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A New Approach Of Multiple Merger And Acquisition (M&A) In AR Time Series Model Under Bayesian Framework

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Abstract Merger and acquisition (M&As) concepts play a pivotal role in fostering economic development and are extensively examined worldwide across various empirical contexts, notably in the banking sector. The primary objective of this study is to introduce a novel approach termed the multiple-merger autoregressive (MM-AR) model, aimed at providing insights into the effects of mergers on model parameters and behaviour. Initially, we propose a comprehensive estimation framework utilizing posterior parameters within the Bayesian paradigm, incorporating diverse loss functions to enhance robustness. The uniqueness of this model is that it will also work for the situation when multiple series get merged at various time points in the same observed series. Bayesian estimation approach is used to record the results of the MM-AR model parameters in terms of MSE, AB, and AE and get good results. Under Bayesian estimation, SELF performs better than the other estimators for most of the parameters. Subsequently, we compute the Bayes factor to quantify the impact of merged series on the overall model dynamics. To further elucidate the efficacy of the proposed model, we conduct both simulation-based analyses and real-world applications focusing on the Indian banking sector. Through this research, we aim to offer valuable insights into the implications of M&A activities. For the purpose of data analysis, we used PCR banking data of ICICI Banks Ltd. for simulation and empirical analysis to verify the models' applicability and purpose.

Keywords Autoregressive model, Merger series, Posterior probability, Bayesian inference

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1. Introduction

Time series plays an important role in spurring the economy's growth and the country's development. Indeed, it has made a significant contribution to strengthening the research field [1, 2, 3]. Models of the time series are hold-out type. "If I know the past, then I know the present. And if I know the present, then I also know the future." The most common type of model used for time-series analysis is the autoregressive (AR). It is a lag-dependent model, If the dependency is linear and the present observation depends only on the observation just before, then it is called an AR(1) process [4, 5, 6]. According to [7] in the discipline of merger and acquisition, our aim is to understand the underlying mechanisms that create time-series of any organization or entity through merger and acquisition. These processes may be explored using a time-series analysis, particularly when possible associated variables are merged into the dependent series. This can reveal insights about merger causation and thus may also contribute to changing the behaviour of any particular series. To overcome these types of problems, we have to work with merger and acquisition (M&A) technique.

M&A technique is not new in research, there are vast amount of literature available for M&A. [8] examine the growth effects of mergers and acquisitions for both domestic and cross-border by sectors and on the overall economy. [9] studied the dynamic impacts of mergers and acquisitions (M&A) on credit institution performance in

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the EU since 2005 to 2013. [10] examined the impact of mergers and acquisitions (M&A) on productivity in the EU from 1996 to 2003. Their empirical findings show that mergers and acquisitions have a beneficial and substantial influence on merging banks. [11] investigate the relationship between shareholder profits and announcement of African bank mergers and acquisitions. [12] discussed the benefits of banking mergers to develop the economy and market of a country. To acquaintance reality, anyone can see the SBI merger studies. This merger came into existence on April 1, 2017. Lots of research works related to analytical and theoretical studies are available on it to see the impact of the merger on SBI and its associated banks and others, like [13] explained about the impact of merger and acquisition on US economy. In India SBI, ICICI banks merger is a real examples of M&A that attracts researchers to work with the M&A technique. Apart from banking mergers, many companies and organizations have used this technique to merge one entity with another [14, 15].

The motivation behind the merger approach is that by merging a weak entity with another strong entity, the outcomes improve, and the weaker body performs better than before ([16]). In time series process, some associated series may work with the dependent series and affect the process equally. All or some of the associated series may merge with the dependent series after a consolidated period of time. Mergers of the all associated series may occur at once or at multiple time points. The dependent series behavior may be affected by the merging of associated series. [17] developed a model when all associated series merge at the same time and examined the impact of merger and acquisition for Real Time Gross Settlement (RTGS) payment system of SBI and its associated banks. When series merges at many different time points, a multi-merger time series model comes into the form. To the best of our knowledge, statistical model of multiple-merger time series has not been explored in the literature yet.

