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Quadratic Programming and Triangular Numbers Ranking to an Optimal Moroccan Diet with Minimal Glycemic Load

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Abstract In this work, we suggest a variety of optimal daily diets for people with diabetes in Morocco. To this end, we model the foods glycemic load in term of triangular fuzzy numbers. Then, we model the problem of daily diet in term of fuzzy quadratic optimization programming. To transform this model into a line model, we use a very performance ranking function. To solve the obtained model, we call particles swarm algorithm due to its performance and simplicity. Various experiments were carried out on the basis of feeds available on the Moroccan market and for different sizes of the PSO swarm. The experimental results show that our system proposes optimal diets with a reasonable glycemic load between, acceptable favorable gaps and unfavorable nutrients excesses. We show how to combine the proposed diets to obtain a balanced weekly regime.

Keywords Glycemic load, Optimal diet, Triangular fuzzy numbers, Ranking function, Particule swarm algorithm, Quadratic optimization.

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1. Introduction

Health conditions including diabetes, cancer, vascular disease, and overweight are strongly associated with dietary imbalances [1], [2], [3] and [4]. To improve the life quality, wellbalanced diets enable to manage the development of chronic pathologies and to mitigate their risks. The aim is to meet the requirements of the person's body, regardless of age, in an optimal manner. The main proposal of our work is to present several optimal daily diets basing on triangular fuzzy numbers, quadratic programming, and on a special ranking function. Over time, the optimal diet problem has attracted the attention of many researchers whose proposals differ only in the objective functions considered. Stigler Dantzig remains the first team to have introduced the first optimization model whose objective function is the cost of the scheme while the constraints represent the requirements on the good balance of the scheme [6]. In [7], the objective function of the proposed model makes a trade-off between different meals using the penalty technique. In this context, the authors consider the three usual meals, a snack, and a portion of fruit.
The proposal of Masset G. [8] is to control the differential of actual and target intake by meeting nutritional requirements. Other studies have proposed supplemental diets and minimal-cost nutritional

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menus for children. To control different targets at the same time, other authors have used multi-objective mathematical models [5] and [9]. V. Mierlo regarded nearly the identical circumstance by replacing the expense of the feeding regime with minimizing the exhaustion of fossil combustible fuels [10]. Cholesterol intake and glycemic load are known to be the major determinants of childhood fatness and were the focus of the two objective functions of the multi-objective model proposed in [11]. In the current study, we propose several optimal daily diets that complement each other. To this end, we model the foods glycemic load in term of triangular fuzzy numbers. Then, we model the problem of daily diet in term of fuzzy quadratic optimization programming. To transform this model into a line model, we use a very performance ranking function. To work out the resulting model, we utilize the particle swarm method for its performance and simplicity.
The remainder of our article is structured in four parts: In the second part, we provide the main notions about triangular fuzzy numbers and the ranking function. In the third part, we transform the feeding issue in terms of constrained quadratic fuzzy programming. In the final part of this paper, we discuss certain of our experimental outcomes.

2. Triangular fuzzy numbers and raking function

Let's us consider a universe X and a set $E \subseteq X$. In classical logic, the characteristic function of A, noted $\mu_E(x)$, equals to 1 if x falls in E, and 0 else. In fuzzy logic, the degree of membership of the element x to E is the positive real number $\mu_{\tilde{E}}(x) \in [0;1]$ as defined in [16] and [18]. The closer x to E, the larger $\mu_{\tilde{E}}(x)$ is . We name $\mu_{\tilde{E}}$ the membership function of E.
We name and denote the triangular number the treplet $\tilde{E} = (m, p, q)$ with $m \leq p \leq q$; in this case, the membership function of E is defined by:

$$\mu_{\tilde{E}}(x) = \begin{cases} \dfrac{x-m}{p-m}, & \text{if } m \leq x \leq p \ , \\ \dfrac{x-q}{p-q}, & \text{if } p < x \leq q \ , \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

