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Mixed convection of a MHD oscillatory laminar flow of a nanofluid (Gold-Kerosene oil) in a vertical channel

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Abstract Analytical study of mixed convection of an oscillating laminar flow of nanofluid (Gold-Kerosene Oil) through the vertical channel in the presence of a magnetic field has been investigated in this article. Assuming that the nanofluid electrically driven due to the magnetic field applied to the transverse direction of the flow and the sides of the channel are subject to thermal radiative flux and high temperature, a system of equations expressed momentum, energy and concentration has been considered under these assumptions. Using the non-dimension variables technique, the system is transformed into differential equations, which are solved analytically by the perturbation method. The velocity and temperature profiles are plotted. Furthermore. The obtained results indicate that the nanofluids has a clear influence on the velocity and temperature behavior. Moreover, the presence of thermal radiation improves the heat transfer rate.

Keywords Magnetic field, mixed convection, Nanofluid, Oscillatory laminar flow, Perturbation method, Vertical channel.

AMS 2010 subject classifications 76-10, 76W99.

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1. Introduction

Heat transfers are the basis of many industrial processes present in our daily life. The intensification of these exchanges and the improvement of their efficiency have become a major issue in the industrial and technological world today. Although they take different forms (radiation, conduction and convection), the latter is the most targeted in some specific fields. The improvement of heat transfer by convection is the main object of several works, and for this purpose, a large number of researchers have conducted a multitude of numerical and experimental tests.
Indeed, the study of mixed convection in channels has been the subject of a very large number of theoretical and experimental works [1], [2]. The interest of these studies lies in its implication in numerous industrial applications such as the cooling of electronic components, building thermic and the metallurgical industry etc. Many researchers have been developed to improve the thermal performance of rectangular channel flows at low Reynolds number, i.e., in the range of 200 to 1200, which implies laminar flow, however only a few of them have considered the oscillatory behavior in channels. Chang and Lin [3] studied the transient oscillatory mixed convection of a low Prandtl number fluid in a symmetrically heated vertical planar channel subjected to opposing thrust assuming discrete heat sources maintained at uniform and equal heat fluxes. They both found that when the buoyancy parameter or Richardson number exceeds a critical value, an oscillatory flow with a unique fundamental frequency appears. The detailed

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characteristics of transient laminar flow and mixed convection in a vertical channel with symmetrical heat input were numerically studied by Lin, Chang, and Chen [4]. They mentioned that under high opposing buoyancy, sudden flow asymmetry and oscillation occur simultaneously in a nearly stable flow after the initial transient. Evans and Greif [5] showed significant buoyancy effects even for small temperature differences and reported time-dependent oscillations, including periodic flow reversals along the channel walls for a downward flow of nitrogen in a high, partially heated vertical channel.

Moreover, today more than ever, when energy consumption is increasing, heat transfer has become a reliable factor in determining the practical efficiency of applications. The growing interest in thermal energy savings, especially in the industrial sectors, has attracted the attention of the scientific community. It has encouraged researchers to seek ways to develop new heat exchanger technologies using heat transfer enhancement techniques. Heat transfer fluids are a crucial parameter that directly affects the size and cost of heat exchangers. Nevertheless, the example of available coolants such as water and oils have low thermal conductivity characteristics, which poses many obstacles to the development of heat transfer technologies to achieve high cooling performance. Recognizing this need, many researchers have utilized the potential of new classes of fluids that could improve heat transfer capabilities. Over the past few decades, modern nanotechnology has enabled the development of nanoparticles, which exhibit unique thermal and electrical properties that can help improve heat transfer using nanofluids. This is a new generation of heat transfer fluids. Indeed, the word nanofluid was initially used by Choi [6], he gave birth to a new term "Nanofluid", the latter had the idea to improve thermal conductivity. Specifically, he experimentally verified that the addition of nanoparticles to conventional-based fluids improves thermal conductivity. However, the development of nanofluid technology has created a niche and sparked a new area of research on improving the heat transfer of convective fluids with different types of nanoparticles in various heat exchange applications, due to the poor thermal performance of conventional fluids. Thus, due to its importance, the study of nanofluid has attracted a lot of interest among researchers as nanotechnology has several practical applications in industry and as a result, many authors have studied and reported results on the improvement of heat transfer in mixed convection with the presence of nanoparticles [7]-[8].

Many investigations conducted to study this topic such as Muthuraj et al [9]. They described the influence of thermal diffusion and viscous dissipation on the hydrodynamic flow of a microscopic fluid in a vertical channel. Similarly, Lourdu et al [10] deduced semi-analytical solutions for mixed convection MHD flows in a plumb channel under the influence of the thermophoresis force. Aaiza et al [11] pointed out that unsteady motion with thermal radiation affects the flow behavior in a vertical channel filled with ferrofluids.

vertical channel filled with ferrofluids. Abu-Nada and Chamkha [12] studied the mixed convection flow of a nanofluid. They considered the cavity led by the lid with the corrugated wall. Their results showed that the presence of nanoparticles has a significant increase in heat transfer for any value of the Richardson number. The heat transfer by mixed convection of a Cu-water nanofluid in a vertical parallel plate channel was studied by Raisi et al. [13]. They found that the increase in the density fraction of the solid leads to an increase in the rate of heat transfer. Khan et al. [14] studied heat transfer in a mixed MHD convection stream of a ferrofluid along a vertical channel. They concluded that the temperature and velocity of ferrofluids are highly dependent on the viscosity and thermal conductivity as well as the magnetic field. Besides, Al-Amri [15] analytically studied the Buongiorno model of mixed convection nanofluid flow inside a vertical channel by considering the non-uniform distribution of nanoparticles. They found that in addition to the mixed convection buoyancy parameter (Gr/Re), the buoyancy parameter of immersed particles also enriches the momentum and enhances the heat transfer inside the channel.

