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Self-Scheduling of a Generation Company with Carbon Emission Trading

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Abstract A carbon emission trading self-scheduling (CETSS) model was proposed. The proposed model considered not only carbon emission allowance constraints but also carbon emission trading. A new method was presented for solving CETSS problems based on piece-wise linearisation and second-order cone linearisation. The effectiveness and validity of the proposed model and method were illustrated by 10-100 unit systems over 24 hours.

Keywords Self-scheduling, Generation Company, Carbon Emission Trading, Piece-wise Linearisation, Second-order Cone Linearisation

AMS 2010 subject classifications 90C06, 90C11, 90C90

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1. Introduction

Climate change has threatened the survival and sustainable development of human beings, and has become one of the major challenges facing the world in the 21st century. In order to cope with climate change at a low cost, carbon market emerged as a market mechanism for emission reduction, which has been applied by more and more countries and regions in their own energy saving and emission reduction practices. The European Union has created the world's first carbon market: the European Union Emissions Trading System. China launched a pilot carbon market in 2010, and established carbon markets in Shenzhen, Shanghai, Fujian and other places from 2013 to 2016, in 2017, the construction of the national carbon market was officially launched, and in 2021, it began to operate formally with the power industry as a breakthrough. The charges and trades for the emissions right trades of carbon dioxide will lead to new challenges and reform of power suppliers owing to the power industry being the largest emitter of greenhouse gases.

Based on power market mechanisms, to maximise profit, generation companies (GENCO) have to forecast loads on the power system and marginal prices during different periods. They need to optimise the scheduling of generation by considering constraints on generator operations and other factors. The mathematical model of self-scheduling (SS) is complex, large-scale, non-linear mix integer programming problem. Since their proposal, many scholars have conducted research thereon and proposed numerous solution methods, such as the binary fireworks algorithm [1], imperialist competitive algorithm [2], Lagrange relaxation-differential evolution algorithm [3], binary successive approach and civilized swarm optimization [4], and memetic binary differential evolution algorithm [5]. In recent years, mix integer programming has been an essential method for solving SS problems with the improvement of computer performance and the development of software packages for solving large-scale optimization problems. For instance, the SS problem was transformed into a non-linear mix integer programming task to be solved using commercial software, including GAMS and CPLEX [6], [7], [8]. In previous research [9]

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the authors established a mix integer linear programming (MILP) model to solve the SS problem through piece-wise linearisation of the quadratic cost function, and achieved beneficial results. An MILP model is used in [10] for self-scheduling of thermal units when focusing on the modelling of hot, warm, and cold start-up types.

Currently, SS is endowed with new connotations due to the introduction of constraints and trading in carbon emission rights. For example, by using the constraint on carbon emission rights, other methods [11] established SS-based hybrid, non-linear, integer programming models. Elsewhere [12] a multi-objective model for solving SS problems based on maximum profit for power suppliers and minimum carbon emissions were developed. However, other work [11], [12] merely considered the limits to carbon emissions in their models and did not focus on traded carbon emission rights.

This research investigated current features of the trading market in carbon emission rights. On this basis, it proposed a CETSS model by considering traded carbon emission rights, according to the management and trade modes for total carbon emissions. The model not only takes into account constraints on carbon emissions, but also traded carbon emission rights. When solving the problem in the model, the second-order cone constraint is approximately regarded as a linear form using the polynomial method to form a new MILP model, thus SS can be solved efficiently and rapidly.

The main contributions of this paper are as follows.

(1)A new CETSS model is presented in order to analyse the impact of carbon trading to GENCO.

(2)A new method is proposed for solving the CETSS model based on piecewise linearization and second-order cone linearization.

This paper is outlined as follows: in the following section, we introduce the carbon emissions trading self-scheduling model; in Section 3, we provide MILP solutions based on piece-wise linearisation and second-order cone linearisation; in Section 4, we present a numerical example; and we provide concluding remarks in Section 5.

2. Formulation of a carbon emission trading self-scheduling problem

2.1. *Profit objective function of GENCO*

The CETSS objective function comprises: generation income, the fuel cost of generating units, the start-up cost, and the carbon emission trading cost:

$$\max\ F_{profit} = R_V - T_C - E_M \tag{1}$$

Where

$$R_V = \sum_{t=1}^{T}\sum_{i=1}^{N} \lambda^t u_i^t P_i^t \tag{2}$$

$$T_C = \sum_{t=1}^{T}\sum_{i=1}^{N} [u_i^t f_i(P_i^t) + u_i^t(1-u_i^{t-1})C_{Ui}^t] \tag{3}$$

$$E_M = \lambda_P E_P - \lambda_S E_S \tag{4}$$

where F_{profit} is the profit; R_V represents generation income; T_C is the total fuel and start-up costs combined; E_M is the emissions cost; T refers to the total time period; N is the number of generated units; λ^t represents the electricity price over time period t; u_i^t is the operation status of unit i during period t. When $u_i^t = 1$, it implied that unit i is in operation, and when $u_i^t = 0$, it was stopped. P_i^t, $f_i(P_i^t)$, and C_{Ui}^t refer to the active generation, fuel cost function, and start-up cost of unit i in period t, respectively; λ_P,λ_S, E_P and E_S represent the carbon purchase price, selling price, purchase quantity and selling quantity.