We denote the set of triangular number by STN. If $m \geq 0$, we say that E is a positive triangular number.
Given $\tilde{E} = (m, p, q)$ and $\tilde{F} = (m', p', q')$. If $m = m', p = p', q = q'$, we said that that E and F are equal and we write $E = F$. In the rest of this section, we consider E and F to introduce the ranking function.
Basing on the inverse function of $\tilde{E}$ and on the the rigth and left integral, we define the ranking transformation (or function) by $R : STN \to \mathbb{R}$. This function transforme triangular representation into a nominal value. In our case, we consider the transformation defined in [17]; that is, given $\tilde{E} = (m, p, q)$, the nominal value associated with E is $R(\tilde{E}) = \dfrac{m + 2p + q}{4}$. The reason behind this choice is that the integral on the left and the integral on the right takes into account any information about the number E.
Let γ be a real number. We define the two basic arithmetic operations that we will need to achieve our goals in this article: the sum and the product by a real [16].
The sum of E and F is defined as follows:

$$\tilde{E} \oplus \tilde{F} = (m + m', p + p', q + q')$$

The multiplication of E by a real number is given by:

$$\gamma\tilde{E} = \begin{cases} (\gamma m, \gamma p, \gamma q), & \text{if } \gamma \geq 0, \\ (\gamma q, \gamma p, \gamma m), & \text{if } \gamma < 0. \end{cases} \tag{2}$$

To illustrate the use of this notion of ranking function in the field of stochastic optimization, we give an example.

Example:
Let's us consider the following optimization problem:

$$\begin{cases} Min\ (4,5,6)x + (2.5,3,3.5)y \\ \quad Subject\ to: \\ \quad (3.5,4,4.1)x + (2.5,3,3.5)y \geq (11,12,13) \\ \quad (0,1,2)x + (2.8,3,3.2)y \geq (5.5,6,7.5) \\ \quad x, y \geq 0 \end{cases} \tag{3}$$

Then the fuzzy model converted to the crisp model by using ranking function as follows:

$$\begin{cases} Min\ 5x + 3y \\ \quad Subject\ to: \\ \quad 3.9x + 3y \geq 12 \\ \quad x + 3y \geq 6 \\ \quad x, y \geq 0 \end{cases} \tag{4}$$

In the next section, we use triangular fuzzy numbers to present the glycemic load of different foods and we utilize the ranking transformation to transform the fuzzy total glycemic load of the diet, in the proposed model, into nominal value that take into a count all the variation of the foods glycemic load.

3. Quadratic optimization modeling to optimal diet

Components of food in nutrients:
A: matrix of the nutrients with a positive effect on the body;
E: matrix of the nutrients with a negative effect on the body;
$\tilde{g} = (gl, gn, gu)$: vector of the triangular fuzzy numbers of the foods glycemic load;
L: column of foods calories.

Nutrients requirements:
b: vector of minimum requirements of the nutrients with a positive effect on the body;
f: vector of maximum requirements of the nutrients with a negative effect on the body;.
ρ_{car}: ratio of calories from carbohydrates estimated by nutrition experts to be 0.55;
ρ_p: ratio of calories from potassium estimated by the experts nutritionniste toestimated by nutrition experts to be 0.18 ;
ρ_{tf}: ratio of calories from total fat estimated by nutrition experts to be 0.29;
ρ_{sf}: ratio of calories from satured fat estimated by nutrition experts to be 0.078.

Variables:
$x = (x_j)_{j=1:nbr\_foods}$ serving sizes from different considred foods; x is in $S = \{0, ..., 6\}^{nbr\_foods}$.

Objective function:
The diet should have a very small total glycemic load, so the following amount should be minimized: $f_1(x) = \tilde{g}^t x$.
The diet must meet the needs of favorable nutrients, so we must minimize the following amount: $f_2(x) = (Ax - b)^t (Ax - b)$.

The diet should not exceed the tolerable amount of unfavorable nutrients, so the following amount should be minimized: $f_3(x) = (Ex - f)^t(Ex - f)$.
The economic function that allows to control these three targets is given by:

$$f(x) = f_1(x) + \alpha f_2(x) + \beta f_3(x)$$

Where α and β are the penalty parameters that achieve a compromise between f_1, f_2, and f_3.