Therefore, in this work, we will conduct an analytical study of the mixed convection of an oscillatory laminar flux of a nanofluid (gold nanoparticles and kerosene oil as a base fluid) in a vertical channel with the presence of a magnetic field. The choice of gold nanoparticles is justified by its high thermal conductivity. The main purpose of this contribution is to determine the effect of the different physical parameters on the concentration, temperature and velocity of the flow. The analytical solutions of the governed equations are calculated using the disturbance technique and discussed in various charts and tables. This analysis provides an estimate of the variation in heat transfer, concentration and velocity with the nanoparticle rate and magnetic field in vertical channels that are of practical interest for technical analysis.

2. Mathematical Modeling

2.1. Statement of the problem

The problem considered in this work is to study the oscillatory laminar flow of a nanofluid in mixed convection in a vertical channel with two parallel plates. The nanofluid is assimilated to an incompressible Newtonian fluid. In addition, an oscillatory pressure gradient was applied in the direction of flow. The nanofluid was electrically driven due to the magnetic field B applied to the transverse direction of the flow. Furthermore, the temperatures $(T_0; T_w)$ and concentrations $(C_0; C_w)$ of the two plates are assumed to be sufficiently high which will generate radiative heat transfer. The geometry of our problem is shown in Figure 1.

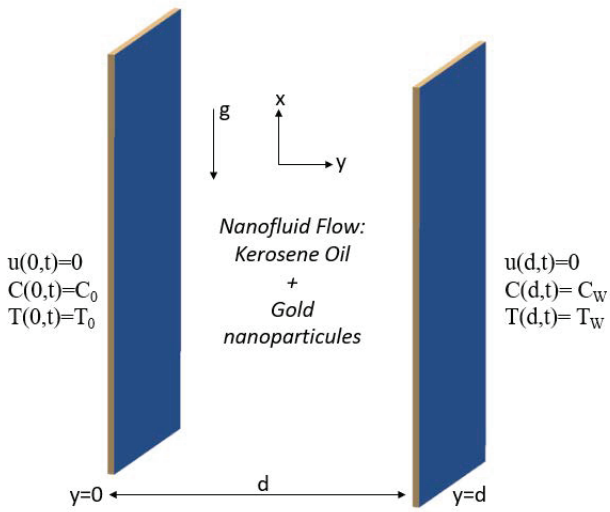

Figure 1. Geometric configuration of the studied problem.

Knowledge of the thermophysical properties of the nanofluids is essential to determine the thermal performance of this flow. We assumed that the nanoparticles are well dispersed in the base fluid and that they are in a state of thermal equilibrium with it. The effective thermophysical properties of the nanofluid will be approximated by different relations taken from the literature. We will use the following models for each thermophysical property:

• **Density:** we will work by the following relation:

$$\rho_{nf} = (1 - \phi)\,\rho_f + \phi\rho_s$$

This formula has been widely used in several theoretical works, it has also been validated by the experimental data of Pack and Cho [16].

• **Heat capacity:** assuming that the nanoparticles and the base fluid are in thermal equilibrium, the heat capacity of the mixture is expressed according to this relation (Xuan and Roetzel [17]):

$$(\rho c_p)_{nf} = (1-\phi)\,(\rho c_p)_f + \phi(\rho c_p)_s$$

• **Coefficient of thermal expansion:** the coefficient of thermal expansion is deduced from the following classical formula:

$$\beta_{nf} = (1-\phi)\,\beta_f + \phi\beta_s$$

• **Dynamic viscosity:** The dynamic viscosity of nanofluids can be calculated using several formulas developed for two-phase mixtures. We take the general case which will be expressed as follows:

$$\mu_{nf} = \mu_f(1 + a\phi + b\phi^2)$$

Where, a and b are constants given by:

Model	Cylinder
a	13.5
b	904.4

Table 1: Empirical form factors

• **Thermal conductivity:** we used the relation developed by Hamilton and Crosser [18]:

$$\frac{k_{nf}}{k_f} = \frac{k_s + (n-1)\,k_f + \phi\,(n-1)\,(k_s - k_f)}{k_s + (n-1)\,k_f - \phi\,(k_s - k_f)}$$

