times are random variables (independent and identically distributed), have an exponential distribution with the rate $\mu_{ij} > 0$, and mean service time $1/\mu_{ij}$, $0 < \lambda_{ij} < \mu_{ij}$.
- Each company Q_i applies the principle of First come, first served (FCFS) to the customer service.
- The customer can move freely from the current company to the other (not only to some neighbor company but any other company on the system).
- The customer should distribute his effort among the companies in order to reduce the searching cost.

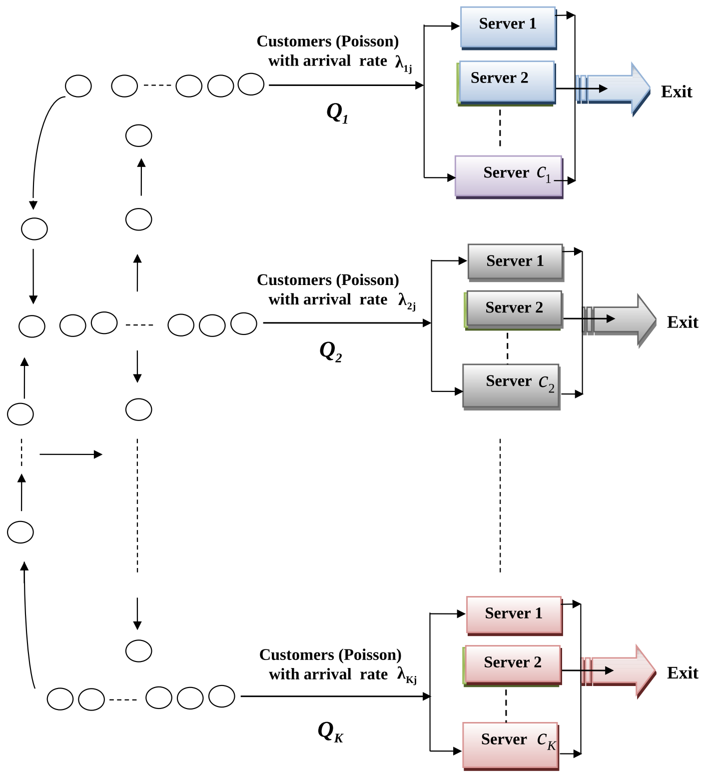


Figure 1. A random movement of the customers to detect the suitable queue from a set of independent $M/M/C_i/N_i,\ i = 1, 2, \ldots, K$ queues at time step $j,\ j = 1, 2, \ldots, T$.

3. Problem Formulation

There are three main rules that move the customers in their dealings with companies or organizations to obtain the service:

1. The customers needs are the starting point in the distinguished service journey, i.e., what the customer wants, particularly the challenge because the failure to realize the reality of what the client needs may lead to serving him in a way that does not achieve his satisfaction. Therefore, he may decide to switch to another organization that is more capable of diagnosing and satisfying his needs.
2. The main driver of the purchasing process for any product (good or service) is the benefits, not the features people usually buy because they have needs they want to satisfy.
3. Expectations are the final selection criterion. The customer's expectations represent the basis on which to reach the final judgment on the organization's ability that owns the product or service to satisfy his needs and achieve his desires, and then continue with them.

Therefore, the customer continues to search for the best company, organization, or institution to achieve his desires. Let us have a set of K-independent institutions (systems) that present the same services. The customer aims to find a suitable company with minimum detection cost to do the needed services. The main feature which the detected company should satisfy is the high quality of the service performance speed. This is well done if the company has a suitable number of servers.
Thus, the customer needs to choose an appropriate company (queue) in terms of the service costs from K-independent queues ($M/M/C_i/N_i,\ i, = 1, 2, \ldots, K$). Figure 1 shows that each queue or company Q_i has

a number of servers C_i, and its capacity is N_i. Thus, as in [43], in the steady-state we have the following system of probabilistic differential-difference equations:

$$-\lambda_{ij}P_{0_{ij}} + \mu_{ij}P_{1_{ij}} = 0, \quad n_{ij} = 0, \tag{1}$$

$$-(\lambda_{ij} + n_{ij}\mu_{ij})P_{n_{ij}} + (n_{ij}+1)P_{n_{ij}+1} + \lambda_{ij}P_{n_{ij}-1} = 0, \quad 1 \leq n_{ij} < C_i, \tag{2}$$

$$-(\lambda_{ij} + C_i\mu_{ij})P_{n_{ij}} + C_i\mu_{ij}P_{n_{ij}+1} + \lambda_{ij}P_{n_{ij}-1} = 0, \quad C_i < n_{ij} < N_i, \tag{3}$$

$$\lambda_{ij}P_{N_i-1} - C_i\mu_{ij}P_{N_i} = 0, \quad n_{ij} = N_i. \tag{4}$$

In the steady-state and as in [43], we found the probability of n_{ij} units in the Q_i at time step j is given by

$$P_{n_{ij}} = \begin{cases} \frac{(r_{ij})^{n_{ij}}}{n_{ij}!}P_{0_{ij}}, & 0 \leq n_{ij} < C_i, \\ \frac{(r_{ij})^{n_{ij}}}{(C_i)^{n_{ij}-C_i}C_i!}P_{0_{ij}}, & C_i \leq n_{ij} \leq N_i, \end{cases} \tag{5}$$

where $r_{ij} = \frac{\lambda_{ij}}{\mu_{ij}C_i}$, and $P_{0_{ij}}$ is the probability of no customers in the Q_i at a time step j, given by

$$P_{0_{ij}} = \left[\frac{r_{ij}^{C_i}}{(C_i-1)!}\left(\frac{1 - r_{ij}^{N_i-C_i+1}}{1-r_{ij}} \right) + \sum_{n_{ij}=0}^{C_i-1} \frac{C^{n_{ij}}(r_{ij})^{n_{ij}}}{n_{ij}!} \right]^{-1}.$$