Constraints:
The calories from carbohydrates and potassium must be sufficiently large in relation to the total calories of the diet, hence the constraints are necessary: $l^t_{car}x \geq \rho_{car}L^t x$ and $l^t_p x \geq \rho_p L^t x$; where l_{car} and l_p are the vectors of calories from carbohydrate and potasium respectively.
Calories from total fat and saturated fat must be sufficiently low in relation to the total calories of the diet, hence the constraints are necessary: $l^t_{tf}x \leq \rho_{tf}L^t x$ and $l^t_{sf}x \leq \rho_{sf}L^t x$; where l_{tf} and l_{sf} are the vectors of calories from total and satured fat, respectively.
These four constraints reflect the guidelines of the Dietary Guidelines for Americans[13].
To ensure a certain degree of devirsity, we limit the number of units of the different foods to 6, from which we obtain the constraint: $0 \leq x \leq 6u$.

Finally, a possible modeling of the optimal regime problem is given by:

$$(P) : \begin{cases} Min\ \tilde{g}^t x + \alpha(Ax - b)^t(Ax - b) + \beta(Ex - f)^t(Ex - f) \\ \quad Subject\ to: \\ \qquad l^t_{car}x \geq \rho_{car}\,(L^t x) \\ \qquad l^t_p x \geq \rho_p\,(L^t x) \\ \qquad l^t_{tf}x \leq \rho_{tf}\,(L^t x) \\ \qquad l^t_{sf}x \leq \rho_{sf}\,(L^t x) \\ \qquad 0 \leq x \leq 6u, x \in S \end{cases} \tag{5}$$

Then the fuzzy model converted to the crisp model by using ranking function as follows:

$$(P) : \begin{cases} Min\ \dfrac{(gl + 2gn + gu)^t}{4}x + \alpha(Ax - b)^t(Ax - b) + \beta(Ex - f)^t(Ex - f) \\ \quad Subject\ to: \\ \qquad l^t_{car}x \geq \rho_{car}\,(L^t x) \\ \qquad l^t_p x \geq \rho_p\,(L^t x) \\ \qquad l^t_{tf}x \leq \rho_{tf}\,(L^t x) \\ \qquad l^t_{sf}x \leq \rho_{sf}\,(L^t x) \\ \qquad 0 \leq x \leq 6u, x \in S \end{cases} \tag{6}$$

Where gl, gn, and gu are the vectors of minimum, mean, and maximum values of glycemic load values of different foods, respectively.
In section five, we utilize the particle swarm optimization method to solve the quadratic diet mathematical model (P').

4. Particle Swarm Algorithm (PSO)

PSO is a heuristic optimization method that simulates the behavior of a cluster. This intuitive mathematical model was introduced by Kennedy and Eberhart to describe the social habits of animals like birds and fish [12]. The framework is mainly derived based on the fundamental concepts of auto-organization, that is widely considered as accounting for process in complex structures. Swarm smartness is the power of these teams to achieve a greater level of cleverness, entirely unreached by the single cells of the community. For example, a cluster of students together in a society has a very complicated way of behaving, exceeding the intelligence of each individual student in the cluster. However, such sophisticated templates are composed of simple, recurrent activities executed by any member of the flock, which can be different from the ones of other members of the group or flock. PSO utilizes a greatly reduced model of social behavior to find the solution of constrained minimization or maximization problem in a cooperative and intelligent collaborative manner. The figure 1 gives different steps of the PSO algorithm.

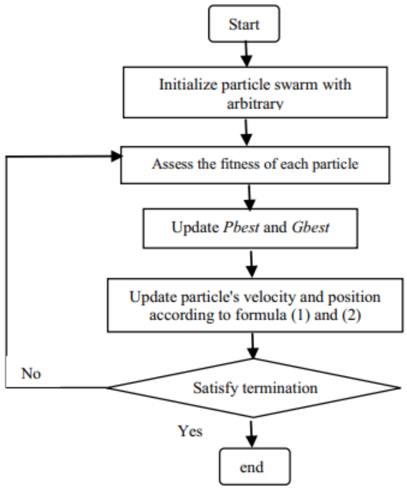

Figure 1. Particle swarm algorithm.

