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Optimal image 2D/3D by Krawtchouk moments and ABC algorithm

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Abstract In our research, we provide an enhanced and effective Krawtchouk moments parameter p optimization approach using the Artificial Bee Colony (ABC) algorithm optimization technique for the purpose of completing tasks including the reconstruction and classification of 3D and 2D images with high quality. The proposed method, which uses the ABC algorithm to generate a suitable parameter based on Krawtchouck moments, aims to accurately characterize the image as a vector known as the descriptor vector of moments, which is then used to reconstruct the image for each order. The simulation and analysis of the findings demonstrate the high quality and effectiveness of the suggested KM-ABC approach for image reconstruction, as well as its excellent accuracy in images with and without noise.

Keywords Artificial Bee Colony algorithm, Krawtchouk moment, image reconstruction, image classification.

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1. Introduction

The use of images on digital devices has increased significantly in recent years. The outcome resulting from this increase is the enormous storing of data across various domains. Several tasks are handled in the field of image processing, including image reconstruction [30], image classification [31], and image segmentation [32], among others. Orthogonal moment methods allow an image to be expressed in a reduced form, i.e. in the form of a descriptor vector for each image that preserves information and eliminates redundancy. This descriptor vector can be used to reconstruct the image and classify it using machine and deep learning methods. Image reconstruction [6, 7, 8] image classification [9], [10], face and pattern recognition [11], [12], image compression [13], [14], image watermarking [15], [16], image registration [17], and image indexing [18] are only a few of the several tasks for which discrete orthogonal moment is useful. Orthogonal moments can be extracted from two types of orthogonal polynomials: discrete orthogonal moment (DOM) and continuous orthogonal moment (COM). The latter category (COM) includes Legendre [20], Fourier-Mellin [21], Zernike [19], Gaussian-Hermite [23], and pseudo-Zernike [22] polynomials. However, using COM kind presents a difficult challenge due to integral calculation errors. On the other hand, the recursive approach of calculating the discrete orthogonal moment can prevent the spread of numerical errors. Numerous Discrete Orthogonal Moments (DOM) kinds exist, including Dual Hahn [27], Tchebichef [24], Krawtchouk [25], Hahn [26], and Racah [28]. While medical image reconstruction has employed a range of transforms, including wavelet, contourlet, and total variation (TV), among others, a method employing an adaptive wavelet tight frame for reconstructing magnetic resonance images [29], reconstructing images from incomplete convolution data using total variation regularization [30]. This work devoted to orthogonal Krawtchouk

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moment (KM) which has a parameter appears in in calculation of orthogonal Krawtchouk polynomials. This parameter has the influence in the distribution of moments. We used the Artificial Bee Colony (ABC) optimization approach to identify the suitable parameter.

Peak Signal-to-Noise Ratio (PSNR) and Structural Similarity Index Measure (SSIM) measurements were utilized to evaluate the quality of the reconstructed images. The cost function to be maximized in optimization problem combine PSNR and SSIM, which provides an optimal parameter . However, the remainder of the work is organized as follows: the generalities of discrete orthogonal Krawtchouk moments are clarified in the second part, which describe the expression of orthogonal Krawtchouk polynomials and the parameters depends on, and method of calculation of orthogonal Krawtchouk moment in two cases, by the direct calculation with hypergeometric, and the recursive method. The third section introduces the optimization method used called Artificial Bee Colony (ABC), and defines the cost function of the Krawtchouk parameter depending on the quantities PSNR and SSIM. The fourth section is devoted to the analysis and results of the experiments, illustrating the intended effect of the method KM-ABC and providing a comparison with the earlier approach. The final section is for the conclusion and perspectives of future works.

2. Generalities of discrete orthogonal Krawtchouk moments

2.1. Discrete orthogonal Krawtchouk polynomial

The classical Krawtchouk polynomials of degree n given by

$$K_n(x;p,N) = \sum_{k=0}^{N} a_{k,n,p} x^k = {}_2F_1\left(-n,-x;-N;\frac{1}{p}\right) \tag{1}$$

Where $x, n = 0, 1, 2, \ldots, N-1$, with $N > 0$ and $0 \leq p \leq 1. {}_2F_1$ represents the hypergeometric function given by:

$${}_2F_1\left(x_1, x_2; y_1; z\right) = \sum_{i=0}^{\infty} \frac{(x_1)_i\,(x_2)_i}{(y_1)_i\, i!} z^i \tag{2}$$

$(x)_i$ is the Pochhammer symbol given by:

$$(x)_i = x(x+1)(x+2)\ldots(x+i-1), \quad k \geq 1 \text{ and } (x)_0 = 1 \tag{3}$$

The weight function is defined by,

$$w(x,p,N) = \left(\begin{array}{c} N \\ x \end{array}\right) p^x (1-p)^{N-x} \tag{4}$$

The orthogonality condition is given by:

$$\sum_{x=0}^{N-1} w(x,p,N-1) K_n(x,p,N-1) K_m(x,p,N-1) = \rho(x,p,N-1)\delta_{nm} \tag{5}$$

