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Discrete Logistic Exponential Distribution with Application

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Abstract In this paper, a new two-parameter discrete logistic exponential distribution is proposed based on the survival discretization approach. Some statistical properties are derived, and it is found that the proposed model can be used to discuss several kinds of failure rates including unimodal, bathtub, and increasing-shaped. Moreover, it can be utilized effectively to model under- and over-dispersed data. The distribution parameters are estimated using the maximum likelihood technique. The behavior of maximum likelihood estimators is assessed utilizing a comprehensive simulation study. In the end, two real data are analyzed to show the usefulness of the new discrete distribution.

Keywords Survival discretization; Logistic exponential; Moments; Simulation; Count observations.

AMS 2010 subject classifications 60E05, 62E10, 62N05

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1. Introduction

Discrete probability distribution plays an important role in data analysis, inference, and forecasting processes. The occurrence times, frequencies, and effects of many events in nature are analyzed by statistical modeling techniques. The majority of natural or scientific phenomena have distinct characteristics. The discrete probability distributions are used to model earthquakes, traffic accidents, landslide counts, and the number of people dying from the disease. To reduce estimation errors in the modeling of large data sets, researchers have proposed more flexible distributions. To introduce a new discrete distribution, there are two common techniques. The commonly used methods for discretization are mixed-Poisson discrete distributions, infinite series, and survival discretization. A widespread method to obtain discrete analogs of continuous random variables is that one based on the survival function of the original distribution. The method of discretization by survival function was proposed by Nakagawa and Osaki (1975). The standard approach of discretizing a continuous variable can be utilized to derive a distribution by incorporating a grouping on the time axis. If the continuous random variable X has the survival function (sf), $S(x) = P(X \geq x)$ and the times are clustered into unit intervals, then the discrete random variable of X will have the probability mass function (pmf) indicated by $dX = \lfloor X \rfloor$; which is the greatest integer less than or equal to x, will have the pmf

$$P(x) = S(x) - S(x+1); \quad x = 0, 1, 2, \ldots . \tag{1}$$

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Several continuous distributions have been discretized based on the survival discretization method for modeling lifetime data some of them are summarized as follows; discrete Weibull distribution (Nakagawa & Osaki, 1975), discrete Rayleigh distribution (Roy, 2004), discrete Pareto distribution (Krishna & Pundir, 2009), discrete Lindley distribution (Gómez-Déniz & Calderín-Ojeda, 2011), discrete Burr type XII distribution (Para, 2014), discrete inverse Weibull distribution (Para & Jan, 2017), discrete generalized Rayleigh distribution (Alamatsaz, Dey, Dey, & Harandi, 2016), discrete Burr III distribution (AL-Huniti & AL-Dayian, 2012), discrete log-logistic distribution (Para, 2016), discrete Gompertz distribution (Al-dayian, 2019), discrete alpha power inverse Lomax distribution (Almetwally & Gamal, 2020), exponentiated discrete Lindley distribution (El-Morshedy, Eliwa & Nagy, 2020), discrete Lindley with three parameters (Eliwa, Altun, Dawoody & El-Morshedy, 2020), discrete Burr-Hatke distribution (El-Morshedy, Eliwa & Altun, 2020), discrete mixture distribution (Eliwa & El-Morshedy, 2021), discrete inverted Topp-Leone distribution (Eldeeb, Ahsan-ul-Haq, & Babar, 2021), discrete Ramous-Louzada distribution (Eldeeb, Ahsan-ul-Haq, & Eliwa, 2021) discrete type-II half-logistic exponential distribution (Ahsan-ul-Haq, Babar, Hashmi, Alghamdi, & Afify, 2021) and discrete power-Ailamujia distribution (Alghamdi et al., 2022). The primary goal of this paper is to apply the method of survival function to derive the discrete analog for the logistic exponential (LE) model, which is a two-parameter lifetime distribution introduced by (Lan & Leemis, 2008) and logistic exponential is generalized logistic exponential was suggested by (Gleaton & Lynch, 2006). A continuous random variable X is said to have LE distribution if its sf can be written as

$$S\left(x\right)=\frac{1}{1+\left(e^{\beta x}-1\right)^{\alpha}};\quad x>0,\ \alpha,\beta>0. \tag{2}$$

The only drawback of this distribution is that it does not yield a closed-form expression for the moments, which can be computed numerically using software like Mathematica, R, or MATLAB. Recently, (Ali, Dey, Tahir & Mansoor, 2020) estimate the parameters considering ten frequent estimation methods namely, maximum likelihood, least square, weighted least square, percentiles, maximum and minimum spacing distance, and a variant of the method of the minimum distances for the LE parameters.