From equation (5) and using the boundary condition $\sum_{n=0}^{N} P_n = \sum_{i=1}^{K}\sum_{j=1}^{T}\sum_{n_{ij}=1}^{N_i} P_{n_{ij}} = 1$, where $\sum_{i=1}^{K} N_i = N$ (the total capacity of the system) and $\sum_{i=1}^{K} C_i = C$ (the total number of the servers in the system), one can get the value of $P_{0_{ij}}$, $i = 1, 2, \ldots, K$, $j = 1, 2, \ldots, T$ by solving the equation

$$\sum_{i=1}^{K}\sum_{j=1}^{T}\left(\sum_{n_{ij}=0}^{C_i-1} \frac{(r_{ij})^{n_{ij}}}{n_{ij}!}P_{0_{ij}} + \sum_{n_{ij}=C_i}^{N_i} \frac{(r_{ij})^{n_{ij}}}{(C_i)^{n_{ij}-C_i}C_i!}P_{0_{ij}} \right) = 1. \tag{6}$$

3.1. Detection Model with Maximum Discount Effort Reward Search Under the Quality Control Process

One of the most important criteria for quality control in any organization is a commitment and fulfillment of customer requirements. Good service contributes to achieving customer satisfaction and improving financial performance by reducing costs through:

- Low budget for advertising and promotion.
- Reduced administrative expenses because of energy utilization; and
- Low cost of attracting new customers.

Also, good service helps to have more freedom in pricing products and contributes to maintaining employment. It contributes to achieving customer satisfaction and consequently their continued interaction with the organization, leading to an increase in its business and the availability of opportunities for promotion, incentives, and job satisfaction for employees.

The client makes an effort to choose a suitable company. The customer distributes his effort randomly to inspect all companies and then select one. On the other hand, the customer always seeks to examine all the institutions that provide the same service in terms of advantages and disadvantages and then choose what suits him. Here, we consider the cost function is

$$L(Z) = \sum_{i=1}^{K} \sum_{j=1}^{T} \sum_{n_{ij}=1}^{N_i} Z_{n_{ij}}, \tag{7}$$

where $Z_{n_{ij}}$ is the amount of cost at time step $j = 1, 2, \ldots, T$, and it is bounded by a Gaussian random variable X such that $0 \leq L(Z) \leq X$. If the service is done then as in [1], then the probability function of the customer service cost is given by

$$W(Z) = \sum_{i=1}^{K} \sum_{j=1}^{T} \sum_{n_{ij}=1}^{N_i} P_{n_{ij}}(1 - b(1, n_{ij}, Z_{n_{ij}})), \tag{8}$$

where $1 - b(1, n_{ij}, Z_{n_{ij}})$ is the conditional probability function of the detection event for the suitable company Q_i which already has n_{ij}, $i = 1, 2, \ldots, K$, $j = 1, 2, \ldots, T$, customers in the service.
In the steady-state and for each Q_i, the discount reward search function at time step j is $\varpi_i^j \in (0, 1)$, see [5]. Here, we need to minimize the searching effort $Z_{n_{ij}}$ and maximize the probability of detection and the discount reward parameter ϖ_i^j. Therefore, the probability function (8) as a function with discounted parameter ϖ_i^j takes the form

$$W(Z) = \sum_{i=1}^{K} \sum_{j=1}^{T} \sum_{n_{ij}=1}^{N_i} \varpi_i^j P_{n_{ij}}(1 - b(1, n_{ij}, Z_{n_{ij}})), \tag{9}$$

and the cost function (7) becomes

$$L(Z) = \sum_{i=1}^{K} \sum_{j=1}^{T} \sum_{n_{ij}=1}^{N_i} \varpi_i^j Z_{n_{ij}}. \tag{10}$$

Since the maximization process of ϖ_i^j and $1 - b(1, n_{ij}, Z_{n_{ij}})$ is equivalent to the minimization process of $1 - \varpi_i^j$ and $b(1, n_{ij}, Z_{n_{ij}})$ (probability of no detection) respectively. Therefore, the probability of not detecting the suitable company over the whole time j, $j = 1, 2, \ldots, T$ is given by

$$\tilde{W}(Z) = \prod_{j=1}^{T} \sum_{i=1}^{K} \sum_{n_{ij}=1}^{N_i} (1 - \varpi_i^j) P_{n_{ij}} b(1, n_{ij}, Z_{n_{ij}}), \tag{11}$$

with the search effort,

$$\tilde{L}(Z) = \sum_{i=1}^{K} \sum_{j=1}^{T} \sum_{n_{ij}=1}^{N_i} (1 - \varpi_i^j) Z_{n_{ij}}. \tag{12}$$

As in [1], we consider the exponential detection function $1 - e^{-Z_{n_{ij}}}$. Thus, from the independence principle of providing service from each queue and by compensation from (5) in (11), we have

$$\tilde{W}(Z) = \prod_{j=1}^{T} \sum_{i=1}^{K} (1 - \varpi_i^j) \left[\sum_{n_{ij}=0}^{C_i - 1} \frac{(r_{ij})^{n_{ij}}}{n_{ij}!} P_{0_{ij}} e^{-Z_{n_{ij}}} + \sum_{n_{ij}=C_i}^{N_i} \frac{(r_{ij})^{n_{ij}}}{(C_i)^{n_{ij} - C_i} C_i!} P_{0_{ij}} e^{-Z_{n_{ij}}} \right]. \tag{13}$$