Where $n, m = 1, 2, \ldots, N-1, \delta_{m,n}$ is the Kronecher function, and the normalization factor $\rho(x,p,N-1)$ presented by:

$$\rho(x;p,N) = (-1)^x \left(\frac{1-p}{p}\right)^x \frac{n!}{(-N)_x} \tag{6}$$

To avoid the computational errors provided by the hypergeometric function, in [4] they presented a recursive relation for classical Krawtchouk polynomials. The formula is given by:

$$\begin{aligned} & K_n(x,p,N-1) = \frac{(N-1)p-2(n-1)p+n-1}{p(N-n)} K_{n-1}(x,p,N-1) \\ & -\frac{1-p}{p}\frac{n-1}{N-n}K_{n-2}(x,p,N-1) \\ & K_0(x,p,N-1)=1 \\ & K_1(x,p,N-1) = 1-\frac{x}{(N-1)p} \end{aligned} \tag{7}$$

To solve the computational error problem in calculation of moments using the classical polynomials provided by (7), authors in [31] present the normalized Krawtchouk polynomials. Equations (4) and (6) provide the normalization factor and weight funtion, which are used to obtain the normalized polynomials that can be obtained by:

$$\bar{K}_n(x,p,N-1) = K_n(x,p,N-1)\sqrt{\frac{w(x,p,N-1)}{\rho(n,p,N-1)}} \tag{8}$$

The recursive relation for normalized Krawtchouk moments is defined as follows:

$$\begin{gathered} \bar{K}_n(x,p,N-1) = A_n\bar{K}_{n-1}(x,p,N-1) - B_n\bar{K}_{n-2}(x,p,N-1) \\ \text{where } A_n = \frac{(N-1)p-2(n-1)p+n-1-x}{\sqrt{p(1-p)n(N-n)}} \text{ and } B_n = \sqrt{\frac{(n-1)(N-n+1)}{(N-n)n}} \\ \bar{K}_0(x,p,N-1) = \sqrt{w(x,p,N-1)} \\ \bar{K}_1(x,p,N-1) = \sqrt{w(x,p,N-1)}\frac{(N-1)p-x}{\sqrt{(N-1)p(1-p)}} \end{gathered} \tag{9}$$

The graphics of the first five orders of Krawtchouk polynomials are presented in Figure 1.

2.2. *Krawtchouk moments (KM)*

The Krawchouk moments are the projections based on orthogonal Krawtchouk polynomials, introduced by the mathematician Ukrainian, Mikhail Krawtchouk [3], which are based on polynomial distributions. The Krawtchouk polynomials' distribution shape is controlled by changing the variable p.

The Krawtchouk moment, for the order $(n+m+l)$ is expressed as a function of Krawtchouk polynomials and pixel intensity values expressed by $f(x,y,z)$, and the formula is given as follows:

$$\begin{aligned} & M_{mnl} = \frac{1}{\rho(n,p_1,N)\,\rho(m,p_2,N)\,\rho(l,p_3,N)} \\ & \times \sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\sum_{z=0}^{N-1} K_n(x,p_1,N)\,K_m(y,p_2,N)\,K_l(z,p_3,N)\,f(x,y,z) \end{aligned} \tag{10}$$

After normalization polynomials, the moment calculation formula for 3DN × N × N image with pixel intensity function $f(x,y,z)$, will be modified as follows:

$$M_{mnl} = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\sum_{z=0}^{N-1} \bar{K}_n(x,p_1,N)\,\bar{K}_m(y,p_2,N)\,\bar{K}_l(z,p_3,N)\,f(x,y,z) \tag{11}$$

The images are reconstructed from the calculated moments of the original image, by the form given as follows:

$$f(x,y,z) = \sum_{n=0}^{N\max}\sum_{m=0}^{N\max}\sum_{l=0}^{N\max} \bar{K}_n(x,p_1,N)\,\bar{K}_m(y,p_2,\mathrm{N})\,\bar{K}_l(z,p_3,N)\,M_{nml} \tag{12}$$