2. Discrete LE distribution

The discrete LE (DLE) distribution with two parameters is derived using the survival discretization approach based on LE model by (Lan & Leemis, 2008). Thus, the pmf of the DLE model can be expressed as

$$P\left(x\right)=\frac{\left(e^{\beta(x+1)}-1\right)^{\alpha}-\left(e^{\beta x}-1\right)^{\alpha}}{\left\{1+\left(e^{\beta x}-1\right)^{\alpha}\right\}\left\{1+\left(e^{\beta(x+1)}-1\right)^{\alpha}\right\}};\ \ x=0,1,2,\ldots, \tag{3}$$

where $\alpha,\beta>0$. Figure 1 shows the behavior of the pmf of the DLE model using selected values for α and β.

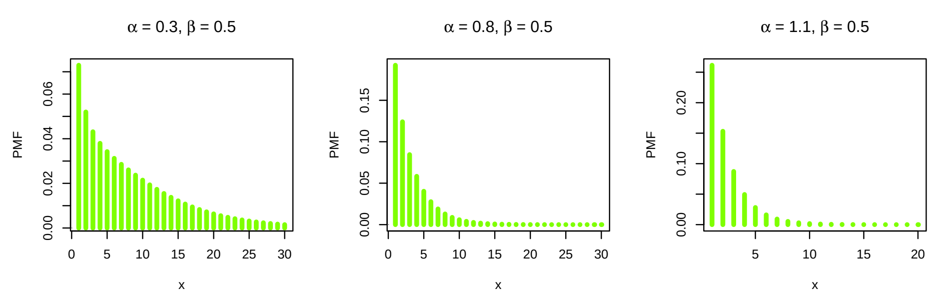

Figure 1. The pmf plots of DLE distribution.

It is noted that the pmf can be used to model positively skewed data. Moreover, it has unimodal-shaped. The corresponding cdf to equation (3) is given by

$$F\left(x\right)=\frac{\left(e^{\beta(x+1)}-1\right)^{\alpha}}{1+\left(e^{\beta(x+1)}-1\right)^{\alpha}};\quad x=0,1,2,\ldots \tag{4}$$

Equation (4) can be formulated as

$$F\left(x\right)=\sum_{k=0}^{\infty}\sum_{j=0}^{k}C_{k}(-1)^{j}\left(\begin{array}{c}k\\ j\end{array}\right)e^{-\beta j(x+1)},$$

where

$$C_{k}=\frac{1}{b_{0}}\left[a_{k}-\frac{1}{b_{0}}\sum_{r=1}^{k}b_{r}C_{k-r}\right];\ \ k\geq1,\ \ C_{0}=\frac{a_{0}}{b_{0}},$$

$$b_{k}=a_{k}+(-1)^{k}\left(\begin{array}{c}\alpha\\ k\end{array}\right)$$

and

$$a_{k}=\sum_{i=k}^{\infty}(-1)^{i+k}\left(\begin{array}{c}\alpha\\ i\end{array}\right)\left(\begin{array}{c}i\\ k\end{array}\right).$$

Note that for $\alpha=1$, we obtain geometric distributions. The hazard rate function (hrf) can be formulated as

$$h\left(x\right)=\frac{\left(e^{\beta(x+1)}-1\right)^{\alpha}-\left(e^{\beta x}-1\right)^{\alpha}}{1+\left(e^{\beta x}-1\right)^{\alpha}};\quad x=0,1,2,\ldots \tag{5}$$

Figure 2 shows the behavior of the hrf of the DLE model using selected values for α and β.

