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A FMKL-GLD Quantile Method for Estimating Economic Growth in Nigeria in the Presence of Multicollinearity and Outliers

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Abstract Nigeria's economic growth has been a serious concern for policymakers, economics, and scholars due to the challenges of economic characteristics, consequences, and contradictions. Despite the effort made by the Central Bank of Nigeria in implementing several policies such as tightening of monetary policy rate and heavy borrowing for infrastructural development to stimulate economic growth in the past few years. The economic growth in Nigeria between 1999-2007 was 6.95 percent, between 2008-2010, it was stood 7.98 percent and it was 4.80 percent between 2011-2015 and from 2016 till day the economic growth stood at 0.81 percent. In this study, we estimate parameters to determine Nigerian economic growth in the presence of multicollinearity and outliers. We employed the FMKL-GLD quantile model to quarterly data from 1986 to 2021 obtained from the Central Bank of Nigeria. Exploratory data analysis (EDA) and diagnostic test carried out ascertained the presence of multicollinearity and outlier. However, it was discovered that in model that INDT, RINR, REXR and OPEN contributed positively to the economic growth to the turn of 0.15%, 0.24%, 0.06% and 1.76% while, it was revealed that EXDT contributed negatively to the economic growth in Nigeria thus, reduced economic growth by 0.02% and as such showed the tendency and potential macroeconomic variables under study as a veritable determinant of economic growth. The location, scale and space parameters $\widehat{\lambda}_1$, $\widehat{\lambda}_2$, $\widehat{\lambda}_3$, and $\widehat{\lambda}_4$ for the fitted FMKL-GLD model were -0.0253, 34.1488 -0.3303, and -0.5758. Therefore, it can be concluded based on the location, scale, and space parameters of FMKL-FGLD and GLD Q,Q plots, as well as the estimated parameter of RGDP, that FMKL-FGLD was the best model that described the economic situation in Nigeria by showing that the economy was growing in retrogressive direction and the need for drastic effort and strong will to curtail the situation. To achieve this, the adoption of economic openness to the development and growth of the economy in Nigeria is a veritable policy direction that must be strictly followed.

Keywords Economic Growth, Internal and External Debt, Interest and Exchange Rate, Trade Openness, Multicollinearity, Outliers, FMKL-GLD Quantile Regression Method.

AMS 2010 subject classifications 62H20, 62H30, 62J10, 62P99

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1. Introduction

Nigeria's economic goals have lingered such that the stability of the production structure and consumption patterns remained challenging. Evidently, the economic growth in Nigeria between 1999-2007 was 6.95 percent, between 2008-2010, it was stood 7.98 percent and it was 4.80 percent between 2011-2015. From 2016 till day the economic growth stood at 0.81 percent. The diversification of the economic base to reduce dependence on oil, with the intention of pushing the economy on a part of viable, all-inclusive and stem the tide of inflationary growth have not yielded the desire result. This causes a great and serious concerns to the scholars, researchers and the policy makers despite all the claimed and huge investments from borrowing both internally and externally sources. The

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implication of this is that, the transformation of the various sectors of the economy and a rapid growth in output measured by the real gross domestic product are important and more critical for development. This has to be consistent with the growth desires of the countries, as the structure of the economy is expected to change as growth progresses.

It was noted that successive governments in Nigeria since independence in 1960, trailed the goal of structural changes without much success. The growth dynamics supposed to be driven by the existence and exploitation of natural resources and primary products. Initially, the agricultural sector, driven by the demand for food and cash crops production was the main focus of the growth process with 54.7 percent contributions to the GDP in the 1960s. Ten years after the independence, the emergence of the oil industry as the main driver of growth overshadowed the agriculture base economy. Since then, the economy has essentially spiraled with the boom-burst cycles of the oil industry. Government expenditure and investment outlays majorly dependent on oil revenues and borrowing in case of shot fall and as such the pace of economic growth were seriously hampered. Thus, it is evidently visible that the economy has not actually performed to its full potential particularly in the face of its rising population.

The economy of Nigeria has unacceptably failed relative to her enormous resource endowment and her peer nations. It has the sixth largest gas reserves and the 8th largest crude oil reserves in the world. It is endowed in commercial quantities with about 37 solid mineral types and has a population of over 200 million persons. Yet economic growth has been rather weak and does not reflect these endowments because Nigeria experienced a retarding growth in 2015 which ultimately deteriorate into harsh economic recession in 2016 and mid 2020 after a partial relief in 2018. An identified shift in global monetary policy cycles as a major reason for the country's economic wretchedness which has seriously impinge on Nigeria's financial market; particularly, the monetary policy rate for Nigeria which was situated at a level of 14% in the first quarter of 2017, as against the 12% mark in the first quarter of 2016 and 13% in 2015. Interest rate is one of the main drivers of gross domestic product, although indirectly through its consequence on investment [1].

Also, various developmental plan and programme, vision and reforms on macroeconomic variables such as trade openness, interest and exchange rate, external and domestic debt management proposed at different time and level generates much hope but actually produces little impact. Therefore, it is important to state that both monetary and fiscal policies influence economic growth. As observed that, economic growth is negatively affected by the external debt and as such the long-term growth was hindered even when it was economic growth-enhancing in the short-term. On the other hand, the effect of the domestic debt on the growth of the economy was in inverse order for both long-term and short-term growth when compared with the external debt [2]. Also, debt servicing constitutes a major hindrance to economic growth as it confirmed the effect of debt overhang. It must be stated that the economic openness can be made to enhance economic growth via a sustainable diversification effort [3]. It can be asserted that in both the short and long run, trade openness had a serious constraint on the growth of the economy. Thus, it can be emphasized that the imports were greater than export in Nigeria and as such government need to engage in export led diversification to spur economic growth [4]. However, the purpose of this study is to investigate the existing relationship and predicting the economic using the macroeconomic variables that includes economic growth (RGDP) as the endogenous variable, internal debt (INDT), external debt (EXDT), interest rate (RINR), an exchange rate (REXR) and trade openness (OPEN) in Nigeria when outlier and multicollinearity assumptions are jointly violated. Also, the study is aimed to estimate the parameters of the model in the presence of outlier and multicollinearity using FMKL-GLD quantile regression model that was dearth in various work and study previously carried out on the aforementioned macroeconomic variables in literatures.

2. Literature Review

In examining economic growth various studied have been carried out using different macroeconomic variables as a determinant. Some of the work done includes: [2] investigated the effect of government debt on Nigeria's economic growth for the period (1980-2018). An Autoregressive Distributed Lag technique was empirically applied for analyzing the collected data. From the findings, it was discovered that the impact of external debt on economic growth was negative and as such the long-term growth of the economy was hindered even when it was growth-enhancing in the short-term. On the other hand, the domestic debt impacted economic growth in inverse order for the long-term and short-term growth when compared with the debt incurred externally. Also, debt servicing hindered the economic growth as it confirmed the effect of debt overhang. [5] examined the impact of external debt on economic growth in Nigeria (1980-2016). Secondary data gathered from the Statistical Bulletin of Central Bank Nigeria were analysed by means of Generalized Method of Moments (GMM). Findings showed that external debt was positively and significantly related to Nigeria's economic growth.