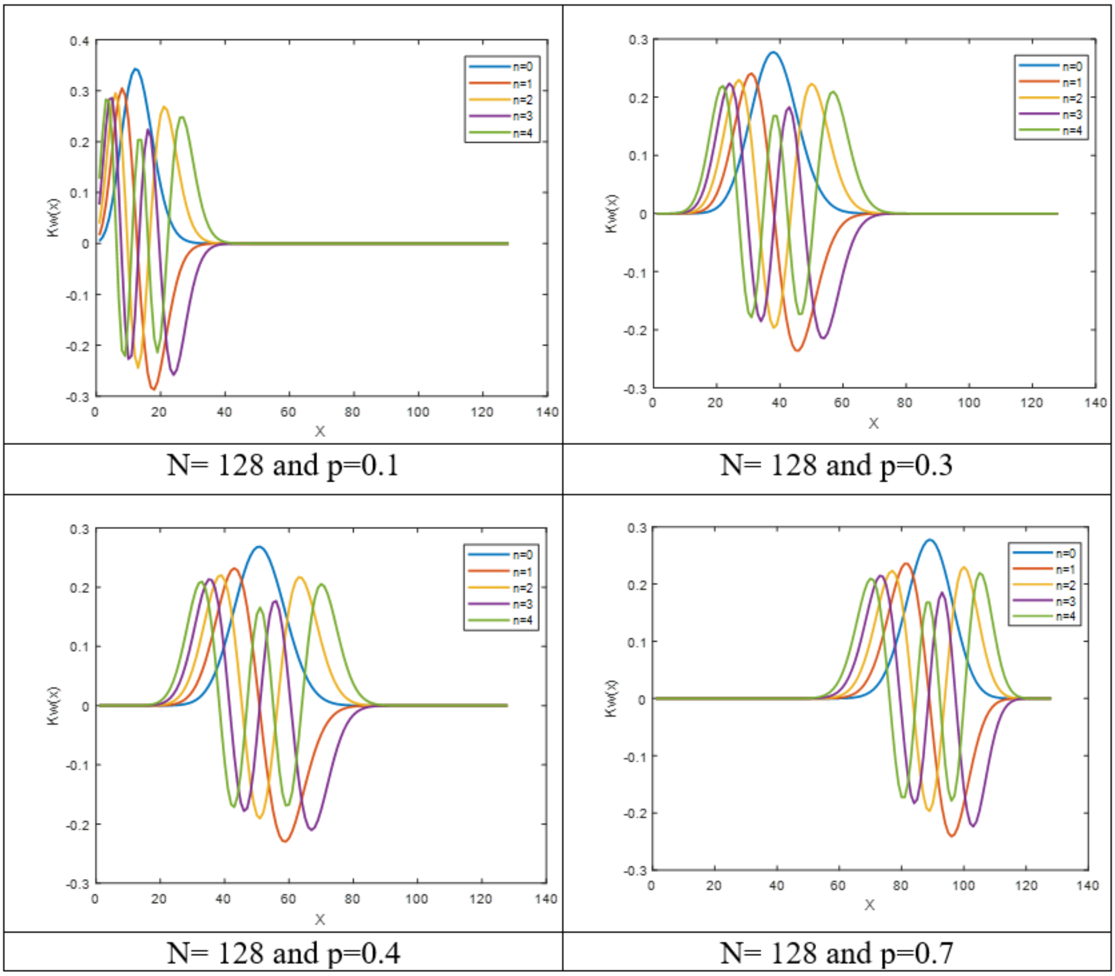


Figure 1. Krawtchouk polynomial graphs for various values of p.

3. Optimal image 3D by Krawtchouk moments and ABC algorithm

3.1. Artificial Bee Colony (ABC)

Among the well-known swarm intelligence algorithms created by Karaboga [5] is the artificial bee colony (ABC). The three kinds of bees in ABC are the employed bee, the spectator (onlooker) bee, and the scout bee. During the search phase, various bees play distinct roles. The following is a presentation of the main search stages:

- **Initialization:** The algorithm starts by initializing a population of artificial bees, where every bee stands for a possible solution to the optimization issue. It's common to refer to these solutions as "food sources."
  With n choice variables, let $v_i = (v_{i1}, v_{i2}, \ldots, v_{in})$ represent the $i-th$ food source in the population. The following is the generation of each food source to initialize the population:

$$v_{ij} = v_{ij}^{\min} + r \times \left(v_{ij}^{\max} - v_{ij}^{\min}\right), \quad j = 1, \ldots, n; i = 1, \ldots, SN \tag{13}$$

  $v_{ij}^{\max}$ and $v_{ij}^{\min}$ represent the upper and lower bounds for the decision variable j, respectively. While r is a uniform random number in $[0, 1]$.
- **Employed bee phase:** The i - th employed bee in the employed bee phase is given the $i-th$ food source (v_{ij}) in the population. It then creates a new nearby solution in the following manner surrounding the assigned

food source.

$$v_{\text{new } j} = v_{ij} + \varphi(-1,1) \times (v_{ij} - v_{kj}) \tag{14}$$

Where $i \in \{1, \ldots, SN\}$, and $k \in \{1, \ldots, i-1, i+1, \ldots SN\}$ randomly selected food source does not equal i.
Following the generation process, a new solution v_{new} is assessed and compared with v_i; the solution with the higher fitness value is chosen.

- **Onlooker bee stage:** Onlooker bees (Observer bees) are in charge of looking for fresh ideas based on the activities of working bees. The neighbor of a few good solutions, chosen using a probability approach, is what the observer bees actively search for, in contrast to the employed bee stage. The onlooker bees employ the probability values to accomplish this.

$$p_i = \frac{\text{fit}_{\ i}}{\sum_{i=n}^{SN} \text{fit}_{\ i}} \tag{15}$$

Where fit $_i$ is the fitness value of the i^{th} food source. The fitness value is used to evaluate the quality of solutions. A higher fitness value denotes a superior option. Below is the calculation for the fitness value.

$$fit\,(X_i) = \begin{cases} \frac{1}{1+f(X_i)}, & \text{if } f\,(X_i) \geq 0 \\ 1 + |f\,(X_i)|\,, & \text{if } f\,(X_i) < 0 \end{cases} \tag{16}$$

Where f is the objective function. According to the probability selection, some good solutions are chosen for further search. The onlooker bee stage employs the same search approach as the employed bee stage, as shown in eq. (14).

- **Scout bee phase:** Scout bees are responsible for diversifying the search. They use eq.(9) to replace stagnant food sources -those that have not improved after a specific number of iterations- with new, random solutions.