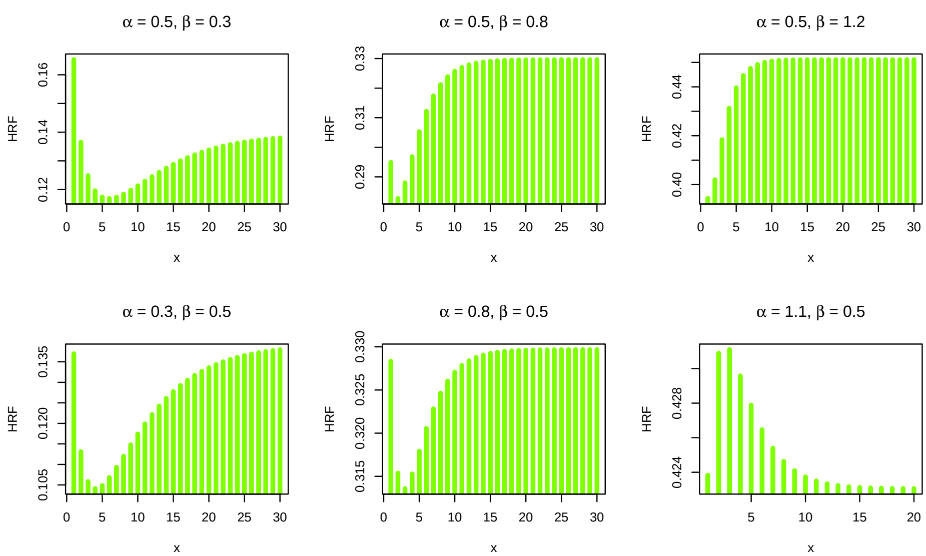

Figure 2. The hrf plots of DLE distribution.

It is noted that the new discrete distribution can be used to model several kinds of failure rates including unimodal, bathtub, and increasing-shaped. The reversed hazard function is as follows

$$r^{*}\left(x\right)=\frac{P(x)}{F(x)}=\frac{\left(e^{\beta(x+1)}-1\right)^{\alpha}-\left(e^{\beta x}-1\right)^{\alpha}}{\left\{1+\left(e^{\beta x}-1\right)^{\alpha}\right\}\ \left(e^{\beta(x+1)}-1\right)^{\alpha}}. \tag{6}$$

The second rate of failure can be defined as $\log\left(1-F(x)/1-F(x+1)\right)$. Then, the second rate of failure of DLE model is as follows

$$r^{**}(x)=\log\left[\frac{1+\left(e^{\beta(x+2)}-1\right)^{\alpha}}{1+\left(e^{\beta(x+1)}-1\right)^{\alpha}}\right]. \tag{7}$$

The recurrence relation for generating the probabilities of DLE distribution is given by

$$P(x+1)=\frac{\left\{1+\left(e^{\beta x}-1\right)^{\alpha}\right\}\left\{\left(e^{\beta(x+2)}-1\right)^{\alpha}-\left(e^{\beta(x+1)}-1\right)^{\alpha}\right\}}{\left\{1+\left(e^{\beta(x+2)}-1\right)^{\alpha}\right\}\left\{\left(e^{\beta(x+1)}-1\right)^{\alpha}-\left(e^{\beta x}-1\right)^{\alpha}\right\}}P(x). \tag{8}$$

3. Probability generating function, moments, and related measures

The probability generating function (pgf) of the DLE distribution is given as follows

$$G_x(Z)=\sum_{x=0}^{\infty}Z^{x}P(X=x). \tag{9}$$

Using equation (9), the pgf of the DLE distribution is obtained as

$$G_x(Z)=1+(Z-1)\sum_{x=1}^{\infty}Z^{x-1}\left[1+\left(e^{\beta x}-1\right)^{\alpha}\right]^{-1}. \tag{10}$$

The mean of the DLE model can be obtained by taking the first derivative of equation (10) and setting $Z=1$, yielding

$$\mu=G_x^{\prime}(1)=\sum_{x=1}^{\infty}\left[1+\left(e^{\beta x}-1\right)^{\alpha}\right]^{-1}. \tag{11}$$

Further derivatives (higher) of the generating function of the DLE model yield the corresponding raw moment around the origin of the distribution. The variance of the DLE distribution is expressed as

$$\delta=\sum_{x=1}^{\infty}(2x-1)\left[1+\left(e^{\beta x}-1\right)^{\alpha}\right]^{-1}-\left(\sum_{x=1}^{\infty}\left[1+\left(e^{\beta x}-1\right)^{\alpha}\right]^{-1}\right)^{2}. \tag{12}$$

Hence, the dispersion index (DI), coefficient of skewness (CS), and coefficient of kurtosis (CK) of the DLE model can be derived from well-known relations. Table 1 lists the nature of the mean, variance, CS, CK, and DI of the DLE distribution for varying values of the parameters.

Table 1: Some computational statistics of the DLE model based on $\beta=0.5$.