Competition between companies has led to increased customer awareness and gives the customer the ability to move from one company to another to get the best services. Therefore, competition between companies based on diversity in the provision of services or the service prices or the innovation and renewal has become a complicated process for the company's management which achieve higher levels of profitability. In addition, the concept of quality control of the service provided emerged as a competitive strategy through which companies can distinguish themselves from the rest of the competitors and thus achieve their marketing objectives. In our model, we seek to provide the right decision to the client to choose an appropriate company in terms of the cost and the service time while at the same time preserving the company's profits. In this regard, [44, 45, 46, 47, 48, 49, 50] presented different models to determine the quality control for many queueing systems. They discussed the development of some mathematical models to determine the optimum quality control policy of the service process provided to each customer.

4. Optimal Decision with Maximum Discount Effort Reward

The optimal decision depends on the minimum probability of not detection (equivalent to the maximum probability of detection), search effort $Z_{n_{ij}}$, and the minimum value of $1-\varpi_i^j$ (equivalent to the maximum discounted effort-reward). In addition, the optimal decision should consider the minimum value $r_{ij}=\frac{\lambda_{ij}}{\mu_{ij}C_i}$ which depends on the maximum value of the servicing cost μ_{ij} of Q_i at time step j. From the convexity of $\tilde{W}(Z)$ and let R^K be a feasible set of constrained decisions, then we have the following optimization problem P1:

$$\text{P1: } \min_{r_{ij},Z_{n_{ij}},\varpi_i^j}\tilde{W}(Z)=\prod_{j=1}^{T}\sum_{i=1}^{K}(1-\varpi_i^j)\left[\sum_{n_{ij}=0}^{C_i-1}\frac{(r_{ij})^{n_{ij}}}{n_{ij}!}P_{0_{ij}}e^{-Z_{n_{ij}}}+\sum_{n_{ij}=C_i}^{N_i}\frac{(r_{ij})^{n_{ij}}}{(C_i)^{n_{ij}-C_i}C_i!}P_{0_{ij}}e^{-Z_{n_{ij}}}\right],$$

$$\text{Sub. to: } \tilde{L}(Z)=\left(Z\in R^K \middle| \tilde{L}(Z)=\sum_{i=1}^{K}\sum_{j=1}^{T}\sum_{n_{ij}=1}^{N_i}(1-\varpi_i^j)Z_{n_{ij}}\le L(X)=X\right),$$

$$\sum_{n=0}^{N}P_n=\sum_{i=1}^{K}\sum_{j=1}^{T}\sum_{n_{ij}=1}^{N_i}P_{n_{ij}}=1,\quad Z_{n_{ij}}\ge 0,\quad 0<r_{ij}<1,$$

$$\varpi_i^j\in(0,1),\ \text{and } i=1,2,\dots,K,\quad j=1,2,\dots,T.$$

Let $P(L(Z)\le X)\le\beta,\ \beta\in[0,1]$, where $L(Z)\le X$ has a Gaussian distribution. Thus, one can obtain $P\left(\frac{L(Z)-E(X)}{\sqrt{\text{Var}(X)}}\le\frac{X-E(X)}{\sqrt{\text{Var}(X)}}\right)\le\beta$ (from central limit theorem). Since $\Gamma_p=fracX-E(X)\sqrt{\text{Var}(X)}$ has a standard normal distribution, then one can get the constraint $\Phi\left(\frac{L(Z)-E(X)}{\sqrt{\text{Var}(X)}}\right)\le\Phi(\Gamma_p)$, at $\Phi(\Gamma_p)=\beta$. Consequently, the constraint $L(Z)-E(X)\le\Gamma_p\sqrt{\text{Var}(X)}$ satisfied, and P1 becomes

$$\text{P2: } \min_{r_{ij},Z_{n_{ij}},\varpi_i^j}\tilde{W}(Z)=\prod_{j=1}^{T}\sum_{i=1}^{K}(1-\varpi_i^j)\left[\sum_{n_{ij}=0}^{C_i-1}\frac{(r_{ij})^{n_{ij}}}{n_{ij}!}P_{0_{ij}}e^{-Z_{n_{ij}}}+\sum_{n_{ij}=C_i}^{N_i}\frac{(r_{ij})^{n_{ij}}}{(C_i)^{n_{ij}-C_i}C_i!}P_{0_{ij}}e^{-Z_{n_{ij}}}\right],$$

$$\text{Sub. to: } \tilde{L}(Z)=\left(Z\in R^K \middle| \tilde{L}(Z)=\sum_{i=1}^{K}\sum_{j=1}^{T}\sum_{n_{ij}=1}^{N_i}(1-\varpi_i^j)Z_{n_{ij}}-E(X)-\Gamma_p\sqrt{\text{Var}(X)}\right)\le 0,$$

$$\sum_{i=1}^{K}\sum_{j=1}^{T}\left(\sum_{n_{ij}=0}^{C_i-1}\frac{(r_{ij})^{n_{ij}}}{n_{ij}!}P_{0_{ij}}+\sum_{n_{ij}=C_i}^{N_i}\frac{(r_{ij})^{n_{ij}}}{(C_i)^{n_{ij}-C_i}C_i!}P_{0_{ij}}\right)-1\le 0,$$

$$\varpi_i^j-1<0,\ r_{ij}-1<0,\quad -Z_{n_{ij}}\le 0,\text{and } i=1,2,\dots,K,\quad j=1,2,\dots,T.$$