In this paper, we have developed a multiple-merger autoregressive (MM-AR) time series model. The specialty of this model is that we can merge solo or multiple series at a time point and it will also work for when the series getting merge at different time point or multiple time points. The MM-AR model is equally applicable to other types of datasets, such as mergers in industries or other types of organizations. This model can be utilized for data analysis in various contexts, including companies, businesses, and any situation where the mergers and acquisitions (M&A) approach is employed, with the condition that the data has the same properties as the model properties.

This paper has been organized in the following sections: In section 1, we discuss briefly about the time series and merger. Section 2 provides the model specification. Section 3 provides the estimation and testing methods, such as inference for the model, and Bayesian estimation, and testing for the problem. By using joint prior distributions and the likelihood function of the model, we calculated the posterior probability and then found out the conditional distributions for all the parameters. Section 4 provides the numerical illustration for the proposed model, where we check the Authenticity and applicability of the model through the simulated and real data, it has adequate scope to validate the study. The last section is related to the conclusion of the paper.

2. Multiple-Merger Autoregressive (MM-AR) model

The proposed model is called a Multiple-Merger Autoregressive (MM-AR) model. Let $(T_1, T_2, \cdots T_s)$ and $(k_1, k_2, \cdots, k_s)$ are the merger times and corresponding merged associated series. After the merger time points, respective associated series merges into the observed series. The same procedure will continue up to all the associated series not merged into the observed series. This reveals that observations of each merged associated series are not recorded due to being merged into the acquired series. But this may change the structure of the series. These associated series also follow the AR model with different orders $(r_m; m = 1, 2, \cdots, k_g;\ g = 1, 2, \cdots, s)$. After each merger time points, the order of MM-AR process is assumed to be different. Finally, the structure of the MM-AR model be in this form.

In this newly developed model $\{y_t, t = 1, 2, \cdots, T\}$ is an observed series, $\{\theta_a; a = 1, 2, \cdots, (s+1)\}$ are the intercept terms. $\{T_1, T_2, \cdots, T_s\}$ represents the merger time points in the series. Here, merger coefficient of the m^{th} series is denoted by δ_m^g.

$$y_t = \begin{cases} \theta_1 + \sum_{i=1}^{p_1} \varphi_i^{(1)} y_{t-i} + \sum_{m=1}^{k_1} \sum_{j=1}^{r_m} \delta_{m_j}^{1} Z_{m,t-j} + \varepsilon_t & 0 < t \le T_1 \\ \theta_2 + \sum_{i=1}^{p_2} \varphi_i^{(2)} y_{t-i} + \sum_{m=1}^{k_2} \sum_{j=1}^{r_m} \delta_{m_j}^{2} Z_{m,t-j} + \varepsilon_t & 0 < t \le T_2 \\ \vdots & \\ \theta_q + \sum_{i=1}^{p_q} \varphi_i^{(q)} y_{t-i} + \sum_{m=1}^{k_q} \sum_{j=1}^{r_m} \delta_{m_j}^{q} Z_{m,t-j} + \varepsilon_t & 0 < t \le T_q \\ \vdots & \\ \theta_s + \sum_{i=1}^{p_s} \varphi_i^{(s)} y_{t-i} + \sum_{m=1}^{k_s} \sum_{j=1}^{r_m} \delta_{m_j}^{s} Z_{m,t-j} + \varepsilon_t & 0 < t \le T_s \\ \theta_{s+1} + \sum_{i=1}^{p_{s+1}} \varphi_i^{(s+1)} y_{t-i} & 0 < t \le T \end{cases} \tag{1}$$

2.1. Graphically Representation of MM-AR model

To clearly explain the working procedure of the MM-AR (Multiple Merger-Autoregressive) model, let's break it down into simple steps. This model involves merging several associated series with a dependent autoregressive series at various time points. Here's how it works. The MM-AR model allows multiple associated series to merge with a dependent series over time. The unique feature of this model is its flexibility to perform these mergers at different time points. The goal is to eventually merge all associated series into the dependent series.

2.2. Step-by-Step Working Procedure

1. **Initial Setup and First Merger (TM1)**
    - **Associate Series:** We start with n associated series.
    - **Time of First Merger (TM1):** At a specific time point, called TM1 (Time of Merger 1), a subset of the associated series merges with the dependent series.
    - **Example:** Suppose there are n associated series. At TM1, 4 of these series merge with the dependent series, while the remaining n-4 series continue independently.