Measure	$\alpha=0.5$	$\alpha=1$	$\alpha=1.5$	$\alpha=2$	$\alpha=2.5$
Mean	2.75690	1.54149	1.20413	1.06911	0.96476
Variance	16.7986	3.91769	1.72052	0.99116	0.35678
DI	6.09330	2.54149	1.42884	0.92708	0.36981
Skewness	2.21527	2.06282	1.74427	1.38948	0.54109
Kurtosis	9.52595	9.25525	8.08973	6.83204	5.22843

According to Table 1, it is found that the proposed model can be used to model over and under dispersion data. Furthermore, it can be utilized to describe asymmetric “positively skewed” data with leptokurtic-shaped.

4. Maximum likelihood estimation (MLE)

Suppose $x = (x_1,\ x_2, \ldots, x_n)$ be a random sample of size n from the DLE model. The log-likelihood function is given by

$$\log L = \sum_{i=1}^{n} \log W(x_i;\alpha,\beta) - \sum_{i=1}^{n} \log\left\{1 + \left(e^{\beta x_i} - 1\right)^{\alpha}\right\} - \sum_{i=1}^{n} \log\left\{1 + \left(e^{\beta(x_i+1)} - 1\right)^{\alpha}\right\}, \tag{13}$$

where $W(x_i;\alpha,\beta) = \left(e^{\beta(x_i+1)} - 1\right)^{\alpha} - \left(e^{\beta x_i} - 1\right)^{\alpha}$. Now, partially differentiate with respect to α and β, we get

$$\begin{aligned}\frac{\partial \log L}{\partial \alpha} &= \sum_{i=1}^{n} \frac{\left(e^{\beta(x_i+1)} - 1\right)^{\alpha} \log\left(e^{\beta(x_i+1)} - 1\right) - \left(e^{\beta x_i} - 1\right)^{\alpha} \log\left(e^{\beta x_i} - 1\right)}{\left(e^{\beta(x_i+1)} - 1\right)^{\alpha} - \left(e^{\beta x_i} - 1\right)^{\alpha}} \\ &\quad - \sum_{i=1}^{n} \frac{\left(e^{\beta x_i} - 1\right)^{\alpha} \log\left(e^{\beta x_i} - 1\right)}{1 + \left(e^{\beta x_i} - 1\right)^{\alpha}} - \sum_{i=1}^{n} \frac{\left(e^{\beta(x_i+1)} - 1\right)^{\alpha} \log\left(e^{\beta(x_i+1)} - 1\right)}{1 + \left(e^{\beta(x_i+1)} - 1\right)^{\alpha}},\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \log L}{\partial \beta} &= \sum_{i=1}^{n} \frac{\alpha\left(x_i + 1\right)\left(e^{\beta(x_i+1)} - 1\right)^{\alpha-1} e^{\beta(x_i+1)} - \alpha x_i \left(e^{\beta x_i} - 1\right)^{\alpha-1} e^{\beta x_i}}{\left(e^{\beta(x_i+1)} - 1\right)^{\alpha} - \left(e^{\beta x_i} - 1\right)^{\alpha}} \\ &\quad - \sum_{i=1}^{n} \frac{\alpha x_i \left(e^{\beta x_i} - 1\right)^{\alpha-1} e^{\beta x_i}}{1 + \left(e^{\beta x_i} - 1\right)^{\alpha}} - \sum_{i=1}^{n} \frac{\alpha\left(x_i + 1\right)\left(e^{\beta(x_i+1)} - 1\right)^{\alpha-1} e^{\beta(x_i+1)}}{1 + \left(e^{\beta(x_i+1)} - 1\right)^{\alpha}}.\end{aligned}$$

The exact solution of these non-linear equations is not possible, so we will solve these using optimization approaches like Newton-Raphson.

5. Simulation

The current section is based on the simulation study to evaluate the behavior of MLEs. The estimates were calculated using different parameter combinations ($\beta = 0.5,\ \ \alpha = 0.5$), ($\beta = 0.5,\ \ \alpha = 1.5$), ($\beta = 0.5,\ \ \alpha = 3.0$), ($\beta = 1.0,\ \ \alpha = 0.5$), ($\beta = 1.0,\ \ \alpha = 1.5$) and ($\beta = 1.0,\ \ \alpha = 3.0$) for various sample sizes ($10,\ 30,\ 60,\ 100,\ 200,\ 300$). Then, $N = 10,000$ samples from the DLE model using MLE derived in the previous subsection. The indices such as to estimate values, mean square errors (MSEs), and 95% convergence probabilities (CP) were generated. All numerical computations are performed using R software. The simulation results are presented in Table 2 and Table 3.