2.2. Governing Equations

Under the above assumptions, our problem is governed by the following partial differential equations:

$$(\rho)_{nf}\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu_{nf}\frac{\partial^2 u}{\partial y^2} - \sigma_{nf}B^2 u + (\rho\beta_T)_{nf}g(T-T_0) + (\rho\beta_c)_{nf}g(C-C_0) \tag{1}$$

$$(\rho c_p)_{nf}\frac{\partial T}{\partial T} = k_{nf}\frac{\partial^2 T}{\partial y^2} + 4(\alpha_0)^2(T-T_0) \tag{2}$$

$$\frac{\partial C}{\partial t} = D_{nf}\frac{\partial^2 C}{\partial y^2} + \frac{D_{nf}K_T}{T_m}\frac{\partial^2 T}{\partial y^2} - k_r(C-C_0) \tag{3}$$

To these equations, the conditions are added to the following limits:

$$u(0,t) = 0, u(d,t) = 0, , T(0,t) = 0, T(d,t) = T_w, C(0,t) = 0, C(d,t) = C_w \tag{4}$$

Where ρ_{nf} is the density of the nanofluid, μ_{nf} is the dynamic viscosity of the nanofluid, k_{nf} is the thermal conductivity of the nanofluid, q is the radiative heat flux in the x-direction, and d is the channel width.

This study adopts the Hamilton and Crosser [18] model for thermal conductivity and dynamic viscosity. Under this model, we will have:

$$\mu_{nf} = \mu_f(1 + a\Phi + b\Phi^2), \frac{k_{nf}}{k_f} = \frac{k_s + (n-1)k_f + \Phi(n-1)(k_s - k_f)}{k_s + (n-1)k_f - \Phi(k_s - k_f)} \tag{5}$$

In equations (2) and (3), the density ρ_{nf}, the thermal expansion coefficient of the nanofluid β_{nf} and the calorific capacity$(\rho c_p)_{nf}$are derived using the relations given by [16],[17] as follows:

$$\rho_{nf} = (1-\Phi)\rho_f + \Phi\rho_s, \qquad (\rho c_p)_{nf} = (1-\Phi)(\rho c_p)_f + \Phi(\rho c_p)_s, \qquad \beta_{nf} = (1-\Phi)\beta_f + \Phi\beta_s \tag{6}$$

Where Φ is the volume fraction of nanoparticles, a and b are constants that depend on the particle shape as given in table 1.
The parameter n appearing in equation (6) is the empirical form factor given by $n = \frac{3}{\Psi}$, where Ψ is the sphericity representing the relationship between the areas of the sphere and the particle having the same volume. It is shown in table 2.

Model	Cylindrical
ψ	0.62

Table 2: Sphericity of nanoparticles

In addition, some physical properties of the base fluid and the gold nanoparticles are given in table 3.

Material	Symbol	$\rho(kg/m^3)$	$c_p(kg^{-1}k^{-1})$	$k(W/MK)$
Gold	Au	19300	129	318
kerosene	-	783	2090	0.145

Table 3: Thermophysical properties of basic fluid and nanoparticles

In order to make the solution applicable at different scales and to get an overview of the physics of the problem, we will create a more practical scale by standardizing each variable using a characteristic value of this variable. This should reduce the number of constants appearing in governing equations and lead to broader conclusions about what controls the thermal performance of the problem at hand. To do this, the dimensions must be removed.

By introducing the following nondimensional parameters, we convert the previous equations into a dimensionless form:

$$x^* = \frac{x}{d}, \quad y^* = \frac{y}{d}, \quad T^* = \frac{T - T_0}{T_w - T_0}, \quad C^* = \frac{C - C_0}{C_w - C_0}, \quad u^* = \frac{u}{U_0}, \quad p^* = \frac{d}{\mu_f U_0}p, \quad t^* = \frac{tU_0}{d} \tag{7}$$

By introducing the above dimensionless numbers in equations 2, 3 and 4, the system becomes:

$$\Phi_1 Re \frac{\partial u^*}{\partial t^*} = -\frac{\partial p^*}{\partial x^*} + \Phi_2 \frac{\partial^2 u^*}{\partial y^{*2}} + \Phi_3 Gr T^* + \Phi_4 Gc C^* - M^2 u^* \tag{8}$$

$$\frac{\Phi_5 Pe}{\lambda_{nf}}\frac{\partial T^*}{\partial t^*} = \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{N^2}{\lambda_{nf}}T^* \tag{9}$$

$$\frac{\mathrm{Re}Sc}{1-\Phi}\frac{\partial C^*}{\partial t^*} = \frac{\partial^2 C^*}{\partial y^{*2}} + ScSr\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{\mathrm{Re}\gamma Sc}{1-\Phi}C^* \tag{10}$$

In the preceding equations, the conditions are associated with the following limits:

$$u(0,t) = 0; u(1,t) = 0; T(0,t) = 0; T(1,t) = 1; C(0,t) = 0; C(1,t) = 1 \tag{11}$$

It appears the following dimensionless numbers such as **the Reynolds number**: $Re = {}^{\rho_f U_0 d}/_{\mu_f}$, **the Grashof number**: $Gr = {}^{g(\beta_T)(T_w - T_0)d^2}/_{\nu_f U_0}$, **the Grashof mass**: $Gc = {}^{g(\beta_c)(C_w - C_0)d^2}/_{\nu_f U_0}$, **the Schmidt number**: $Sc = {}^{\nu_f}/_{D_f}$, **the Magnetic number**: $M^2 = {}^{\delta {B_0}^2 d^2}/_{\mu_f}$, and **the radiative number**: $N^2 = {}^{4{\alpha_0}^2 d^2}/_{k_f}$.