    The diagrammatically representation below shows how 4 of the n associated series merge with the dependent series at TM1 in fig-1.
2. **Second Merger (TM2) and Subsequent Mergers**
    - **Next Merger (TM2):** After the first merger, the model allows for another merger at a later time point, TM2.
    - **Additional Mergers:** At TM2, another subset of the remaining associated series merges with the dependent series.
    - **Example:** After the first merger at TM1, n-4 series remain. At TM2, 3 more of these series merge with the dependent series, leaving n-7 associated series continuing independently.

    The process repeats multiple times (up to T-q times or more), allowing for additional mergers of the remaining associated series into the dependent series at each new time point.

2.2.1. Final Merger

- **Completion:** The model continues this process until all associated series have merged with the dependent series. The final state of the model is that all series have become part of the dependent series.

2.2.2. Key Features of the MM-AR Model

- **Flexible Merger Timing:** The model allows for multiple mergers at different time points, offering flexibility in how and when series are combined.
- **Sequential Integration:** The associated series can merge in stages, not all at once, making the process adaptable to different scenarios.

- **Final Outcome:** Eventually, all associated series merge into the dependent series, which is the ultimate goal of the model.

The MM-AR model is designed to handle situations where multiple associated series need to be integrated with a dependent series over time. By allowing for flexible merger points and sequential integration, the model provides a structured yet adaptable approach to combining multiple time series data.

Figure 1 to Figure 3 below visually illustrate the merger process at each time point (TM1, TM2, etc.), showing how the associated series gradually merge into the dependent series. Figure 3 shows the complete working procedure of the developed MM-AR model.

