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Economic growing quantity: Broiler chicken in Morocco

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Abstract In this work, we introduce an innovative mathematical model for growing items, with a particular emphasis on broiler chicken production within the context of Moroccan agriculture. The main goal of this study is to determine the optimal order quantity of items that the inventory should purchase to satisfy customer demand and the corresponding cycle duration. For the first time, a staircase function is used to depict the gradual reduction in item quantities over time due to the mortality during the rearing period. Furthermore, and for the first time, our model incorporates three distinct feeding types—Starter Feed, Grower Feed, and Finisher Feed-bringing it in line with real-world agricultural practices. A numerical example is provided, based on data extended from a Moroccan farm project, along with an analytic solution.

Keywords Economic growing quantity, staircase mortality function, variable cost feeding

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1. Introduction

The Economic Order Quantity (EOQ) is a widely used inventory management technique adopted by all types of businesses, from large corporations to small family-owned shops, to increase profits and minimize costs. This technique aims to secure the appropriate quantity of inventory is ordered for each batch, thereby preventing frequent orders of inventory and avoid of excess stock on hand. This is achieved by determining the optimal quantity of units a company should order to meet customer demand and providing the length of the ordering cycle. The inception of this methodology started in the second decade of the 19th century, with Harris introducing the first inventory model [1]. Despite the simplicity of the model, it revolutionized the way businesses manage their inventory. The model was refined and gained significant attention when it was independently rediscovered and popularized by Wilson [2] who developed the model by deriving mathematical formulas to calculate economic order quantities. The model has been used for years as a fundamental tool in inventory management, proving its value by offering a structured approach to balancing holding costs and ordering costs. However, when we apply the basic Economic Order Quantity (EOQ) model to real-life inventory management scenarios, various limitations emerge due to its assumptions. For instance, there exists a research stream on (EOQ) models applicable to perishable products such as poultry, fruit, and meat [3, 4, 5, 6], the traditional (EOQ) model is inadequate in handling these cases, as it inherently assumes that inventory items can be stored indefinitely. However, for perishable products, this assumption needs to be reconsidered. Deteriorated products (see [7, 8, 9, 10, 11, 12, 13]) present another challenge for the classical EOQ model, the implicit assumption is that stored items maintain their physical characteristics and remain in perfect condition throughout their tenure in inventory and remain with perfect quality all the time

