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Application of NURBS in the Fracture Mechanics Framework to Study the Stress Intensity Factor

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Abstract In this investigation, we successfully employed the Non-Uniform Rational B-spline (NURBS) to study the fracture mechanics. the NURBS functions are very popular in the design field (CAD model) and it is used as an alternative to Lagrange interpolation polynomials. The extended isogeometric analysis based on this basis function is used to evaluate the stress intensity factors (SIFs) in order to control the crack propagation. For various crack lengths, SIFs were calculated to validate the accuracy of this technique. The results obtained are in good agreement with the available techniques as CFEM, X-FEM.

Keywords Crack, Fracture, NURBS, X-IGA

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1. Introduction

To ensure structural integrity and estimate failure life, cracking problems must be examined. The crack-type surface defect has widely appeared on engineering structures, and until today, this issue attracts the attention of researchers in this field. The focus of these studies is to find a numerical solution that can provide a realistic simulation of the crack. Comprehensive analytical solutions based on fracture mechanics law have been obtained [1, 2, 3]. However, because analytical approaches are insufficiently flexible in dealing with complicated geometric issues, numerical techniques have been discovered to be effective alternatives. The Conventional Finite Element Approach (CFEA) is extensively employed to solve engineering issues, although it suffers from simulation of crack growth analysis, which requires remeshing and the crack must conform with the mesh. To solve the limitations of CFEA in the simulation of crack propagation, [4] developed the Extended Finite Element Method (X-FEM). by using this method, the crack can be represented by discontinuous enrichment functions with their degree of freedom to the finite element approximation displacement field following the unit partition concept [5]. This approach allows the crack to be independently simulated. Using different basis functions to approximate the geometry and the solution by these techniques, introduce geometry discretization errors into the analysis [6]. This error has been eliminated by the development of a new approach termed Isogeometric Analysis (IGA) [7]. The IGA was suggested with the aim to approximat the geometry (design process) and the solution (analysis) by the same basis function. The basic principle of this method is represented by the Fig 1. Recently, IGA

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was extended to simulate crack problems. This extension aims to combine the features of the X-FEM and IGA methods, it is termed X-IGA. it has been employed for the first time by [8] to address the problems of linear fracture mechanics.

In the present study, we employed a Non-Uniform Rational Basis Spline (NURBS) to simulate the crack. The use of the advantages of these functions allows improving the X-IGA accuracy. Two problems were used and the results obtained were compared with Analytical solution, X-FEM, and CFEM. As a result, X-IGA shows a good agreement with these methods.

The details of this article are organized as follows: In section 2, we introduce a recent overview of the IGA and XIGA concept. In section 3, we present the theoretical background of NURBS. In section 4, the efficiency of using NURBS in XIGA technique is compared with the XFEM, Contour integral and analytical solution. Finally, the conclusion of the present study are discusssed in section 5.

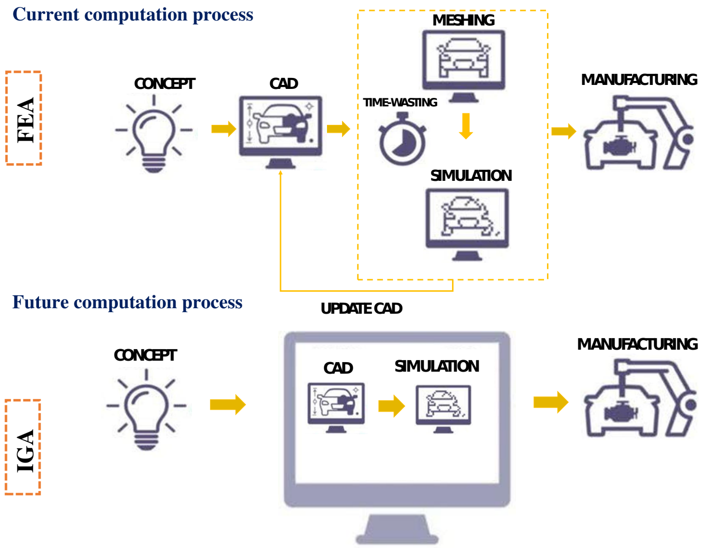

Figure 1. the basic of the Isogeometric method [9]