[6] investigated the effect of external debt on economic growth in Jordan from 2010 to 2017. A descriptive statistic and least square method were adopted for the study during the period under examination revealed that external debt negatively and significantly affected economic growth. [7] examined the effect of external debt on the economic growth of Nigeria (1981-2017). The secondary data collected on the identified economic variables from CBN Statistical Bulletin and NBS were used for the study. The Granger Causality test and Error Correction Mechanism techniques revealed that debt stock from foreign nation and spending on capital projects by the government positively and significantly influenced the growth of the economy. It was further revealed from the study that the influence of the cost of servicing foreign debt on economic growth in Nigeria was insignificant. [8] carried out a study to investigate the impact of external debt on economic growth in Nigeria (1999-2015). In the study, an econometric technique such as Johansen Co-integration and Vector Error Correction Mechanism were employed to analyse the collected data. Findings showed that external debt and economic growth were inversely related in Nigeria.

[9] in their study examined the relation between external borrowing by the government and economic growth between 1990 and 2015. A continuous increase in Oman's external debt in the financing budget on yearly basis served as the motivation for the study. In the study, an Autoregressive Distributed Lag cointegration method was adopted in analyzing the data. Thus, it was discovered from the result that external borrowing by the government negatively and significantly affected the economic growth in Oman. It was further discovered that gross fixed capital positively and significantly impacted the Oman's economic growth during the period examined. [10] analyzed the impact of external debt on economic growth in Nigeria between 1985 and 2015. The data gathered for the study were analyzed using ordinary least square regression method, unit root test, Johansen cointegration and error correction test. From the results, a negative and insignificant was discovered as the impact of debt servicing on the growth of the economy in Nigeria. Also, it was discovered that the stock of external debt positively and significantly affects the index of economic growth. The effects of other control variables used in the study were positive and significant on economic growth. A long-run relationship and unidirectional causality were established between external debt and economic growth during the period under consideration.

[11] carried out study to assess the impact of trade openness on economic growth among ECOWAS countries (1975-2017). A non-stationary heterogeneous dynamic panel models through the application of Pooled Mean Group (PMG) and Mean Group (MG) estimators were employed to explore the secondary data gathered for the study. In the findings, it was revealed that trade openness had positive effects on growth in ECOWAS countries in the long-run but mixed effects in the short-run. Thus, it was emphasized that the cooperation among ECOWAS member countries need to be improved as this would help economic actors in the region to access international markets and to be more strategic in term of trade and competitiveness through export consortia. [4] investigated the dynamic impact of trade openness on the economic growth in Nigerian economy (1980-2016). The secondary data that were sourced from the CBN Statistical Bulletin were used. The techniques and diagnostic test carried

out in the studied were: unit root, cointegration and error correction model. As a result, it was discovered trade openness had a negative impact on the economic growth, both in the short run and the long run. Thus, it can be emphasized that the imports were greater than export in Nigeria and as such, they need the government to engage in export led diversification to spur economic growth.

[12] conducted research on the impacts of international trade on Nigeria's economic growth for the period 1985-2015. The variables considered in the study were: interest rate, the balance of trade, exports and trade openness. The analytic techniques adopted were unit root, cointegration and vector error correction model and it was found that the relationship between the variables under investigation were insignificant in the long run. Also, it was found that economic growth and the trade openness were unidirectionally related. [13] Da'Silva (2014) examined the nexus between the economic openness and productivity growth of Nigeria (1970-2010). Secondary data collected on real gross domestic product, openness, real interest rate, exchange rate and unemployment were analysed using Ordinary Least Square (OLS) Method. The result revealed a positive and statistically significant relationship between trade openness and economic growth.

[3] examined the impact of the exchange rate, as an important determinant of economic growth in Nigeria between 1980 and 2019. Secondary data used were sourced from the CBN Statistical Bulletin and an econometric technique such as: Unit Root, Cointegration, and Error Correction Model were employed. In the result, it was indicated that exchange had a positive and significant impact on economic growth. The result further indicated that economic openness had a negative impact on economic growth. Thus, government should make more effort to redesign the monetary policies so as to ensure stable exchange rate. [14] empirically studied the contribution of interest rate and exchange rate on economic competitiveness in Nigeria. The data collected on interest rate, exchange rate and gross domestic product the proxy for economic competitiveness spanning the period 1981-2016 were examined. The result obtained from the OLS technique employed revealed that both interest rate and exchange rate had a significant impact on economic competitiveness.

[15] investigated the effect of exchange rate fluctuation on economic growth in Nigeria using time series data spanning (1970-2012). The study adopted exchange rate, inflation, money supply and oil revenue as the explanatory variables, while gross domestic product was used to proxy economic growth which was the response variable. Secondary data for the study were obtained from CBN Statistical Bulletin. A multiple linear regression technique was used for the analysis of the collected data. A mixed finding was obtained which indicated that a floating exchange rate was better than fixed exchange rate in determining a sustainable economic growth.

[16] investigated the use of GLD quantile regression using three datasets, namely the Belgian Engel dataset, pipeline repair cost data, and the simulated motor cycle acceleration data. The Belgian dataset consisted of 235 observations, the simulated motorcycle dataset had 133 observations and the pipeline repair cost data had 532 observations. A FMKL-GLD quantile regression was used for the analysis. The result revealed that the simplicity of GLD quantile regression when compared to standard quantile regression. The results also showed that FMKL-GLD quantile regression provided a robust reference line to outliers and produced zero mean residuals. Besides, it provided a reference line with smooth regression coefficients across different quantiles. [17] investigated an efficient estimation technique for economic growth and its determinants for Nigeria in the presence of multicollinearity. The macroeconomic variables considered in the study were economic growth (RGDP), internal debt, external debt, interest rate, exchange rate and trade openness. An exploratory data analysis and the variance inflation factor carried out revealed the presence of multicollinearity. Thus, a ridge regression method was adopted and it was found that a ridge regression technique with appropriate ridge constant was a robust method that was efficient to estimate economic growth in Nigeria.

However, the summary of various literature reviews, the presence of outlier and interdependency of the explanatory variables as common assumptions violation in classical linear model have not been examined together or jointly put into consideration in various analyses been carried out. As such, the detection of outlier

and interdependency among the explanatory macroeconomic variables thus, becomes an important problem in modelling, analysis and inference about the fitted regression model. Specifically, we aim to find an estimate of the parameters for RGDP considering INDT, EXDT, RINR, REXR and OPEN as the drivers when there is interdependency among the explanatory and outliers using granger causality and FMKL GLD quantile regression model based on causality as an efficient estimation technique that was lacking in various work and study previously carried out in relation to the macroeconomic variables under investigation.

3. Material and Method

In this study, the macroeconomic data gathered from the CBN statistical bulletin that were used to examine the existing relationship among economic growth (RGDP) as endogenous variable, internal debt (INDT), external debt (EXDT), interest rate (RINR), exchange rate (REXR), and trade openness (OPEN) between 1986Q1-2021Q1. The variables assumed a linear model stated in both functional and econometrical form as (1) and (2)

$$RGDP = F(INDT,\ EXDT,\ RINR,\ REXR,\ OPEN) \tag{1}$$

$$RGDP = \alpha_0 + \alpha_1 INDT + \alpha_2 EXDT + \alpha_3 RINR + \alpha_4 REXR + \alpha_5 OPEN + \epsilon_i \tag{2}$$

where, α_i and ϵ_i are the parameters to be estimated and error term respectively.