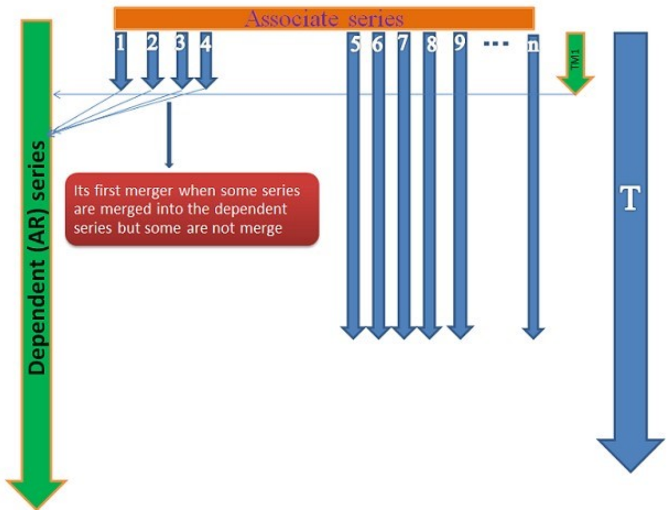


Figure 1

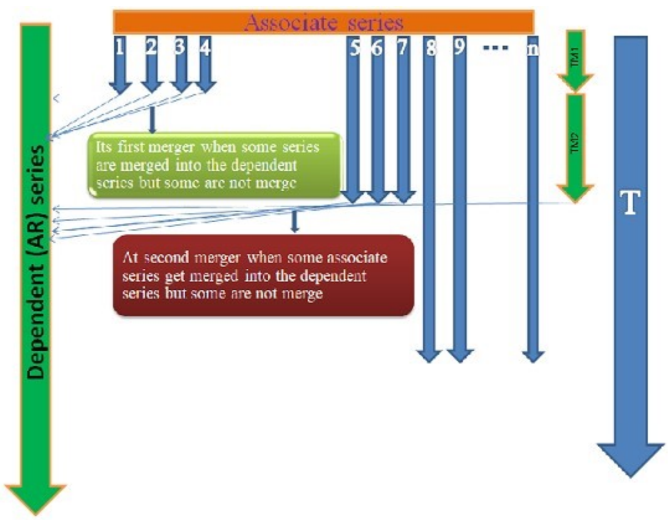


Figure 2

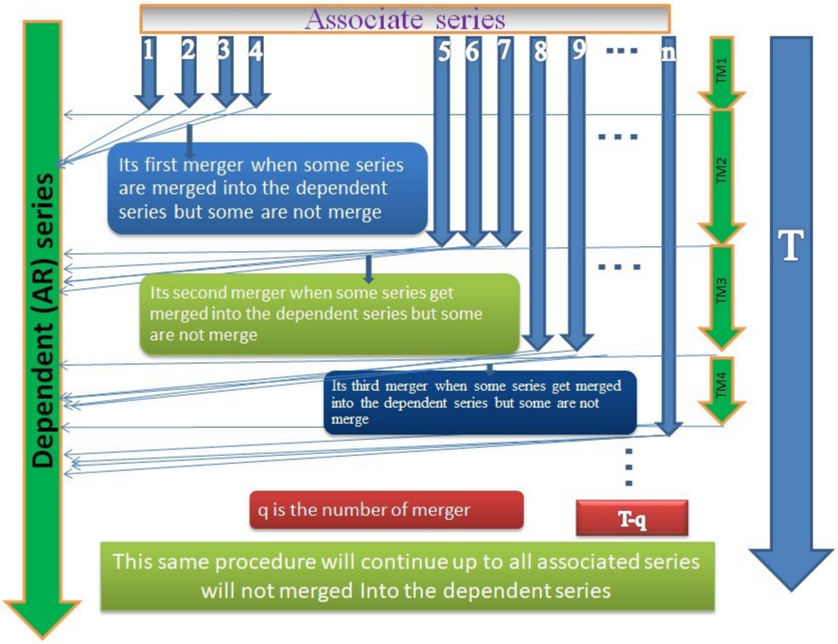


Figure 3

The model (1) can be written in the form of matrix notation as given below.

$$y_{T_1} = \theta_1 l_{T_1} + \omega_{T_1} x_{T_1} + \delta_{T_1} Z_{T_1} + \varepsilon_{T_1} \tag{2}$$
$$y_{T-T_1} = \theta_2 l_{T-T_1} + \omega_{T-T_1} x_{T-T_1} + \delta_{T-T_1} Z_{T-T_1} + \varepsilon_{T-T_1} \tag{3}$$
$$y_{T-T_{q-1}} = \theta_q l_{T-T_{q-1}} + \omega_{T-T_{q-1}} x_{T-T_{q-1}} + \delta_{T-T_{q-1}} Z_{T-T_{q-1}} + \varepsilon_{T-T_{q-1}} \tag{4}$$
$$y_{T-T_{s-1}} = \theta_s l_{T-T_{s-1}} + \omega_{T-T_{s-1}} x_{T-T_{s-1}} + \delta_{T-T_{s-1}} Z_{T-T_{s-1}} + \varepsilon_{T-T_{s-1}} \tag{5}$$
$$y_T = \theta_{s+1} l_{T-T_s} + \omega_{T-T_s} x_{T-T_s} + \varepsilon_{T-T_s} \tag{6}$$

Combining equation (2) to (6)

$$Y = l\theta + X\omega + Z\delta + \varepsilon \tag{7}$$

where,

$$x_{T_1} = \begin{pmatrix} y_0 & y_{-1} & \cdots & y_{1-p_1} \\ y_1 & y_0 & \cdots & y_{2-p_1} \\ \vdots & & \ddots & \\ y_{T_1-1} & y_{T_1-2} & \cdots & y_{T_1-p_1} \end{pmatrix}; \; x_{T-T_2} = \begin{pmatrix} y_{T_1} & y_{T_1-1} & \cdots & y_{T_1+1-p_2} \\ y_{T_1+1} & y_{T_1} & \cdots & y_{T_1+2-p_2} \\ \vdots & & \ddots & \\ y_{T_2-1} & y_{T_2-2} & \cdots & y_{T_2-p_2} \end{pmatrix}$$