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and that is not a line with reality. All these issues underscore the need to relax the assumptions inherent in the basic EOQ model. These imperative drives the exploration and formulation of models that are closer to reality, addressing the complexities of real-life inventory management scenarios. The Economic Growing Quantity model (EGQ) is one of the relaxed models of (EOQ) models. Jafar Rezaei [14] was the first researcher to work on this new extension model. He dealt with items that grow during the storage period, he incorporated several parameters in his model such as holding cost, feeding cost, production cost, setup cost, and purchasing cost. He used the logistic function (Richards' growth function) to describe the evolution of weight per age and the polynomial function to describe item feeding per time. The goal of this work was to determine the ideal quantity to order of newborn chick should the inventory buy and the most favorable day for slaughtering. Afterward, Zhang et al. [15] developed a novel model for growing items under carbon emissions. They kept the same functions used in [14] to describe their model. Later, extending from [14], Sebatjane [16] formulated lot sizing models for growing items based on three distinct hypotheses: in the first model, he considered imperfect quality items; in the second, he considered a limit capacity constraint. The last model dealt with incremental quantity discounts for larger quantities. For the first time, a screening period was considered. A year later, Sebatjane and Adetunji [17] formulated a novel model for growing items under the assumption of perishability, items of inferior quality were determined at the screening stage and sold at a discounted price in a single batch. Later, Sebatjane and Adetunji [18] formulated another EOQ model for growing items, this time with three echelons. They devised inventory control for three stages: farming, processing, and retail operations. The farming stage continued from the time the items were newborn until they reached the ideal weight for slaughter. In the processing stage, they converted the items into consumable products (slaughtering, cleaning, processing, and packaging). The last stage of this supply chain was the retail stage, where the product was offered for sale. Khalilpourazari and Pasandideh [19] generalized Rezaei's model and created the first multi-objective model for growing items. Different constraints were considered, encompassing on-hand budget, warehouse capacity, and total allowable holding cost constraints. They proposed two metaheuristic algorithms for solving the model. Malekitabar et al. [20] provided a suitable mathematical inventory model for growing-mortal items. The model consisted of a two-echelon supply chain, one for the supplier and the other for the farmer. For the first time, the Von Bertalanffy growth function was used as the growth function, and an exponential function was used to describe feeding. The model also took into consideration the mortality of items and deteriorating items during the holding period. Gharaei and Almehdawe [21] created an Economic Growing Quantity (EGQ) inventory model. They provided a general inventory model by considering both dead and live-grown items. Two uniform density functions were incorporated to represent the survival and mortality rates within the model. Makoena Sebatjane and Olufemi Adetunji [22] created a new mathematical model for growing items, this time with four echelons. They added a new stage named the screening period, during which they separated items with good quality from items with poorer quality. Items with poorer quality were sold at a discount, with considering the cost of disposal for mortality. Mokhtari, H., Salmasnia, A., & Asadkhani, J. [23] focused on the perishability of items during the consumption period and demonstrated its significant effect on total profit. They also considered the reduction in weight after the items were slaughtered. Unfortunately, previous models used survival percentages or continuous functions to approximate mortality, which may not provide a precise description because, in reality, mortality occurs in discrete steps, decreasing in an integer-based manner. Therefore, it's better to use a staircase function to depict mortality, which allows a very precise approximation of the whole time series. Moreover, previous models assumed that the cost of feeding is constant, but in reality, it depends on the age of the items; in other words, for every age, the item needs a specific type of food. In this research, we introduce a novel mathematical model for growing items. This work offers a comprehensive representation of broiler chicken growth stages and the consumption stage. For the first time, we employ a staircase function to describe the decrease in the number of items over time due to mortality during the rearing period. Additionally, we introduce a variable cost for feeding in the model to align with realworld conditions. We utilize the logistic function to illustrate the evolving weight of broiler chickens over time. Furthermore, in contrast to various models, we consider the holding cost during the growth period, which includes all expenses such as cleaning and security. When the items reach the targeted weight, A screening process is implemented to segregate items based on their quality. Items of inferior quality are then collectively sold in a single batch at a discounted price. the proposed model is used to assist Moroccan firms specializing in broiler chickens. The results obtained are satisfactory, and the proposed model has demonstrated its ability to faithfully represent the phenomenon under study. The organization of this paper is outlined as follows: The initial section offers a concise overview of broiler chicken. The notations and assumptions employed in formulating the mathematical model are also presented. Following that, we present the main concepts of our model by depicting the different costs and revenues that are considered in our study, in section 3 , we give the general mathematical alongside the constraint that needs to be verified. an algorithm is illustrated in section 4 followed by a numerical result.

2. Assumptions and notations

2.1. Broiler chicken

Figure 1. Broiler chicken

Broiler chicken (Figure 1) is the primary source of meat in Morocco and the broader African continent [24], in 2021, there were around 217 million chickens in Morocco, an increase of approximately four million heads compared to the previous year. In comparison, the chicken stock amounted to 170 million heads in 2010 . This prevalence returns to multiple reasons such as its rapid growth and efficient conversion of feed into meat, it's cheap price & being an excellent source of high-quality protein. All these factors make Broiler chicken an attractive option in terms of investment and consumption, it contributes to meeting dietary requirements and promoting overall health, which makes it a factor that resonates with consumers seeking wholesome food options, especially for people suffering from some diseases such diabetes whose need protein-rich food with minimal carbohydrates like broiler chicken [25].

Our research is concentrated on broiler chicken reared in Morocco exactly at Casablanca, the favorite weight of broiler chicken at that zone is $2\mathrm{Kg}$, this study includes all costs needed to establish a project of broiler chicken from the start to the finish. All data used in this application is extracted from [26].

2.2. Assumptions

The model derivation is based on the following assumptions:

- The annual demand for the item remains constant throughout time.
- Shortages are not permitted.
- The items ordered have the ability to undergo growth before the slaughtering process
- The cost of feeding the items is assumed to be directly correlated with both the weight gained by the items and the specific type of food.
- A random fraction of the slaughtered items is of poorer quality.
- Items of inferior quality are sold in the market as a single batch.
- Mortality of items is considered during the growth period.
- Disposal of mortality is not considered.
- Workers are compensated for every chick that reaches the targeted weight.