Hence, the necessary Kuhn-Tucker conditions are,

$$\frac{\partial \tilde{W}(Z)}{\partial Z_{n_{ij}}} + \sum_{\Xi=1}^{5} U_{i\Xi} \frac{\partial f_{i\Xi}}{\partial Z_{n_{ij}}}(r_{ij}, Z_{n_{ij}}, \varpi_i^j) = 0,$$

$$\frac{\partial \tilde{W}(Z)}{\partial r_{ij}} + \sum_{\Xi=1}^{5} U_{i\Xi} \frac{\partial f_{i\Xi}}{\partial r_{ij}}(r_{ij}, Z_{n_{ij}}, \varpi_i^j) = 0,$$

$$\frac{\partial \tilde{W}(Z)}{\partial \varpi_i^j} + \sum_{\Xi=1}^{5} U_{i\Xi} \frac{\partial f_{i\Xi}}{\partial \varpi_i^j}(r_{ij}, Z_{n_{ij}}, \varpi_i^j) = 0,$$

$$\partial f_{i\Xi}(r_{ij}, Z_{n_{ij}}, \varpi_i^j) \leq 0, \quad U_{i\Xi} \partial f_{i\Xi}(r_{ij}, Z_{n_{ij}}, \varpi_i^j) = 0, \; U_{i\Xi} \geq 0.$$

This leads to

$$(\varpi_i^\sigma - 1)\left(\sum_{n_{i\sigma}=0}^{C_i-1} \frac{(r_{i\sigma})^{n_{i\sigma}}}{n_{i\sigma}!} P_{0_{i\sigma}} e^{-Z_{n_{i\sigma}}} + \sum_{n_{i\sigma}=C_i}^{N_i} \frac{(r_{i\sigma})^{n_{i\sigma}}}{(C_i)^{n_{i\sigma}-C_i} C_i!} P_{0_{i\sigma}} e^{-Z_{n_{i\sigma}}}\right)$$
$$\times \prod_{\substack{j=1 \\ j\neq\sigma}}^{T} \sum_{i=1}^{K} (1-\varpi_i^j) \left[\sum_{n_{ij}=0}^{C_i-1} \frac{(r_{ij})^{n_{ij}}}{n_{ij}!} P_{0_{ij}} e^{-Z_{n_{ij}}} + \sum_{n_{ij}=C_i}^{N_i} \frac{(r_{ij})^{n_{ij}}}{(C_i)^{n_{ij}-C_i} C_i!} P_{0_{ij}} e^{-Z_{n_{ij}}}\right] + U_{\sigma 1}(1-\varpi_i^\sigma) - U_{\sigma 3} = 0, \tag{14}$$

$$U_{\sigma 2} \sum_{\sigma=1}^{K} \left(\frac{(r_{i\sigma})^{n_{i\sigma}-1} P_{0_{i\sigma}}}{(n_{i\sigma}-1)!} + \frac{n_{i\sigma}(r_{i\sigma})^{n_{i\sigma}-1} P_{0_{i\sigma}}}{(C_\sigma)^{n_{i\sigma}-C_\sigma} C_\sigma!}\right) + U_{\sigma 4} + (\varpi_i^\sigma - 1)\left(\sum_{n_{i\sigma}=1}^{C_i-1} \frac{(r_{i\sigma})^{n_{i\sigma}-1}}{(n_{i\sigma}-1)!} P_{0_{i\sigma}} e^{-Z_{n_{i\sigma}}} + \right.$$
$$\left.\sum_{n_{i\sigma}=C_i}^{N_i} \frac{n_{i\sigma}(r_{i\sigma})^{n_{i\sigma}-1}}{(C_i)^{n_{i\sigma}-C_i} C_i!} P_{0_{i\sigma}} e^{-Z_{n_{i\sigma}}}\right) \prod_{\substack{j=1 \\ j\neq\sigma}}^{T} \sum_{i=1}^{K} (1-\varpi_i^j) \left[\sum_{n_{ij}=0}^{C_i-1} \frac{(r_{ij})^{n_{ij}}}{n_{ij}!} P_{0_{ij}} e^{-Z_{n_{ij}}} + \sum_{n_{ij}=C_i}^{N_i} \frac{(r_{ij})^{n_{ij}}}{(C_i)^{n_{ij}-C_i} C_i!} P_{0_{ij}} e^{-Z_{n_{ij}}}\right], \tag{15}$$

$$-\left(\sum_{n_{i\sigma}=0}^{C_i-1} \frac{(r_{i\sigma})^{n_{i\sigma}}}{n_{i\sigma}!} P_{0_{i\sigma}} e^{-Z_{n_{i\sigma}}} + \sum_{n_{i\sigma}=C_i}^{N_i} \frac{(r_{i\sigma})^{n_{i\sigma}}}{(C_i)^{n_{i\sigma}-C_i} C_i!} P_{0_{i\sigma}} e^{-Z_{n_{i\sigma}}}\right)$$
$$\times \prod_{\substack{j=1 \\ j\neq\sigma}}^{T} \sum_{i=1}^{K} (1-\varpi_i^j) \left[\sum_{n_{ij}=0}^{C_i-1} \frac{(r_{ij})^{n_{ij}}}{n_{ij}!} P_{0_{ij}} e^{-Z_{n_{ij}}} + \sum_{n_{ij}=C_i}^{N_i} \frac{(r_{ij})^{n_{ij}}}{(C_i)^{n_{ij}-C_i} C_i!} P_{0_{ij}} e^{-Z_{n_{ij}}}\right] - U_{\sigma 1}(Z_{n_{i\sigma}}) + U_{\sigma 5} = 0, \tag{16}$$