2. Recent Overview of the of IGA and XIGA

Isogeometric Analysis (IGA) is a recent computational method that has received a lot of interest. Since the development of this technique, various papers have been presented to provide implementation specifics of IGA technology in various computing contexts. A detailed description of the method of integration of this new technology in a finite element code was presented by [10]. Because of the need to build a geometric model for the IGA, [11] describe many popular tools and approaches that can be used for building models for IGA. Due to the lack of an isogeometric NURBS modeling toolbox necessitates the creation of implemen-tation codes from beginning, which may not be a simple process for a novice in this field. The integration of the IGA methodology within the current classical finite element analysis code is shown with a simplified explanation and implementation details by [12, 13]. The success of IGA led to the creation of a new approach, which is the extension of the formulation of this technique, it is termed XIGA. It is developed to solve fracture mechanics problems. [14] provide a comprehensive summary of the research that has been done in the XIGA field. Various recent works that have used this technique, [15] present an analysis of cracks subjected to thermo-mechanical loads. [16] Evaluate the stability of a pressurized cracked structure. [17] Used XIGA to study a cracked functionally graded magneto electro elastic material.

3. Theoretical Background

B-spline concept and X-IGA implementation for cracks in the LEFM framework, as well as the computation technique of SIFs, are briefly discussed in this section.

3.1. Rational B-spline concept

The Non-Uniform Rational B-splines are the basis functions used to describe the model in computer aided design (CAD). Given a Knot vector

$$L_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \leq \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$L_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i}\, L_{i,p}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} L_{i+1,p-1}(\xi) \tag{2}$$

NUBRS [18] are an extension of the B-spline functions, and they are de-fined as:

$$B_{i,p}(\xi) = \frac{L_{i,p}(\xi) w_i}{\sum_{i=1}^{n} L_{i,p}(\xi) w_i} \tag{3}$$

Where $L_{i,p}$, W_i are the B-spline basis functions and the weights functions respectively. A NURBS curve is constructed by a knot vector, a set of rational B-spline $B_{i,p}$, and a set of control points P_i as:

$$C(\xi) = \sum_{i=1}^{n} L_{i,p} P_i \tag{4}$$

To build a NURBS surface, a knot vector must be introduced in two directions;$M = \{\xi_1, ..., \xi_{n+p+1}\}$ and $\aleph = \{\eta_1, ..., \eta_{n+p+1}\}$. As well as the control points $P_{i,j}$:

$$S(\xi, \eta) = \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{L_i^p(\xi) K_j^q w_{i,j}}{\sum_{i=1}^{n} \sum_{j=1}^{m} L_i^p(\xi) K_j^j(\eta) w_{i,j}} P_{i,j} \tag{5}$$

Where, $L_i^p(\xi)$ and $K_i^p(\xi)$ are shape functions respectively. Then, p and q are the order of the basis function in the two directions and m and n are the numbers of control points in the two directions.

3.2. B-spline for linear elastic fracture mechanics

The crack modeling problem relates to the way of capturing the discontinuity and the crack tip. To overcome this issue, X-FEM appeared and it is based on enriching the approximation displacement [19, 20, 21]. In this study, the same concept was applied but with the X-IGA. The enriched approximation displacement is given as follow:

$$U^h \xi = \sum_{i=1}^{n_{en}} B_i(\xi) u_i + \sum_{j=1}^{n_d} B_j(\xi) \{H(\xi) - H(\xi_j)\} a_j + \sum_{k=1}^{n_t} B_k(\xi) \sum_{\beta=1}^{4} [F_\beta(\xi) - F_\beta(\xi_k)] b_k^\beta \tag{6}$$