With : $\left\{\begin{array}{ll} \Phi_1 = [(1-\Phi) + \Phi\frac{\rho_s}{\rho_f}], & \Phi_2 = [1 + a\Phi + b\Phi^2] \\ \Phi_3 = [(1-\Phi) + \Phi\frac{(\rho\beta_T)_s}{(\rho\beta_T)_f}], & \Phi_4 = [(1-\Phi) + \Phi\frac{(\rho\beta_c)_s}{(\rho\beta_c)_f} \end{array}\right\}; \Phi_5 = (1-\Phi_0) + \frac{(\rho c_p)_{nf}}{(\rho c_p)_f}$

2.3. Analytical Resolution by Perturbation Method:

An analytical solution of the mixed convection in a vertical channel with two vertical parallel plates uniformly heated in the presence of a heat source is presented. This solution is deduced from the perturbation method which consists in perturbing the system of equations obtained previously to determine the general expressions of the temperature and velocity profiles of the flow as well as the concentration profiles.
In order to solve the equations (8, 9 et10), with the boundary conditions (11), we used the perturbation method:

$$u(y,t) = u_0(y) + \varepsilon\exp(i\omega t)u_1(y) \tag{12}$$

$$T(y,t) = T_0(y) + \varepsilon\exp(i\omega t)T_1(y) \tag{13}$$

$$C(y,t) = C_0(y) + \varepsilon\exp(i\omega t)C_1(y) \tag{14}$$

The pressure gradient, sinusoidal type, applied in the direction of the flow is imposed in the following form:

$$-\frac{\partial p}{\partial x} = \lambda\varepsilon\exp(i\omega t) \tag{15}$$

With ω is the pulsation and λ is the amplitude of the oscillation.

By substituting the equations (12) - (14) in the equations (8) - (10), we obtain the following system of equations:

$$\begin{aligned}
&u(y,t)=\frac{\sin(my)}{\sinh(m)}\left[-\frac{A}{\left(b_1^2+m^2\right)}+B\left\{\frac{1}{\left(b_4^2-m^2\right)}\left(1+\frac{b_1^2b_3^2}{b_1^2+b_4^2}\right)+\frac{b_1^2b_3^2}{\left(b_1^2+b_4^2\right)\left(b_1^2+m^2\right)}\right\}\right]\\
&-B\left\{\frac{\sinh(b_4y)}{\left(b_4^2-m^2\right)\sinh(b_4)}\left(1+\frac{b_1^2b_3^2}{b_1^2+b_4^2}\right)+\frac{b_1^2b_3^2\sin(b_1y)}{\left(b_1^2+m^2\right)\left(b_1^2+b_4^2\right)\sin(b_1)}\right\}\\
&+\varepsilon\exp(i\omega t)\left[\frac{\lambda}{m_1^2}\left(1-\cosh\left(m_1y\right)\right)+\frac{\lambda}{m_1^2}\frac{\sinh(m_1y)}{\sinh(m_1)}\left(\cosh(m_1)-1\right)\right]+\frac{A}{\left(b_1^2+m^2\right)\sin(b_1)}\sin(b_1y)
\end{aligned} \tag{16}$$

$$T(y)=\frac{\sin(b_1y)}{\sin(b_1)} \tag{17}$$

$$C(y)=\frac{\sin(b_4y)}{\sinh(b_4)}\left(1+\frac{b_3^2b_1^2}{b_1^2+b_4^2}\right)-\frac{b_3^2b_1^2}{(b_1^2+b_4^2)\sin(b_1)}\sin(b_1y) \tag{18}$$

With :

$b_0^2=\frac{\phi_5 Pe}{\lambda_{nf}}$ $\quad b_1=\sqrt{\frac{N^2}{\lambda_{nf}}}$ $\quad b_2=\sqrt{\frac{\mathrm{Re}Sc}{1-\phi}}$

$b_3=\sqrt{ScSr}$ $\quad b_4=\sqrt{\frac{\mathrm{Re}\gamma Sc}{1-\phi}}$

$m_1=\sqrt{M^2+i\omega\phi_1\mathrm{Re}}$ $\quad m^2=\frac{M^2}{\phi_2}$

$A=\frac{\phi_3 Gr}{\phi_2}$ $\quad B=\frac{\phi_4 Gc}{\phi_2}.$

One can note that the nanofluid are employed with a volume fraction of gold nanoparticles (AuNP) are less than 4in this case the base fluid of kerosene oil is considered as single phase together with AuNP particles. Thus, to investigate the physical performance of AuNP- kerosene nanofluid in a vertical channel, most laminar flows are considered at Reynolds numbers in the range $Re \simeq 20-1000$. Moreover, the typical values of Schmidt number Sc, the Magnetic number M and the Radiative number N in all applications of Kerosene oil in liquid state are known to be small (of the order $\simeq 10^{-1}-10^{0}$). So, it is clear that the analytical solutions given by (16)-(18) converge and do not have a critical value in this range of hyperparameters of our studied problem, and therefore the current analytical solution of the mixed convection laminar flow will be presented and discussed extensively in the next paragraph.

