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Time truncated double acceptance sampling plan for the Nadarajah-Haghighi distribution

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Abstract In this article, we design a double acceptance sampling plan for the Nadarajah-Haghighi (NH) distribution when the lifetime is truncated. The minimum sample sizes necessary to ensure a certain mean lifetime for selected acceptance numbers and consumer's confidence levels are obtained. The operating characteristic function and the associated producer's risks are studied. We also analyze the minimum ratios of the mean life to the specified life. Real data and simulated examples are provided to illustrate the results of the paper.

Keywords Double acceptance sampling plan; Consumer and producer's risk; Nadarajah-Haghighi distribution; Operating characteristic function.

AMS 2010 subject classifications 62N10; 62P30

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1. Introduction

Reliability sampling plans are used to determine the acceptability of a product with respect to its lifetime. Due to various resterictions like time and cost, complete inspection is not possible. Then, a random sample is drown from the lot by consumer and a decision (acceptance or rejection) is made by a pre-specified time t. Let $P^\star$ be the consumer's confidence level, then the goal is to find a confidence limit on the mean life of products, μ and to set a specified mean life, μ_0, with a probability of at least $P^\star$.

Many types of sampling plans are available in the area of quality control like single, two stage (double) and group acceptance sampling plans. In single acceptance sampling plans, a lot of products is accepted if and only if the observed number of failures is not greater than the acceptance number c. The single sampling plan has been proposed for many underlying lifetime distributions, see for example [31], [16], [14], [15], [27], [9], [20] and [3] for the exponential, Weibull, gamma, log-normal, Marshall-Olkin extended Lomax, generalized exponential, half normal and exponentiated Fréchet models, respectively. Recently, [12], [5], [4], [21], [2], [1], [30] and [33] discussed the single sampling plan for the weighted exponential, extended exponential, Tsallis q-exponential, Topp-Leone Gompertz and power Lomax, exponentiated momemt exponential, logistic Rayleigh and transmuted Rayleigh models, respectively.

If with the help of the first sample, it cannot lead to the final decision about the lot, the second sample is selected from it. This reduces the sample size or producer's risk. [7], [8], [26], [28], [13], [18], [32] and [29] developed double acceptance sampling plans for the truncated life test based on the Rayleigh, the generalized log-logistic,

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the Marshall-Olkin extended exponential, generalized exponential, half exponential power, compound Weibull-exponential, inverse log-logistic and transmuted Rayleigh distribuions, respectively.

We explored the literature and found a gap that has not been addressed by any previous attempts to develop the double sampling plan for NH distribution. In this paper, we design a double acceptance sampling plan when the lifetime distribution follows an NH distribution with the following probability density function (pdf)

$$f(t,\alpha,\lambda)=\alpha\lambda(1+\lambda t)^{\alpha-1}\exp\{1-(1+\lambda t)^{\alpha}\},\quad t>0, \tag{1}$$

whose corresponding cummulative density function (cdf) is given by

$$F(t,\alpha,\lambda)=1-\exp\{1-(1+\lambda t)^{\alpha}\},\quad t>0, \tag{2}$$

where $\alpha>0$ and $\lambda>0$ are the shape and scale parameters, respectively. We write $T\sim NH(\alpha,\lambda)$ if T has the pdf (1). The NH model reduces to the exponential distribution when $\alpha=1$.

[24] have introduced the NH distribution which is a special case of a formerly introduced distribution by [11]. The hazard rate function of the NH model can be increasing or decreasing depending on the value of parameter α. The NH distribution has a closed form for its cdf that makes it preferable in comparison with some other two parameter models like the gamma distribution. These advantages as well as the others tempted the researchers to focus on modelling life data using the NH model and many authors worked on the inferential aspects of the NH model, see for instance [23] and [6].

The kth moment of the NH distribution is given by $E(T^k)=\alpha\,\lambda\,\mathrm{e}\,I(k,0,1)$, where

$$I(k,0,1)=\frac{1}{\alpha\lambda^{k+1}}\sum_{i=0}^{k}\binom{k}{i}(-1)^{k-i}\Gamma(\frac{i}{\alpha}+1,1),$$

in which $\Gamma(\cdot,\cdot)$ is the incomplete gamma function defined as $\Gamma(a,x)=\int_x^{\infty}t^{a-1}\mathrm{e}^{-t}\mathrm{d}t$, see [24]. Consequently, the mean of the NH distribution is given by

$$\mu=\frac{1}{\lambda}[\mathrm{e}\,\Gamma(1+\frac{1}{\alpha},1)-1]. \tag{3}$$

The paper is organized as follows. The double acceptance sampling plan is presented in Section 2. Also, the minimum sample sizes to be tested for a pre-assigned time necessary to assert the mean life to exceed a given value is calculated. In Section 3, the operating characteristic values of the plan are computed and reported. The minimum ratio of μ/μ_0 for the acceptability of a lot with a certain producer's risk are intended in Section 4. Three examples are given to show the applicability of double acceptance sampling plan in Section 5. Finally, the paper ends with some concluding remarks.