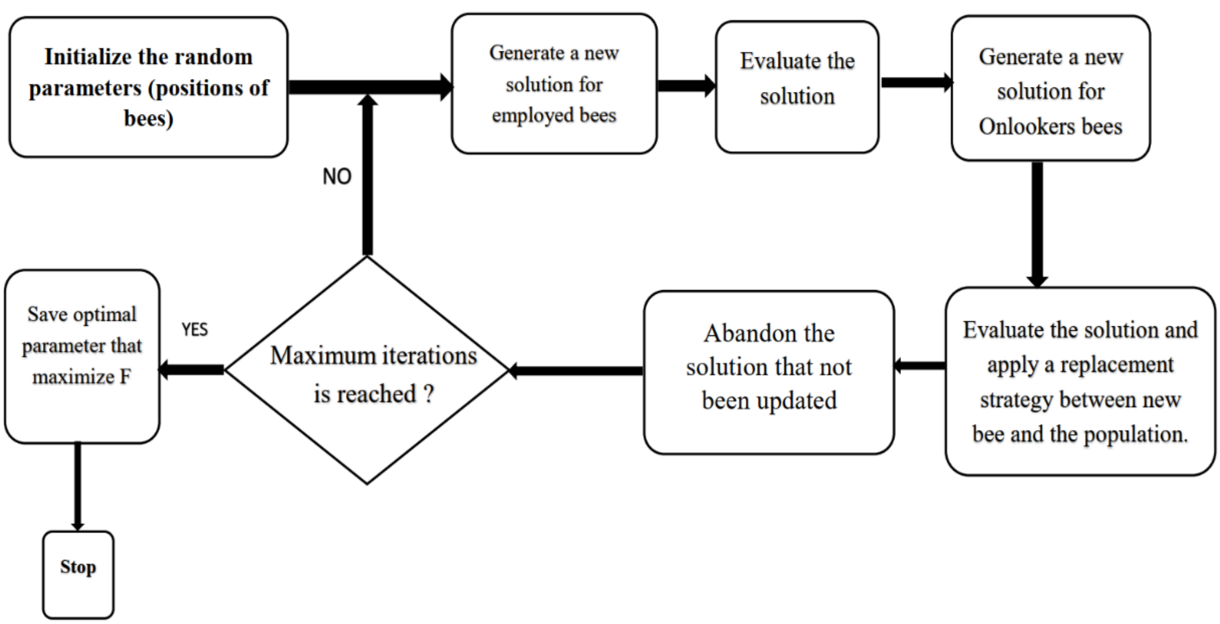


Figure 2. Flowchart of ABC algorithm.

3.2. Cost function optimization

In this section, we present the cost (objective) function that we use in the optimization problem (maximization) to find suitable parameters p_1, p_2, p_3 of Krawtchouk polynomials, using the optimization method ABC. This function depends on the quality measures of an image. The following describes the processes involved in finding the best scaling factor in the suggested method; Figure 2 displays the related flow chart, and Algorithm 1 displays the pseudo-code.

Algorithm 1. Optimization of KM parameter via ABC algorithm

Inputs: Image function $f(x, y, z)$, the moment order $Nmax$
Outputs: Optimized parameters p_1, p_2, p_3.

Step 1	Initialization of the parameters of ABC	• nVar =3, (variable size); • number of iterations, MaxIt ; population size, NP; • VarMin = 0; VarMax = 1; (Decision Variables Bounds)
Step 2	Generate a random parameters in [0,1]	Using eq. (13). (Create Initial Population)
Step 3	Evaluate the objective function for each solution and update	BestSol = 0; for i = 1← NP pop(i)=unifrnd(VarMin,VarMax, VarSize); if F(pop(i)) >= BestSol BestSol = pop(i); end end

	Employed Bees stage:	for i = 1← NP
		K = [1:i-1 i+1:nPop];
		k = K(rand([1 numel(K)]));
		phi = unifrnd(-1, +1, VarSize);
		newbee = pop(i) +phi.*(pop(i -pop(k));
		if F(newbee) >= F(pop(i))
		pop(i) = newbee;
		else
		C(i) = C(i)+1; (Update trial abandonment)
		End

	Calculate Fitness values and selection probabilities using eq. (16).	
Step 4	Onlooker Bees stage:	Choose a solution according to the probability generate a new solution with the aid of eq. (13) and apply a probability selection between the solutions.
Step 5	Scout bee stage:	Test a trial limit and memorize a best solution. for i = 1← NP if C(i) >= L pop(i)=unifrnd(VarMin,VarMax,VarSize); C(i) = 0; end end
Step 6	Test maximum iterations criterion.	
Step 7	Repeat until maximum iterations. Return $p_{optimal}$.	

In order to achieve the performance of the reconstructed image, the metrics used are the Peak Signal Noise Ratio (PSNR) and structural similarity index for measuring image quality (SSIM).

PSNR is defined by,

$$\begin{gathered}\operatorname{PSNR}\left(I, I^*\right)=10 \lg \frac{I_{\max }^2}{\operatorname{MSE}}, \\ \operatorname{MSE}=\frac{1}{M^3} \sum_{i=1}^M \sum_{j=1}^M \sum_{k=1}^M\left(I_{i, j, k}-I_{i, j, k}^*\right)^2,\end{gathered} \tag{17}$$

Where MSE is the mean square error between the original image and the reconstructed image and $I_{\max}$ is the maximum pixel value in the original image.