The standard DOFs are represented by u_i, the enrichment DOFs for capturing cracks faces are represented by a_j, and the enrichment DOFs for simulating the crack tip singularities are represented by b_k^β. The basis functions whose support is divided by the crack are designated as n_d, whereas those whose support contains crack tip are designated as n_T. $H(\xi)$ defines the Heaviside jump functions, which is equivalent to +1 above the crack and -1 on the other side of the crack. $F_\beta(\xi)$ represent the singular functions for the crack tip and for an isotropic material, they are introduced as follows:

$$F_\beta(\xi) = \sqrt{r}[sin(\frac{\theta}{2}), cos(\frac{\theta}{2}, sin(\frac{\theta}{2}) sin(\theta), cos(\frac{\theta}{2}) sin(\theta)] \tag{7}$$

With r and θ are the polar coordinate.

Table 1. Material properties [23]

Young's modulus	207 GPa
Poisson's ratio	0.3
Fracture toughness	95 MPa$\sqrt{m}$

3.3. SIFs computation technique

The study of the model response is performed by calculating displacement, stresses, and strains. However, the large amount of data that are generated by these quantities are difficult to understand. Stress intensity factors (SIFs) reduce such data into an understandable format and making it easier to analyze. To compute SIFs, a variety of methods have been devised. In the current study, the contour interaction integral is used. This method is calculated by superimposing auxiliary fields onto the real fields generated by the boundary value problem solution. When the auxiliary fields are filled out correctly, the contour interaction energy integral can be translated from the general 2D crack tip integral, which is given as [22].

$$I = \int_{\Gamma} (\sigma_{i,j} u_{i,1}^{aux} + \sigma_{i,j}^{aux} u_{i,1} - \sigma_{i,k} \varepsilon_{i,k}^{aux} \delta_{1,j}) q_{,j} dA \tag{8}$$

The auxiliary displacement, stress, and strain fields are represented by $u_{i,1}^{aux}$, σ_{ij}^{aux}, and ε_{ik}^{aux}, respectively. q define a scalar weighting function, which takes 1 at the crack tip and 0 at the edge of the domain region. as seen below, the interaction integral can be connected to the SIFs:

$$I = \frac{2}{E^*}(K_I^{aux} K_I + K_{II}^{aux} K_{II}) \tag{9}$$

With,

$$E^* = \begin{cases} E & \text{for plane stress} \\ \dfrac{E}{1-\mu^2} & \text{for plane strain} \end{cases} \tag{10}$$

4. Results and discussion

In this section, we focus on linear elastic fracture mechanics in some mechanical problems including an edge cracked plate under uniform tension and a center cracked plate under uniform load. The present study shows the interest in using a higher interpolation polynomial (NURBS) in the XIGA framework. The accuracy of this technique has been checked by comparison with the exact solution, Contour integral, and XFEM. The mechanical characteristics used for the simulation are mentioned in the table 1. In this study, the control points in the x-direction are taken to be 18 and in the y-direction are taken to be 36. The two problems are subjected to the uniform load of σ=2.5 MPa to remain in the elastic field [24, 25]. The stress intensity factor (SIFs), KI, is calculated to determine the fracture parameter, i.e. to assess the stress state near a crack tip. For the three techniques used in this work, the interaction integral method was implemented, about the X-IGA technique, it is implemented in MATLAB code. The Abaqus-Software was used for the CFEM and X-FEM to extract the KI but for X-FEM, it is important to mention that software does not support the computation of this parameter in the 2-D domain that leads to the use of a subroutine UEL [26].