The function $f_i(P_i^t)$ in (3) is denoted by:

$$f_i(P_i^t) = \alpha_i + \beta_i P_i^t + \gamma_i (P_i^t)^2$$

where α_i, β_i and γ_i are parameters of the unit.

The start-up cost C_{Ui}^t in (3) is specified as:

$$C_{Ui}^t = \begin{cases} C_i^{hot} : \underline{T}_i^{off} \le -T_i^t \le \underline{T}_i^{off} + T_i^{cold} \\ C_i^{cold} : -T_i^t > \underline{T}_i^{off} + T_i^{cold} \end{cases}$$

Where C_i^{hot} is the warm start-up cost of unit i; C_i^{cold} refers to its cold start-up cost; $\underline{T}_i^{off}$ is its minimum down time; T_i^t is its consistent operating time (positive) or stop time (negative) in period t, and T_i^{cold} is the cold start-up time.

2.2. Constraints

The constraint conditions on the CETSS model included: system constraints, unit generation and spinning reserve constraints, unit ramp rate constraints, the minimum on/off time constraints, and carbon emission quota constraints given by (5) to (16):

(1) System constraints

$$\sum_{i=1}^{N} P_i^t \le P_D^t \tag{5}$$

$$\sum_{i=1}^{N} R_i^t \le S_R^t \tag{6}$$

(2) Unit generation and spinning reserve constraints

$$0 \le R_i^t \le u_i^t (P_i^{max} - P_i^{min}) \tag{7}$$

$$u_i^t P_i^{min} \le P_i^t + R_i^t \le u_i^t P_i^{max} \tag{8}$$

$$u_i^t P_i^{min} \le P_i^t \le u_i^t P_i^{max} \tag{9}$$

(3) Unit ramp rate constraints

$$P_i^t - P_i^{t-1} \le u_i^{t-1} P_i^{up} + (u_i^t - u_i^{t-1}) P_i^{start} + (1 - u_i^t) P_i^{max} \tag{10}$$

$$P_i^{t-1} - P_i^t \le u_i^{t-1} P_i^{down} + (u_i^{t-1} - u_i^t) P_i^{shut} + (1 - u_i^{t-1}) P_i^{max} \tag{11}$$

(4) Minimum on/off time constraints

$$(u_i^{t-1} - u_i^t)(T_i^{t-1} - \underline{T}_i^{on}) \ge 0 \tag{12}$$

$$(u_i^t - u_i^{t-1})(-T_i^{t-1} - \underline{T}_i^{off}) \ge 0 \tag{13}$$

(5) Carbon emission quota constraints

$$\sum_{t=1}^{T} \sum_{i=1}^{N} u_i^t g_i(P_i^t) \le E_C + E_P - E_S \tag{14}$$

$$0 \le E_P \le z E_C \tag{15}$$

$$0 \le E_S \le (1 - z) E_C \tag{16}$$

3. MIP on CETSS problem

Judging from the mathematical model, the CETSS problem is a complex, large-scale, non-linear programming model which consists of not only the discrete variables but also the continuous variables. The former variable contains the starting up and powering off states for the generator while the latter includes power output and reserve. This non-linear model is mainly characterised by the cost of power generation, the starting up of the generator in the objective function, as well as constrains including the shortest time for starting up and shutting down, and the quota for carbon emissions. It is difficult to solve CETSS problems in any direct sense: this research transforms the problem into an MILP model based on the idea of linearisation of a second-order cone. The research indicates that the MILP model presents perfect computational efficiency and accuracy in the solution of large-scale problems.

3.1. MIQP model of CETSS problem

(1) Generation cost

In the objective function (1),$u_i^t P_i^t$ is the power generation cost, and:

$$u_i^t f_i(P_i^t) = \alpha_i u_i^t + \beta_i P_i^t u_i^t + \gamma_i (P_i^t)^2 u_i^t \tag{17}$$

So (17) is a non-convex term which is unfavourable for solving the function. In terms of (9), relating to the constraints of power output, if $u_i^t = 0$, $P_i^t = 0$. When $u_i^t = 1$, P_i^t is between the minimum and maximum power output. It indicates that u_i^t closely corresponds to P_i^t. Regardless of whether $u_i^t = 0$ or $u_i^t = 1$, we obtain:

$$P_i^t u_i^t = P_i^t \tag{18}$$

$$(P_i^t)^2 u_i^t = (P_i^t)^2 \tag{19}$$

Thus, (17) can be constructed as the lower convex form as follows:

$$u_i^t f_i(P_i^t) = \alpha_i u_i^t + \beta_i P_i^t + \gamma_i (P_i^t)^2 \tag{20}$$

Likewise, $\lambda^t u_i^t P_i^t$- the power generation income, as the first term of the objective function is denoted by $\lambda^t P_i^t$.