2.3. Notations

The subsequent notations, encompassing the parameters, are employed for the mathematical formulation of the EGQ model:

Table 1. Parameters and decision variables

Parameters	
$\mathbf{w_0}$	New-born chicks' weight (g)
$\mathbf{w_1}$	Item's weight at time (g)
D	Annual demand rate of item (g/year)
$\boldsymbol{\alpha}$	Asymptotic weight (g)
$\boldsymbol{\beta}$	Exponential growth rate per unit time
$\boldsymbol{\lambda}$	Exponential growth rate per unit time
$\mathrm{E(y)}$	Fraction of Items of inferior quality
$\mathrm{f(.)}$	Probability density function of a variable
Decision Parameter	
y	Number of new-born chicks ordered per cycle
$\mathrm{E(T)}$	Duration of the consumption period (years)
y_i	Number of items that still alive at $[\mathrm{t'}_i, \mathrm{t'}_{\mathrm{i+1}}]$
$\mathrm{y_n}$	Number of items that still alive at the end of rearing period
Cost components	
p	Cost of purchasing a single unit item (Dh/g)
s	Cost of selling a single unit good quality item (Dh/g)
v	Cost of selling a single unit Items of inferior quality (Dh /g)
H	Cost of holding a single unit item at consumption period (Dh /g/year)
$\mathrm{h_i}$	Cost of holding a single unit item at growth period (Dh)
K	Cost of setting up per growing cycle (Dh)
z	Cost of screening of each unit (Dh /g)
r	Rate of screening (g/year)
q	Vaccination cost
$\mathrm{c_1}$	Cost of Starter Feed (Dh /g)
$\mathrm{c_2}$	Cost of Grower Feed (Dh /g)
$\mathrm{c_3}$	Cost of Finisher Feed (Dh /g)
$\mathrm{m_i}$	Mortality rate between $[t'_i, t'_{i+1}]$
Time period:	
$\mathbf{t_1}$	Length of the growing period (years)
$\mathbf{t_2}$	Screening time (years)
$\mathbf{t_3}$	Duration of the selling period for the items (years)
$\mathbf{t'_i}$	subdivision of the time interval $[0, \mathrm{t_1}]$
$\mathbf{t_s}$	Time of setup (years)

it's essential to emphasize that all subsequent sections are built upon these assumptions and adhere to the notational framework.

3. General model: main concepts

At the initiation of the cycle, the inventory procures y items. The initial weight of each of these items at that moment is w_0. These items are reared until they reach the target weight w_1 during a period t_1. Once they attain the desired weight, the items are then slaughtered, and mortality during the feeding period is considered.

To separate the items with poorer quality from the items with good quality, a screening period takes place with a screening rate r. The fraction of items with poorer quality is sold in a single batch at a discounted price. It is assumed that items with poorer quality follow a random variable with a known distribution, $f(y)$, with an expectation denoted as $E(y)$.

At the conclusion of the screening period t_2, customer demand arises, and the inventory endeavors to meet customers' requests. This stage persists for a duration of T, which represents the anticipated duration of the consumption period, represented as $E(T)$.

$$E(T) = t_2 + t_3$$

The general mathematical model that gives the total profit is written as follows:

$$\text{Total profit } (TP) = \text{ Total Revenue } (TR) - \text{ Total Cost } (TC)$$

In short, we can write $E(TP) = TR - TC$. In the following, we give the formula of the two main components of TP which are Total revenue and total cost.

Total revenue

Given the selling price of a good-quality item as s and the selling price of Items of inferior quality as v, the total revenue can be expressed as follows:

TR: total revenue $= \mathrm{s} \cdot \mathrm{y_n} \cdot w_1 \cdot (1 - E(y)) + v \cdot y_n \cdot w_1 \cdot E(y)$

Total cost

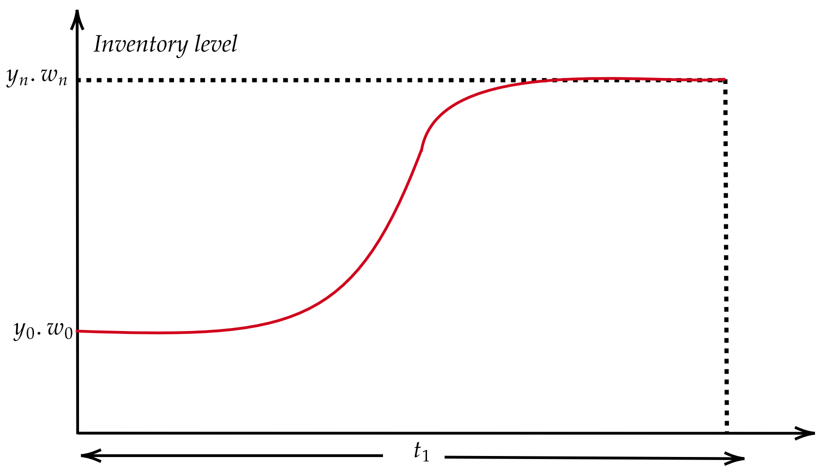

Figure 2. The evolution of the weight of broiler chicken per time..