SSIM is defined by

$$\operatorname{SSIM}\left(I, I^*\right)=\frac{\mu_I \mu_{I^*}+d_1}{\mu_I^2+\mu_{I^*}^2+d_1} \cdot \frac{\sigma_{I^*}+d_2}{\sigma_I^2+\sigma_{I^*}^2+d_2} \tag{18}$$

where μ_C and μ_{C^*} are the average of original image C and reconstructed image C^*, σ_I^2 and $\sigma_{I^*}{}^2$ are the variance of original image I and reconstructed image I^*, σ_{II^*} is the covariance of I and I^*, d_1 and d_2 are two variables which are used to stabilize the division with a weak denominator. In the objective to find an optimal parameters p_1, p_2, p_3 we can define the function objective as follows,

$$F\left(p_1, p_2, p_3\right)=\operatorname{PSNR}\left(I, I^*\right)+\operatorname{SSIM}\left(I, I^*\right) \tag{19}$$

4. Results analysis and discussions

4.1. Optimal reconstruction of 2D image

In the first, we apply the proposed method on 2D image color. For this task, we are choosing a color image (Brain 256×256). Figure 3 describes the quality measures arranging various orders from 50 to 256. The proposed Krawtchouk moments' effectiveness in image reconstruction is demonstrated by the high values of the PSNR and SSIM measurements. Figure 4 shows the graphs of MSE and PSNR that highlight the comparison of measures of 2D reconstruction with existing meta-heuristics methods, while Figure 4 displays the host image "Brain.jpg" along with a collection of reconstructed images that the KMs based on the ABC algorithm's results for various orders were used to reconstruct, in comparison with conventional meta-heuristic methods including ACO and FA. Additionally, the value of the KPs' local parameters, $p1$ and $p2$, as determined by compared methods for each order, are displayed in Figure 4.

As the KMs' order grows, so does the quality of the reconstruction of these images, and the values of the polynomial parameter p vary. it is evident from the findings that the suggested technique is dependable for the best reconstruction of colored images.

4.2. Optimal 3D reconstruction

In this subsection, we display 3D images together with a collection of reconstructed images that the KMs optimized for various orders used to reconstruct. With the value of the local parameter p generated by the ABC method for each order, Figure 5 displays the 3D image "Four" that was reconstructed using KM-ABC algorithm and existing methods like ACO and FA algorithms, for moment orders 50, 100, 150, and 256. Furthermore, each reconstruction's MSE and PSNR values are computed.

4.3. Optimal classification of 2D image

In this subsection, we describe the classification of 2D images using KM generated by the proposed method as features of training and test data, and the classifier used is the KNN model. The images used are both free noise and noisy conditions evaluated in database [33] showed in Figure 7. The task passes by two stages: first, the extraction of descriptor vector moments of each image using the proposed method, and second applying the KNN classifier to train the model and testing with free noise and noisy images. The model's accuracy demonstrates how well the features extracted using the method suggested work.