However, the multiple linear regression model stated in (1) and (2) were transformed and expressed in general form as given in (3) where $\mathbf{Y}$) represent the dependent variable $RGDP$ and $\mathbf{X}$ represent the explanatory variables $INDT$, $EXDT$, $RINR$, $REXR$ and $OPEN$.

$$\mathbf{Y} = \mathbf{X}'\boldsymbol{\beta} + \boldsymbol{\epsilon} \tag{3}$$

The ordinary least square estimator of $\boldsymbol{\beta}$ is given in (4)

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-\mathbf{1}}\mathbf{X}'\mathbf{Y} \tag{4}$$

The covariance matrix of $\boldsymbol{\beta}$ can be obtained as given in (5)

$$\mathrm{Cov}(\hat{\boldsymbol{\beta}}) = \sigma^2(\mathbf{X}'\mathbf{X})$$

where $\mathbf{Y}$ is an observational vector of dimension $n \times p$, $\mathbf{X}$ is an $n \times p$ data matrix of regressors, $\boldsymbol{\beta}$ is a $p \times 1$ vector of regression coefficient and $\boldsymbol{\epsilon}$ is an $n \times 1$ disturbance vector.

Outlier and multicollinearity problem as common assumptions violation in classical linear model were simultaneously addressed in this study using a robust estimation technique. An outlier is noted to be an observation that appears to be inconsistent with other observations in a given dataset which can influences or causes a substantial change of some important aspects of the regression analysis such as estimated parameters and the variance or standard error. Also, the presence of outliers can lead to biased estimation of the parameters, misspecification of the model and inappropriate predictions. Thus, the need for diagnostics in order to detect the presence of outlier and multicollinearity. Testing for outliers can be done using any of the following test: Dixon's test, Grubbs' test, Cochran's C test, and Bartlett test while, correlation analysis, variance inflation factor, tolerance level, eigenvalue and condition number, can be used to detect the presence of multicollinearity. In this study, Grubbs' test and variance inflation factor were used to establish the violation of these assumptions.

Grubbs' test

The Grubbs' test for outliers was recommended by International Statistical Organisation (ISO) and as such it will be used in this study. This test compares the deviation of the suspect value from the sample mean with the standard deviation of the sample. The suspect value is the value that is furthest away from the mean. In order to

use Grubbs' test for an outlier, the null hypothesis (H_0) is there no outliers in the dataset under investigation. The statistic G_m is calculated as expressed in (6):

$$\hat{G}_m = \frac{|\mathbf{X}_s^* - \mathbf{X}|}{\mathbf{S}} \tag{6}$$

$$\hat{G}_m = \frac{|\text{Suspect value} - \mathbf{X}|}{\mathbf{S}}$$

Under the assumption multicollinearity, correlation coefficients of independent variables are computed even though strong correlation coefficients do not necessarily imply the presence of multicollinearity, it can be a suspect and as such it can be ascertained by checking the variance inflation factor (VIF) and condition number (CN).

Variance Inflation Factor

The variance Inflation factor (VIF) is given by (7)

$$VIF = \frac{1}{1 - R_i^2},\ i = 1, 2, ..., r, \tag{7}$$

and R_i^2 represents the squared multiple correlation coefficients when X_1 is regressed on the remaining X_{1+i} independent variables. It can be opined that VIF increased the variability of the estimated coefficients. In other words, it inflated the variance of the coefficient in comparison to what can be obtained when the variables were uncorrelated with any other variable in the model. Thus, VIF greater than 10 indicates a statistically significant multicollinearity. In such situation, the ordinary least square estimator does not possess the optimum statistical property hence, the need for alternative estimator that can address the situation.

Generalized Lambda Distribution (GLD)

The four-parameter (quantile) GLD family is known for its high flexibility of producing distributions with a range of different shapes. According to [18] and [19] it was showed that the six regions in which the shape parameters can lie are similar to the shapes of the GLD. [20], [21] and [22] introduced the notation for generalized lambda distribution. [23] placed restriction only restriction on $\lambda_4 > 0$. Thus, [24] emphasized that the ultimate drive for the development of FMKL GLD is that the distribution is defined over all λ_3 and λ_4. The quantile probability density function of the GLD which is known as RS GLD is given as the inverse distribution function of Tukey's lambda distribution (TLD) as expressed in (8)

$$F^{-1}(P|\lambda) = F^{-1}(P|\lambda_i),\ \forall\, i = 1, 2, ..4$$

$$F^{-1}(P|\lambda_i) = \lambda_1 + \frac{P^{\lambda_3} - (1-P)^{\lambda_4}}{\lambda_2} \tag{8}$$

where P are the probabilities, $P \in \{0, 1\}$, λ_1 and λ_2 are the location and scale parameters while λ_3 and λ_4 are the shape parameters which define the strengths of the lower and upper tail respectively. According to [ ] Chalabi et al. (2010), it can be stated that the original one-parameter TLD results in the limiting case $\lambda_1 = 0$ and $\lambda_2 = \lambda_3 = \lambda_4 = \lambda$.

The FMKL-GLD Quantile Regression Model

This model is divided into two parts and is adopted to explore and predict the economic growth ($RGDP$) in Nigeria in relation to the macroeconomic variables such as internal debt ($INDT$), external debt ($EXDT$), interest rate ($RINR$), exchange rate ($REXR$), and trade openness ($OPEN$). The first part deals with the FMKL-GLD regression reference line generation and the second part concentrates on finding the quantile regression coefficients based on the reference line that will be obtained from the FMKL GLD as discuss below. [23] introduced this

parameterization to improve the [18] parameterization given in (1). Thus, the quantile function for the distribution of FMKL-GLD is defined as stated in (9):

$$Q(P) = \lambda_1 + \frac{\frac{P^{\lambda_3-1}}{\lambda_3} - \frac{(1-P)^{\lambda_4-1}}{\lambda_4}}{\lambda_2} \tag{9}$$

λ_1 and λ_2 are the location and scale parameters whereas λ_3 and λ_4 are the shape parameters. This distribution is most favourable because it is valid for all values of λ_3 and λ_4 where $\lambda_2 > 2$. If $\lambda_3 = \lambda_4 = 0$ the FMKL parameterisation has the following quantile function given in (10):

$$F^{-1}(P) = \lambda_1 + \frac{\ln(p) - \ln(1-p)}{\lambda_2} \tag{10}$$

The FMKL-GLD takes different quantile forms if either $\lambda_3,\ \lambda_4$ or both are equal to zero. These forms can be obtained as follows in (11), (12), (13) and (14):

if $\lambda_3 = 0,\ \lambda_4 \neq\ 0$

$$Q(P) = \lambda_1 + \frac{1}{\lambda_2}\left(\ln(p) - \frac{(1-P)^{\lambda_4-1}}{\lambda_4}\right),\ 0 \leq p \leq 1. \tag{11}$$

If $\lambda_3 \neq 0$ and $\lambda_4 = 0$,

$$Q(P) = \lambda_1 + \frac{1}{\lambda_2}\left(\frac{(P)^{\lambda_3-1}}{\lambda_3} - \ln(1-p)\right),\ 0 \leq p \leq 1 \tag{12}$$

If $\lambda_3 = 0$ and $\lambda_4 = 0$,

$$Q(P) = \lambda_1 + \frac{1}{\lambda_2}\left(\ln(p) - \ln(1-p)\right),\ 0 \leq p \leq 1 \tag{13}$$