2. Double acceptance sampling plans

Suppose that the lifetime follows an $NH(\alpha,\lambda)$, where α is fixed. A lot is considered good if $\mu\geq\mu_0$, otherwise it is considered a bad lot. Thus, the consumers' risk is defined as the probability of accepting a bad lot,whereas the producer's risk is the probability of rejecting a good lot. In this study, we fixed the consumer's risk not to become greater than $1-P^{\star}$, where $0<P^{\star}<1$. We suppose that size of the lot is so large that we can use the binomial distribution in our work. The acceptance or rejection of the lot corresponds to the acceptance or rejection of the hypothesis $\mu\geq\mu_0$. From (3), we can say that the hypothesis $\mu\geq\mu_0$ corresponds to $\lambda\leq\lambda_0$, where $\lambda_0=m/\mu_0$ and $m=\mathrm{e}\,\Gamma(1+\frac{1}{\alpha},1)-1$.

A double acceptance sampling plan is designed as follows:

1. Draw a first sample of size n_1 from the lot and put on the test during time t . If there are c_1 or less failures observed before time t , accept the lot. If c_2+1 failures are recorded, the test is terminated before t and the lot is rejected $(c_1<c_2)$.

2. If the number of failures before t is between c_1+1 and c_2, then select a second sample of size n_2 and put them on the test during time t. The lot is accepted if at most c_2 failures are observed from the two samples. Otherwise, the lot is rejected and the test is terminated.

We search the minimum sample sizes n_1 and n_2 satisfying the consumer's risk, the probability of accepting a bad lot, is fixed not to exceed $1-P^\star$, when $\mu=\mu_0$. Let X_1 and X_2 are the numbers of failures from n_1 and n_2, respectively. Then, the probability of accepting a lot at the first and the second stage are given respectively by

$$P_a^{(1)} = p(X_1 \leq c_1) = \sum_{i=0}^{c_1} \binom{n_1}{i} p^i (1-p)^{n_1-i},$$

and

$$\begin{aligned} P_a^{(2)} &= P(c_1+1 \leq X_1 \leq c_2, X_1+X_2 \leq c_2) \\ &= \sum_{i=c_1+1}^{c_2} \binom{n_1}{i} p^i (1-p)^{n_1-i} \left(\sum_{j=0}^{c_2-i} \binom{n_2}{j} p^j (1-p)^{n_2-j} \right), \end{aligned}$$

where $p=F(t,\alpha,\lambda)=$probability of a failure before time t that is given by

$$\begin{aligned} p &= 1-\exp\{1-(1+\lambda t)^\alpha\} \\ &= 1-\exp\{1-\big(1+\frac{mt/\mu_0}{\mu/\mu_0}\big)^\alpha\} \end{aligned} \tag{4}$$

Therefore, The probability of accepting a lot is given by

$$P_a = P_a^{(1)} + P_a^{(2)} \tag{5}$$

We consider the zero and one failure schemes, that is, $c_1=0$ and $c_2=1$ for the proposed double sampling plan. Therefore, the lot acceptance probability (5) reduces to

$$P_a = (1-p)^{n_1} + n_1(1-p)^{n_1} p (1-p)^{n_2-1}. \tag{6}$$

The minimum sample sizes n_1 and n_2 ensuring $\mu \geq \mu_0$ can be found as the solution to the following inequality

$$(1-p_0)^{n_1} + n_1(1-p_0)^{n_1} p_0 (1-p_0)^{n_2-1} \leq 1-P^\star \tag{7}$$

where p_0 is the probability (4) at $\mu=\mu_0$ as $p_0 = 1-\exp\{1-(1+mt/\mu_0)^\alpha\}$. There are multiple solutions for sample sizes n_1 and n_2 satisfying the specified confidence level in inequality (6), Therefore, the minimum of the average sample number (ASN) is used to find the optiml sample sizes. The ASN for the proposed double sampling plan is obtained as

$$ASN = n_1 P_1 + (n_1+n_2)(1-P_1),$$

where P_1 is the probability of acceptance or rejection based on the first sample,

$$P_1 = 1 - \sum_{i=c_1+1}^{c_2} \binom{n_1}{i} p^i (1-p)^{n_1-i}.$$

For the zero and one failure schemes , i.e., $c_1=0,\ c_2=1$ the ASN is

$$ASN = n_1 + n_1 n_2 p_0 (1-p_0)^{n_1-1}, \tag{8}$$

Therefore, minimum sample sizes n_1 and n_2 can be calculated from (8) subject to

$$(1-p_0)^{n_1} + n_1(1-p_0)^{n_1} p_0 (1-p_0)^{n_2-1} \leq 1-P^*, \quad n_1 \geq n_2 \geq 1. \tag{9}$$

Table 1. Minimum sample sizes n_1 and n_2 for the NH distribution.