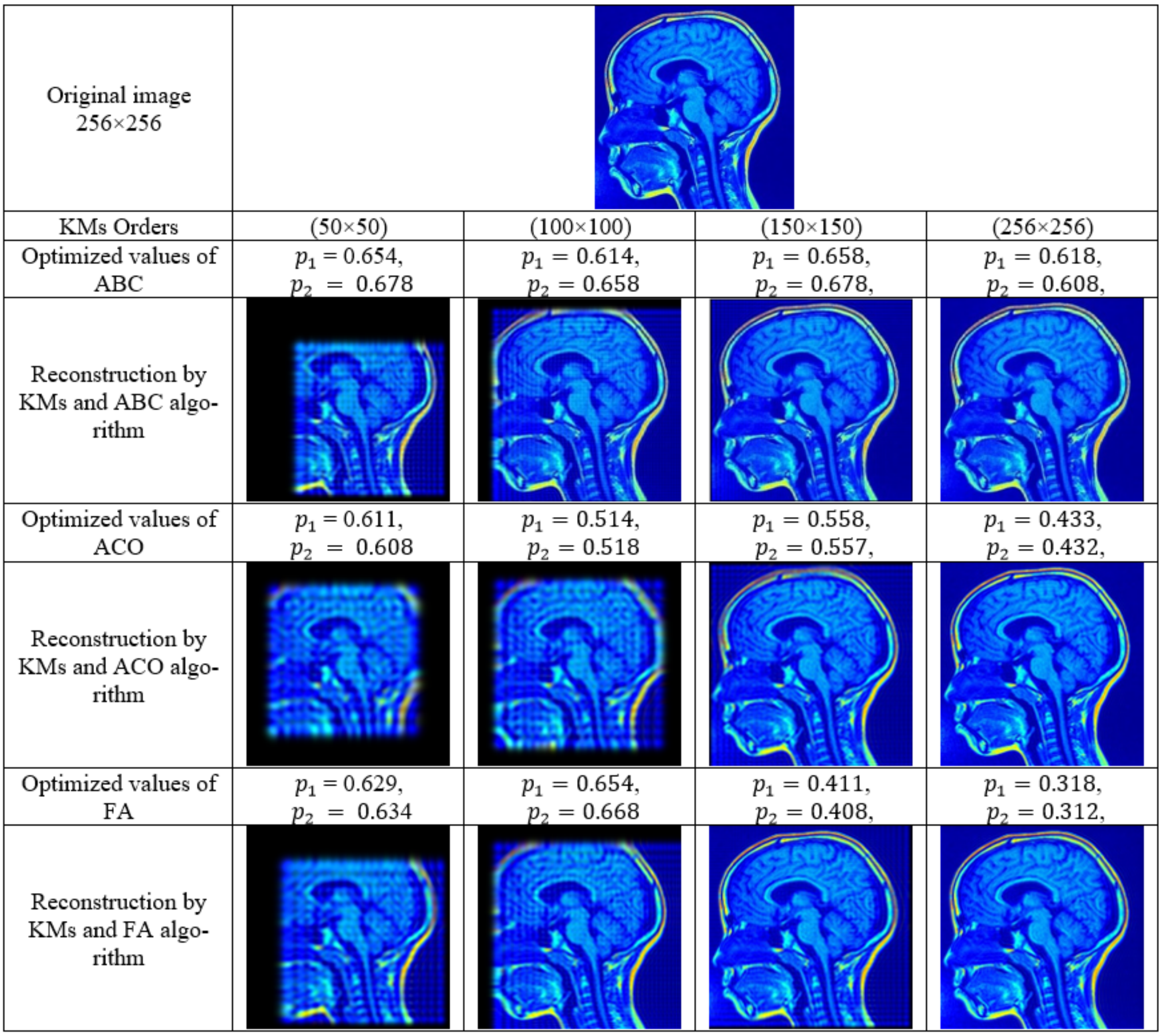


Figure 3. The 2D reconstructed image “brain” using moments optimized by different methods.

Table 1 provides the average of accuracy with various noises. This experiment presents a comparison with parameters chosen randomly and presents the dominance parameters optimized, which guarantees the efficiency and the quality of the reconstructed image by the proposed method.

Table 1. Accuracy average of 2D with salt and peppers noise.

Moments KM	Noise free	Salt and Pepper noise					Average
		1%	2%	3%	4%	5%	
$p_1 = 0.4$, $p_2 = 0.4$,	93.94	87.14	81.28	80.47	78.69	76.920	**83.07**
$p_1 = p_2 = 0.5$	94.15	88.76	82.12	80.65	78.15	77.305	**83.52**
$p_1 = 0.6$, $p_2 = 0.4$,	94.73	87.92	82.96	81.61	80.55	82.962	**85.12**
Proposed KM-ABC	98.43	95.21	93.11	90.03	88.47	86.708	**91.99**

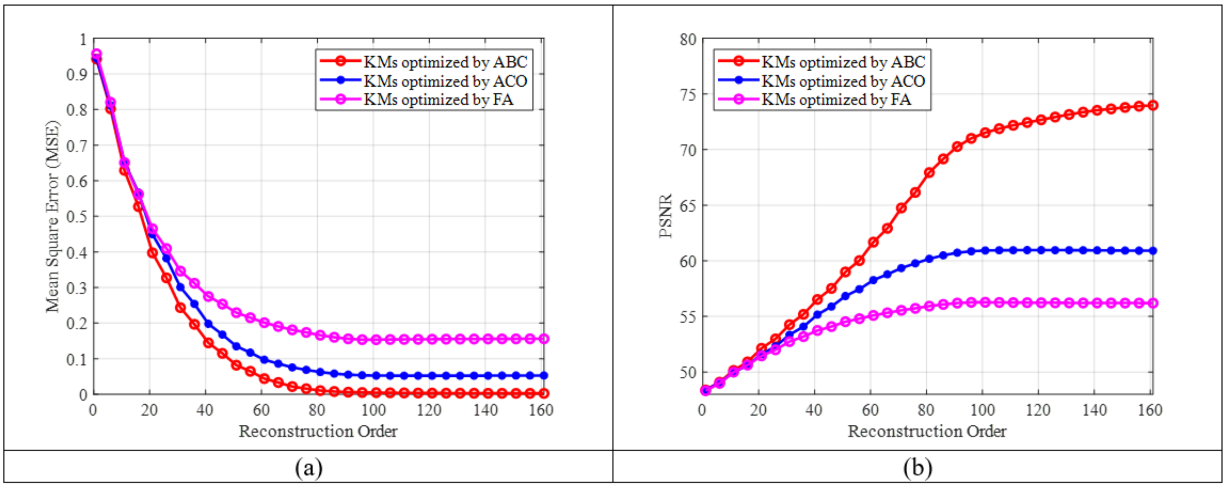


Figure 4. A comparison of measures of 2D reconstruction with existing meta-heuristics methods: (a) MSE, (b) PSNR.

4.4. Optimal classification of 3D image

In this experiment, we present the task of classification of 3D images using KM generated by the proposed method as features of training and test data, and the classifier used is the KNN model. The images used are both free noise and noisy conditions evaluated in the database [33]. The task passes by two stages: first, the extraction of descriptor vector moments of each image using the proposed method, and second applying the KNN classifier to train the model and testing with free noise and noisy images. The precision of the model indicates the effectiveness of features extracted by the proposed method.

We evaluate the optimal parameters of method KM-ABC with different parameters selected at random to demonstrate the efficiency of the proposed approach. Table 2 provides the high average of accuracy with various noises, which guarantees the efficiency and the quality of the reconstructed image by the proposed method.

Table 2. Accuracy average of 3D with salt and pepper noise.