4.1. 2-D plate with an edge crack

For the first problem, a 2-D finite plate with dimension (100 mm × 200 mm) with an axial crack is submitted for analysis. The boundary conditions assumed for this study are imposed as follows: the top edge of the model was subjected to uniform load 2.5 MPa, while we have embedded the bottom of the model as illustrated in Fig 3(a). CFEM, X-FEM, and the X-IGA methodologies were used to investigate the presented problem. For the first studied problem, in each of the three numerical procedures, the domain was discretized into a uniform mesh of 18 $\times$ 36. The whole domain is discretized into conventional finite elements using CFEM. Then, the whole domain is discretized into conventional finite elements and the crack domain is discretized into enrichment finite elements using X-FEM. Finally, the entire domain is discretized by the present study into a set of control points as presented in Fig 2. Fig 3(b). shows an example of σ_{xx} stress contour plot using X-IGA technique for a=35 mm. For this configuration, the stress intensity factor exact solution is given as:

$$K_I = F(\mu)\sigma\sqrt{\pi a} \tag{11}$$

Where σ, a, $F(\mu)$ are the applied stress, the crack length, and the factor reflect the geometry effect given by:

$$F(\mu) = 1.12 - 0.231\mu + 10.55\mu^2 - 21.72\mu^3 + 30.39\mu^4 \tag{12}$$

With μ is the ratio of the crack length to the plate width, expressed as aW. Various crack lengths are used here, Fig. 4 shows the variation in stress intensity factors as a function of the crack dimension. The results derived from the present study are compared with SIF exact solution, Contour integral, and XFEM. Therefore, they showed a good agreement. It is observed that the Mode I, SIF, KI gradually increase with crack length. Also To check, the accuracy of the XIGA technique, Mode I, SIFs are presented with applied load Fig 5. We did not exceed 4.5 MPa to remain in the elastic field. It's the same here, the SIFs increase gradually with applied load. The results obtained show that the use of a high interpolation polynomial order allows controlling the crack propagation with accuracy. Since the SIFs values do not exceed the material critical value so the studied model remains stable.

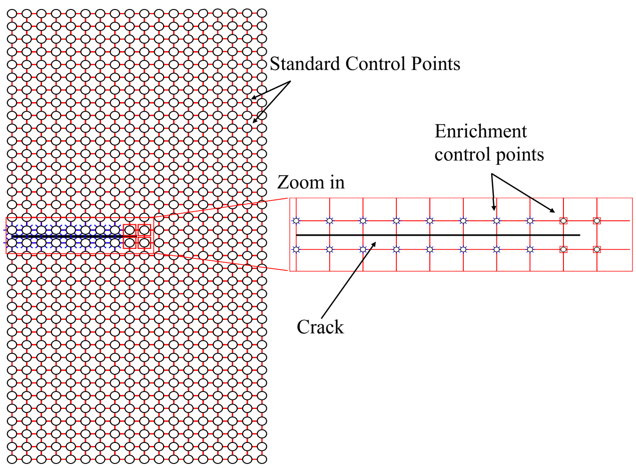

Figure 2. Domain discretization in the present study with crack enrichment.

4.2. 2-D plate with center crack

For the second problem, a 2-D finite plate with dimension (100 mm $\times$ 100 mm) with center crack is submitted for analysis. The boundary conditions assumed for this study are imposed as follows: The top edge of the model was subjected to uniform load 2.5 MPa, while we have embedded the bottom of the model. For the seconde problem, in each of the three numerical procedures, the domain was discretized