Table 2: MLEs, MSEs and CP for DLE model.

n	Estimates		MSE		$\mathbf{CP}_{95\%}$	
	β	α	β	α	β	α
	Case 1: $\beta = 0.5$ **and** $\alpha = 0.5$					
10	1.5829	0.6088	8.0982	0.3234	0.8681	0.8624
30	0.9234	0.5118	2.5686	0.0472	0.9012	0.9212
60	0.6696	0.4988	0.7694	0.0236	0.9197	0.9537
100	0.5788	0.4968	0.2527	0.0138	0.9323	0.9616
200	0.5226	0.4987	0.0183	0.0065	0.9450	0.9553
300	0.5142	0.4985	0.0067	0.0041	0.9501	0.9537
	Case 2: $\beta = 0.5$ **and** $\alpha = 1.5$					
10	0.6167	1.8573	0.7300	2.2595	0.9782	0.9783
30	0.5135	1.5746	0.0183	0.1301	0.9403	0.9596
60	0.5062	1.5350	0.0035	0.0575	0.9484	0.9513
100	0.5038	1.5218	0.0019	0.0339	0.9497	0.9474
200	0.5023	1.5103	0.0010	0.0164	0.9470	0.9481
300	0.5014	1.5063	0.0006	0.0104	0.9515	0.9533
	Case 3: $\beta = 0.5$ **and** $\alpha = 3.0$					
10	0.5104	4.6922	0.0079	2.3587	0.8740	0.9753
30	0.5025	3.2048	0.0020	0.8533	0.9284	0.9612
60	0.5015	3.0848	0.0010	0.1824	0.9424	0.9581
100	0.5010	3.0500	0.0006	0.1014	0.9461	0.9534
200	0.5007	3.0253	0.0003	0.0485	0.9471	0.9533
300	0.5005	3.0145	0.0002	0.0316	0.9502	0.9520

Table 3: MLEs, MSEs and CP for DLE model.

n	Estimates		MSE		$\mathbf{CP}_{95\%}$	
	β	α	β	α	β	α
	Case 4: $\beta = 1.0$ **and** $\alpha = 0.5$					
10	2.5070	0.8765	13.462	2.3637	0.7463	0.8413
30	1.9730	0.5516	7.4242	0.0904	0.8474	0.8511
60	1.6086	0.5185	4.0927	0.0500	0.8723	0.8844
100	1.4237	0.5014	2.4798	0.0331	0.8941	0.9102
200	1.1876	0.4942	0.7693	0.0179	0.9134	0.9440
300	1.1074	0.4938	0.3254	0.0116	0.9293	0.9617
	Case 5: $\beta = 1.0$ **and** $\alpha = 1.5$					
10	1.8790	6.5630	8.9570	57.964	0.7211	0.8784
30	1.2680	2.6400	2.1190	11.980	0.8140	0.9767
60	1.1150	1.7960	0.5860	2.5330	0.8912	0.9863
100	1.0472	1.5885	0.1561	0.3025	0.9073	0.9814
200	1.0168	1.5357	0.0168	0.0848	0.9284	0.9604
300	1.0091	1.5239	0.0089	0.0513	0.9403	0.9591
	Case 6: $\beta = 1.0$ **and** $\alpha = 3.0$					
10	0.9397	11.118	1.3118	70.971	0.9283	0.9793
30	1.2120	9.2410	2.6540	57.466	0.7784	0.9524
60	1.1310	7.9390	1.4050	45.001	0.5861	0.9530
100	1.0443	6.5806	0.3943	34.547	0.5752	0.9542
200	1.0134	4.7041	0.0488	17.616	0.7563	0.9584
300	1.0022	4.2149	0.0208	12.256	0.8544	0.9433

As we see from the simulation results, the MSE tends to zero when sample size grows. Thus, the maximum likelihood approach can be used effectively in estimating the model parameters.

6. Applications

In this section, DLE model is fitted on real data sets and compared with discrete Ryleigh (DR), discrete Burr II (DBu-II), discrete Lindley (DLi), discrete Log-Logistic (DLogL), discrete Bilal (DBl), geometric (Geo), and Poisson (Poi) distributions.

The model parameters are estimated using the maximum likelihood method. For model comparison, we have considered the log-likelihood and the likelihood-based statistics measures, Akaike information criterion (AIC), corrected AIC (CAIC), and the Bayesian information criterion (BIC). We also consider the Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) test for the selection of best-fitted probability distribution. A distribution with the maximum log-likelihood and lowest AIC, and BIC is said to be a better fit for the data. The **fitdistrplus** R package is used for all computations. Further, the fits of these distributions are presented in the form of a comparative density plot and plot of the distribution functions.