It will be very important to recall the kernel version of PSO. Initially, the particles are randomly positioned in the realisable solution area. Then, the location of every indidual k is updated according to three components::

1. The current speed of k denoted V_k;
2. The best position of k denoted P_i;
3. The best solution obtained in the neighborhood of k.

The new location of the individual k is updated as follows:
$future\_speed(k+1) = inertie \times current\_speed(k) + rand_1 \times (best\_position(i) - current\_position(k)) + rand_2 \times (best\_voisin(g) - current\_position(k))$;
$future\_position(k+1) = future\_position(k) + future\_speed(k+1)$.
There are a multitude of approaches to improving the structure of the PSO algorithm that can be categorized in several sub-approaches: Adoption of multi-subpopulations, enhancing the individual selection method, modifying the particle method, changing the particle maintenance scheme, modifying the momentum update strategy, modifying the velocity, and hibridation of PSO with different technics [23], and [24].
PSO is being used successfully for various NP-complet issues. The main difficulty of PSO lies in the setting of the parameters to resolve the different NP-complet combinatoty problems. In our case, we experimentally select inertie, $rand_1$, and $rand_2$.

5. Experimental results

We employed PSO to resolve the problem (P') with $\alpha = \frac{2}{3}$ and $\beta = \frac{4}{3}$. We give between briquettes the favorable and unfavorable nutrients daily requirements [14] and [15]: Calories (2000 kcal), Protein (91g), Carbohydrate (271g), Potassium (4044mg), Magnesium (380mg), Calcium (1316mg), Iron (18mg), Phosphorus (1740mg), Zinc (14mg), Vb6 (2.4mg), Vb12 (8.3µg), VC (155mg), VA (1052µg), VE (9.5mg), Satured fat (17 g), Sodium (1779mg), Total fat (65 g), and Cholesterol (230mg).

In our case, the optimal diets are constructed on the basis of the foods that are currently present on the Moroccan marketplace. We consider the problem (P') whose the fuzzy glycemic load is transformed into line number using the ranking function. In the following, we give the configuration of particles swarm algorithm:

- The adaptive inertia is taken in the interval: $[0.1, 1.1]$;
- Maximum iterations: $100\times$ number of foods;
- Initial range of particle positions: 2000;
- Minimum adaptive neighbourhood size: 0.25;

Number of particles in the swarm: varied between 100 and $10\times$ number of foods .

Number of particles	TGL	TLF	TEU
100	92.7635	54.5845	21.9734
150	51.4619	12.1393	0.0282
200	88.8279	142.4581	34.1516
250	38.9450	9.0727	6.9537
300	98.8133	8.8525	7.8380
350	50.4502	5.6472	2.9460
400	40.6457	3.1082	7.0209
450	55.8223	4.6913	0.0282
500	41.5074	3.7363	2.0070
550	61.1312	4.6488	5.3201
600	35.0593	8.4611	41.5126
650	35.3437	13.5047	5.0299
700	49.7421	6.1629	7.3933
750	43.9454	3.8439	0.7238
800	69.5175	8.2794	2.5037
850	27.8898	6.7627	5.9549
900	41.4315	7.8400	7.9241
950	31.7798	4.0842	0.0096
1000	38.5936	14.2285	3.6163

Table 1. Daily diets produced by PSO for different swarm sizes.

Table 1 and Figure 2 give the Total, in Glycemic Load (TGL), in Lack of Favorable nutrients (TLF), and in Excess of Unfavorable nutrients (TEU) vs swarm size. In general, PSO begins to produce acceptable solutions, in terms of glycemic load, when the swarm size exceeds 350. The diets produced by PSO all have an acceptable total glycemic load. So it is the pareto plan (total gap favorable nutrients, total excess unfavorable nutrients) that will decide about the quality of these diets. On the one hand, we reject the two diets produced by PSO_100 and PSO_200 because of the fatal lack of positive nutrients; in fact, the accumulation of such lacks can have bad repercussions on health.