α	$P^\star$	t/μ_0 0.3	0.5	0.7	0.9	1.1	1.5	1.7	1.9
0.5	0.75	4, 3	3, 2	2, 2	2, 1	2, 1	2, 1	2, 1	2, 1
	0.9	6, 4	4, 3	3, 2	3, 1	2, 2	2, 1	2, 1	2, 1
	0.95	7, 5	5, 3	4, 2	3, 3	3, 2	2, 2	2, 2	2, 2
	0.99	10, 7	7, 4	5, 5	5, 2	4, 2	3, 3	3 , 2	3, 2
1	0.75	6, 5	4, 3	3 2	2, 2	2, 2	2, 1	2, 1	2, 1
	0.9	9, 7	6, 3	4, 3	3, 3	3, 2	2, 2	2, 1	2, 1
	0.95	11, 8	7, 4	5, 3	4, 3	3, 3	3, 1	2, 2	2, 2
	0.99	16, 11	10, 6	7, 5	6, 3	5, 2	4, 2	3, 2	3, 2
1.5	0.75	7, 6	4, 4	3, 3	3, 1	2, 2	2, 1	2, 1	2, 1
	0.9	10, 9	6, 5	5, 2	4, 2	3, 2	2, 2	2, 1	2, 1
	0.95	13, 9	8, 5	6, 3	4, 4	4, 2	3, 1	2, 2	2, 2
	0.99	19, 11	11, 7	8, 4	6, 4	5, 3	4, 2	3, 2	3, 2
2	0.75	8, 5	5, 3	3, 3	3, 1	2, 2	2, 1	2, 1	2, 1
	0.9	11, 9	7, 4	5, 3	4, 2	3, 2	2, 2	2, 1	2, 1
	0.95	14, 9	8, 6	6, 3	4, 4	4, 2	3, 1	2, 2	2, 2
	0.99	20, 14	12, 7	8, 6	6, 5	5, 3	4, 2	3, 2	3, 2

We consider the constraint $n_1 \geq n_2$ because if the sample size in the second stage is greater than that in the first stage, it may not be desirable due to cost, time, loss of statistical power and so on. The minimum values of n_1 and n_2 satisfying (9) are computed and summarized in Table 1 for $\alpha = 0.5, 1, 1.5, 2$, $P^\star = 0.75, 0.90, 0.95, 0.99$, and $t/\mu_0 = 0.3, 0.5, 0.7, 0.9, 1.1, 1.5, 1.7, 1.9$.

It is observed from Table 1 that the sample sizes increase as shape parameter α increases when the experiment time t/μ_0 is shorter, but the two sample sizes are close regardless of α when the experiment time is longer. Figure 1 shows the plot of the first sample sizes versus t/μ_0 for $P^\star = 0.95$.

3. Operating characteristic function

The operating characteristic (OC) function is the probability of accepting the lot. A sampling plan is more preferable if its operating characteristics approach more rapidly to one. For the proposed double sampling plan $(n_1, n_2, c_1, c_2, t/\mu_0)$, the OC is defined in (5). Note that increasing the ratio of the true mean life to the specified life (μ/μ_0) will lead to increasing the acceptance probability. So, we want to obtain the operating characteristics with respect to μ/μ_0.

Table 2 provides the OC values for the NH distribution with $\alpha = 2$. We see that the OC values increase as the quality levels (μ/μ_0) increases. The trend of OC values versus μ/μ_0 for values of $\alpha = 0.1, 1, 1.5, 2$ when $t/\mu_0 = 0.9$ and $P^\star = 0.95$ is plotted in Figure 2. It is observed that the OC values increases to one more rapidly at a higher value of α.