Data Set I: The first dataset represents the counts of cysts of kidneys using steroids. This data set originated from a study by Chan et al. (2009). Some nonparametric plots including relative frequencies, box, violin, Q-Q, strip and scatter | scores plots for this data are presented in Figure 3. The MLEs along with model comparison measures of all fitted distributions are listed in Table 4. We also plot the estimated pmfs on the observed dataset in Figure 4.

Data Set II: The second dataset represents the number of European corn borer larvae Pyrusta in the field (Bodhisuwan & Sangpoom, 2017). It was an experiment conducted randomly on eight hills in 15 replications, and the experimenter counted the number of borers per hill of corn. The nonparametric plots for this data are reported in Figure 5. The MLEs along with model comparison measures of all fitted distributions are reported in Table 5. We also plot the estimated pmfs on the observed dataset in Figure 6.

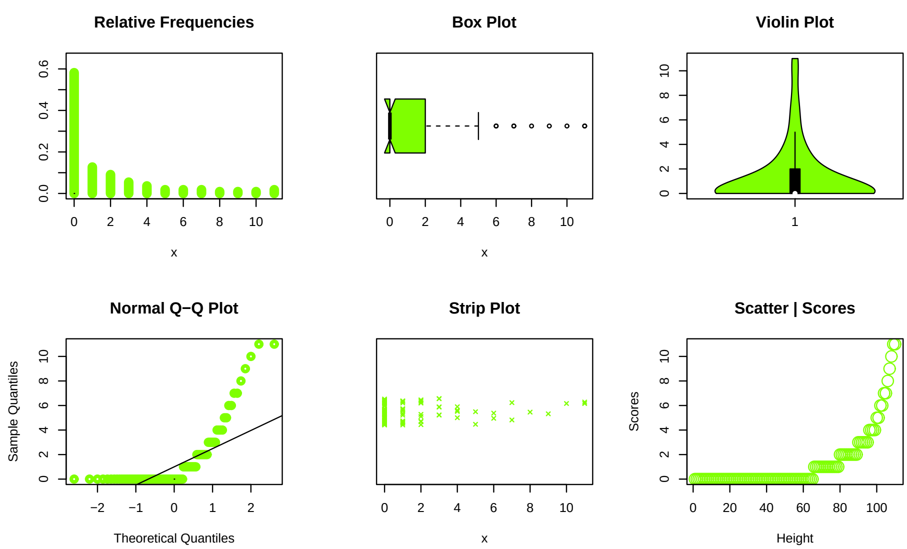

Figure 3. Some nonparametric plots for dataset I.

Table 4: The MLEs and goodness-of-fit measures for dataset I.

X	**Observed**	**DLE**	**Geo**	**DR**	**DBu-II**	**DLi**	**DLogL**	**DBl**	**Poi**
0	65	62.84	45.98	11.00	64.74	40.25	63.18	32.08	27.42
1	14	13.13	26.76	26.83	19.18	29.83	20.09	37.10	38.08
2	10	9.49	15.57	29.55	8.48	18.36	8.64	21.66	26.47
3	6	7.18	9.06	22.23	4.63	10.35	4.66	10.63	12.26
4	4	5.32	5.28	12.49	2.86	5.53	2.87	4.84	4.26
5	2	3.82	3.07	5.42	1.92	2.86	1.92	2.12	1.18
6	2	2.67	1.79	1.85	1.36	1.44	1.37	0.91	0.27
7	2	1.83	1.04	0.52	1.01	0.71	1.02	0.38	0.05
8	1	1.24	0.61	0.11	0.78	0.35	0.79	0.16	0.01
9	1	0.83	0.35	0.02	0.61	0.17	0.62	0.07	0
10	1	0.55	0.21	0	0.49	0.08	0.50	0.03	0
11	2	1.1	0.28	0	3.94	0.07	4.34	0.02	0
Total	**110**	**110**	**110**	**110**	**110**	**110**	**110**	**110**	**110**
α	MLE	0.358	0.582	0.900	0.278	0.436	0.780	0.643	1.390
	MSE	0.162	0.162	0.009	0.045	0.026	0.136	0.020	0.112
β	MLE	1.172	–	–	1.053	–	1.208	–	–
	MSE	0.551	–	–	0.167	–	0.159	–	–
	-L	167.546	178.821	277.821	171.139	189.101	171.717	207.436	246.211
	AIC	339.092	359.512	557.652	346.278	380.223	347.434	416.871	494.432
	CAIC	339.204	359.601	557.601	346.391	380.301	347.546	416.908	494.501
	BIC	344.493	362.203	560.301	351.679	382.924	352.835	419.572	497.124
	HQIC	341.282	360.613	558.711	348.469	381.304	349.625	417.967	495.521
	χ^2	0.771	22.840	321.112	2.592	43.481	2.941	61.366	294.1
	D.F	3	4	4	2	4	2	3	4
	P.Value	0.856	< 0.001	< 0.001	0.274	< 0.001	0.229	< 0.001	< 0.001