As shown in figure 2, the weight undergoes a slow initial increase, gradually escalating over time until the items achieve the targeted weight. Subsequently, the weight gain decelerates, and a logistic function is employed to depict this progression. It's written as follows:

$$w_t = \frac{\alpha}{1 + \beta e^{-\lambda t}}$$

Noted that $\mathrm{w_t(t=0) = w_o}$.
The duration time needed to reach the targeted weight w_1 is t_1 which is equal to:

$$\mathrm{t_1} = -\ln\left(\frac{1}{\mathrm{b}} \cdot \left(\frac{\alpha}{\mathrm{w_1}} - 1\right)\right)/\lambda$$

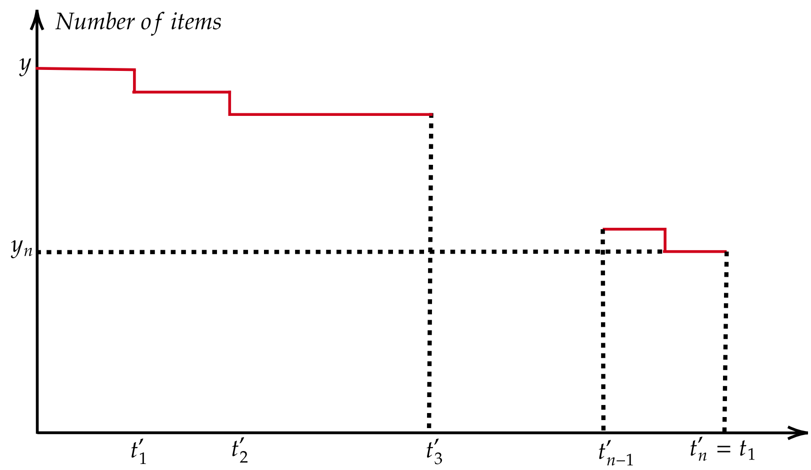

Figure 3. Number of items per time.

The figure 3 illustrates the number of items that are still alive at the period t_i, the mortality rate between $[\mathrm{t'_i, t'_{i+1}}]$ is equal to $\mathrm{m_i}$. The staircase function is used to depict this decreasing over time due to the mortality.

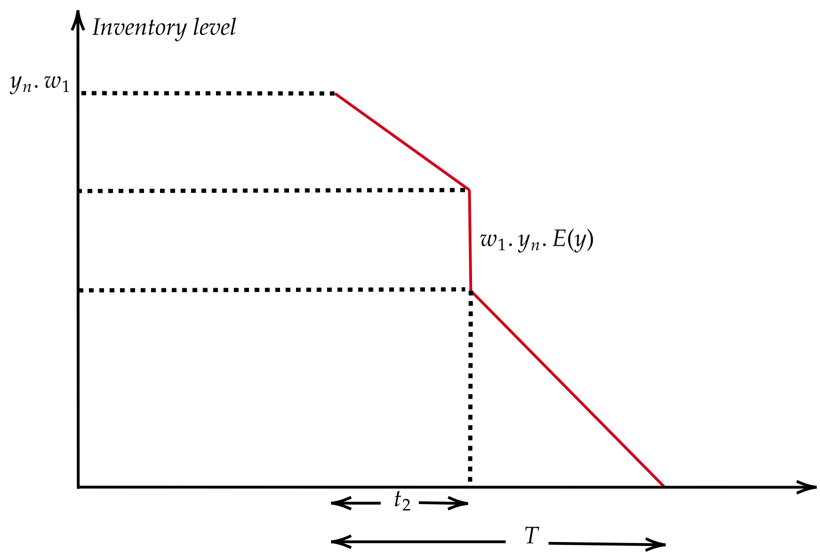

Figure 4. The consumption period.

the figure above (4) represents the consumption period in which the slaughter items are prepared to be served to customers. During this period, a screening process is considered in which they separate items with good quality from items with poorer quality, this screening period is extended to t_2 which equal to $\frac{y_n \cdot w_1}{r}$. The expected length of the consumption period is denoted as E(T) and is assumed to equal

$$\mathrm{E(T)} = \frac{\mathrm{y_n \cdot w_1 \cdot (1 - E(y))}}{\mathrm{D}} \tag{1}$$

Number of items

Certainly, over time, the number of items has reduced due to mortality. Let's denote the mortality rate as m_i. It's reasonable to assume that the mortality rate varies with age. Based on this, we divide the growth period into n intervals, and in each interval, we represent the number of items that are still alive as y_i (as shown in 3).