4. Minimum ratio of μ/μ_0 for the acceptability of a lot

For a specified producer's risk and a sampling plan $(n_1, n_2, c_1, c_2, t/\mu_0)$, one may seek the value of quality level μ/μ_0 that will ensure the producer's risk not to become greater than γ. Recall that the producer's risk is the probability of rejection of the lot when it is good i.e. $\mu > \mu_0$, or equivalently $\lambda < \lambda_0$. Then, we seek the minimum values of μ/μ_0 that satisfy the prodecure's risk is less than γ or equivalently

$$PA \geq 1 - \gamma, \tag{10}$$

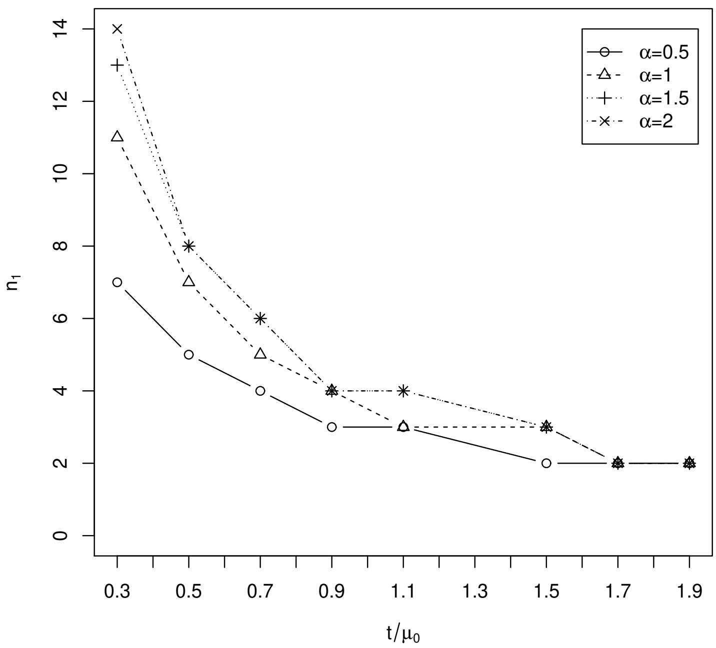

Figure 1. The plot of first sample size againest t/μ_0 for $P^{\star} = 0.95$.

The minimum values of μ/μ_0 satisfying (10) are presented in Table 3 with producer's risk of $\gamma = 0.05$ and $\alpha = 0.5, 1, 1.5, 2$.

Hypothetical example. Assume that the lifetime distribution of the products is Nadarajah-Haghighi with shape parameter $\alpha = 2$. We wish to estabilish that the mean lifetime μ_0 is at least 1000 hours with propbability $P^{\star} = 0.95$. Suppose the experimenter wants to terminate the life test at $t = 700$ hours under the zero and one failure schemes of the double sampling plan $(c_1 = 0, c_2 = 1)$. The required sample sizes in Table 1 corresponding to the value $t/\mu_0 = 0.7$ are $n_1 = 6$ and $n_2 = 3$. That is, 6 items have to be put on test for 700 hours. If no failures observed during this time, then the experimenter can accept the lot. Reject the lot for more than one failure. If one failure occures during the test, a second sample of size 3 is selected and put on test for 700 hours. We accept the lot if no failure occurs in the second sample and reject it otherwise. Suppose that the producer wants to know what quality level will lead to the producer's risk less than $\gamma = 0.05$. From Table 3 , the minimum ratio for $\alpha = 2$, $t/\mu_0 = 0.7$ and consumer's risk 0.05 (or $P^{\star} = 0.95$) is 12.241. Therefore, the true mean required of the product should be at least 12241 hours.

The ASN for this plan is found to be 6.52, so it may be interesting to compare this with a single sampling plan having the sample size of 7 and the acceptance number of 0. The operating characteristic values according to μ/μ_0 are shown as in Table 4.

Table 2. OC values of the sampling plan for $\alpha = 2$.