$$U_{\sigma 1}\left[\sum_{i=1}^{K} \sum_{\sigma=1}^{T} \sum_{n_{i\sigma}=0}^{N_i} (1-\varpi_i^\sigma) Z_{n_{i\sigma}} - E(X) - \Gamma_p \sqrt{\mathrm{Var}(X)}\right] = 0, \tag{17}$$

$$U_{\sigma 2}\left[\sum_{i=1}^{K} \sum_{\sigma=1}^{T} \left[\sum_{n_{i\sigma}=0}^{C_i-1} \frac{(r_{i\sigma})^{n_{i\sigma}}}{n_{i\sigma}!} P_{0_{i\sigma}} + \sum_{n_{i\sigma}=C_i}^{N_i} \frac{(r_{i\sigma})^{n_{i\sigma}}}{(C_i)^{n_{i\sigma}-C_i} C_i!} P_{0_{i\sigma}}\right] - 1\right] = 0, \tag{18}$$

$$U_{\sigma 3}[-Z_{n_{i\sigma}}] = 0, \tag{19}$$

$$U_{\sigma 4}[r_{i\sigma} - 1] = 0, \tag{20}$$

$$U_{\sigma 5}[\varpi_i^\sigma - 1] = 0. \tag{21}$$

When the detection process starts, we have $Z_{n_{i\sigma}} > 0$ (for each Q_i, $i = 1, 2, \ldots, K$ at time step σ, $\sigma = 1, 2, \ldots, T$), then in (19) we have $U_{\sigma 3} = 0$. Logically, the company continues to provide the service when its service rate is greater than the arrival rate (i.e., $\lambda_{i\sigma} < \mu_{i\sigma}$). Consequently, $r_{i\sigma}$ should be greater than zero and less than one for each Q_i, $i = 1, 2, \ldots, K$ at any time σ, $\sigma = 1, 2, \ldots, T$. From (20) one can find $U_{\sigma 4}$ must be equal zero. When the detection process starts, we have $\sum_{i=1}^{K}\sum_{\sigma=1}^{T}\sum_{n_{i\sigma}=0}^{N_i}(1-\varpi_i^\sigma)Z_{n_{i\sigma}} - E(X) - \Gamma_p\sqrt{\mathrm{Var}(X)} < 0$, then from (17) we conclude that $U_{\sigma 1} \neq 0$. Also, $\varpi_i^\sigma > 0$ this leads to $U_{\sigma 5} = 0$. Throughout the service and the detection processes, the probability condition $1 - \sum_{i=1}^{K}\sum_{\sigma=1}^{T}\left[\sum_{n_{i\sigma}=0}^{C_i-1}\frac{(r_{i\sigma})^{n_{i\sigma}}}{n_{i\sigma}!}P_{0_{i\sigma}} + \sum_{n_{i\sigma}=C_i}^{N_i}\frac{(r_{i\sigma})^{n_{i\sigma}}}{(C_i)^{n_{i\sigma}-C_i}C_i!}P_{0_{i\sigma}}\right] > 0$ should be satisfied, thus from (18) we have $U_{\sigma 2} = 0$. Consequently, from (14) and (16) we get:

$$
\begin{aligned}
&(Z_{n_{i\sigma}} + \varpi_i^\sigma - 1)^{-1}\Bigg[(\varpi_i^\sigma - 2)\left(\sum_{n_{i\sigma}=0}^{C_i-1}\frac{(r_{i\sigma})^{n_{i\sigma}}}{n_{i\sigma}!}P_{0_{i\sigma}}e^{-Z_{n_{i\sigma}}} + \sum_{n_{i\sigma}=C_i}^{N_i}\frac{(r_{i\sigma})^{n_{i\sigma}}}{(C_i)^{n_{i\sigma}-C_i}C_i!}P_{0_{i\sigma}}e^{-Z_{n_{i\sigma}}}\right)\\
&\times\prod_{\substack{j=1\\ j\neq\sigma}}^{T}\sum_{i=1}^{K}(1-\varpi_i^j)\left[\sum_{n_{ij}=0}^{C_i-1}\frac{(r_{ij})^{n_{ij}}}{n_{ij}!}P_{0_{ij}}e^{-Z_{n_{ij}}} + \sum_{n_{ij}=C_i}^{N_i}\frac{(r_{ij})^{n_{ij}}}{(C_i)^{n_{ij}-C_i}C_i!}P_{0_{ij}}e^{-Z_{n_{ij}}}\right]\Bigg] = U_{\sigma 1},
\end{aligned} \tag{22}
$$

from equation (15)

$$
\begin{aligned}
&(\varpi_i^\sigma - 1)\left(\sum_{n_{i\sigma}=1}^{C_i-1}\frac{(r_{i\sigma})^{n_{i\sigma}-1}}{(n_{i\sigma}-1)!}P_{0_{i\sigma}}e^{-Z_{n_{i\sigma}}} + \sum_{n_{i\sigma}=C_i}^{N_i}\frac{n_{i\sigma}(r_{i\sigma})^{n_{i\sigma}-1}}{(C_i)^{n_{i\sigma}-C_i}C_i!}P_{0_{i\sigma}}e^{-Z_{n_{i\sigma}}}\right)\\
&\times\prod_{\substack{j=1\\ j\neq\sigma}}^{T}\sum_{i=1}^{K}(1-\varpi_i^j)\left[\sum_{n_{ij}=0}^{C_i-1}\frac{(r_{ij})^{n_{ij}}}{n_{ij}!}P_{0_{ij}}e^{-Z_{n_{ij}}} + \sum_{n_{ij}=C_i}^{N_i}\frac{(r_{ij})^{n_{ij}}}{(C_i)^{n_{ij}-C_i}C_i!}P_{0_{ij}}e^{-Z_{n_{ij}}}\right] = 0
\end{aligned} \tag{23}
$$