(2) Start-up cost

Initially, the start-up cost and the minimum on/off time constraints were linearised. The research applied a method reported elsewhere [13] to linearise the start-up cost. For convenience $S_i^t = u_i^t(1 - u_i^{t-1})C_{Ui}^t$, and S_i^t was approximately transformed into:

$$\begin{cases} S_i^t \geq K_i^\tau (u_i^t - \sum\limits_{j=1}^{\tau} u_i^{t-j}) \\ S_i^t \geq 0, i = 1, ..., N; t = 1, ..., T; \tau = 1, .., N_{Di} \end{cases} \tag{21}$$

where coefficient K_i^τ refers to

$$K_i^\tau = \begin{cases} C_i^{hot} : \tau = 1, ..., \underline{T}_i^{off} + T_i^{cold} \\ C_i^{cold} : \tau = \underline{T}_i^{off} + T_i^{cold} + 1, ..., N_{Di} \end{cases}$$

and N_{Di} is the given parameter.

(3) Minimum on/off time

The minimum on/off time constraints of (12) and (13) were non-linear and were transformed to the following linear versions using the method suggested in the literature [14]:

$$\sum_{\tau=t}^{\theta(\underline{T}_i^{on})} u_i^\tau \geq (u_i^t - u_i^{t-1})\varsigma(\underline{T}_i^{on}) + \nu(t-1)\theta_i^0 \tag{22}$$

$$\sum_{\tau=t}^{\theta(\underline{T}_i^{off})}(1-u_i^\tau) \geq (u_i^{t-1}-u_i^t)\varsigma(\underline{T}_i^{off}) + \nu(t-1)\varsigma_i^0 \tag{23}$$

where $\nu(t-1)$ is the unit impulse function; and

$$\begin{cases} \theta(\omega) = min\{t+\omega-1, T\} \\ \varsigma(\omega) = min\{\omega, T-t+1\} \\ \theta_i^0 = u_i^1 u_i^0 max\{0, \underline{T}_i^{on} - T_i^0\} \\ \varsigma_i^0 = (1-u_i^1)(1-u_i^0)max\{0, \underline{T}_i^{off} + T_i^0\} \end{cases}$$

where u_i^0 and T_i^0 are known quantities.

(4) Carbon emissions quota

Based on ways of dealing with power generation costs mentioned in Section 3.1, the constraint of a carbon emission quota is established as (24):

$$\sum_{t=1}^{T}\sum_{i=1}^{N}[u_i^t a_i + b_i P_i^t + c_i(P_i^t)^2] \leq E_C + E_P - E_S \tag{24}$$

This is a non-linear constraint. Based on the linearisation idea proposed by [13], the constraint of a carbon emission quota (Eq. (24)) can be transformed into a linear constraint system as follows:

$$\sum_{t=1}^{T}\sum_{i=1}^{N}A_i u_i^t + \sum_{t=1}^{T}\sum_{i=1}^{N}\sum_{l=1}^{L}F_{li}\delta_{li}^t \leq E_C + E_P - E_S \tag{25}$$

$$P_i^t = \sum_{l=1}^{L}\delta_{li}^t + P_i^{min}u_i^t \tag{26}$$

$$\delta_{li}^t \leq q_{1i} - P_i^{min} \tag{27}$$

$$\delta_{li}^t \leq q_{li} - q_{l-1i}, l = 2, \ldots, L-1 \tag{28}$$

$$\delta_{Li}^t \leq P_i^{max} - q_{L-1i} \tag{29}$$

$$\delta_{li}^t \geq 0 \tag{30}$$

Where $A_i = a_i + b_i P_i^{min} + c_i(P_i^{min})^2$; L denotes the linearised piece-wise number; F_{li} refers to the slope of the l piece-wise segment of generator i; q_{li} represents the power output of the l piecewise segment of generator i; and δ_{li}^t is an instrumental variable.

(5) MIQP formulation

According to the methods of dealing with the cost of power generation and start-up of the generator, the minimum on/off time, as well as the carbon emission quota, give a MIQP formulation of CETSS problem

$$\begin{aligned} &\max \sum_{t=1}^{T}\sum_{i=1}^{N}[\lambda^t P_i^t - \alpha_i u_i^t - \beta_i P_i^t - \gamma_i(P_i^t)^2 - S_i^t] \\ &s.t. \begin{cases} (5)-(11), (15)-(16), (22)-(23), (25)-(30) \\ u_i^t \in \{0,1\}, z \in \{0,1\} \end{cases} \end{aligned} \tag{31}$$

3.2. MISOCP model of the CETSS problem

(1) A more compact second-order conic relaxation

To consider a simple mix integer set, we obtain:

$$C = \{(s,y,z)|s^2 - y \leq 0, z\underline{s} \leq s \leq z\overline{s}, 0 < \underline{s} < \overline{s}, z \in \{0,1\}\} \tag{32}$$

where $\{s^2 - y \leq 0\}$ can be $\{s^2 - 2py \leq 0, p = \frac{1}{2}\}$: a rotation second-order cone form.
The continuous relaxation of set C is expressed as:

$$C_R = \{(s, y, z) | s^2 - y \leq 0, z\underline{s} \leq s \leq z\overline{s}, 0 < \underline{s} < \overline{s}, 0 \leq z \leq 1\} \tag{33}$$