3. Results et Discussion :

In this section, we will graphically illustrate the influence of various parameters on velocity, temperature, and concentration. Thus, we also presented a comparison of our results with those present in the literature for particular cases of flow in order to study the validation of the obtained analytical solution.

The figure 2(a) shows the variation of the temperature as a function of the cross-sectional component y for different values of the radiation parameter N.

It can be seen that in the case where $N=0$ the temperature varies linearly as a function of y.While increasing this parameter, the temperature begins to vary in a sinusoidal way between the two plates, this is due to the presence of radiative transfer.

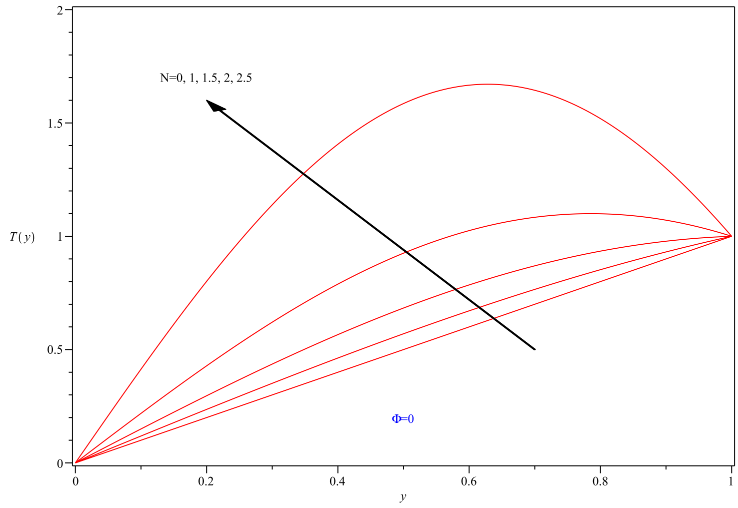

Figure 2(a). Temperature variation as a function of the radiation parameter N.

Thus, the increase in the radiative parameter will divide the temperature profile, as shown in figure 2(b), into two regions: the first is characterized by the decrease in temperature, the radiative transfer reacts in this case as an attenuation in the temperature. While in the second region is characterized by the increase in temperature due to the absorption of energy, in this case the radiative transfer plays the role of a source.

However, it is obvious from Figure 2(b) that the sinusoidal effect maximizes with the increase of the density fraction and radiation parameter N. This result is in good agreement with that achieved by Aiza et al. [19].

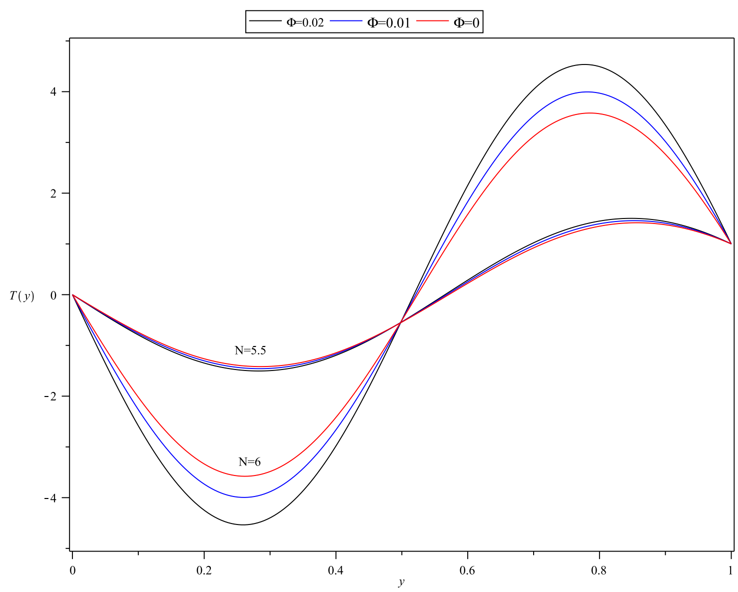

Figure 2(b). Temperature variation as a function of the radiation parameter N.

However, the increase in this radiative parameter N leads to a large decrease in concentration for each fixed value of the number of density fraction is exposed in Figure 3. Due to the presence of the velocity field, the concentration profiles become more curved.

By increasing the number of density fractions of the nanoparticles, the effect of the radiative parameter becomes more important, the concentration will have a sinusoidal behavior due to the fact that the temperature field in this case varies sinusoidal depending on N. This result is well in line with the result obtained by Sidra Aman [20].