$$x_{T-T_{q-1}} = \begin{pmatrix} y_{T_{q-1}} & y_{T_{q-1}-1} & \cdots & y_{T_{q-1}+1-p_q} \\ y_{T_{q-1}+1} & y_{T_{q-1}} & \cdots & y_{T_{q-1}+2-p_q} \\ \vdots & & \ddots & \\ y_{T_q-1} & y_{T_q-2} & \cdots & y_{T_q-p_q} \end{pmatrix}; \; x_{T-T_{s-1}} = \begin{pmatrix} y_{T_{s-1}} & y_{T_{s-1}-1} & \cdots & y_{T_{s-1}+1-p_s} \\ y_{T_{s-1}+1} & y_{T_{s-1}} & \cdots & y_{T_{s-1}+2-p_s} \\ \vdots & & \ddots & \\ y_{T_s-1} & y_{T_s-2} & \cdots & y_{T_s-p_s} \end{pmatrix}$$

$$x_{T-T_s} = \begin{pmatrix} y_{T_s} & y_{T_s-1} & \cdots & y_{T_{s-1}+1-p_{s+1}} \\ y_{T_s+1} & y_{T_s} & \cdots & y_{T_s+2-p_{s+1}} \\ \vdots & & \ddots & \\ y_{T-1} & y_{T-2} & \cdots & y_{T-p_{s+1}} \end{pmatrix}; \; Z_{T_1}^m = \begin{pmatrix} Z_{m,0} & Z_{m,-1} & \cdots & Z_{m,1-r_m} \\ Z_{m,1} & Z_{m,0} & \cdots & Z_{m,2-r_m} \\ \vdots & & \ddots & \\ Z_{m,T_1-1} & Z_{m,T_1-2} & \cdots & Z_{m,T_1-r_m} \end{pmatrix}$$

$$X = \begin{pmatrix} x_{T_1} & 0 & 0 & 0 & \cdots & 0 & \\ 0 & x_{T-T_1} & 0 & 0 & \cdots & 0 & \\ \vdots & & & & \ddots & & \\ 0 & 0 & 0 & 0 & \cdots & x_{T-T_{s-1}} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & x_{T-T_s} \end{pmatrix}; \; Z_{T_1} = \begin{pmatrix} Z^1_{T_1} & Z^2_{T_1} & \cdots & Z^{K_1}_{T_1} \end{pmatrix}$$

$$Z_{T-T_1} = \begin{pmatrix} Z^1_{T-T_1} & Z^2_{T-T_1} & \cdots & Z^{K_2}_{T-T_1} \end{pmatrix}; \; Z_{T-T_{q-1}} = \begin{pmatrix} Z^1_{T-T_{q-1}} & Z^2_{T-T_{q-1}} & \cdots & Z^{K_q}_{T-T_{q-1}} \end{pmatrix}$$

$$Z_{T-T_{s-1}} = \begin{pmatrix} Z^1_{T-T_{s-1}} & Z^2_{T-T_{s-1}} & \cdots & Z^{K_s}_{T-T_{s-1}} \end{pmatrix}; \; Z_{T-T_s} = \begin{pmatrix} Z^1_{T-T_s} & Z^2_{T-T_s} & \cdots & Z^{K_{s+1}}_{T-T_s} \end{pmatrix}$$

$$Z_{T_m} = \begin{pmatrix} Z_{T_1} & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & Z_{T-T_1} & 0 & 0 & \cdots & 0 & \\ \vdots & & & & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & Z_{T-T_{s-1}} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & Z_{T-T_s} \end{pmatrix}; \; l = \begin{pmatrix} l_{T_1} & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & l_{T-T_1} & 0 & 0 & \cdots & 0 & \\ \vdots & & & & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & l_{T-T_{s-1}} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & l_{T-T_s} \end{pmatrix}$$

$$Z = \begin{pmatrix} Z_{T_m} \\ 0 \end{pmatrix}; \; \theta = \begin{pmatrix} \theta_1 & \theta_2 & \cdots & \theta_s & \theta_{s+1} \end{pmatrix}'; \; \omega_{T_1} = \begin{pmatrix} \phi^{(1)}_1 & \phi^{(1)}_2 & \cdots & \phi^{(1)}_{p_1} \end{pmatrix}'$$