The continuous relaxation C_R is loose. To overcome the shortfall of looseness, the literature [15] proposes a much more compact continuous relaxation:

$$C_J = \{(s, y, z) | s^2 - zy \leq 0, z\underline{s} \leq s \leq z\overline{s}, 0 < \underline{s} < \overline{s}, 0 \leq z \leq 1\} \tag{34}$$

To discuss the compactness of set C_J , the set is defined as:

$$C_S = \{(s, y, z) | s^2 - y \leq 0, z\underline{s} \leq s \leq z\overline{s}, 0 < \underline{s} < \overline{s}, 0 < z < 1\} \tag{35}$$

Research [15] pointed out the satisfaction relations of the sets defined by Eqns (33) to (35):

$$\begin{cases} C_S \subset C_R, C_S \cap C_J = \emptyset \\ C \subset C_J \subset C_R \end{cases} \tag{36}$$

Eq. (36) shows that C_J at least excluded the subset C_S of C_R. Therefore, C_J can more compactly conclude the mix integer set C than C_R.
(2) MISOCP formulation
According to Section 3.2, C_J is able to more closely conclude the mix integer set C than C_R. We can also obtain a much more compact MISOCP model, regarding CETSS in Eq. (31), by means of the compactness of C_J.
By introducing instrumental variable v_i^t, (31) can be converted to:

$$\begin{aligned} &\max \sum_{t=1}^{T} \sum_{i=1}^{N} (\lambda^t P_i^t - \alpha_i u_i^t - \beta_i P_i^t - \gamma_i v_i^t - S_i^t) \\ &s.t. \begin{cases} (5)-(11), (15)-(16), (22)-(23), (25)-(30) \\ u_i^t \in \{0,1\}, z \in \{0,1\} \\ (P_i^t)^2 \leq v_i^t \end{cases} \end{aligned} \tag{37}$$

Each couple of indices (i, t) can be defined as the set:

$$C_i^t = \{(P_i^t, v_i^t, u_i^t) | (P_i^t)^2 - v_i^t \leq 0, u_i^t P_i^{min} \leq P_i^t \leq u_i^t P_i^{max}, u_i^t \in \{0,1\}\}$$

So C_i^t is the concrete form of set C in (32). From (34), we can obtain:

$$C_{Ji}^t = \{(P_i^t, v_i^t, u_i^t) | (P_i^t)^2 - u_i^t v_i^t \leq 0, u_i^t P_i^{min} \leq P_i^t \leq u_i^t P_i^{max}, v_i^t \geq 0, 0 \leq u_i^t \leq 1\}$$

Hence, the more compact MISOCP formulation of CETSS problem is obtained.

$$\begin{aligned} &\max \sum_{t=1}^{T} \sum_{i=1}^{N} (\lambda^t P_i^t - \alpha_i u_i^t - \beta_i P_i^t - \gamma_i v_i^t - S_i^t) \\ &s.t. \begin{cases} (5)-(11), (15)-(16), (22)-(23), (25)-(30) \\ u_i^t \in \{0,1\}, z \in \{0,1\} \\ (P_i^t)^2 \leq u_i^t v_i^t \\ v_i^t \geq 0 \end{cases} \end{aligned} \tag{38}$$

where $(P_i^t)^2 \leq u_i^t v_i^t$ is a rotational second-order conic constraint.

3.3. *MILP model of CETSS problem*

(1) Second-order cone linearisation

The solution algorithm for linear programming has been mature and efficient. Thus, transforming the second-order conic programming into its linear form provides another method for second-order conic programming. It can, in particular, be converted to an MILP model when the constraint contains integer variables.

Taking into account the second-order cone constraint, we obtain:

$$x_1 \geq \sqrt{x_2^2 + x_3^2} \tag{39}$$

Based on other research [16], (39) can be converted to the following linear system:

$$\xi_0 \geq |x_2| \tag{40}$$

$$\eta_0 \geq |x_3| \tag{41}$$

$$\xi_i = cos(\frac{\pi}{2^{i+1}})\xi_{i-1} + sin(\frac{\pi}{2^{i+1}})\eta_{i-1} \tag{42}$$

$$\eta_i \geq \left| -sin(\frac{\pi}{2^{i+1}})\xi_i + cos(\frac{\pi}{2^{i+1}})\eta_i \right|, i = 1, 2, ..., v \tag{43}$$

$$\xi_v \leq x_1 \tag{44}$$

$$\eta_v \leq tan(\frac{\pi}{2^{v+1}})\xi_v \tag{45}$$

In (40) to (45), ξ_0, η_0, ξ_i, and η_i are added variables and the total number of added variables is $2(v + 1)$.