If $\lambda_3 \neq 0$ and $\lambda_4 \neq 0$,

$$Q(P) = \frac{\lambda_2}{P^{\lambda_3-1} + (1-P)^{\lambda_4-1}},\ 0 \leq p \leq 1 \tag{14}$$

The probability density function of the FMKL-GLD can be obtained by using the relationship in (15), where $F(y) = p$ and $y = Q(P)$.

$$y = F^{-1}(y) = Q(P) \tag{15}$$

By differentiating $Q(P)$ with respect to y the density function of FMKL-GLD can be obtained as given in (16):

$$\frac{\mathrm{d}p}{\mathrm{d}y} = f(y) \text{ and } \mathrm{d}y = \mathrm{d}(Q(p)) \tag{16}$$

These two relationships give us $f(y) = \frac{\mathrm{d}p}{\mathrm{d}(Q(p))}$. From (2), the expression in (17) can be obtained

$$\frac{\mathrm{d}(Q(p))}{\mathrm{d}p} = \frac{\lambda_3 P^{\lambda_3-1} - \lambda_4(1-P)^{\lambda_4-1}}{\lambda_2} \tag{17}$$

substituting the result in (17) into $f(y) = \frac{\mathrm{d}p}{\mathrm{d}(Q(p))}$ the density function given in (18) can be obtained

$$f(y) = \frac{\lambda_2}{\lambda_3 P^{\lambda_3-1} - \lambda_4(1-P)^{\lambda_4-1}} \tag{18}$$

where $0 \leq p \leq 1$. Thus, the probability density function of the FMKL-GLD when $\lambda_3 \neq 0$ and $\lambda_4 \neq 0$, is as follows in (19), (20), (21), (22):

$$f(y) = \frac{\lambda_2}{\lambda_3 P^{\lambda_3-1} - (1-P)^{\lambda_4-1}},\ 0 \leq p \leq 1 \tag{19}$$

The probability functions of the other forms of the FMKL-GLD can be in a similar fashion as: if $\lambda_3 = 0$ and $\lambda_4 \neq 0$

$$f(y) = \frac{\lambda_2}{P^{\lambda_3 - 1} - \lambda_4 (1-P)^{\lambda_4 - 1}},\ 0 \leq p \leq 1 \tag{20}$$

The probability functions of the other forms of the FMKL-GLD can be in a similar fashion as: if $\lambda_3 = 0$ and $\lambda_4 = 0$

$$f(y) = \frac{\lambda_2}{P^{\lambda_3 - 1} - (1-P)^{\lambda_4 - 1}},\ 0 \leq p \leq 1 \tag{21}$$

if $\lambda_3 \neq 0$ and $\lambda_4 \neq 0$

$$f(y) = \frac{\lambda_2}{\lambda_3 P^{\lambda_3 - 1} - \lambda_4 (1-P)^{\lambda_4 - 1}},\ 0 \leq p \leq 1 \tag{22}$$

The Maximum Likelihood Estimation

To use this method, the quantiles $Q(p_i)$ for every observation y_i, for $i = 1, 2, 3, ...n$ observations under a set of initial values. This involves solving expression in (9) numerically. This can be done by using the Newton-Raphson method through GLD. Having obtained the $Q(p_i)$, the substitution in the log-likelihood equation in (23) can be stated and transformed as given in (24)

$$ML = \prod_{i=1}^{n} \ln \left(\frac{\lambda_2}{\lambda_3 P^{\lambda_3 - 1} - \lambda_4 (1-P)^{\lambda_4 - 1}} \right) \tag{23}$$

$$ML = n \ln \lambda_2 - \prod_{i=1}^{n} \ln (\lambda_3 P^{\lambda_3 - 1} - \lambda_4 (1-P)^{\lambda_4 - 1}) \tag{24}$$

This idea is valid as its maximize the likelihood in (10) and (11) using the Nelder-Mead also known as simplex search algorithm.

The Empirical Likelihood Goodness of Fit Test

The hypothesis to test the goodness of fit for the GLD distribution is given in (25) and (26) as follow:

$$H_0 : f = f_0 \sim \text{ GLD}\,(\lambda_1, \lambda_2, \lambda_3, \lambda_4) \tag{0.1}$$

$$H_1 : f = f_1 \sim \text{ GLD}\,(\lambda_1, \lambda_2, \lambda_3, \lambda_4) \tag{0.2}$$

The definition of the likelihood ratio test statistic for this hypothesis is given in (27) and (28) as:

$$LR = \frac{\prod_{i=1}^{n} f H_1(y_i)}{\prod_{i=1}^{n} f H_0(y_i)} \tag{27}$$

$$LR = \frac{\prod_{i=1}^{n} f H_1(y_i)}{\prod_{i=1}^{n} f(y_i|\lambda)} \tag{28}$$

where $y_1, y_2, y_3, \ldots, y_n$ follows a GLD distribution with the parameter $\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ under the null hypothesis. Since f_0 and f_1 are unknown, the maximum likelihood method estimates λ of a GLD under the null hypothesis. Thus, it was observed that the maximum empirical likelihood method to estimate the numerator can be written as in (29):

$$L_f = \prod_{i=1}^{n} f H_1(y_i) = \prod_{i=1}^{n} f_i \tag{29}$$

where $y_{(1)} \leq y_{(2)} \leq y_{(3)} \leq \ldots \leq y_{(n)}$ are the order statistics of the observations $y_1, y_2, y_3, \ldots, y_n$. The values of f_i to maximize L_f were obtained by using the constraint $f(s)\mathrm{d}s = 1$ corresponding to the alternative hypothesis.

The value obtained is as follows in (30):

$$f_j = \frac{2m}{n(Y_{j+m} - Y_{j-m})} \tag{30}$$

where $Y_j = Y_1$, if $j \leq 1$ and $Y_j = Y_n$ if $j \geq n$

The likelihood ratio statistic based on the maximum likelihood empirical method is given in (31):

$$\mathrm{GLD}_{mn} = \frac{\prod_{j=1}^{n} \frac{2m}{n(Y_{j+m} - Y_{j=m})}}{\max \prod_{j=1}^{n} f(y_j|\lambda)} \tag{31}$$

Where $\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ and $0 < \lambda < 1$

Thus, in the next session, data exploration, descriptive analysis, test for outliers, multicollinearity, and granger causality as well as fitting FMKL-GLD quantile regression models and its associated diagnostics to determine the most efficient FMKL-GLD quantile regression models for estimating parameters economic growth in Nigeria based on the identified macroeconomic variables in this study.

4. Data Exploration

The characteristics of the economic growth (RGDP) and drivers such as internal debt (INDT), external debt (EXDT), interest rate (RINR), exchange rate (REXR) and trade openness (OPEN) were described in this section using trend so as to explore the existing relationship among the variables mentioned. Thus, it was revealed that INDT, EXDT, RINR, REXR, and OPEN had either inverse or direct relationship with the RGDP in Nigeria and as such the need to examine the descriptive and diagnostic properties of the model such as outliers and multicollinearity among the variables. In view of this, a linear model is fitted and the presence of outliers and multicollinearity as assumptions violation among macroeconomic variables is examined so as to ensure efficient estimate of the parameters.