The function that describes the number of items can be written as:

$$\begin{cases} \mathrm{y_0 = y} \quad 0 < \mathrm{t} < \mathrm{t}'_1 \\ \mathrm{y_i = y \cdot \prod_{j=1}^{i} (1 - m_j)} \quad \mathrm{t}'_\mathrm{i} < \mathrm{t} < \mathrm{t}'_{\mathrm{i+1}} \end{cases}$$

Purchasing cost

The inventory purchases y items at the beginning of the cycle, and it is assumed that the price of the newborn items is p each, with an initial weight of w_0. Therefore, the purchasing cost can be calculated as follows:

$$\text{PC: the purchasing cost } = \mathrm{p \cdot y} \cdot w_0$$

setup cost

This cost is referred to as all the expenses associated with actually ordering the inventory. It includes costs such as packaging, delivery, shipping, and other related expenses. Additionally, there is a fixed setup cost, denoted as K.

$$SC: \text{ The setup cost } = K$$

Feeding cost

As newborn animals undergo growth, the feeding costs fluctuate over time. We assume that the feeding cost is influenced by the number of items, their weight, and the type of food they require. To compute the feeding cost, we use the previous subdivisions to determine the number of items that are still alive in each period. Then, we multiply this number by the weight of the items from the same period and multiply it by the cost associated with the type of food required during that period.

To specify the weight, various functions can be used, and one of the most commonly employed models is the logistic function [27]. This function enables us to describe the weight gain per unit of time for poultry. Setting up this function requires three parameters denoted as α, β, and λ, which represent the asymptotic weight of the items, the integration constant, and the exponential growth rate, respectively. The growth function of the items is given by the equation shown in Fig. 1.

The logistic function is written as:

$$w_t = \frac{\alpha}{1 + \beta \cdot e^{-rt}}$$

$$FC: \text{ Feeding cost } = \sum_{i=0}^{n} c_{[t'_i, t'_{i+1}]} \cdot y_i \cdot \int_{t'_i} w_t dt \quad i = 0, \ldots, n-1$$

holding cost

We distinguish between two types of holding costs: one during the growing period and the other during the consumption period. The holding cost during the growing period includes all costs incurred to rear the items, such as shelter, cleaning, and the cost of caretakers, among others. The holding cost during the consumption period consists of the costs incurred while storing the mature items and any additional charges during the screening phase. It appears that the holding cost during the growth period is influenced by the number of items in each distinct stage of the growth process. The holding cost can be expressed as:

$$H_g = \sum_{i=1}^{n-1} h_i \cdot y_i$$

Where h_i represents the holding cost at $t'_i < t < t'_{i+1}$
Based on the figure 4, the holding cost at the selling period could be written as:

$$H_s = H \cdot \left(E(T) \cdot \frac{y_n \cdot w_1 \cdot (1 - E(y))}{2} + \frac{w_1^2 \cdot y_n^2 \cdot E(y)}{r} \right)$$

The total expects holding cost is:

$$H_g + H_s$$

screening cost

This refers to all fees associated with evaluating the quality of items, with the aim of distinguishing between items of good quality and those of inferior quality.

$$Zc = z \cdot y_n \cdot w_1$$

Vaccination cost

it's referred to the cost of vaccinating each item at the beginning of the cycle.

$$VC = q \cdot y$$

4 Proposed mathematical model

The anticipated total profit value can be expressed as:

$$E(TP) = TR - ZC - SC - H_g - PC - H_s - FC - VC$$

Substitute the total revenue and each cost by its values to find

$$E(TP) = \text{ s.}\, y_n \cdot w_1 \cdot (1 - E(y)) + v \cdot y_n \cdot w_1 \cdot E(y) - z \cdot w_1 \cdot y_n - K - h \cdot y_n -$$
$$\text{p.y.}\, w_0 - H \cdot \left(\frac{E(T) \cdot y_n \cdot w_1 \cdot (1 - E(y))}{2} + \frac{w_1^2 \cdot y_n^2 \cdot E(y)}{r} \right) - \sum_{i=0}^{n} c_{[t'_i, t'_{i+1}]} \cdot y_i \cdot \int_{t'_i}^{t'_{i+1}} w_t dt - \text{q} \cdot \text{y}$$