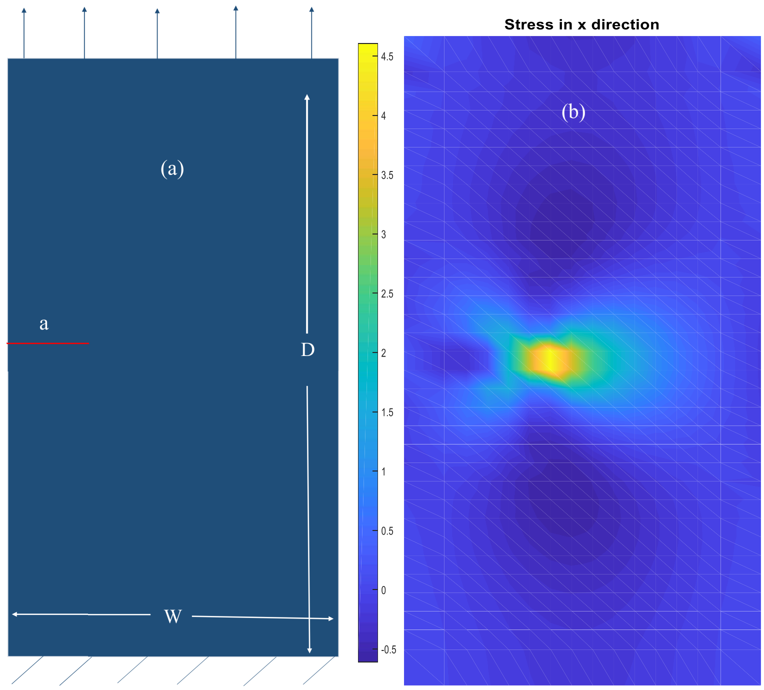

Figure 3. Representation of the model: (a) represent the boundary conditions, (b) show the σ_{xx} distribution around the crack.

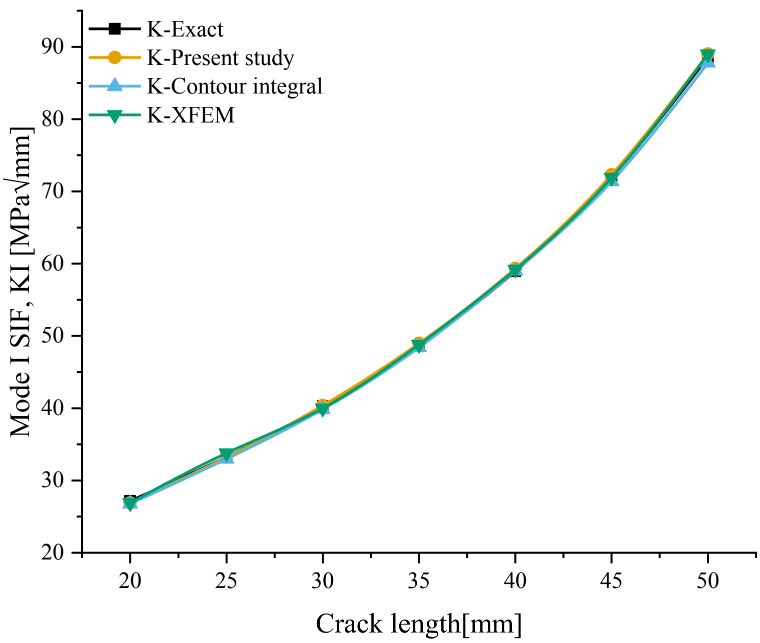

Figure 4. Variation of Stress Intensity factor with various crack length

into a uniform mesh of 18 × 36. The whole domain is discretized into conventional finite elements using CFEM. Then, the whole domain is discretized into conventional finite elements and the crack domain is discretized into enrichment finite elements using X-FEM. Finally, the entire domain is discretized by the