Thus, in Figure 1, the plot that showed the relation between the macroeconomic variables under consideration was displayed. This was done to show the direction of relationship between the economic growth (RGDP) in Nigeria and the drivers mentioned in this study.

In Figure 1, the scattered plot that showed various trends in mean and variance of the independent variables such as INDT, EXDT, RINR, REXR and OPEN and how they were related to the RGDP during the period under study. Thus, in this study, a linear relationship was found between the identified drivers and economic growth in Nigeria. To further establish the linear relationship of the economic growth and the aforementioned drivers, we carried out correlation analysis, variance inflation factor and Grubb's test for the presence of multicollinearity and outliers among the macroeconomic variables which is presented in Table 2 and Table 3. Also, in Table 1 the test to examine the nature of distribution of the macroeconomic variables were considered.

5. Result and Discussion

In Table 1, the descriptive statistic such as mean, minimum, maximum, standard deviation, variance, skewness, kurtosis and others were presented for the macroeconomic variables under consideration in this study. Table 1 showed the descriptive analysis of results of the economic variables such as RGDP, INDT, EXDT, RINR, REXR, and OPEN under investigation in this study. The average values of RGDP during the period under study stood at 10.3046 and it ranged from 9.6315 to 11.1422. The mean value of INDT and EXDT were 6.6288 and 6.6572 which were ranged between 3.3478 to 9.0867 and 3.7245 to 8.4950 respectively. While, the average values of RINR, REXH and OPEN were 3.1051, 4.3175 and 0.16645 respectively. It was observed that RINR, REXH and

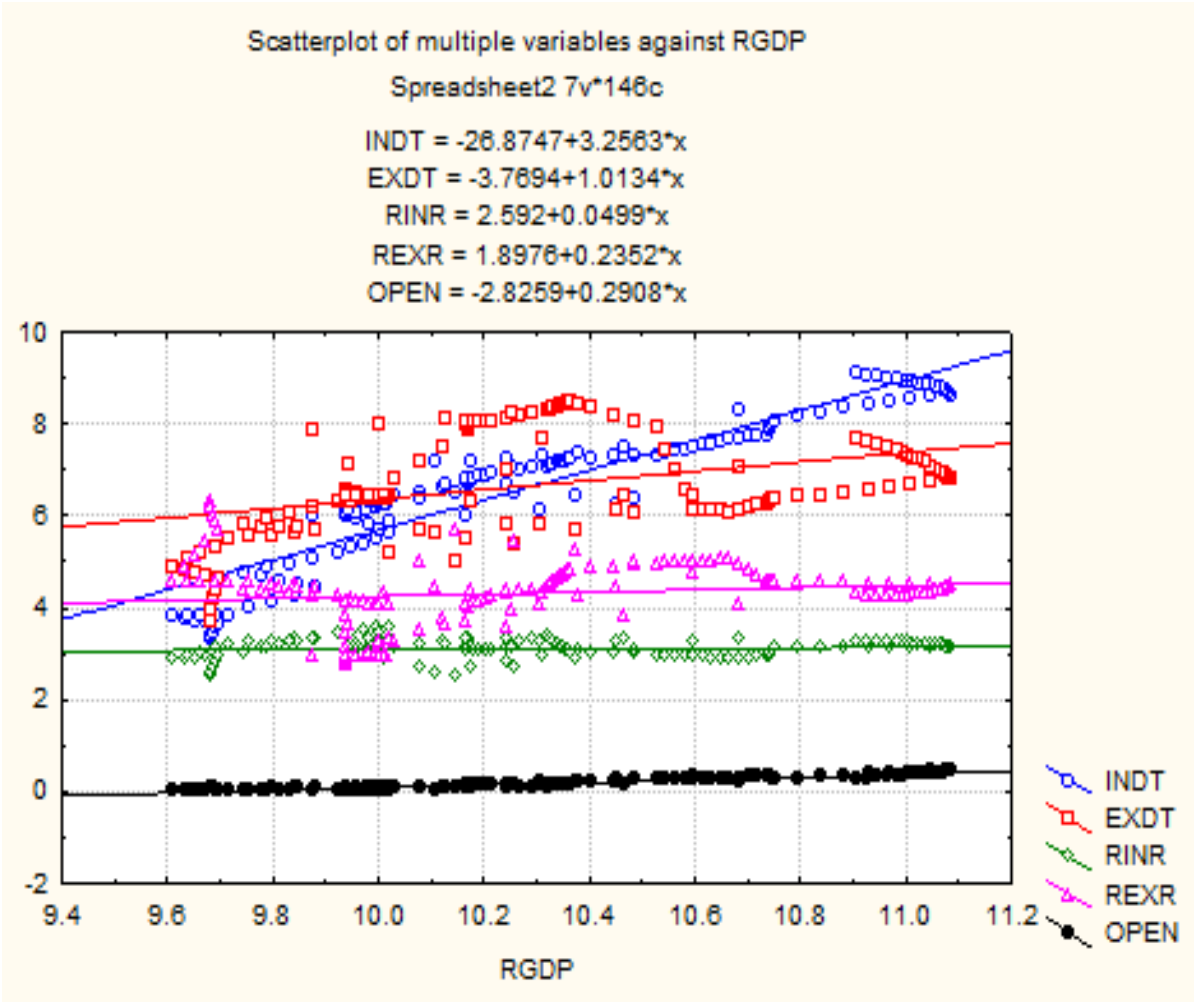

Figure 1. Trend of independent variables in relation to RGDP

Table 1. Descriptive Analysis

	RGDP	INDT	EXDT	RINR	REXR	OPEN
Mean	10.3046	6.6288	6.6573	3.1051	4.3175	0.1665
Median	10.2054	6.7393	6.4629	3.1139	4.3803	0.1200
Maximum	11.1422	9.0867	8.4950	3.5860	6.3197	0.4600
Minimum	9.6316	3.3478	3.7246	2.4849	2.7763	0.0100
Std. Dev.	0.4503	1.4988	1.0655	0.1929	0.6951	0.1302
Skewness	0.3710	-0.4095	-0.1634	-0.5734	-0.1748	0.6710
Kurtosis	1.8572	2.5576	2.7076	4.1714	3.6271	2.1246
Jarque-Bera (P-value)	10.9083 (0.0043)	5.090 (0.0785)	1.1299 (0.5684)	15.7879 (0.0004)	3.0287 (0.2120)	15.0811 (0.0005)
Sum	1452.950	934.6548	938.6716	437.8212	608.7708	23.4700
Sum Sq. Dev.	28.3891	314.4906	158.9331	5.2110	67.6489	2.3744
Observations	141	141	141	141	141	141

OPEN were ranged from 2.4849 to 3.5860, 2.7763 to 6.3197 and 0.01 to 0.46 in that order in the study period. The values 0.4503, 1.4988, 1.0655, 0.1929, 0.6951 and 0.1302 revealed the rate at which RGDP, INDT, EXDT, RINR, REXR and OPEN deviated from their respective mean values.

The skewness and kurtosis as shown in the result gave explanation about the distribution and shape of the economic variables under investigation. The skewness result showed that the RGDP (0.3710) and OPEN (0.6709) were positively skewed that is, skewed to the right of the mean and it was also discovered that INDT (-0.4095), EXDT (-0.1634), RINR (-0.5734) and REXR (-0.1748) were negatively skewed that is skewed to the left of the mean. The kurtosis results revealed that all the macroeconomic variables under consideration were platykurtic with the kurtosis coefficient index less than 3 except RINR and REXR which were mesokurtic thus, emphasized the flattering beyond the level of normal distribution.