$$\omega_{T-T_1} = \begin{pmatrix} \phi^{(2)}_1 & \phi^{(2)}_2 & \cdots & \phi^{(2)}_{p_2} \end{pmatrix}'; \; \omega_{T-T_{q-1}} = \begin{pmatrix} \phi^{(q)}_1 & \phi^{(q)}_2 & \cdots & \phi^{(q)}_{p_q} \end{pmatrix}';$$

$$\omega_{T-T_{s-1}} = \begin{pmatrix} \phi^{(s)}_1 & \phi^{(s)}_2 & \cdots & \phi^{(s)}_{p_s} \end{pmatrix}'; \; \omega_{T-T_s} = \begin{pmatrix} \phi^{(s+1)}_1 & \phi^{(s+1)}_2 & \cdots & \phi^{(s+1)}_{p_{s+1}} \end{pmatrix}'$$

$$\omega = \begin{pmatrix} \omega_{T_1} & \omega_{T-T_1} & \cdots & \omega_{T-T_s} \end{pmatrix}'$$

3. Bayesian estimation

In the Bayesian framework, prior information is about the unknown parameters which are equally important as the likelihood function of the model [18, 19]. To determine the posterior probability, the prior function is needed. For all parameters of the MM-AR model, we consider the informative conjugate priors function and adopt a multivariate normal (MVN) distribution with a different mean and common variance for intercept, autoregressive, and merger coefficients. Assume an inverted gamma prior where a and b are the hyper parameters of gamma prior. These priors are chosen the same as the priors taken in the paper [20]. Priors are:

$$\theta \sim MVN(\mu, I_A\sigma^2); \; \omega \sim MVN(\gamma, I_p\sigma^2)$$
$$\delta \sim MVN(\alpha, I_R\sigma^2); \; \sigma^2 \sim IG(a, b)$$

Where,
$A = \sum_{i=1}^{s+1} i; \; p = \sum_{i=1}^{s+1} p_i; \; R = \sum_{j=1}^{s} (\sum_{m=1}^{k_j} r_m)$
Using the prior of σ^2 as inverse gamma with parameters "a" and "b" Joint prior is:
$\Pi(\Theta) = f(\theta)f(\omega)f(\delta)f(\sigma^2)$

$$\Pi(\Theta) = \left(\frac{b^a (\sigma^2)^{-\left(\frac{A+p+R}{2}+a+1\right)}}{(2\pi)^{\frac{A+p+R}{2}} \Gamma a} \right) exp\left[-\frac{1}{2\sigma^2} \{ (\theta-\mu)' I_A^{-1} (\theta-\mu) + (\omega-\gamma)' I_p^{-1} (\omega-\gamma) + (\delta-\alpha) + 2b \} \right] \tag{8}$$

The likelihood function for the observed series can be expressed under the assumption of a given error is,

$$L(\Theta|y) = \frac{(\sigma^2)^{-\frac{T}{2}}}{(2\pi)^{\frac{T}{2}}} exp\left[-\frac{1}{2\sigma^2} \left\{ (Y - l\theta - X\omega - Z\delta)'(Y - l\theta - X\omega - Z\delta) \right\} \right] \tag{9}$$

Calculated the posterior distribution by using the likelihood function and joint distribution ,

$$\Pi(\Theta|Y) = \left(\frac{b^a (\sigma^2)^{-\left(\frac{T+A+p+R}{2}+a+1\right)}}{(2\pi)^{\frac{T+A+p+R}{2}} \Gamma a} \right) exp \left[-\frac{1}{2\sigma^2} \{ (Y - l\theta - X\omega - Z\delta)^{'} (Y - l\theta - X\omega - Z\delta) \right.$$
$$\left. + (\theta - \mu)^{'} I_A^{-1} (\theta - \mu) + (\omega - \gamma)^{'} I_p^{-1} (\omega - \gamma) + (\delta - \alpha) + 2b \} \right] \tag{10}$$

By using the posterior distributions one can find the Conditional distributions.