The anticipated total profit per unit time, E[TPU]:

$$E(TPU) = \frac{E(TP)}{E(T)} = s \cdot D + \frac{v \cdot D \cdot E(y)}{1 - E(y)} - \frac{z \cdot D}{1 - E(y)} - \frac{K}{E(T)} - \frac{D \cdot h}{w_1(1 - E(y))}$$
$$- \frac{p \cdot D \cdot w_0}{w_1(1 - E(y)) \prod_{i=0}^{n-1} (1 - m_i)} - H \cdot \left(\frac{E(T) \cdot E(y) \cdot D^2}{r(1 - E(y))^2} + \frac{D \cdot E(T)}{2} \right)$$
$$- \frac{D \cdot \sum_{i=0}^{n-1} c_{[t'_i, t'_{i+1}]} \cdot \prod_{j=0}^{i} (1 - m_j) \cdot \int_{t'_i}^{t'i+1} w_t dt}{w_1(1 - E(y)) \prod_{i=0}^{n-1} (1 - m_i)} - \frac{q \cdot D}{w_1(1 - E(y)) \prod_{i=0}^{n-1} (1 - m_i)}$$

To determine the optimal $E(T)$, we solve $\frac{\partial E(TPU)}{\partial E(T)} = 0$

We find that:

$$E(T) = \sqrt{\frac{K}{\frac{H \cdot E(y) \cdot D^2}{r(1-E(y))^2} + \frac{H \cdot D}{2}}} \tag{2}$$

Constraint

Two constraints are essential to guarantee the feasibility of our model. The first constraint is designed to ensure that the items are prepared for consumption at the designated time, while the second ensures the prevention of shortages during the screening period.

- Contraints 1: $t_1 + t_s \leq E(T)$

$$E(T) \geq -\frac{\ln\left(\frac{1}{\beta}\left(\frac{\alpha}{w_1} - 1\right)\right)}{\lambda} + t_s = T_{\min}$$

- Constraint 2: To avoid a shortage, the quantity of items with good quality must be at least equal to demand at the screening period.

Assume that the quantity of goods is $N\left(y \cdot w_1, E(y)\right) = y \cdot w_1 \ E(y) \cdot y \cdot w_1$.
So, the constraint written as follow:
So

$$N\left(y \cdot w_1, E(y)\right) \geq D \cdot t_2$$
$$E(y) \leq 1 - \frac{D}{r}$$

By calculating the second derivative of $E(TPU)$ we find:

$$\frac{\partial^2 E(TPU)}{\partial E(T)^2} = \frac{-K}{E(T)^3} < 0$$

So, our problem is concave that implied that E(T) is the maximum point that maximizes E(TPU)

5 Numeric application

The proposed model is used to assist Moroccan firms specializing in broiler chickens. The data were extracted from the website agrimaroc.ma [26].

5.1 Algorithm

The proposed model was solved using the following algorithm :

Step 1. Calculate the values of $\boldsymbol{t_1}$ and $\boldsymbol{T}_{\text{min}}$.
Step 2. Check the feasibility of the problem by examine the validity of the two constraints.
If $\boldsymbol{T}_{\text{min}} \geq \mathbf{0}$& if $\boldsymbol{E}(\boldsymbol{y}) \leq \mathbf{1} - \frac{\boldsymbol{D}}{\boldsymbol{r}}$ are verified, the problem is feasible.
Go to step 3. else go to step 6
Step 3. Compute E[T].
Step 4. If $\boldsymbol{E}(\boldsymbol{T}) \geq \boldsymbol{T}_{\text{min}}$, $\boldsymbol{T}^* = \boldsymbol{E}(\boldsymbol{T})$ else $\boldsymbol{T}^* = \boldsymbol{T}_{\text{min}}$.
Step 5. Compute $\boldsymbol{y}^*$ and $\boldsymbol{E}(\boldsymbol{TPU})$ using equation 1 & 2 .
Step 6. End.