Table 2. Correlation Matrix

	INDT	EXDT	RINR	REXR	OPEN
INDT	1.0000	0.6073	0.1276	-0.0924	0.8201
EXDT	0.6073	1.0000	0.3876	-0.2898	0.2534
RINR	0.1276	0.3876	1.0000	-0.4594	-0.0090
REXR	-0.0924	-0.2898	-0.4594	1.0000	0.2416
OPEN	0.8201	0.2534	-0.0090	0.2416	1.0000

The correlation coefficients presented in Table 2 showed the extent of relationship that exist among the explanatory variables under consideration such as INDT, EXDT, RINR, REXR and OPEN. From the Table 2, it was discovered that INDT was positively correlated with EXDT, RINR and OPEN with correlation coefficient of 0.61, 0.13 and 0.82 respectively. The study also revealed a positive correlation between the EXDT and RINR, EXDT and OPEN, REXR and OPEN with correlation coefficient of 0.39, 0.25 and 0.24 respectively. Thus, the high or strong correlation between the INDT and OPEN revealed the need to test for the presence of multicollinearity problem.

Therefore, in order to check for the outliers and multicollinearity among the macroeconomic variables INDT, EXDT, RINR, REXR, and OPEN used as the drivers of economic growth (RGDP), we carried out Grubb's test which is an International Statistical Organisation (ISO) recommended test for large data in detecting outliers and variance inflation factor (VIF) to detect multicollinearity among the identified variables used in this study and in Table 3, the results were presented.
The result presented in Table 3 showed the Grubb's test carried out with the null hypothesis (H_0) stated as "there

Table 3. Test for Outliers and Multicollinearity using Grubb's Test and VIF

Variable	Grubb's Value	G (Critical value)	VIF
INDT	2.1890	3.4970	14.2657
EXDT	2.7520	3.4970	3.5290
RINR	3.2150	3.4970	1.4369
REXR	2.8800	3.4970	1.7571
OPEN	2.2540	3.4970	9.5644

is outliers in the data set under consideration". The rule of thumb for this test is that if the calculated Grubb's value is less than the critical value, we do not reject H_0. Therefore, from the result presented in Table 3, it was discovered that the Grubb's value of 1.860, 2.189, 2.752, 3.215, 2.880 and 2.254 were less than the Grubb's critical value of 3.497 for RGDP, INDT, EXDT, RINR, REXR, OPEN respectively. Thus, it can be affirmed from the result that outliers were in the macroeconomic data set under investigation. Also, in Table 3, the result of variance inflation

factor (VIF) for examining the presence of multicollinearity was presented and the VIF for the variables revealed that INDT, EXDT, RINR, REXR and OPEN were 14.2657, 3.5290, 1.4369, 1.7571, and 9.5644 respectively. Thus, according to Allison (1999) and Freund and Littell (2000) as cited in Khalaf and Iguernane (2016), the result of VIF 14.2657 $>$ 10.00 indicated the presence and statistical significance of multicollinearity caused by INDT. Based on this evidence that affirmed the presence of outlier and multicollinearity as an assumption violation for linear model, a robust principal component analysis was adopted to address the problems in order to obtain an estimation of the parameters of linear model that can efficiently predict the economic growth.

FMKL-GLD Quantile Regression Results

The results are displayed in the form of tables showing parameter estimates for FMKL-GLD, GLD Q-Q plots for the FMKL-GLD quantile models, and FMKL-GLD quantile model estimated parameter and diagnostics properties such as goodness of fit, best predictors model and their quantile plots for the quarterly data on identified macroeconomic variables for investigating and predicting economic growth in Nigeria.

Table 4. Estimated GLD Parameters for the Variables Return to Fit FKML-GLD Model

Returns	λ_1	λ_2	λ_3	λ_4	p-value of the AD statistic
RGDP	10.2499	1.2035	1.3437	0.9304	0.9999
INDT	6.8491	0.4215	0.6635	1.0603	0.9995
EXDT	6.9516	0.8196	0.3336	0.7856	0.9997
RINR	3.1222	9.7941	-0.0920	0.0713	0.9954
REXR	4.3947	4.4019	-0.4068	-0.2142	0.9999
OPEN	5.0623	5.0622	2.0101	0.5312	0.9999

Table 4 showed the estimates for the generalized lambda distribution (GLD) $\lambda_1, \lambda_2, \lambda_3$ and λ_4 to the fit the FMKL-GLD quantile model for investigating RGDP in relation with INDT, EXDT, RINR, REXR and OPEN as an economic growth drivers in Nigeria. From the result, λ_1 and λ_2 represent the location and scale parameter while λ_3, and λ_4 represent the space parameters generated from GLD fit of the variables under investigation. The *p*-value of the AD statistic > 0.05 showed the normality of the macroeconomic variables or that the macroeconomic variables under consideration were from normally distributed population. Thus, in Table 5 and Figure 2, FMKL-GLD quantile regression result and FMKL-GLD Q-Q plots for the FMKL-GLD quantile regression model

Table 5. FMKL-GLD quantile Regression Results

Model	Estimate	p-value
Constant	8.1492	0.0000
INDT	0.1514	0.0000
EXDT	-0.0247	0.0000
RINR	0.2449	0.0000
REXR	0.0608	0.0000
OPEN	1.7597	0.0192
$\widehat{\lambda_1}$	-0.0253	
$\widehat{\lambda_2}$	34.1488	
$\widehat{\lambda_3}$	-0.3303	
$\widehat{\lambda_4}$	-0.5758	
KS test *p*-value	0.7949	
data driven smooth test *p*-value	0.6850	
Resample KS test$>$ 0.05	91.4000	