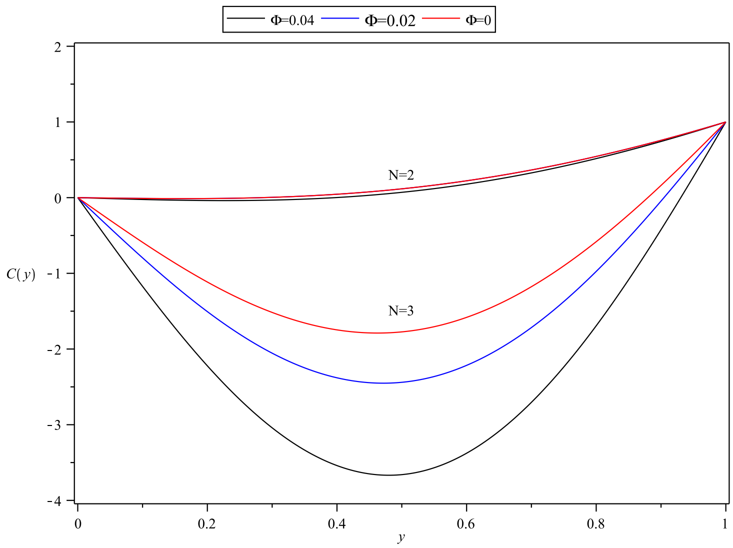

Figure 3. Concentration variation as a function of radiation parameter N.

The velocity profile in the absence of the magnetic field for different values of the Grashof number Gr is shown in Figures 4(a) and 4(b). In the absence of the natural convection characterized by the Grashof number Gr, the velocity field is similar to the flow between two planes (Poiseuille flow). It has been observed that an increase in the Gr parameter has led to an increase in the velocity within the channel due to the phenomenon of natural convection, thus illustrating the theory of dissipative system that results in the destabilization of the nanofluid medium. This increase in the number leads to the appearance of parabolic curves slightly inclined towards the direction of destabilization caused by the gravitational force (Boussinesq term).

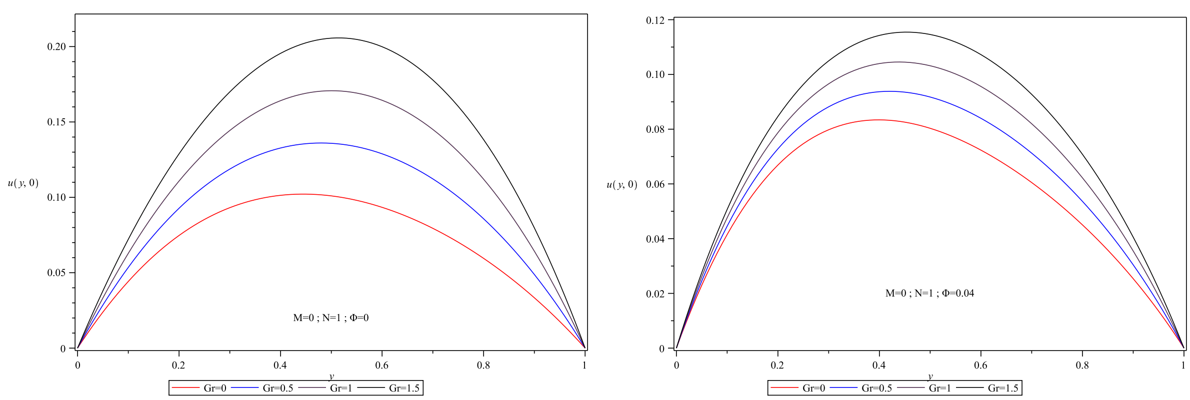

Figure 4 (a),(b). The variation of the velocity as a function of the transverse component y for different values of Gr in the absence of the magnetic field.

By increasing the volume fraction of the nanoparticles, we can see that the velocity field decreases considerably for each fixed value of the Grashof number Gr; this decrease is due to the increase of the viscosity of the medium.This phenomenon can clearly be seen from figure 5 where we represent the effect of the volume fraction of nanoparticles on the velocity field for fixed values of Gr, M and N ($Gr = 0.5$, $M = 0$, $N = 1$).

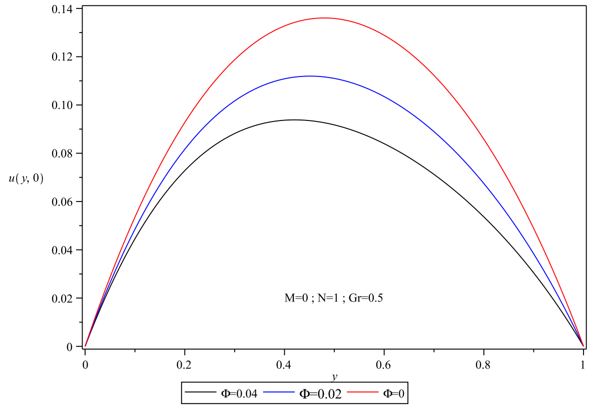

Figure 5. The variation of the velocity in the presence of nanoparticles for values of M=0, N=1, Gr=0.5 and Gc=0.3.

The velocity profile for different values of the magnetic parameter M is shown in Figure 6(a). It was observed that an increase in the magnetic M parameter resulted in a decrease in the the nanofluid velocity.
This is justified by the fact that the effect of a transverse magnetic field on the fluid gives rise to a resistive force known as the Lorentz force, which is like the drag force, so by increasing the value of M, the drag force increases and causes the fluid to move slowly and slow its velocity. As can be seen from this figure, when the influence of the magnetic field was taken as zero ($M = 0$), the velocity was found to be greater than when the magnetic parameter was present.
However, it can be seen from figure 6(b) that when injecting the nanoparticles, the effect of the magnetic field becomes more important. The velocity field will considerably decrease within the channel due to the increase in viscosity. Therefore, the flow of the nanofluid, frequently called magnetic fluid since gold belongs to the ferromagnetic metals, will be largely slowed down according to the increase of both the volume fractions of the nanoparticles and the magnetic field intensity.