On the other hand, we reject the diet produced by PSO_600 because of the great excess of negative nutrients, the accumulation of which can cause chronic diseases.

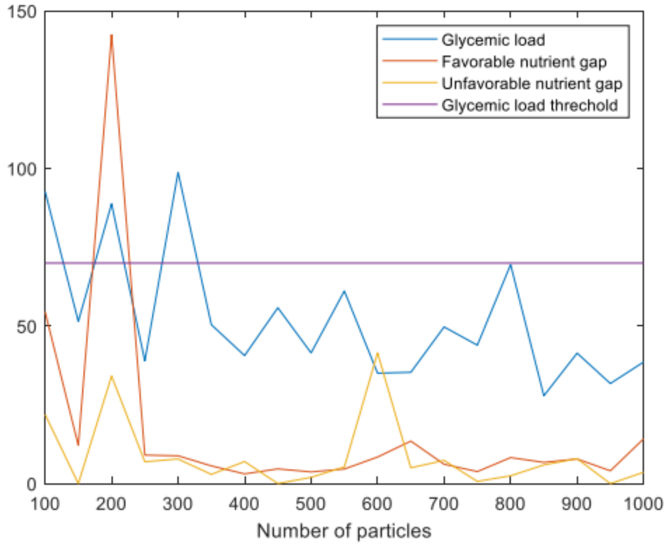

Figure 2. PSO diet glycemic load, favorable nutrients gap, and unfavorable nutrients excess vs swarm size.

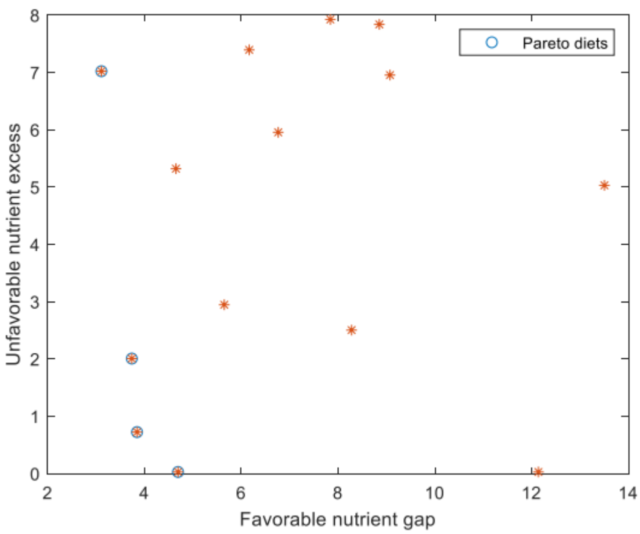

Figure 3. Pareto PSO diets on favorable nutrients and unfavorable gap plan.

Figure 3 shows that there is no dominance between the corresponding regimes produced by PSO with 400 particles, PSO with 500 particles, PSO with 750 particles, and PSO with 950 particles. Each of these regimes dominates a set of regimes that are not members in Pareto regimes.