The linear system expressed by (40) to (45) is the polyhedron in $(v + 3)$ dimensional space. Its projection on the Cartesian coordinate system with x_2 and x_3 as axes is a polygon of the inscribed circle. The radius of the circle is x_1, and the number of edges of the polygon is 2^{v+1}.

In terms of the second-order conic constraint $x_1 \geq \sqrt{x_2^2 + x_3^2}$, if the polyhedron is approximately processed to be a linear constraint, the error is [16]:

$$\delta(v) = \frac{1}{\cos(\frac{\pi}{2^{v+1}})} - 1 \tag{46}$$

(2) MILP formulation

According to Section 3.3, the linearisation of the second-order conic constraint is more accurate. The MILP formulation of CETSS problem is acquired through linearisation as mentioned in Section 3.3.

Based on the rotational second-order conic constraint of (38), we get

$$(P_i^t)^2 \leq u_i^t v_i^t \tag{47}$$

Eq. (47) can be equivalently transformed into the following second-order conic constraint:

$$\sqrt{\left(\frac{v_i^t - u_i^t}{2}\right)^2 + (P_i^t)^2} \leq \frac{v_i^t + u_i^t}{2} \tag{48}$$

Eq. (48) is converted to a linear constraint through linearisation (as in Section 3.3) as follows:

$$\xi_{0i}^t \geq \left|P_i^t\right| \tag{49}$$

$$\eta_{0i}^t \geq \left|\frac{v_i^t - u_i^t}{2}\right| \tag{50}$$

$$\xi_{ki}^t = \cos\left(\frac{\pi}{2^{k+1}}\right)\xi_{k-1i}^t + \sin\left(\frac{\pi}{2^{k+1}}\right)\eta_{k-1i}^t \tag{51}$$

$$\eta_{ki}^t \geq \left| -\sin\left(\frac{\pi}{2^{k+1}}\right)\xi_{ki}^t + \cos\left(\frac{\pi}{2^{k+1}}\right)\eta_{k-1i}^t \right|, k = 1, 2, \cdots, K \tag{52}$$

$$\xi_{Ki}^t \leq \frac{v_i^t + u_i^t}{2} \tag{53}$$

$$\eta_{Ki}^t \leq \tan\left(\frac{\pi}{2^{K+1}}\right)\xi_{Ki}^t \tag{54}$$

ξ_{0i}^t, η_{0i}^t, ξ_{ki}^t and η_{ki}^t in (49) to (54) are instrumental variables.

For the sake of ease of programming, the symbols of absolute value in (49), (50), and (52) are removed. Eq. (9) reveals that $P_i^t \geq 0$, so the absolute value in (49) can be directly eliminated as follows:

$$\xi_{0i}^t \geq P_i^t \tag{55}$$

For (50), if $u_i^t = 0$, it is seen that $v_i^t = 0$ according to both (9) and (47). Similarly, if $u_i^t = 1$, $v_i^t > u_i^t$.Therefore, the absolute value in (50) can also be directly eliminated as:

$$\eta_{0i}^t \geq \frac{v_i^t - u_i^t}{2} \tag{56}$$

Eq. (52) is transformed into the constraint, according to the inequality property of the absolute value, as follows:

$$\eta_{ki}^t \geq -\sin\left(\frac{\pi}{2^{k+1}}\right)\xi_{k-1i}^t + \cos\left(\frac{\pi}{2^{k+1}}\right)\eta_{k-1i}^t \tag{57}$$

$$\eta_{ki}^t \geq \sin\left(\frac{\pi}{2^{k+1}}\right)\xi_{k-1i}^t - \cos\left(\frac{\pi}{2^{k+1}}\right)\eta_{k-1i}^t \tag{58}$$

Hence, the MILP formulation of CETSS problem is obtained:

$$\begin{aligned} &\max \sum_{t=1}^{T}\sum_{i=1}^{N}(\lambda^t P_i^t - \alpha_i u_i^t - \beta_i P_i^t - \gamma_i v_i^t - S_i^t) \\ &s.t. \begin{cases} (5)-(11), (15)-(16), (22)-(23), (25)-(30) \\ u_i^t \in \{0,1\}, z \in \{0,1\} \\ (51), (53)-(58) \\ v_i^t \geq 0 \end{cases} \end{aligned} \tag{59}$$

4. Numerical results

The hardware platform for the numerical calculation is a Pentium® dual-core CPU T4300 (2.1 GHz, 2.09 GHz) PC, 1.93 GB RAM, running the software MOSEK 6.0 under MATLAB 2010a environment. The generator parameters, and the load data over each 24 hour period, for 10 unit systems are taken from previous literature [17]; the ramp rate constraint data is obtained from [18]; the emissions parameter from [6]; and the electricity price data from [6].

4.1. Self-scheduling without carbon emission trading

The MISOCP formulation of SS problem does not consider the cost or quota constraint of carbon emissions in the objective function in (38). While The MILP formulation of SS problem does not take into account the cost or quota constraint of carbon emissions in the objective function in (59). The model obtained through linearisation of the second-order cone in this work (defined as MILP-II) is compared with the model obtained through piece-wise linearisation in [13] (defined as MILP-I): the solution process is shown in more detail elsewhere [13]. The piece-wise number of linearisations for MILP-I model is 6 and $N_{Di} = 10$; while the linearised parameters of the MILP-II model are $K = 3$ and $N_{Di} = 10$. The relatively optimal clearance is designed as 0.3% when solving the MILP model.