$P^\star$	n_1	n_2	t/μ_0	μ/μ_0 2	4	6	8	10	12
0.75	8	5	0.3	0.6095004	0.8549149	0.9260927	0.9554597	0.9702836	0.9787814
	5	3	0.5	0.5951169	0.8501540	0.9238838	0.9542087	0.9694844	0.9782287
	3	3	0.7	0.6077241	0.8565614	0.9275096	0.9565116	0.9710699	0.9793847
	3	1	0.9	0.6351761	0.8732755	0.9372568	0.9627670	0.9753976	0.9825487
	2	2	1.1	0.5837653	0.8489504	0.9241022	0.9546306	0.9698907	0.9785808
	2	1	1.5	0.5336923	0.8295379	0.9144701	0.9489528	0.9661642	0.9759526
	2	1	1.7	0.4600510	0.7916113	0.8936009	0.9359607	0.9573428	0.9695849
	2	1	1.9	0.3921567	0.7523634	0.8713117	0.9218829	0.9477056	0.9625916
0.9	11	9	0.3	0.4080109	0.7364747	0.8561209	0.9100870	0.9386606	0.9555371
	7	4	0.5	0.4224925	0.7525118	0.8672257	0.9178326	0.9442901	0.9597894
	5	3	0.7	0.4132038	0.7493794	0.8659457	0.9171996	0.9439339	0.9595698
	4	2	0.9	0.4219762	0.7589075	0.8724593	0.9217075	0.9471934	0.9620230
	3	2	1.1	0.4269462	0.7637010	0.8757007	0.9239440	0.9488085	0.9632378
	2	2	1.5	0.4091970	0.7546065	0.8709011	0.9210524	0.9468926	0.9618803
	2	1	1.7	0.4600510	0.7916113	0.8936009	0.9359607	0.9573428	0.9695849
	2	1	1.9	0.3921567	0.7523634	0.8713117	0.9218829	0.9477056	0.9625916
0.95	14	9	0.3	0.3125911	0.6667882	0.8118330	0.8802998	0.9174397	0.9397090
	8	6	0.5	0.3135651	0.6699077	0.8144626	0.8823041	0.9189724	0.9409058
	6	3	0.7	0.3370426	0.6962070	0.8330545	0.8954228	0.9285761	0.9481953
	4	4	0.9	0.3202799	0.6787623	0.8211720	0.8872107	0.9226430	0.9437325
	4	2	1.1	0.3063871	0.6791096	0.8236907	0.8896645	0.9247109	0.9454320
	3	1	1.5	0.3469493	0.7171407	0.8491167	0.9071022	0.9372568	0.9548442
	2	2	1.7	0.3357606	0.7054717	0.8414453	0.9019225	0.9335729	0.9521040
	2	2	1.9	0.2723437	0.6562486	0.8106472	0.8815188	0.9192026	0.9414780
0.99	20	14	0.3	0.1430784	0.4828710	0.6777250	0.7835915	0.8455802	0.8845787
	12	7	0.5	0.1531242	0.5058988	0.6980721	0.7996227	0.8581152	0.8945194
	8	6	0.7	0.1539247	0.5066641	0.6988143	0.8002608	0.8586443	0.8949566
	6	5	0.9	0.1543651	0.5092818	0.7015058	0.8025210	0.8604747	0.8964407
	5	3	1.1	0.1736821	0.5462133	0.7320639	0.8257676	0.8782588	0.9103356
	4	2	1.5	0.1488876	0.5250796	0.7190706	0.8173621	0.8724593	0.9061185
	3	2	1.7	0.1853677	0.5711703	0.7529753	0.8417218	0.8904730	0.9198793
	3	2	1.9	0.1355895	0.5108112	0.7097572	0.8111719	0.8681219	0.9029330

Table 3. Minimum ratio of μ/μ_0 for the acceptability of a lot with producer's risk of $\gamma = 0.05$.

α	$P^\star$	t/μ_0 0.3	0.5	0.7	0.9	1.1	1.5	1.7	1.9
0.5	0.75	9.806	11.429	11.731	11.539	14.103	19.231	21.795	24.359
	0.9	14.576	16.342	16.000	16.670	18.434	19.231	21.795	24.359
	0.95	17.514	19.367	20.139	23.808	25.143	25.137	28.489	31.840
	0.99	25.224	27.308	32.076	31.130	31.647	39.680	38.857	43.429
1	0.75	7.974	8.414	8.336	7.967	9.738	10.315	11.690	13.065
	0.9	11.831	11.414	11.780	12.338	13.100	13.279	11.690	13.065
	0.95	14.225	13.900	13.899	15.145	15.080	14.605	15.049	16.819
	0.99	20.471	20.340	20.777	20.544	19.559	22.303	20.245	22.626
1.5	0.75	7.846	7.746	8.036	7.372	8.206	8.735	9.899	11.064
	0.9	11.458	11.066	10.395	11.196	10.989	11.189	9.899	11.064
	0.95	13.791	13.612	13.318	13.943	13.684	12.287	12.681	14.173
	0.99	19.254	18.934	17.924	18.581	18.223	18.660	16.982	18.980
2	0.75	7.499	7.618	7.403	6.808	7.583	8.092	9.171	10.250
	0.9	11.234	10.628	10.665	10.310	10.132	10.340	9.171	10.250
	0.95	13.347	13.192	12.241	12.826	12.601	11.346	11.719	13.098
	0.99	19.603	18.520	18.468	18.298	16.758	17.183	15.659	17.501