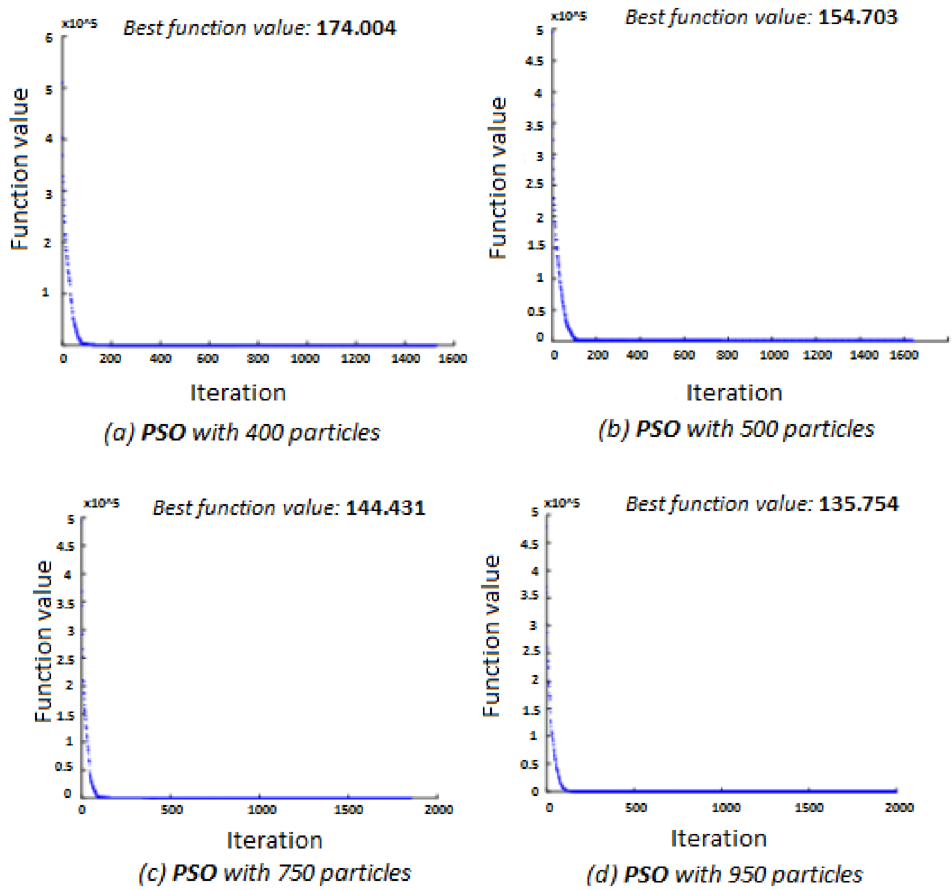

Figure 4. PSO Best fitness for pareto optimal diets.

The Figure 4 gives the research process for pareto optimal diets in function of the number of iterations. For small swarms' size, PSO is early attracted by local minima (from 200 iteration the fitness begins to be almost constant). For a large swarm's size, the solution begins to be constant from iteration 500. The remaining 16 diets are accepted because their TGL, TLF, and TEU are acceptable. These diets are a liability for decision makers in that they can be altered so that no two diets with high TGL or TLF or TEU should be used in succession to avoid cumulative defects of the same kind for a long period of time. For example, we suggest the following nutritional plan:

Diet_PSO_150⟶Diet_PSO_500⟶Diet_PSO_750⟶Diet_PSO_900
⟶Diet_PSO_950⟶Diet_PSO_350

In addition, one should never have Diet_PSO_150⟶ Diet_PSO_1000 in his diet, because the patient will accumulate almost 26mg shortage in two days in favorable nutrients. As you should never have Diet_PSO_250⟶ Diet_PSO_300 in your diet, if not the patient will accumulate almost 14.5mg in negative nutrients in two days and a sum of glycemic load of almost 140.

6. Conclusion

In this work, we present the foods glycemic load in term of triangular numbers. Then, we modeled the problem of daily diet in term of fuzzy quadratic optimization programming. To transform this model

into a line model, we used a very performance ranking function. To solve the obtained model, we use the particles swarm algorithm with swarms of different sizes. Several experimentations were realised based were carried out on the basis of feeds available on the Moroccan market and for different size of the PSO swarm. Our system proposes 16 optimal diets with a reasonable glycemic load, acceptable TLF, and acceptable TEU. These diets must be used intelligently to build up adequate diets over a month (for example) so that two successive diets must complement each other. It is possible to manually correct these diets by switching to equivalent foods that may correct the inadequacies and excessivity of these plans.
In future, we will propose a hybrid heuristics methods to improve the quality of the regimes basing on well-known heuristics methods [22].

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