KM's Parameters	Noise Free	Salt and Pepper noise					Average
		1%	2%	3%	4%	5%	
$p_1 = 0.4$ $p_2 = 0.5$ $p_3 = 0.6$	93.82	83.29	71.32	60.52	56.29	49.73	69.16
$p_1 = p_2 = p_3 = 0.5$	95.85	84.83	72.75	69.32	54.23	40.65	69.605
$p_1 = 0.6, p_2 = 0.5$ $p_3 = 0.4$	93.89	85.54	74.35	60.15	52.73	49.09	69.29
Proposed KM-ABC	98.91	93.23	86.67	81.64	79.18	76.89	86.09

4.5. Discussions

In general, Krawtchouk moments (KMs) are the most extracted features used to describe both 2D and 3D images. As well known, the KMs depend on the variable p which plays a crucial role in the distribution of pixel intensity in images. Our work, proposed KMs optimized using ABC algorithms focuses on the finding of the best parameter p, in order to present each image by the descriptor vector of moments. As shown in figure 3 and figure 5, the discrete moments optimized by various classical methods make a good reconstruction of 2D/3D images from lower orders, while the difference between these methods of optimization appeared in the graphs of MSE and PSNR displayed in

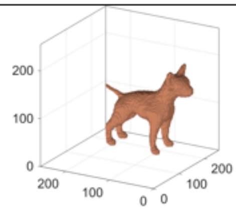
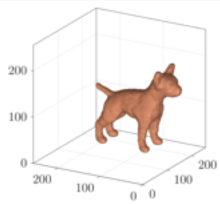
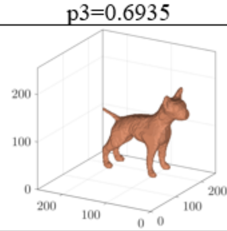
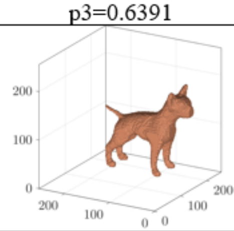
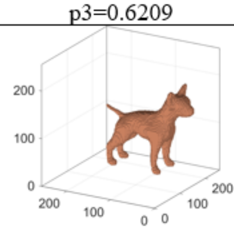
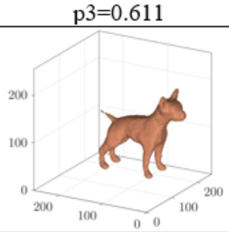
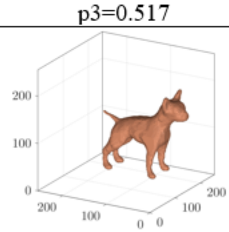
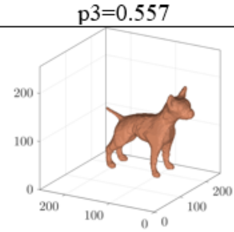
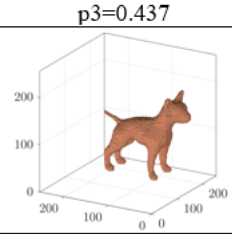
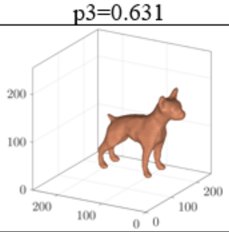
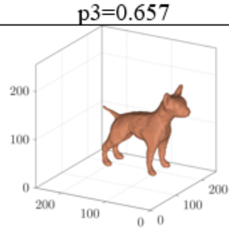
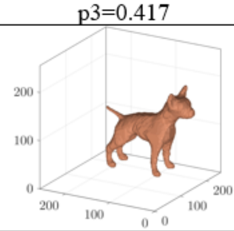
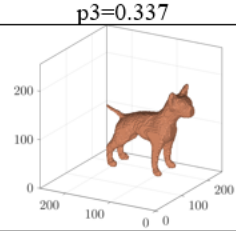
3D image " Four " size 256×256×256				
Moment Orders	(50×50×50)	100×100×100	150×150×150	256×256×256
Optimized values of ABC	p1=0.6013, p2=0.6021, p3=0.6101	p1=0.6544, p2=0.6978, p3=0.6935	p1=0.6515, p2=0.6421, p3=0.6391	p1=0.6154, p2=0.6098, p3=0.6209
Reconstruction by KMs and ABC algorithm				
Optimized values of ACO	p1=0.611, p2=0.602, p3=0.611	p1=0.514, p2=0.512, p3=0.517	p1=0.554, p2=0.558, p3=0.557	p1=0.433, p2=0.332, p3=0.437
Reconstruction by KMs and ACO algorithm				
Optimized values of FA	p1=0.629, p2=0.623, p3=0.631	p1=0.654, p2=0.652, p3=0.657	p1=0.414, p2=0.428, p3=0.417	p1=0.313, p2=0.342, p3=0.337
Reconstruction by KMs and FA algorithm				

Figure 5. The reconstructed 3D image “Four” using moments optimized by different methods.

figure 4 and figure 6. We remark that a proposed method based on KMs optimized by ABC presents an efficiency of reconstruction because their MSE is less than the MSE of the existing methods, in the same PSNR of KMs-ABC is upper in comparison with ACO and FA methods. For the classification task, Table 1 and Table 2 present both cases of free noise and noise, in comparison with values chosen randomly, the proposed method KMs-ABC has good accuracy.