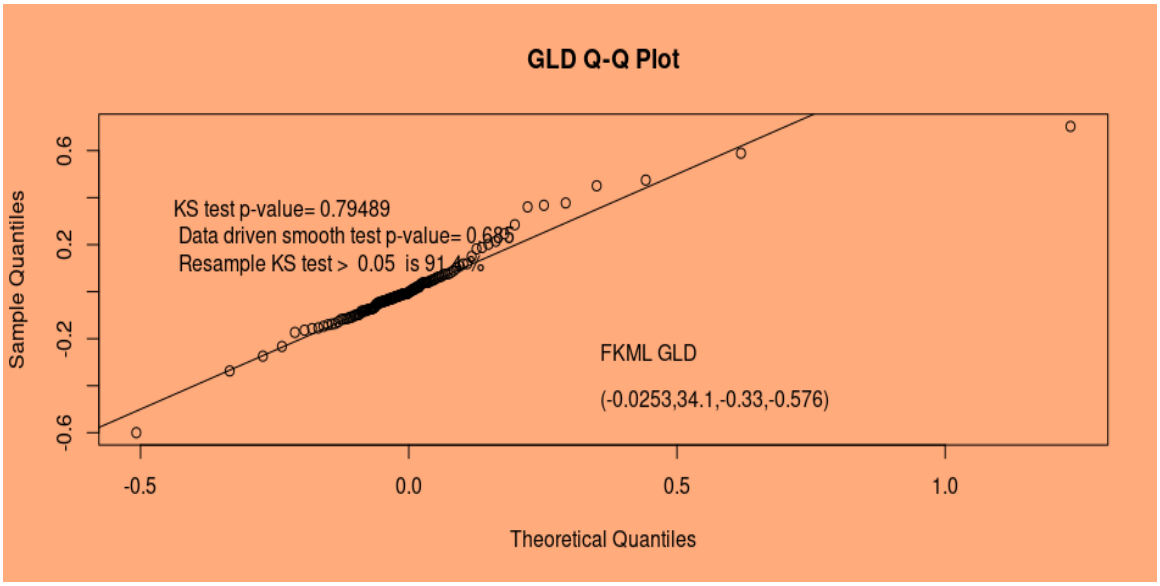

Figure 2. QQ plot of the FMKL-FGLD Quantile Model

Table 5 showed the estimated intercepts and coefficients of the independent variables for fitted FMKL-GLD model and the lambda parameters $\widehat{\lambda}_1$, $\widehat{\lambda}_2$, $\widehat{\lambda}_3$, and $\widehat{\lambda}_4$ for the fitted FMKL-GLD model. Specifically, in the Table 5, it was revealed that in model that INDT, RINR, REXR and OPEN contributed positively to the economic growth to the turn of 0.15%, 0.24%, 0.06% and 1.76% while, it was discovered that EXDT contributed negatively to the economic growth in Nigeria thus, reduced economic growth by 0.02% and as such showed the tendency and potential macroeconomic variables under study as a veritable determinant of economic growth in Nigeria. The location, scale and space parameters $\widehat{\lambda}_1$, $\widehat{\lambda}_2$, $\widehat{\lambda}_3$, and $\widehat{\lambda}_4$ for the fitted FMKL-GLD model were -0.0253, 34.1488 -0.3303, and -0.5758. Also, in Table 5 the Kolmogorov-Smirnov goodness of fit tests for the FMKL-GLD quantile regression model presented. The result showed that the *p*-values of the K-S test, data driven smooth test and Resample KS test. All the *p*-values are above the 5% level of significance which indicated that the FMKL-GLD fits the data moderately well. The GLD Q-Q plot as shown in Figure 2 evidently revealed this since most of the data points were in line with the data distribution line. Therefore, it can be emphasized that the FMKL-GLD regression model performed well in estimating the parameters of the economic growth in Nigeria under the violation of linear model assumptions considered in this study. Thus, based on the scale parameter, it can be emphasized that FMKL-GLD quantile model was efficient model that better described the economic situation in Nigeria. However, in Table 6, we the FKML-GLD maximum likelihood estimation method that helped us to get the parameter values for the 25%, 50% and 75% of the model were presented. Source: Researcher's Computation, 2024

Table 6 showed the result of FMKL-GLD quantile estimated parameters for 25%, 50% and 75% of the model were fitted. Thus, from the result presented in Table 6, the coefficients of the FMKL-GLD quantile regression for model revealed the negative and significant influence of EXDT on RGDP, the negative and insignificant influence of OPEN on RGDP for 75% FMKL-GLD model. It was observed that INDT, RINR, REXR, and OPEN were positive and significantly influenced RGDP for the 25%, 50% and 75% FMKL-GLD quantile regression This shows the importance of macroeconomic variables under consideration to the development and the growth of economy in Nigeria. Having obtained the contributions of the economic growth drivers identified in this study to the economic growth by the various fitted FMKL-GLD quantile model, it is imperative to examine the goodness of fit of the fitted FMKL-GLD quantile model as presented in Table 7.

Table 6. ML parameter estimates of the FKML-GLD Quantile Regression model at different quantile levels

Model	parameter	Estimate	p-value	lower 95 Interval	Upper 95 interver
	C	8.1465	0.0000	7.7753	8.5177
	INDT	0.2035	0.0005	0.0912	0.3158
0.25	EXDT	-0.0679	0.0433	-0.1337	-0.0021
	RINR	0.2294	0.0005	0.1016	0.3573
	REXR	0.0599	0.0043	0.0191	0.1006
	OPEN	1.2848	0.0082	0.3377	2.2318
	C	7.8604	0.0000	7.2521	8.4686
	INDT	0.1794	0.0000	0.1017	0.2571
0.50	EXDT	-0.0392	0.1380	-0.0911	0.0127
	RINR	0.2941	0.0014	0.1161	0.4722
	REXR	0.0799	0.0003	0.0369	0.1230
	OPEN	1.4510	0.0007	0.6201	2.2818
	C	6.7134	0.0000	5.7058	7.7210
	INDT	0.3180	0.0000	0.2379	0.3981
0.75	EXDT	-0.0922	0.0007	-0.1446	-0.0399
	RINR	0.4901	0.0002	0.2337	0.7465
	REXR	0.1628	0.0000	0.0961	0.2296
	OPEN	-0.2183	0.6320	-1.1176	0.6810

Table 7. Goodness of Fit for the Fitted FMKL-GLD Quantile Model

Model	FMKL-GLD 25Q	FMKL-GLD 50Q	FMKL-GLD 75Q
Adj. R-Square	0.7313	0.7659	0.7539
Quasi-LR Stat	582.5285	751.7937	581.2553
Prob(Quasi-LR stat)	0.0000	0.0000	0.0000
RMSE	0.1581	0.1445	0.1701
MAE	0.1034	0.0875	0.1212
MAPE	0.9911	0.8417	‘0‘999
Bias P	0.2319	0.0118	0.2871

In Table 7, the goodness of fit of the FMKL-GLD quantile model was presented. This was determined by root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE) and bias proportion (Bias P). Thus, for the model, the result the RMSE for the fitted FMKL-GLD 25Q, FMKL-GLD 50Q and FMKL-GLD 75Q quantile model were 0.1581, 0.1445 and 0.1701 respectively. The MAE for the fitted FMKL-GLD 25Q, FMKL-GLD 50Q, and FMKL-GLD 75Q quantile model were 0.1034, 0.0875 and 0.1212 respectively. The same results were also obtained for other measure of goodness of fit. The adjusted R-square of the fitted FMKL-GLD 25Q, FMKL-GLD 50Q and FMKL-GLD 75Q quantile model were further revealed that 73.13%, 76.59% and 75.39% variations in RGDP and be explained by the macroeconomic growth drivers under consideration. Thus, it can be emphasized that FMKL-GLD 50Q regression model was the most efficient FMKL-GLD quantile model for examining and predicting the economic growth and trade openness in Nigeria based on the smallest RMSE and MAE of the FMKL-GLD quantile model.

6. Conclusion

In this study, an estimation of economic growth's parameters in Nigeria in relation to the identified drivers of economic growth such as INDT, EXDT, RINR, REXR, and OPEN in the presence of multicollinearity and

outliers as assumptions violation were thoroughly examined. An exploratory and diagnostics analysis established a relationship between the economic growth (RGDP) and the aforementioned economy drivers. The presence of multicollinearity and outlier were established via VIF and Grubb's test carried out on the macroeconomic data set under consideration. Consequently, to simultaneously address these problems and obtain efficient parameter estimate a granger causality test was carried out in order to fit appropriate FMKL-GLD quantile model.