Table 1 shows the comparison of results obtained by three models: MISOCP, MILP-I, and MILP-II. The values in the models are the profit achieved by power suppliers in the scheduling period. It is difficult to solve the large-scale MISOCP model directly. Therefore, the authors firstly solve the continuous relaxation problem in the MISOCP model using the method proposed in the literature [19]. Then, the state of the generator is regulated to make it satisfy all constraints. On this basis, the start-up and shut-down states of the generator for the SS problem can be obtained. Consequently, the quadratic programming problem is solved. Table 2 shows the comparison of computing times using three models: MISOCP, MILP-I, and MILP-II. Table 3 lists the number of variables and constraints in MILP-I and MILP-II.

Table 1. Comparison of three models ($)

Number of units	10	20	40	60	80	100
MISOCP	711261	1422522	2845045	4267567	5690090	7112612
MILP-I	**715193**	1434189	2869324	4304600	5735846	7170291
MILP-II	715119	**1434305**	**2870803**	**4306049**	**5744867**	**7179209**

Table 2. Execution time comparison of three models (s)

Number of units	10	20	40	60	80	100
MISOCP	3.25	5.98	10.88	15.91	21.15	26.87
MILP-I	49.43	141.58	169.76	463.12	688.3	868.87
MILP-II	40.37	111.62	134.56	245.96	429.94	605.11

Tables 1 and 2 indicate that it is quite difficult to solve the MISOCP model directly. However, the solution using continuous relaxation is not accurate even though less computing time is required. Thus, how to establish a more compact continuous relaxation problem needs to be investigated further. Except for the 10 generator system over 24 hours, the MILP-II models in other systems are more optimal than the MILP-I model. Moreover, the computing time needed by MILP-II is less than that for MILP-I: this indicates that a better MILP formulation can be obtained by adopting the linearisation technology of second-order conics as mooted here.

Table 3. Variable and constraint number of MILP-I model

Number of units	10	20	40	60	80	100
Total variables	2400	4800	9600	14400	19200	24000
Integer variables	240	480	960	1440	1920	2400
Constraints	4848	9648	19248	28848	38448	48048

Table 3 and 4 indicate that the linearisation method of second-order conics does not increase the number of integer variables but the numbers of continuous variables and constraints. The number of increased continuous variables and constraints depends on the parameter. The number of increased continuous variables and constraints in the MILP-II model is greater than for MILP-I, but the computing time of the former does not increase.

Table 4. Variable and constraint number of MILP-II model

Number of units	10	20	40	60	80	100
Total variables	3120	6240	12480	18720	24960	31200
Integer variables	240	480	960	1440	1920	2400
Constraints	7728	15408	30768	46128	61488	76848

4.2. *Self-scheduling with carbon emissions trading*

The MILP formulation of CETSS problem is indicated in (59). Taking the 10 generator over 24 hours as an example, it is assumed that the prices for purchasing and selling carbon emissions quotas are the same: the virtual price scene ranges from \$1 per tonne to \$29 per tonne in 15 scenarios in total with a tax rate difference of \$2 per tonne among contiguous scenarios. Neglecting the carbon emission rights trade, the total emissions of the 10th generator over 24 hours is 26,702 tonnes with a profit of \$71,5191, and the capacity of power generation is 26,970 MW. The total emissions are defined as Em = 26702 tonnes and the carbon emissions quota is assumed to be 0.5Em. Fig. 1 shows the total carbon dioxide emissions for different price scenarios; Fig. 2 shows the varied carbon trade costs; Fig. 3 shows the total profits for each scenario; and Fig. 4 shows the power outputs for different carbon price scenarios.

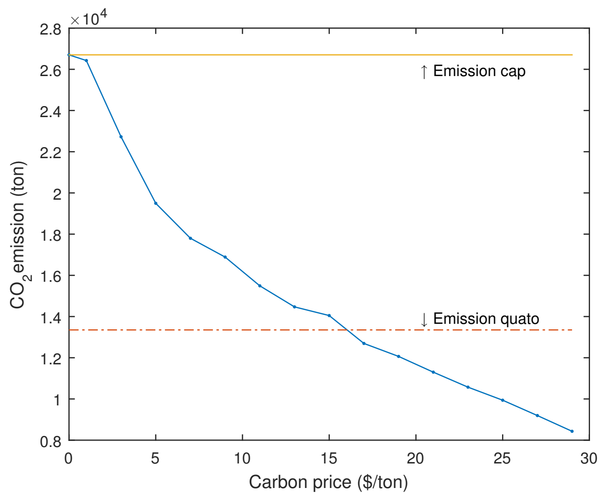

Figure 1. CO2 emission of different carbon price scenarios.