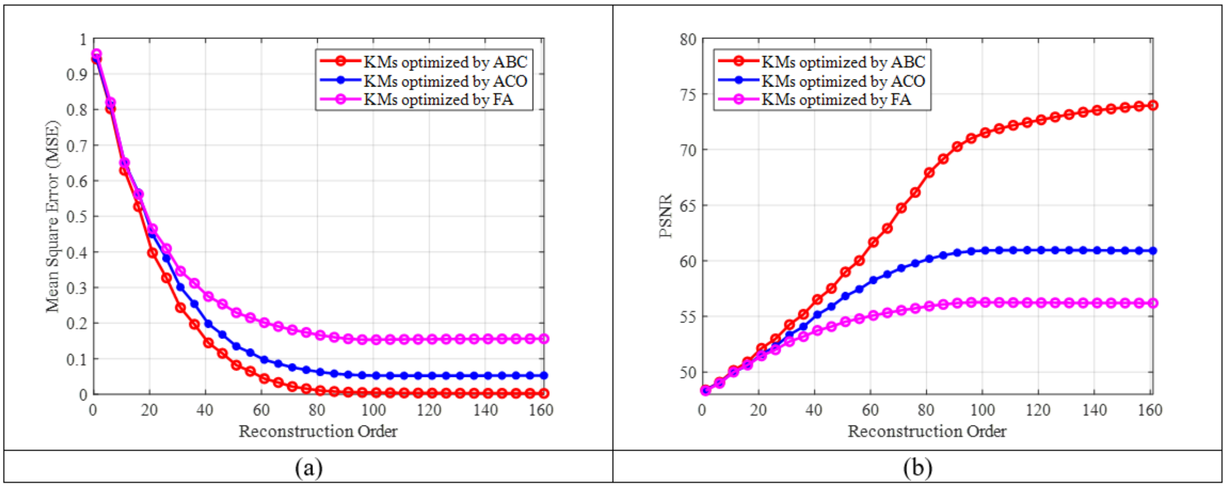


Figure 6. A comparison of measures of 3D reconstruction with existing meta-heuristics methods: (a) MSE, (b) PSNR.

Figure 7. Sample of 2D images extracted from the COIL-100 [33] database.

5. Conclusion

In this work, we presented a novel method of Krawtchouk orthogonal moments ex-traction based on the optimization algorithm called Artificial Bee Colony (ABC). This technique of optimization allowed choosing the optimal parameter for presenting the image in the form of a descriptor vector of orthogonal moments. The moment's orthogonal descriptor has several objectives. In this study, we concentrate on image reconstruction and classification tasks. Using optimal parameters showed good reconstruction results measured by the quantities MSE, PSNR, and SSIM. For the classification, we used the k-nearest neighbors' classifier to draw the model, the features we used the moments of the Krawtchouk images extracted from the database. The evaluation of the classification model is done by calculating the accuracy value. The high value of accuracy and similarity of the test database target shows the dominance and effectiveness of our proposed method.

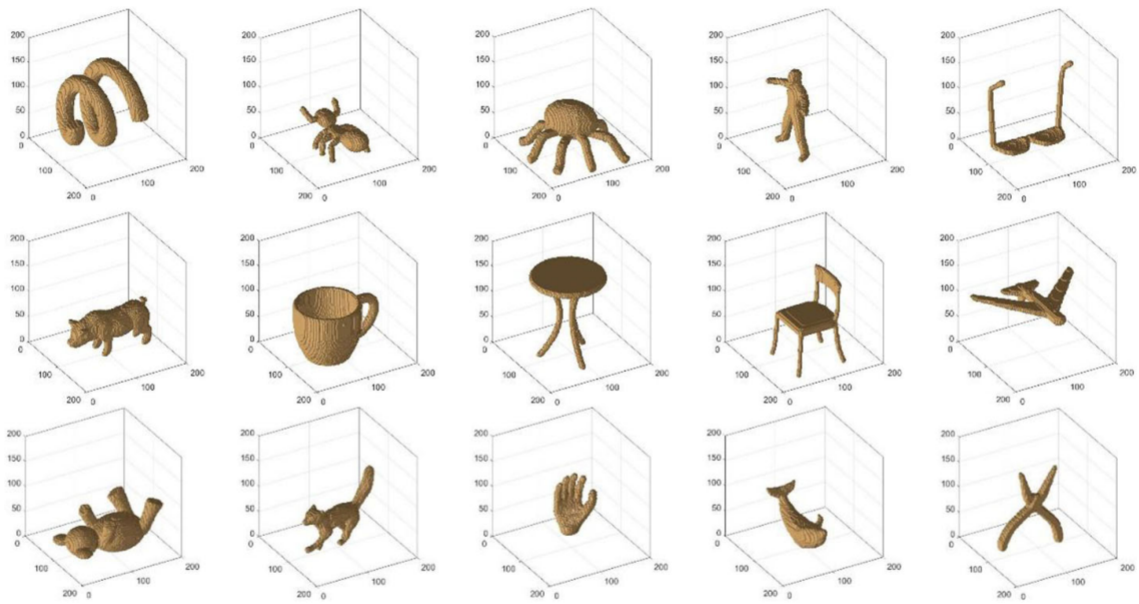


Figure 8. Sample of 15 objects from the PSB [34] database.

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