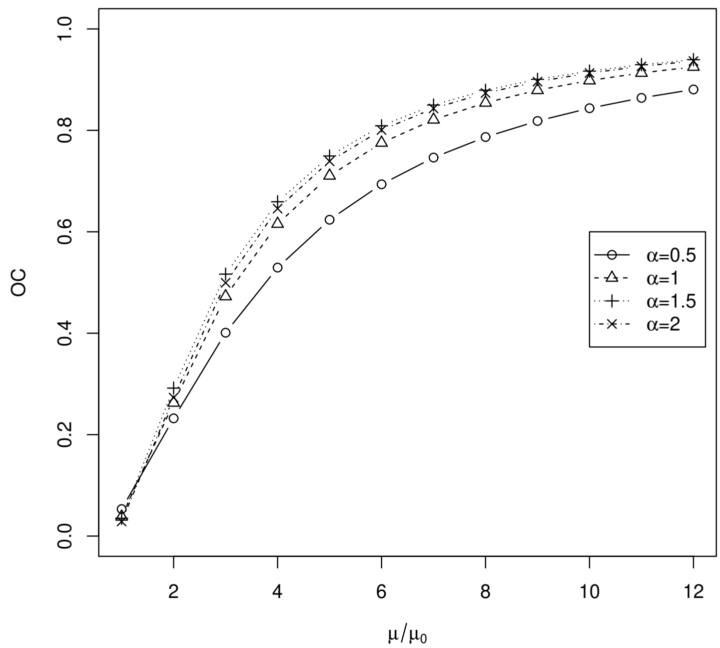

Figure 2. OC values versus μ/μ_0 for $t/\mu_0 = 0.9$ and $P^{\star} = 0.95$.

Table 4. OC values for single and double acceptance plans.

μ/μ_0	1	2	4	6	8	10	12
Single plan	0.04145847	0.2036135	0.4512356	0.5883046	0.6717407	0.7273800	0.7670101
Double plan	0.04932907	0.3370426	0.6962070	0.8330545	0.8954228	0.9285761	0.9481953

5. Applications

Example 1. We consider a real data regarding the lifetimes of 20 small electric carts (in months) used by the manufacturing company for internal transportation and delivery services. The data are as follows (see [34] and [2])

$$0.9,\ 1.5,\ 2.3,\ 3.2,\ 3.9,\ 5.0,\ 6.2,\ 7.5,\ 8.3,\ 10.4,\ 11.1,\ 12.6,\ 15.0,\ 16.3,\ 19.3,$$
$$22.6,\ 24.8,\ 31.5,\ 38.1,\ 53.0.$$

A Kolmogorov-Smirnov (K-S) test is used to check that the NH distribution fits the above data well. The K-S statistic is computed as $D = 0.052344$ with the corresponding p-value that equals 1. The maximum likelihood (ML) estimates of the scale and shape parameters are calculated as $\hat{\lambda} = 0.04518172$ and $\hat{\alpha} = 1.31386385$, respectively. Thus, we conclude that the two parameter NH distribution can model the above data perfectly.

Now, We test the null hypothesis $H_0 : \alpha = 2$ versus $H_1 : \alpha \neq 2$ using the likelihood ratio test (LRT). The ML estimate of λ under the null hypothesis is obtained as $\lambda_0 = 0.02512025$. The asymptotic LRT statistic $-2\lambda^*(\boldsymbol{x}) = 0.2657774$ with corresponding p-value 0.6061785 accept H_0.

Now, we check if the NH distribution with $\alpha = 2$ and $\lambda = \lambda_0$ fits the data. Several test such as the K-S test, Anderson-Darling (A-D) test and Cramér-von Mises (C-M) test are used to detect the suitability of $NH(2, \lambda_0)$ for this data. Also, the Akaike information criterion (AIC), Bayesian information criterion (BIC) and Hannan-Quinn information criterion (HQIC) are computed. The numerical results are given in Table 5. From Table 5, we observe that a $NH(\lambda_0, 2)$ distribution has a good fit the data.

Table 5. Goodness of fit statistics and the information criteria for the electric carts data.

	K-S	A-D	C-M	AIC	BIC	HQIC
Stataistic	0.08099	0.17115	0.02649	149.5264	150.5221	149.7208
p-value	0.9980	0.9965	0.9882			

Assume that we wish to estabilish the the mean lifetime μ of the product is at least 10 months at the confidence level of $P^\star = 0.95$. An experimenter wants to terminate the life test at $t = 7$ months under the zero and one failure schemes of the double sampling plan. From Table 1 , the minimum sample sizes are $n_1 = 6$ and $n_2 = 3$ corresponding to the value $t/\mu_0 = 0.7$. That is, first, 6 carts are put on test for 7 months and the lot will be accepted if no failure observed during the time. Reject the lot if more than one failure occurs during the test. When there is only one failure observed, a second sample of size 3 is drawn and put on test for 7 months. Accept the lot if there is one failure from the second sample and reject the lot, otherwise. Suppose that we observe the first 6 items as 0.9, 2.3, 5, 6.2, 11.1, 12.6, then lot should be rejected.