From the result, it was revealed that in model that INDT, RINR, REXR and OPEN contributed positively to the economic growth to the turn of 0.15%, 0.24%, 0.06% and 1.76% while, it was discovered that EXDT contributed negatively to the economic growth in Nigeria thus, reduced economic growth by 0.02% and as such showed the tendency and potential macroeconomic variables under study as a veritable determinant of economic growth. The location, scale and space parameters $\widehat{\lambda}_1$, $\widehat{\lambda}_2$, $\widehat{\lambda}_3$, and $\widehat{\lambda}_4$ for the fitted FMKL-GLD model were -0.0253, 34.1488 -0.3303, and -0.5758. It was observed that INDT, RINR, REXR, and OPEN were positive and significantly influenced RGDP for the 25%, 50% and 75% FMKL-GLD quantile regression. The goodness of fit of the FMKL-GLD quantile models were done using RMSE, MAE, MAPE and bias proportion. Thus, for models, the result the RMSE and MAE for the fitted FMKL-GLD 25Q, FMKL-GLD 50Q and FMKL-GLD 75Q quantile model presented.

Hence, it can be emphasized that FMKL-GLD 50Q regression model was the most efficient FMKL-GLD quantile model for examining and predicting the economic growth and trade openness in Nigeria based on the smallest RMSE and MAE of the FMKL-GLD quantile model. FMKL-GLD quantile regression technique was produced efficient and optimal parameter estimate when multicollinearity and outliers were jointly present in data set under investigation were fitted. Therefore, it can be concluded based on the location, scale and space parameters from FMKL-FGLD and GLD Q,Q plots as well as the estimated parameter of RGDP, it can be emphasized that FMKL-FGLD was the most efficient model that better described the economic situation in Nigeria that the economy is growing in inverse direction and the need for drastic effort and strong willed to curtail the situation. To achieve this, the adoption of economic openness to the development and the growth of economy in Nigeria is a veritable policy direction that must be strictly followed.

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REFERENCES

[1] T. Agbanobi, Looking inwards: Will Nigeria tread the path of economic recovery and growth in 2017. *Economic and Policy Review*, vol.16 no. 1, pp. 3-11, 2017.

[2] A. Yusuf and S. Mohd, The impact of government debt on economic growth in Nigeria, *Cogent Economics & Finance*, vol.9 no.1, 2021. 1946249, DOI: 10.1080/23322039.2021.1946249

[3] B. Z. Lawali, P. E. Machief and T. K. Aliyu, Exchange rate and economic growth nexus: An impact analysis of the Nigerian economy, *Journal of Research in Emerging Markets*, vol. 2 no. 4, pp. 58-67, 2020.

[4] E. Sunday and B. M. Ahmed, B. M. Dynamic Impact of Trade Openness on the Economic Growth in Nigeria. *International Journal of Engineering and Advanced Technology*, vol. 8 no. 5C, pp. 606-616, 2019.

[5] J. B. Obayori, W. S. Krokeyi and S. Kakain, External debt and economic growth in Nigeria. *International Journal of Science and Management Studies*, vol. 2 no. 2, pp. 1-6, 2019.

[6] K. A. Moh'd AL-Tamimi and M. S. Jaradat, Impact of external debt on economic growth in Jordan for the period (2010-2017). *International Journal of Economics and Finance*, vol. 11 no. 4, pp. 114-118, 2019.

[7] A. C. Odubuasi, P. U. Uzoka and A. S. Anichebe, External debt and economic growth in Nigeria. *Journal of Accounting and Financial Management*, vol. 4 no. 6, pp. 98-108, 2018.

[8] S. S. Ademola, A. O. Tajudeen and Z. A. Adewumi, External debt and economic growth of Nigeria: An empirical investigation. *South Asian Journal of Social Studies and Economics*, vol. 1 no. 2, pp. 1-11, 2018.

[9] S. Al Kharusi and M. S. Ada, External debt and economic growth: The case of emerging economy. *Journal of Economic Integration*, vol. 33 no. 1, pp. 1141-1157, 2018.

[10] A. C. Odubuasi, P. U. Uzoka and A. S. Anichebe, External debt and economic growth in Nigeria. *Journal of Accounting and Financial Management*, vol. 4 no. 6, pp. 98-108, 2018.

[11] V. U. Ijirshar, Impact of trade openness on economic growth among ECOWAS Countries: 1975-2017, *CBN Journal of Applied Statistics*, vol. 10 no. 1, pp. 75-96, 2019.

[12] K. Ezeuchenne, International Trade and Economic Growth in Nigeria. *Journal of Humanities and Social Science*, vol. 22 no. 6, pp. 33-43, 2017.

[13] C. Da'Silva, Impact of Trade Openness on the Output Growth in Nigeria. *British Journal of Economics, Management and Trade*, vol. 4 no. 5, pp. 755-768, 2014.

[14] K. F. Oguntegbe and V. Alexander, Enhancing Economic Competitiveness in Nigeria: The Roles of Exchange Rate and Interest Rate, *Global Scientific Journal*, vol. 7 no. 6, pp. 526-536, 2019.

[15] L. O. Ufoeze, C. N. Okuma, C. Nwakoby and U. B. Alajekwu, Effect of Foreign exchange rate fluctuations on Nigerian economy, *Annals of Spiru Haret University Economic Series*, no. 1, pp. 105-122, 2018.

[16] S. A. Su, Discretized Approach to Flexibly Fit Generalized Lambda Distributions to Data. *Journal of Modern Applied Statistical Methods*, vol. 4 no. 2, 2005 Retrieved from http://digitalcommons.wayne.edu/jmasm/vol4/iss2/7

[17] A. Ebiwonjumi, R. Chifurira and K. Chinhamu, An Efficient Estimation Technique for investigating Economic Growth and its Determinants for Nigeria in the Presence of Multicollinearity, *International Journal of Finance & Banking Studies*, Vol. 11, no. 1, pp., 107-119, 2022.

[18] J. S. Ramberg and B. W. Schmeister, An approximate method for generating asymmetric random variables. *Communication of the ACM*, vol. 17, pp. 78-82, 1974.

[19] R. A. R. King and H. L. MacGillivray, A Starship Estimation Method for the Generalized Distributions. *Australian & New Zealand Journal of Statistics*, vol. 41 np. 3, pp. 353-374, 1999.

[20] J. S. Ramberg, E. J. Dudewicz, P. R. Tadikamalla and E. F. Mykytka, A probability distribution and its uses in fitting data. *Technometrics*, vol. 21 no. 2, pp. 201-214, 1979.

[21] Y. Chalabi, D. J. Scott and D. Wuertz, Flexible distribution modeling with the generalized lambda distribution (2012) Retrieved from https://mpra.ub.uni-muenchen.de/43333/

[22] Y. Chalabi, D. J. Scott and D. Würtz, *The generalized lambda distribution as an alternative to model financial returns*. Working paper, Eidgenössische Technische Hochschule and University of Auckland, Zurich and Auckland. (2010). Retrieved from https://www.rmetrics.org/sites/default/files/glambda_0.pdf

[23] M. Freimer, G. Kollia, G. S. Mudholkar, and C. T. Lin, A study of the generalized Tukey lambda family. *Communications in Statistics-Theory and Methods*, vol. 17 no. 10, pp. 3547-3567, 1988.

[24] S. Su, Fitting single and mixture of generalized lambda distributions to data via discretized and maximum likelihood methods: GLDEX in R. *Journal of Statistical Software*, vol. 21 no. 9, pp. 1-17, 2007a.