As seen from Fig. 1, the carbon emissions decrease with increasing carbon price. When the price ranges from \$1 per tonne to \$15 per tonne, the carbon emissions are higher than the corresponding carbon emission quota; hence, power suppliers have to purchase quota in the carbon market. However, power suppliers need to change their market strategy as the carbon price rises, which leads to an increasing carbon trade cost. When the price ranges from \$17 per tonne to \$29 per tonne, the carbon emissions are lower than the given carbon emission quota; power suppliers sell redundant quota instead of purchasing so as to achieve profit from the carbon market.

Fig. 2 shows that when the carbon price is zero, the carbon trade cost for power suppliers is zero. In the case of a low carbon price, power suppliers purchase carbon emission rights; the trade cost shows an increasing tendency as the price increases. When the price is higher than \$9 per tonne, the carbon emissions purchased decreases: this further reduces the trade cost. When the price is roughly \$16 per tonne, the carbon emissions decrease to a level

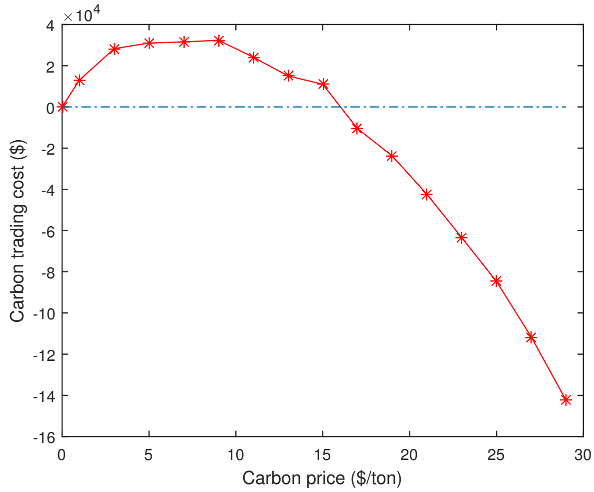

Figure 2. Carbon trading costs of different carbon price scenarios.

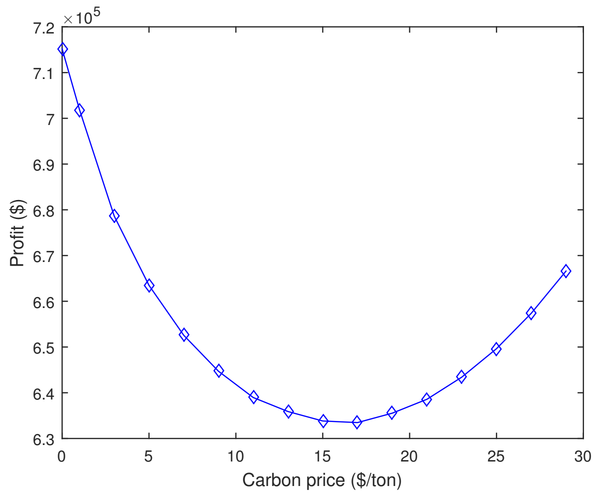

Figure 3. Profit of different carbon price scenarios.

equal to the quota, which produces no trade cost. Afterwards, power suppliers sell their carbon emission rights as the price is higher than $16 per tonne, to profit from the carbon market (the carbon trade cost is now negative).

Fig. 3 shows that when the carbon price is low, power suppliers pay extra carbon trade costs; the total profit therefore shows a decreasing trend. As the price rises to $16 per tonne, profit declines and reaches its lowest level. Afterwards, as the price rises from $16 per tonne to a higher level, the profit increases; the elevated price produces more profit for power suppliers.

Fig. 4 shows that using the CETSS model, power suppliers ensure that power output is lower than load demand. The power output of the generator has also been reduced as the carbon price rises. This reveals the more apparent effect of increasing carbon price on curbing power demand.

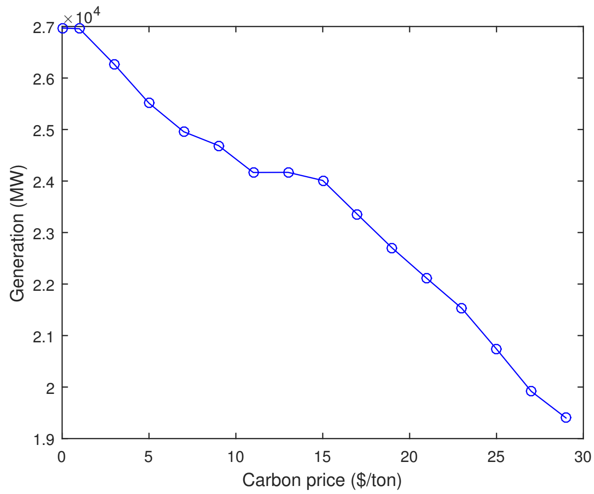

Figure 4. Generation of different carbon price scenarios.

5. Conclusion

This research proposes a carbon emission trading self-scheduling problem. The CETSS model is solved by transforming it into an MILP formulation based on piece-wise linearisation, and second-order conical, linearisation methods. The simulation results from 10 to 100 unit systems over a 24-hour period show that the MILP-II process is superior to MILP-I. Overall, we conclude that the carbon price has a greater effect on the unit's emissions, profits, and outputs.