Example 2. Consider two simulated samples from $NH(1.5, 0.2)$ as follows:
First sample:

$$0.8182777,\ 1.6648015,\ 2.3456478,\ 3.5433390,\ 4.6009261,\ 7.6340567.$$

Second sample:

$$0.3660405,\ 1.4341841,\ 1.5433351,\ 4.5945382,\ 15.8271727.$$

Suppose the experimenter wants to set the mean lifetime of an item is $\mu_0 = 2.76$ unit for any quality characteristic at the confidence level of $P^\star = 0.9$. Consider the termination time of the life test at $t = 1.38$ under the zero and one failure schemes of the double sampling plan. From Table 1, the minimum sample sizes are $n_1 = 6$ and $n_2 = 5$ corresponding to the value $t/\mu_0 = 0.5$. Then, the design of double sampling plan for the time truncated life test is as follows:

1. Select a random sample of size $n_1 = 6$ from the lot and put the selected items on test for time $t = 1.38$.
2. Accept the lot if no failure observed during the time. The lot is rejected if more than one failure occurs during the test.
3. When there is only one failure observed, a second sample of size $n_2 = 5$ is drawn and put on test for time t.
4. Accept the lot if there are no failures from the combined sample of size $(n_1 + n_2 = 11)$ and reject the lot, otherwise.

We observe from the first 6 items that only one failure observed (there is one failure before time $t = 1.38$). Then, we draw a second sample of size 5. We see that the total number of failures from the combined samples is 2, Hence, we can reject the lot at this stage.

6. Concluding remarks

In this paper, we developed a dounle acceptance sampling plan based on the truncated life test for NH distribution. This distribution as a generalization of the exponential distribution is actually a special case of a three parameter model that had been introduced by [11] and has an increasing and decreasing hazard rate functions depending on the value of its shape parameter. The minimum sample sizes required to guarantee a certain mean lifetime of the test items is calculated. The values of operating characteristic function are obtained and analyzed. Also, the minimum ratio of true mean life to specified life for the acceptability of a lot with certain procedure's risk is provided. Three examples are provided for illustration the results. The computations of the paper were done using packages in R [25] such as AdequacyModel [22], nleqslv [19], goftest [17] and ADGofTest [10]. Designing of double sampling plans for some compound Nadarjah-Haghighi distributions is a future work.