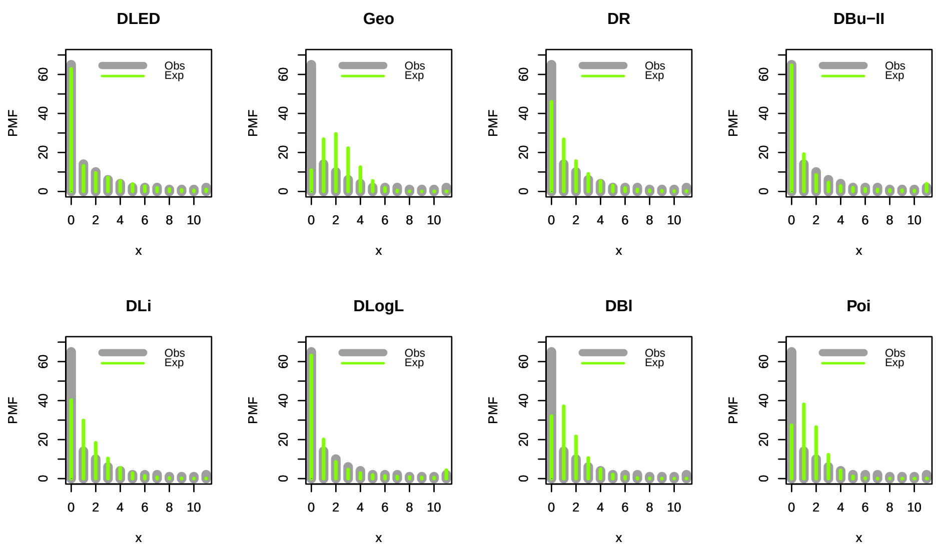

Figure 4. The fitted pmfs of tested distributions for data set I.

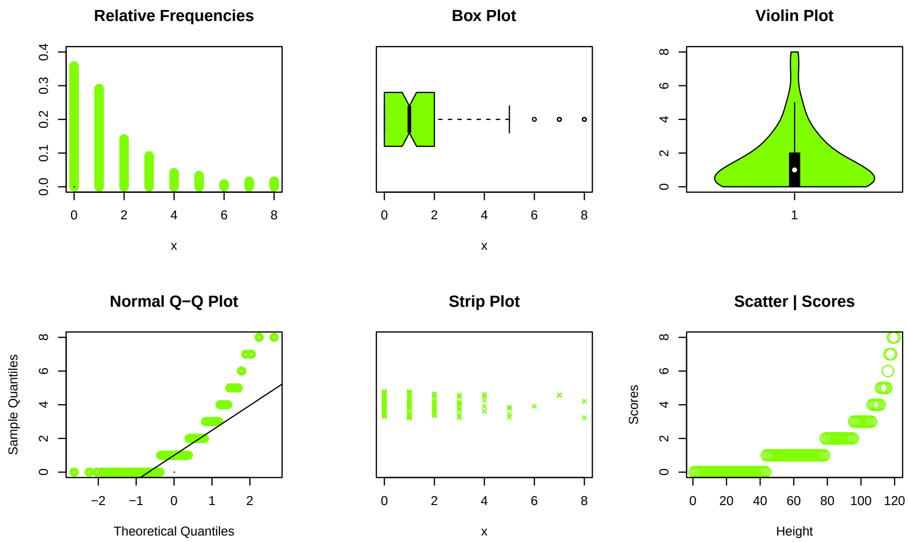

Figure 5. Some nonparametric plots for dataset II.

Table 5: The MLEs and goodness-of-fit measures for dataset II.