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REFERENCES

1. Reddy, K Srikanth and Panwar, Lokesh Kumar and Kumar, Rajesh and Panigrahi, BK, *Binary fireworks algorithm for profit based unit commitment (PBUC) problem*, International Journal of Electrical Power & Energy Systems, vol. 83, pp. 270–282, 2016.
2. Ghadi, M Jabbari and Baghramian, Alfred and Imani, M Hosseini, *An ICA based approach for solving profit based unit commitment problem market*, Applied Soft Computing, vol. 38, pp. 487–500, 2016.
3. Sudhakar, AVV and Karri, Chandram and Laxmi, A Jaya, *Profit based unit commitment for GENCOs using Lagrange relaxation–differential evolution*, Engineering Science and Technology, an International Journal, vol. 20, no. 2, pp. 738–747, 2017.
4. Anand, Himanshu and Narang, Nitin and Dhillon, JS, *Profit based unit commitment using hybrid optimization technique*, Energy, vol. 148, pp. 701–715, 2018.
5. Dhaliwal, Jatinder Singh and Dhillon, JS, *Profit based unit commitment using memetic binary differential evolution algorithm*, Applied soft computing, vol. 81, pp. 105502, 2019.
6. Kumar, Vineet and Naresh, R and Singh, Amita, *Scheduling of Energy on Deregulated Environment of Profit based Unit Commitment Problem*, 2020 IEEE 9th Power India International Conference (PIICON). 1–6, 2020.
7. Elkamel, Marwen and Ahmadian, Ali and Diabat, Ali and Zheng, Qipeng P, *Stochastic optimization for price-based unit commitment in renewable energy-based personal rapid transit systems in sustainable smart cities*, Sustainable Cities and Society, vol. 65, pp. 102618, 2021.
8. Jabari, Farkhondeh and Mohammadi-ivatloo, Behnam and Mohammadpourfard, Mousa, *Robust optimal self-scheduling of potable water and power producers under uncertain electricity prices*, Applied Thermal Engineering, vol. 162, pp. 114258, 2019.

9. Liu, Sidong and Chevallier, Julien and Zhu, Bangzhu, *Self-scheduling of a power generating company: Carbon tax considerations*, Computers & Operations Research, vol. 66, pp. 384–392, 2016.
10. Pousinho, Hugo Miguel Inácio and Silva, H and Mendes, VMF and Collares-Pereira, Manuel and Cabrita, C Pereira, *Self-scheduling for energy and spinning reserve of wind/CSP plants by a MILP approach*, Energy, vol. 78, pp. 524–534, 2014.
11. Yang, Linfeng and Li, Wei and Chen, Guo and Fang, Beihua and Tang, Chunming, *A quick center-point algorithm for unit commitment with carbon emission trading*, International Journal of Electrical Power & Energy Systems, vol. 120, pp. 105996, 2020.
12. Reddy, Srikanth and Panwar, Lokesh and Panigrahi, Bijaya Ketan and Kumar, Rajesh and Goel, Lalit and Al-Sumaiti, Ameena Saad, *A profit-based self-scheduling framework for generation company energy and ancillary service participation in multi-constrained environment with renewable energy penetration*, Energy & environment, vol. 31, no. 4, pp. 549–569, 2020.
13. Carrión, Miguel and Arroyo, José M, *A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem*, IEEE Transactions on power systems, vol. 21, no. 3, pp. 1371–1378, 2006.
14. Zheng, Haiyan and Jian, Jinbao and Yang, Linfeng and Quan, Ran, *A deterministic method for the unit commitment problem in power systems*, Computers & Operations Research, vol. 66, pp. 241–247, 2016.
15. Quan, Ran and Jian, Jin-bao and Mu, Yun-dong, *Tighter relaxation method for unit commitment based on second-order cone programming and valid inequalities*, International Journal of Electrical Power & Energy Systems, vol. 55, pp. 82–90, 2014.
16. Ben-Tal, Aharon and Nemirovski, Arkadi, *On polyhedral approximations of the second-order cone*, Mathematics of Operations Research, vol. 26, no. 2, pp. 193–205, 2001.
17. Pang, Xiali and Zheng, Haiyan and Huang, Liying and Liang, Yumei, *Outer Approximation Method for the Unit Commitment Problem with Wind Curtailment and Pollutant Emission*, Mathematics, vol. 9, no. 21, pp. 2686, 2021.
18. Ostrowski, James and Anjos, Miguel F and Vannelli, Anthony, *Tight mixed integer linear programming formulations for the unit commitment problem*, IEEE Transactions on Power Systems, vol. 27, no. 1, pp. 39–46, 2011.
19. Yuan, Xiaohui and Tian, Hao and Zhang, Shuangquan and Ji, Bin and Hou, Yanhong, *Second-order cone programming for solving unit commitment strategy of thermal generators*, Energy conversion and management, vol. 76, pp. 20–25, 2013.