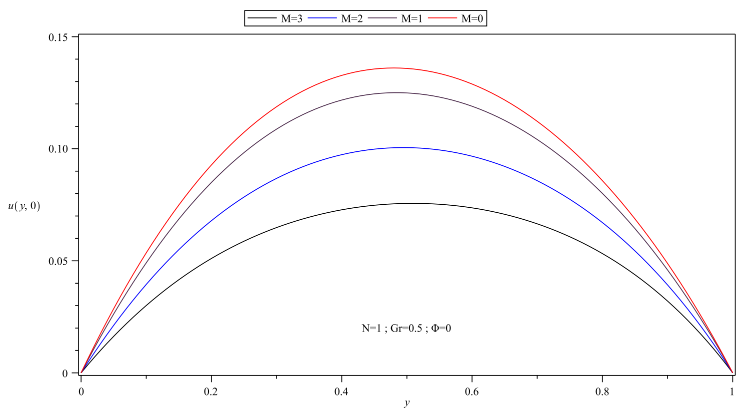

Figure 6 (a). The variation of the velocity for different values of magnetic parameter M.

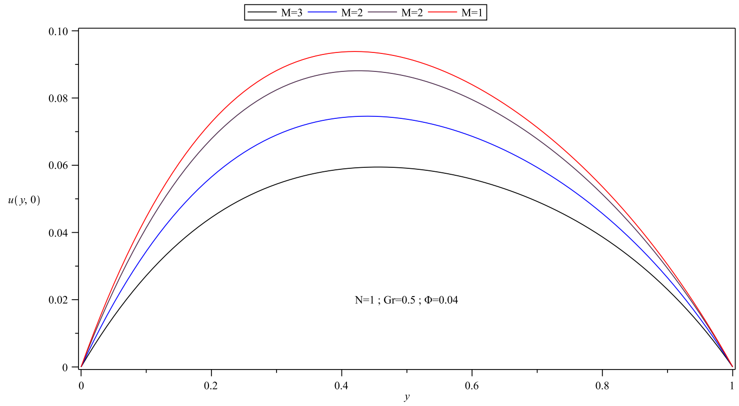

Figure 6(b). The variation of the velocity for different values of magnetic parameter M.

The influence of the N radiation parameter on velocity is presented in Figure 7. Evidently, an increase in the radiation parameter N, contributes to the increase in the velocity of the nanofluid, which physically means that the fluid emits more and more heat thus the energy transfer rate increases.

In addition, it can also be observed that an increase in the N parameter, thus a decrease in the viscosity of the nanofluid, which resulted in an increase in its velocity. This result agrees well with the result obtained by Makinde and Mhone [21].

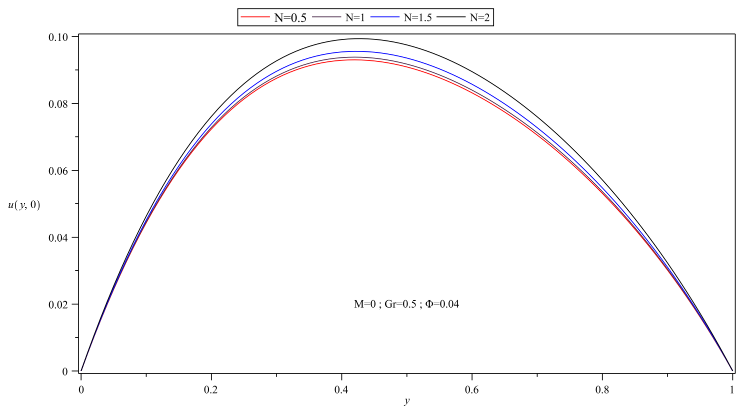

Figure 7. The variation of the velocity for different values of radiation parameter N in the presence of nanoparticles.

The figure 8 shows the temporal evolution of the velocity field within the vertical channel with two parallel plates. This figure shows that the velocity field accelerates considerably close to the right wall and is totally slowed down near the left wall. This observed behavior is due to the destabilization effects caused by the gravitational force (Boussinesq term) which will be amplified due to the oscillatory behavior of the flow.