X	Observed	DLE	Geo	DR	DBu-II	DLi	DLogL	DBl	Poi
0	43	43.53	48.32	15.92	41.03	41.91	43.83	32.74	27.23
1	35	32.46	28.86	36.17	32.07	32.07	39.61	39.59	40.38
2	17	19.19	17.24	34.58	17.78	20.39	15.62	24.28	29.95
3	11	10.84	10.29	21.03	8.43	11.87	7.21	12.50	14.81
4	5	6.07	6.15	8.89	4.48	6.56	3.91	5.97	5.49
5	4	3.41	3.67	2.70	2.63	3.50	2.38	2.74	1.63
6	1	1.93	2.19	0.60	1.66	1.82	1.56	1.23	0.40
7	2	1.09	1.31	0.09	1.12	0.93	1.09	0.54	0.09
8	2	1.48	1.97	0.02	3.93	0.95	4.79	0.41	0.02
Total	**120**	**120**	**120**	**120**	**120**	**120**	**120**	**120**	**120**
α	MLE	1.155	0.597	0.867	1.401	0.451	0.519	0.657	1.483
	MSE	0.132	0.028	0.012	0.121	0.025	0.051	0.019	0.025
β	MLE	0.479	–	–	1.943	–	2.358	–	–
	MSE	0.047	–	–	0.188	–	0.366	–	–
	-L	200.167	200.882	235.232	202.630	200.641	204.293	204.675	219.192
	AIC	404.335	403.751	472.451	409.261	403.290	412.587	411.351	440.384
	CAIC	404.437	403.793	472.490	409.363	403.323	412.689	411.384	440.413
	BIC	409.909	406.540	475.243	414.836	406.071	418.162	414.138	443.177
	HQIC	406.599	404.891	473.597	411.525	404.421	414.851	412.483	441.512
	χ^2	0.796	2.162	70.688	2.246	1.744	4.649	6.998	38.478
	D.F	3	4	4	3	4	3	3	4
	P.Value	0.850	0.706	< 0.001	0.523	0.783	0.199	0.072	< 0.001

According to Tables 3 and 4, we can say that the DLE model is the best distribution among all tested models for analyzing data sets I and II. Figures 4 and 6 support our empirical results which have been mentioned in Tables 3 and 4.

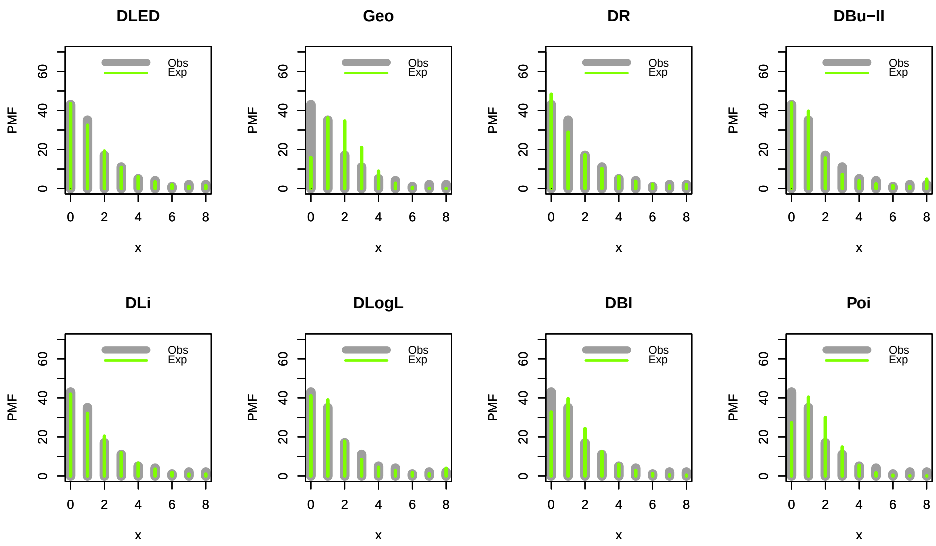

Figure 6. The fitted pmfs of tested distributions for data set II.

7. Conclusion

In this study, a discrete analogue of the continuous logistic exponential distribution is proposed. The resultant probability distribution is called discrete logistic exponential (DLE) distribution. Some statistical properties have been derived. The parameters of DLE model have been estimated using the maximum likelihood approach, and the behavior of these estimates has been assessed via a Monte-Carlo simulation study. Two datasets were considered to show the efficiency of the proposed distribution. The proposed distribution provides the best fit as compared to competitive distributions. We hope the DLE model will be an interesting alternative probability to analyze count observations.

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