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REFERENCES

1. Z. AL-Husseini, M. Naghizadeh Qomi, and S.M.T.K. MirMostafaee, Single acceptance sampling plan Based on truncated-life tests for the exponentiated moment exponential distribution with application in bladder cancer data, Iranian Journal of Health Sciences, vol. 11, no. 3, pp. 217-228, 2023.
2. A. D. Al-Nasser, and M. Ahsan ul Haq, Acceptance sampling plans from a truncated life test based on the power Lomax distribution with application to manufacturing, Statistics in Transition, New Series, vol. 22, no. 3, pp. 1-13, 2021.
3. A. D. Al-Nasser, and A. I. Al-Omari, Acceptance sampling plan based on truncated life tests for exponentiated Fréchet distribution, Journal of Statistics and Management Systems, vol. 16., no. 1, pp. 13-24, 2013.
4. A. D. Al-Nasser, and M. A. Obeidat, Acceptance sampling plans from truncated life test based on Tsallis q-exponential distribution, Journal of Applied Statistics, vol. 47, no. 4, pp. 685-697, 2020.
5. A. I. Al-Omari, and S. A. Al-Hadhrami, Acceptance sampling plans based on truncated life tests for extended exponential distribution. Kuwait Journal of Science, vol. 45, no. 2, pp. 30-41, 2018.
6. S.K. Ashour, A.A. El-Sheikh, and A. Elshahhat, Inferences and optimal censoring schemes for progressively first-failure censored Nadarajah-Haghighi distribution. Sankhya A, https://doi.org/10.1007/s13171-019-00175-2, 2020.
7. M. Aslam, Double acceptance sampling based on truncated life tests in Rayleigh distribution, European Journal of Scientific Research, vol. 17, no. 4, pp. 605-610, 2005.
8. J. Aslam, and C. H. Jun, A double acceptance sampling plan for generalized log–logistic distributions with known shape parameters, Journal of Applied Statistics, vol. 37, no. 3, pp. 405–414, 2010.
9. M. Aslam, D. Kundu, and M. Ahmad, Time truncated acceptance sampling plans for generalized exponential distribution, Journal of Applied Statistics, vol. 37, no. 4, pp. 555-566, 2010.
10. C.J.G. Bellosta, ADGofTest: Anderson-Darling GoF test, R package version 0.3, https://CRAN.R-project.org/package=ADGofTest, 2015.
11. T. Dimitrakopoulou, K. Adamidis, and S. Loukas, A lifetime distribution with an upside-down bathtub-shaped hazard functions, IEEE Transactions on Reliability, vol. 56, no. 2, pp. 308-311, 2007.
12. W. Gui, and M. Aslam, Acceptance sampling plans based on truncated life tests for weighted exponential distribution, Communications in Statistics - Simulation and Computation, vol. 46, pp. 2138-2151, 2017.
13. W. Gui, and M. Xu, Double acceptance sampling plan based on truncated life tests for half exponential power distribution, Statistical Methodology, vol. 27, pp. 123-131, 2015.
14. S. S. Gupta, and S. S. Gupta, Gamma distribution in acceptance sampling based on life tests, Journal of the American Statistical Association, vol. 56, no. 296, pp. 942-970, 1961.
15. S. S. Gupta, Life test sampling plans for normal and lognormal distributions, Technometrics, vol. 4, no. 2, pp. 151-175, 1962.
16. H. P. Goode, and J. H. K. Kao, Sampling procedures and tables for life and reliability testing based on the Weibull distribution (mean life criterion). TR-3, DTIC Document: US Department of Defense, Washington, D.C, 1961.
17. J. Faraway, G. Marsaglia, J. Marsaglia, and A. Baddeley, goftest: Classical goodness-of-fit tests for univariate distributions, R package version 1.2-3, https://CRAN.R-project.org/package=goftest, 2021.
18. C. Hamurkaroglu, A. Yigiter, N. Danacıoglu, Single and double acceptance sampling plans based on the time truncated life tests for the compound Weibull-exponential distribution. *Journal of the Indian Society for Probability and Statistics*, 21: 387-408.
19. B. Hasselman, nleqslv: Solve systems of nonlinear equations, R package version 3.3.2, https://CRAN.R-project.org/package=nleqslv, 2018.
20. X. Lu, W. Gui, and J. Yan, Acceptance sampling plans for half normal distribution under truncated life tests, American Journal of Mathematical and Management Sciences, vol. 32, no. 2, 133-144, 2013.
21. Y. Mahmood, S. Fatima, H. Khan, H. Amir, M. Khoo, and S. Teh, Acceptance sampling plans based on Topp-Leone Gompertz distribution, Computers and Industrial Engineering, vol. 159, p. 107-526, 2021.
22. P.R.D. Marinho , M. Bourguignon, C.R.B. Dias, AdequacyModel: Adequacy of probabilistic models and general purpose, R package version 2.0.0, https://CRAN.R-project.org/package=AdequacyModel, 2016.
23. M. Minić, Estimation of parameters of Nadarajah-Haghighi extension of the exponential distribution using perfect and imperfect ranked set sample, Yugoslav Journal of Operations Research, vol. 30, no. 2, pp. 177-198, 2020.
24. S. Nadarajaha, and F. Haghighi, An extension of the exponential distribution, Statistics, vol. 5, no. 6, 543-558, 2011.
25. R Core Team, R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria, 2022
26. G. S. Rao, Double acceptance sampling plans based on truncated life tests for the Marshall-Olkin extended exponential distribution, Austrian Journal of Statistics, vol. 40, no. 3, pp. 169–176, .

27. G.S. Rao, M. E. Ghitany, and R.R.L. Kantam, Acceptance sampling plans for Marshall-Olkin extended Lomax distribution, International Journal of Applied Mathematics, vol. 21, no. 2, pp. 315-325, 2008.
28. A. M. Ramaswamy, and P. Anburajan, Double acceptance sampling based on truncated life tests in generalized exponential distribution, Applied Mathematical Sciences, vol. 6, no. 64, pp. 3199-3207, 2012.
29. M. Saha, H. Tripathi, and S. Dey, Single and double acceptance sampling plans for truncated life tests Based on transmuted Rayleigh distribution, Journal of industrial and production engineering, vol. 38, pp. 356-368, 2021.
30. M. Saha, H. Tripathi, A. Devi, and P. Pareek, Applications of Reliability Test Plan for Logistic Rayleigh Distributed Quality Characteristic, Annals of Data Science, doi: 10.1007/s40745-023-00473-5, 2023.
31. M. Sobel, and J. Tischendrof, Acceptance sampling with new life test objectives. In: Proceedings of Fifth National Symposium on Reliability and Quality Control, pp. 108-118, 1959, Philadelphia.
32. H. Tripathi, S. Dey, and M. Saha, Double and group acceptance sampling plan for truncated life test based on inverse log-logistic distribution, Journal of Applied Statistics, vol. 48, no. 7, pp. 1227-1242, 2021.
33. H. Tripathi, M. Saha, and S. Halder, Single acceptance sampling inspection plan based on transmuted Rayleigh distribution. Life Cycle Reliability and Safety Engineering, vol. 12, pp. 111-123, 2023.
34. W. J. Zimmer, J. B. Keats, and F. K. Wang, The Burr XII distribution in reliability analysis, Journal of Quality Technology, vol 30, pp. 386-394, 1998.