from equation (22) into equation (17), we have

$$
\begin{aligned}
&(Z_{n_{i\sigma}} + \varpi_i^\sigma - 1)^{-1}\Bigg[(\varpi_i^\sigma - 2)\left(\sum_{n_{i\sigma}=0}^{C_i-1}\frac{(r_{i\sigma})^{n_{i\sigma}}}{n_{i\sigma}!}P_{0_{i\sigma}}e^{-Z_{n_{i\sigma}}} + \sum_{n_{i\sigma}=C_i}^{N_i}\frac{(r_{i\sigma})^{n_{i\sigma}}}{(C_i)^{n_{i\sigma}-C_i}C_i!}P_{0_{i\sigma}}e^{-Z_{n_{i\sigma}}}\right)\\
&\times\prod_{\substack{j=1\\ j\neq\sigma}}^{T}\sum_{i=1}^{K}(1-\varpi_i^j)\left[\sum_{n_{ij}=0}^{C_i-1}\frac{(r_{ij})^{n_{ij}}}{n_{ij}!}P_{0_{ij}}e^{-Z_{n_{ij}}} + \sum_{n_{ij}=C_i}^{N_i}\frac{(r_{ij})^{n_{ij}}}{(C_i)^{n_{ij}-C_i}C_i!}P_{0_{ij}}e^{-Z_{n_{ij}}}\right]\Bigg]\\
&\times\left[\sum_{i=1}^{K}\sum_{\sigma=1}^{T}\sum_{n_{i\sigma}=0}^{N_i}(1-\varpi_i^\sigma)Z_{n_{i\sigma}} - E(X) - \Gamma_p\sqrt{\mathrm{Var}(X)}\right] = 0.
\end{aligned} \tag{24}
$$

By incorporating equations (23) and (24), we have

$$
\begin{aligned}
&\Bigg((\varpi_i^\sigma - 1)\left(\sum_{n_{i\sigma}=1}^{C_i-1}\frac{(r_{i\sigma})^{n_{i\sigma}-1}}{(n_{i\sigma}-1)!}P_{0_{i\sigma}}e^{-Z_{n_{i\sigma}}} + \sum_{n_{i\sigma}=C_i}^{N_i}\frac{n_{i\sigma}(r_{i\sigma})^{n_{i\sigma}-1}}{(C_i)^{n_{i\sigma}-C_i}C_i!}P_{0_{i\sigma}}e^{-Z_{n_{i\sigma}}}\right)\\
&+(Z_{n_{i\sigma}} + \varpi_i^\sigma - 1)^{-1}\Bigg[(\varpi_i^\sigma - 2)\left(\sum_{n_{i\sigma}=0}^{C_i-1}\frac{(r_{i\sigma})^{n_{i\sigma}}}{n_{i\sigma}!}P_{0_{i\sigma}}e^{-Z_{n_{i\sigma}}} + \sum_{n_{i\sigma}=C_i}^{N_i}\frac{(r_{i\sigma})^{n_{i\sigma}}}{(C_i)^{n_{i\sigma}-C_i}C_i!}P_{0_{i\sigma}}e^{-Z_{n_{i\sigma}}}\right)\\
&\times\left[\sum_{i=1}^{K}\sum_{\sigma=1}^{T}\sum_{n_{i\sigma}=0}^{N_i}(1-\varpi_i^\sigma)Z_{n_{i\sigma}} - E(X) - \Gamma_p\sqrt{\mathrm{Var}(X)}\right]\Bigg]\Bigg)\\
&\times\prod_{\substack{j=1\\ j\neq\sigma}}^{T}\sum_{i=1}^{K}(1-\varpi_i^j)\left[\sum_{n_{ij}=0}^{C_i-1}\frac{(r_{ij})^{n_{ij}}}{n_{ij}!}P_{0_{ij}}e^{-Z_{n_{ij}}} + \sum_{n_{ij}=C_i}^{N_i}\frac{(r_{ij})^{n_{ij}}}{(C_i)^{n_{ij}-C_i}C_i!}P_{0_{ij}}e^{-Z_{n_{ij}}}\right] = 0.
\end{aligned} \tag{25}
$$

Table 1. The optimal values μ^*_σ, $(\varpi^\sigma_\sigma)^*$, $Z^*_{n_\sigma}$, $P^*_{0_\sigma}$, and $P^*_{(n_\sigma)\sigma}$

σ	N_σ	n_σ	λ_σ	ξ^*_σ	μ^*_σ	$(\varpi^\sigma_\sigma)^*$	$Z^*_{n_\sigma}$	$P^*_{0_\sigma}$	$P^*_{(n_\sigma)\sigma}$
2	11	6	4.780291372	0.01481011141	4.880291372	0.843	0.036	0.3821677061	0.4866848476
4	19	11	6.834132513	0.3962100023	6.934132513	0.129	0.0189	0.3799917987	0.4792122844