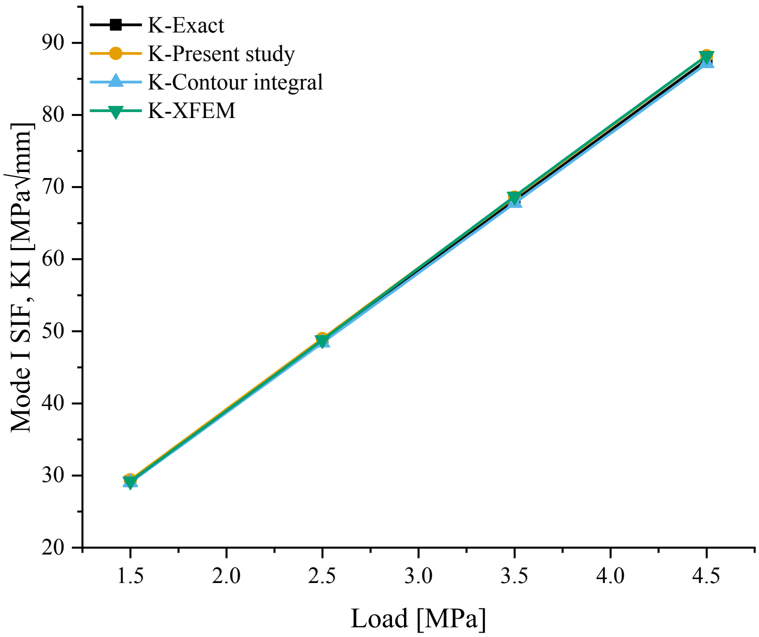

Figure 5. Variation of Stress Intensity factor with various load

present study into a set of control points as presented in Fig 6 For a = 35 mm, Fig 7 presents an example of distribution stress around defect plot with XIGA using MATLAB and Contour integral using ABAQUS Software. A repre-sentation of the stress intensity factor as a function of crack length and applied load are shown in Fig 8 and Fig 9,respectively. The results of the present study have been compared with the exact solution, Contour integral, and XFEM. For both representations, we notice that the SIFs increase with the crack size, and the model is stable with these various crack sizes but it should be noted that as we increase the crack size, we are approaching the critical SIF value of the material.

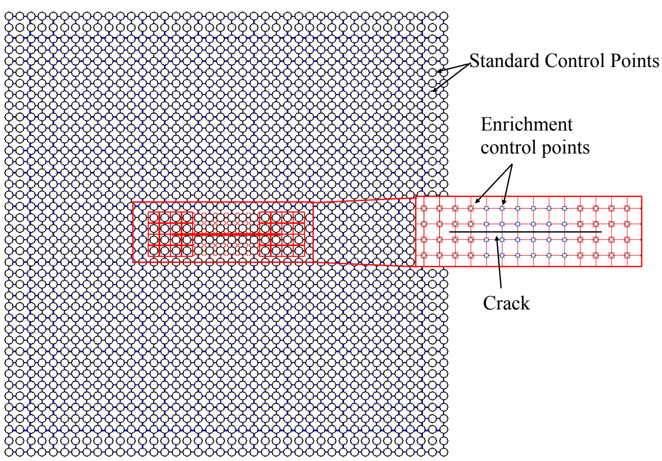

Figure 6. Domain discretization in the present study with crack enrichment.

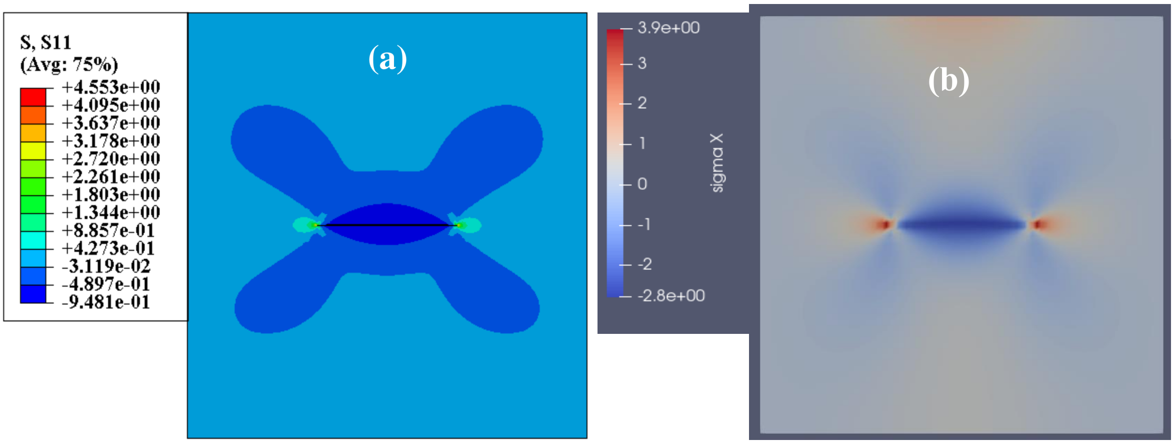

Figure 7. the σ_{xx} distribution around the crack: (a) result obtained with CFEM, (b) result obtained with X-IGA.