Table 2. Algorithm

First, we calculate t_1 and T_{min}; then, we check the feasibility of the problem by examining the validity of the two constraints; after, we compute the mean of T which we compare to T_{min}; as the last step, we compute y^* and $E(TPU)$ using the equation 1 & 2

5.2 Numerical results

From the collected data we estimate different parameters of the proposed model :
$\alpha = 4500$ g $\lambda = 51/$ year $s = 0.015\text{Dh/g}$ $h = 0.0015\text{Dh}$
$w_1 = 2$ kg$w_0 = 43$ g$D = 1.0178.10^{10}$ g/ year $\beta = 104\text{H} = 0.004\text{DH/g/}$
year $K = 3.3\text{MDh}$ $p = 0.003\text{dh/g}$ $v = 0.01\text{Dh/g}$ $z = 0.001\text{DH/g}$ $r =$ 3.2.10 ^{10} g/ year $E(y) = 0.02$ $\text{t}_\text{s} = 0.1; \text{q} = 0.0005\text{Dh}$

Table 3. Mortality rate per week.

days	rate
$0 - 7$	1% to $\mathbf{3}\%$
$7 - 14$	1% to $\mathbf{2}\%$
$15 - 25$	0.5% to $\mathbf{1}, \mathbf{5}\%$
$25 - 32$	0.5% to $\mathbf{1}, \mathbf{5}\%$

Table 4. Type of Food.

Type of Food	Duration	Coste
Starter Feed	$0 - 14$ day	9.10^{-5}DH/g
Cost of Grower Feed	$14 - 25$ day	$3.64.10^{-4}\text{DH/g}$
Cost of Finisher Feed	$25-$ day	$4.30.10^{-5}\text{DH/g}$

Table 3 provides the mortality rate per week. The mortality rate decreases with the age of the items because the more mature the broiler chickens become, the more resistant they are to disease and climate.

The logistic function:

$$w_t = \frac{4500}{1 + 104.e^{-51t}}$$

It's need $t_1 = 0.087$ to reach to the targeted weight 2Kg

Table 5. Optimal solution of the proposed model in the broiler chickens' case of study

Variable	Unit	Value
$\boldsymbol{t_1}$	Year	0.087 year ($\cong$ 32 day)
$\boldsymbol{t_2}$	Year	0.13(48 day)
$\boldsymbol{E(T)}$	Year	0,4 year ($\cong$ 146 days)
$\boldsymbol{y_n}$	Items	2077142.85($\approx$ 2077142)
$\boldsymbol{y}$	Items	2195501
$\boldsymbol{E(TPU)}$	DH/YEAR	127098534.9 DH/year

First, the optimal solution informs the investor that he needs 32 days to reach the targeted weight. In addition, 48 days is sufficient to start the screen actions. As our model recommends a number of items of 2195501, the proposed strategy concerns a big firm. The optimum total investment period is almost 146 days.

6 Conclusion

In summary, the main objective of this paper was to create a more realistic model that accurately represents real-world scenarios. This involved incorporating two new parameters: the utilization of a staircase function to illustrate mortality over time and the consideration of a non-constant feeding cost. The use of percentage of dead items to model mortality simplifies the theoretical framework but has significant implications for overall cost estimation. Additionally, mortality, being non-continuous and unpredictable, presents challenges for precise representation with traditional continuous functions like Gompertz or Weibull. These challenges highlight the suitability of the staircase function for depicting mortality and capturing sudden changes, providing a more faithful representation overall. Moreover, our model incorporated three distinct feeding types — Starter Feed, Grower Feed, and Finisher Feed-, where the feeding type varied based of the age of items. This addition enhances the realism of our model, aligning it more closely with the complexities of real-world scenarios . In conclusion, our model represents a significant improvement over traditional methods, as demonstrated through a practical case study from a Moroccan farm project. This application showcases the model's effectiveness in better reflecting the intricacies of real-world situations, reinforcing its value in enhancing the accuracy of mortality and cost modeling.

Future scop

We suggest considering an EOQ (Economic Order Quantity) model with multiple cycles and one purchase, where items from the first cycle will supply us with items for the subsequent cycle. This approach aims to reduce costs and increase overall profit, taking into account other products such as eggs, milk, etc. In addition, we can generalize the model to multiple items considering uncertain demand. By incorporating some algorithms such as FP-Growth (see [28]), recurring patterns and association rules can be discovered in transactional datasets. Therefore, choosing the most requested items and most related.

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