Customer expectations represent the basis on which to reach the final judgment on the ability of the service provider to satisfy his needs and achieve his desires, and then continue with them. Therefore, during the searching process and at time step σ, $\sigma = 1, 2, \ldots, T$), it is expected that a suitable company will be detected. At this moment, the following formula,

$$\prod_{\substack{j=1\\ j\neq\sigma}}^{T}\sum_{i=1}^{K}(1-\varpi_i^j)\left[\sum_{n_{ij}=0}^{C_i-1}\frac{(r_{ij})^{n_{ij}}}{n_{ij}!}P_{0_{ij}}e^{-Z_{n_{ij}}}+\sum_{n_{ij}=C_i}^{N_i}\frac{(r_{ij})^{n_{ij}}}{(C_i)^{n_{ij}-C_i}C_i!}P_{0_{ij}}e^{-Z_{n_{ij}}}\right] \tag{26}$$

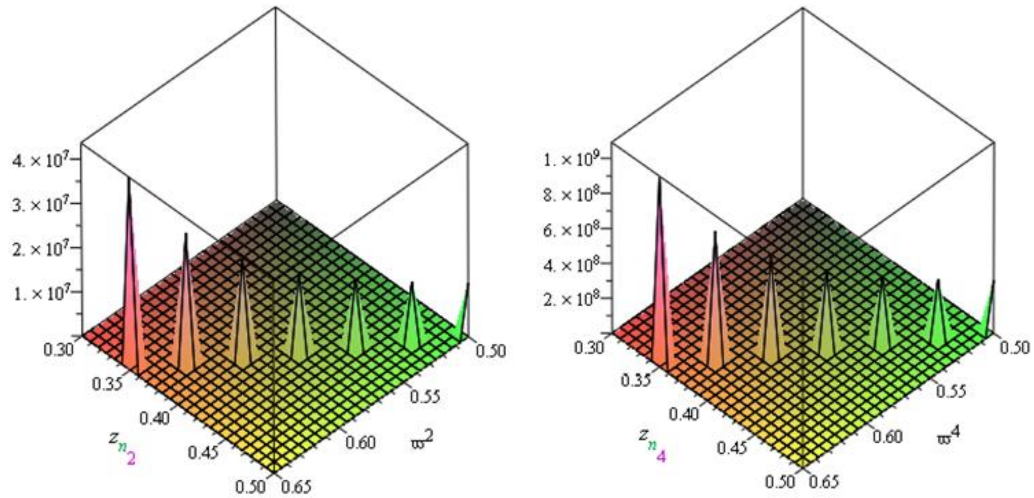
not exists. Consequently, (25) will become,

$$\begin{aligned}&(\varpi_i^\sigma-1)\left(\sum_{n_{i\sigma}=1}^{C_i-1}\frac{(r_{i\sigma})^{n_{i\sigma}-1}}{(n_{i\sigma}-1)!}P_{0_{i\sigma}}e^{-Z_{n_{i\sigma}}}+\sum_{n_{i\sigma}=C_i}^{N_i}\frac{n_{i\sigma}(r_{i\sigma})^{n_{i\sigma}-1}}{(C_i)^{n_{i\sigma}-C_i}C_i!}P_{0_{i\sigma}}e^{-Z_{n_{i\sigma}}}\right)\\ &+(Z_{n_{i\sigma}}+\varpi_i^\sigma-1)^{-1}\left[(\varpi_i^\sigma-2)\left(\sum_{n_{i\sigma}=0}^{C_i-1}\frac{(r_{i\sigma})^{n_{i\sigma}}}{n_{i\sigma}!}P_{0_{i\sigma}}e^{-Z_{n_{i\sigma}}}+\sum_{n_{i\sigma}=C_i}^{N_i}\frac{(r_{i\sigma})^{n_{i\sigma}}}{(C_i)^{n_{i\sigma}-C_i}C_i!}P_{0_{i\sigma}}e^{-Z_{n_{i\sigma}}}\right)\right.\\ &\left.\times\left[\sum_{i=1}^{K}\sum_{\sigma=1}^{T}\sum_{n_{i\sigma}=0}^{N_i}(1-\varpi_i^\sigma)Z_{n_{i\sigma}}-E(X)-\Gamma_p\sqrt{\mathrm{Var}(X)}\right]\right]=0.\end{aligned} \tag{27}$$

For each Q_i, $i = 1, 2, \ldots, K$ and at time step σ, $\sigma = 1, 2, \ldots, T$), we let the values of C_σ and N_σ be known such that $C_\sigma < N_\sigma$. We use Maple 13 to solve (23) and (24). We can use the random generation method to get the optimal $Z^*_{n_{i\sigma}}$, $r^*_{i\sigma}$, and $(\varpi^\sigma_i)^*$ (minimum values). The minimum value of $r^*_{i\sigma}$ will give the maximum value of $\mu^*_{i\sigma}$. This maximum service rate will reduce the crowding and hence accelerate the service. These optimal measures contribute to considering the suitable company under the quality control process.

5. An Application with Optimal Measures for a Detected Company

There is an optimal standard, which the client seeks to find in the required company to achieve his services at reasonable costs. The institution aims to provide a distinguished service to fulfill the customer¡¯s desires. The failure to accomplish this will lead to a failure to achieve customer satisfaction with the service. For this reason, he decides to switch to another institution that is better in its ability to diagnose and satisfy his needs. There is an optimal standard, which the client seeks to find in the required company to achieve his services at reasonable costs. To achieve these criteria, it is necessary to find the optimal values $Z^*_{n_{i\sigma}}$, $\mu^*_{i\sigma}$, and $(\varpi^\sigma_i)^*$. These values can be found by solving (27). We consider the values of $\lambda_{i\sigma}$, $n_{i\sigma}$, C_i, and N_i are generated randomly by using Maple 13 as in Table 1. Also, we consider the optimal values $\xi^*_{i\sigma} = \sum_{i=1}^{K}\sum_{\sigma=1}^{T}\sum_{n_{i\sigma}=0}^{N_i}(1-\varpi_i^\sigma)Z_{n_{i\sigma}}-E(X)-\Gamma_p\sqrt{\mathrm{Var}(X)}$, which have been obtained in [1] (values without discount effort-reward). Using these values in (27), one can get the maximum probability of



(a) $N_2 = 11$ **(b)** $N_4 = 19$

Figure 2. 3D plot of equation (27).