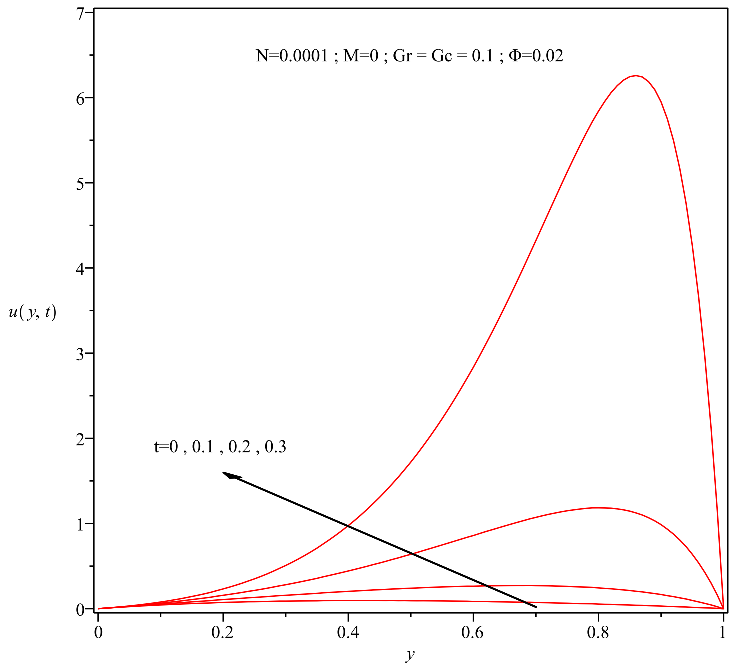

Figure 8. Velocity variation as a function of time.

4. Application :

The mixed convection problem of oscillating laminar nanofluid flow through a vertical channel is ubiquitous in many technical and biomedical applications. Therefore, the analytical solution presented in section 2.3 finds interest in many different fields such as device cooling for electronic equipment, food products, chemical processes, tribological applications, medical applications, etc. Among the application areas, some are mentioned in the following.

In the electronics sector, the cooling of electronic devices is one of the main challenges of new generation technology and different techniques have been used in the past to improve heat transfer performance, such as fins and louvers. However, these techniques have reached their limits with the extreme miniaturization of electronic devices and, at the same time, the increase of their energy efficiency. For this reason, special attention has been paid to the development of a new type of coolants incorporating particle suspensions in order to improve the thermal properties of coolants, and despite the fact that the use of nanofluids is not yet common, many researches have deal with this topic reporting experimental and theoretical results and suggested that the thermal design of cooling systems for electronic equipment should be thoroughly revised using nanofluids [22]- [24].

In medical field, fluid flows driven by travelling sinusoidal waves on channel has fundamental importance due to physiological and many other applications such as urine transportation, spermatozoa transport, vasomotion of small blood vessels, heart–lung machine, sanitary fluid transport. Several studies [25]- [27] have pointed out that the use of nanofluid involves destruction of undesirable cancer tissues by improving the heat transfer exchange in channels, researches additionally trimmed down for the case of non-Newtonian fluids with nanoparticles of gold. It should be noted that gold nanoparticles are an important class of materials because their unique physicochemical properties, such as near-infrared light absorption releasing thermal energy, offer new opportunities in the treatment of diseases [28]. The analytical solution presented in this paper is further explored.

5. Conclusion

An investigation has been carried out for the mixed convection of nanofluid through a vertical channel in the presence of a magnetic field. The results obtained using the perturbation method, the observations deduced are:
• The sinusoidal effect of the temperature profile maximizes with the increase of the density fraction and radiation parameter N;
• The concentration profile increases by maximizing the density fraction of the nanoparticles and decreases with the increase in N-radiation parameter;
• Increasing the N-radiation parameter contributes to increasing the speed of the nanofluid, which physically means the emission of the nanofluid from the heat;
• The speed of the nanofluid decreases with the increase in the magnetic parameter due to the increase in drag force that tends to slow the movement of the fluid.

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Nomenclature

y	Cartesian coordinate along the vertical channel	$[m]$
x	Cartesian coordinate along the normal to y	$[m]$
d	Width of channel	$[m]$
(u,v)	Components of the velocity vector	(m/s)
(u^*, v^*)	Dimensionless components of the velocity vector	
V	Volume	(m^3)
T	Temperature	$[K]$
T^*	Dimensionless temperature	
ΔT	Temperature variation	
C	Concentration	
C^*	Dimensionless concentration	
c_p	Specific heat capacity	$[J/(kg.K)]$
g	Gravitational acceleration	(m/s^2)
k	Thermal conductivity	$[W/mK]$
P^*	Dimensionless pression	
Greek symbols		
α	Thermal diffusivity	(m^2/s)
μ	Dynamic viscosity	(Ns/m^2)
ν	Kinematic viscosity	(m^2/s)
ρ	Density	(kg/m^3)
β	Coefficient of thermal expansion of the fluid	(K^{-1})
Φ	Nanoparticles volume fraction	
σ	Electrical conductivity	(m^2kg/s^2k)
δ	Constant	(K/m)
ρC_p	Volumetric heat capacity	$(J/m^3.K)$
Dimensionless parameters		
$\Pr = \frac{\nu_f}{\alpha_f}$	Prandtl number	
$Gr = \frac{(\beta_T)_f g(T_w - T_0)d^2}{U_0 \nu_f}$	Thermal Grashof number	
$Gr = \frac{(\beta_T)_f g(C_w - C_0)d^2}{U_0 \nu_f}$	Mass Grashof number	
$\mathrm{Re} = \frac{U_0 d}{\nu_f}$	Reynolds number	
Sc	Schmidt Number	
Subscripts		
f	Fluid phase	
nf	Nanofluid	
s	Solid particles	