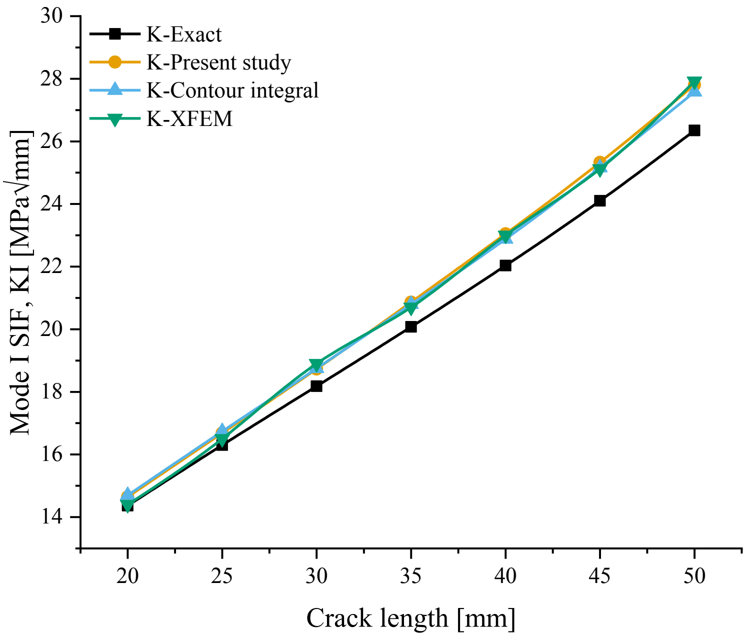

Figure 8. Variation of Stress Intensity factor with various crack length

5. Conclusion

In this study, we have employed the advantage of NURBS in the crack evaluation. This technique used the enrichment function of the X-FEM methodology and the IGA concept. We have proved the accuracy of this technique by SIFs calculation. The results obtained show a good agreement with the recent technique developed in this field. For the future work, we want to use this new technology to simulate the crack in a complex geometry. X-IGA may have become an alternative to FEA in the future, the integration of this method in computation software remains a complicated subject, and until today research is in progress in this field.

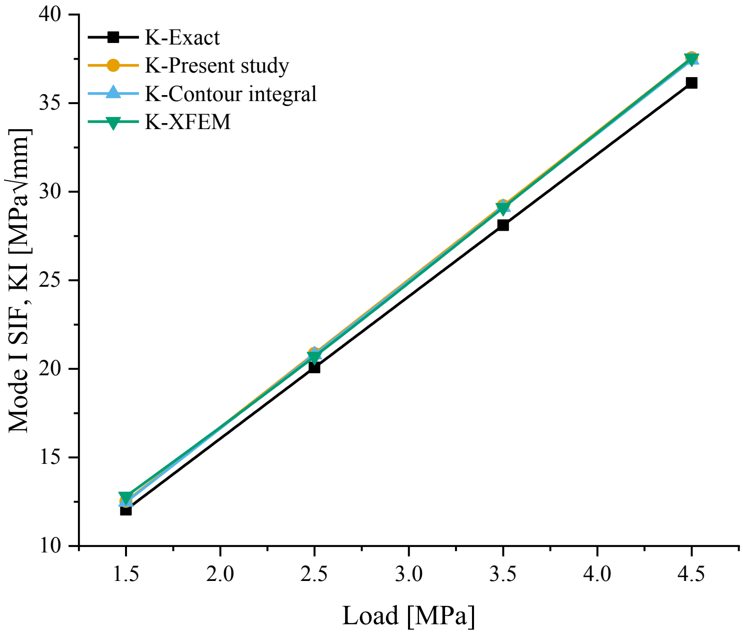

Figure 9. Variation of Stress Intensity factor with various load

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