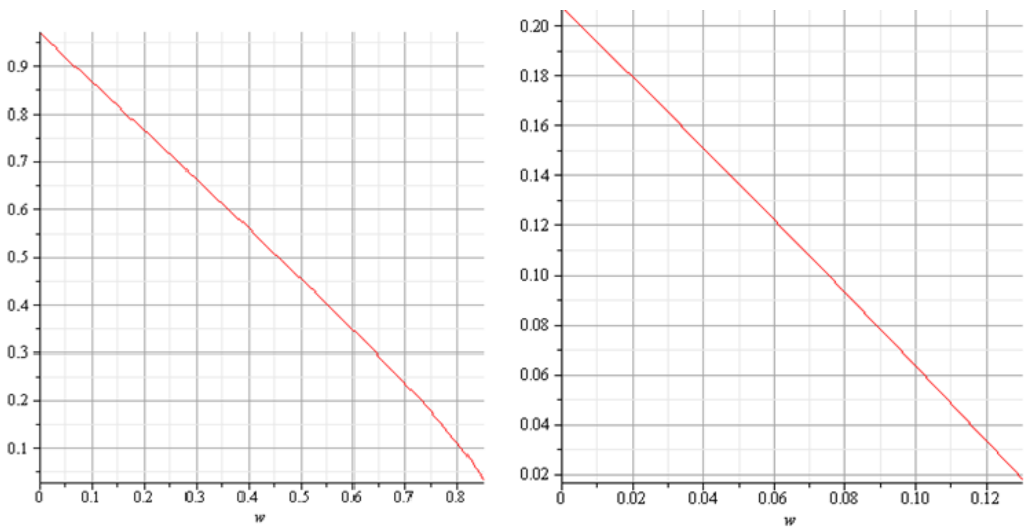
$n_{i\sigma}$ customers in the company Q_i, which have several service channels $C_i = 4$ at time step σ, $\sigma = 2\&4$ see Table 1. The plotting 3D of (27) (Figure 2), presents the relationship between $(\varpi_\sigma^\sigma)^*$, and $Z_{n_\sigma}^*$ for different capacities N_σ when $\sigma = 2\&\ 4$, where all peaks of the cones give the needed optimal values which satisfied (27). By using Maple 13, we can obtain an equation of $(\varpi_\sigma^\sigma)^*$, and $Z_{n_\sigma}^*$ that present the needed optimal solution, see Figure 3. In addition, one can obtain the maximum value μ_σ^* for all N_σ, $\sigma = 2\&\ 4$ after getting the minimum value $Z_{n_\sigma}^*$ where $0 < \lambda_\sigma < \mu_\sigma$. These optimal values will be used in $P_{0_{ij}} = \left[\frac{r_{ij}^{C_i}}{(C_i-1)!} \left(\frac{1-r_{ij}^{N_i-C_i+1}}{1-r_{ij}} \right) + \sum_{n_{ij}=0}^{C_i-1} \frac{C^{n_{ij}}(r_{ij})^{n_{ij}}}{n_{ij}!} \right]^{-1}$ and (5) to give the maximum probabilities $P_{0_\sigma}^*$, and $P_{(n_\sigma)\sigma}^*$, respectively, for the appropriate company at time step $\sigma = 2\ \&\ 4$, see Table 1.

Now, from (13) one can obtain the probability of not detection $\tilde{W}(Z) = 0.01186284413$. Thus, the maximum value of detecting the suitable company is $W^*(Z) = 1 - \tilde{W}(Z) = 0.9881371559$.

Depending on the optimal values $Z_{in_\sigma}^*$, μ_σ^* and $(\varpi_\sigma^\sigma)^*$ that are obtained at time step $\sigma = 2\&\ 4$, it is easy to get the optimal expected value of the customer's number, the minimum expected waiting time and the minimum expected service time for the appropriate company, see [1].

6. Conclusion and Future Work

A new mathematical model has been presented. This model helps the customers to take a good decision with minimum cost to choose an appropriate company from K independent queues $(M/M/C_i/N_i, i = 1, 2, ..., K$, where each one of them has a Poisson arrival process and exponential service time. The



(a) $N_2 = 11$ **(b)** $N_4 = 20$

Figure 3. The optimal values $(\varpi_{\sigma}^{\sigma})^*$, and $Z_{n_\sigma}^*$ for different values of N_σ.

customers jump between these queues according to the Markov process. We get a complex discrete stochastic optimization problem using the maximum discount effort-reward function and the exponential detection function in the steady-state. Applying the Kuhn-Tucker conditions to solve this problem, we get the minimum detection effort, maximum service rate, and maximum discount effort-reward. We consider the searching attempt to be a random variable bounded by a known Gaussian distribution. The optimal which obtained are used to get the optimal performance measures of a suitable company. An illustrative example has been presented to show the effectiveness of this model.
For more optimization of this model, one can study it under the fuzziness of the discount effort-reward parameter as future work. Also, a novel detection algorithm for this model can be constructed after obtaining the probability of n customers in the system under the transient solution.

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