$$\Pi(\theta|Y,\omega,\delta,\sigma^2) \sim MVN \left(l^{'} \left((Y - X\omega - Z\delta) + \mu I_2^{-1} \right) \left(l^{'} l + I_A^{-1} \right), \sigma^2 \left(l^{'} l + I_A^{-1} \right)^{-1} \right) \tag{11}$$

$$\Pi(\omega|Y,\theta,\delta,\sigma^2) \sim MVN \left(X^{'} \left((Y - l\theta - Z\delta) + \gamma I_{p_1+p_2}^{-1} \right) \left(X^{'} X + I_p^{-1} \right), \sigma^2 \left(X^{'} X + I_p^{-1} \right)^{-1} \right) \tag{12}$$

$$\Pi(\delta|Y,\theta,\omega,\sigma^2) \sim MVN \left(Z^{'} \left((Y - l\theta - X\omega) + \alpha I_R^{-1} \right) \left(Z^{'} Z + I_R^{-1} \right), \sigma^2 \left(Z^{'} Z + I_R^{-1} \right)^{-1} \right) \tag{13}$$

$$\Pi(\sigma^2|Y,\theta,\omega,\delta) \sim IG \left(\frac{(A + P + R + T)}{2} + a + 1, K \right) \tag{14}$$

where,

$$\begin{aligned} K &= (\theta - \mu)^{'} I_A^{-1} (\theta - \mu) + (\omega - \gamma)^{'} I_p^{-1} (\omega - \gamma) \\ &+ (\delta - \alpha) + 2b + (Y - l\theta - X\omega - Z\delta)^{'} (Y - l\theta - X\omega - Z\delta) \end{aligned}$$

Equations (11) to (14) contain the standard and closed forms of the conditional posterior distributions. Hence, we can use the Gibbs sampler algorithm to obtain posterior samples from the specified conditional posterior distribution ([21]).

3.1. Bayesian testing for Merger Coefficient

From a Bayesian perspective, we are presenting a procedure using Bayes factors for testing the impact of merger/acquire series on models, intending to analyze the impact on the model as associate series may affect the model [22]. The merger may have favourable or unfavourable consequences. This testing procedure is completed with the help of different hypothetical procedures. For this one can consider the null hypothesis $H_0 : \delta = 0$ i.e there is no significant impact of merger series, and against the alternative hypothesis is $H_1 : \delta \neq 0$. The reduced forms of the model under the null and alternative hypothesis are given below;
Under $H_0 : Y = l\theta + X\omega + \varepsilon$
Under $H_1 : Y = l\theta + X\omega + Z\delta + \varepsilon$
Using the approach of [23] calculated the posterior probabilities and the Bayes factor for both the above models.

$$p(Y|H_0) = \frac{b^a |D_1|^{-\frac{1}{2}} |D_2|^{-\frac{1}{2}} \Gamma\left(\frac{T}{2} + a\right)}{(2\pi)^{\frac{T}{2}} \Gamma(a) \left(\frac{N_0}{2}\right)^{\frac{T}{2}+a}} \tag{15}$$

Posterior probability under alternative hypothesis is,

$$p(Y|H_1) = \frac{b^a |D_1|^{-\frac{1}{2}} |D_2|^{-\frac{1}{2}} |D_3|^{-\frac{1}{2}} \Gamma\left(\frac{T}{2} + a\right)}{(2\pi)^{\frac{T}{2}} \Gamma(a) \left(\frac{N_1}{2}\right)^{\frac{T}{2}+a}} \tag{16}$$

Using equation (15) and (16) calculate the BF_{10}.

$$BF_{10} = |D_3|^{-\frac{1}{2}} \left(\frac{N_0}{N_1}\right)^{\frac{T}{2}+a} \tag{17}$$

Where,

$$\begin{aligned} D_1 &= (ll + I_2^{-1}) \\ D_2 &= XX + I_{P_1+P_2}^{-1} - X^{'} l D_1^{-1} l^{'} X \\ D_3 &= Z^{'} Z + I_R^{-1} - Z^{'} l D_1^{-1} l^{'} Z - (Z^{'} X - Z^{'} l D_1^{-1} l^{'} X)^{'} D_2^{-1} (Z^{'} X - Z^{'} l D_1^{-1} l^{'} X) \end{aligned}$$

$$\begin{aligned} C_{21}^{'} &= Y^{'} X + \gamma^{'} I_{P_1+P_2}^{-1} - (Y^{'} l + \mu^{'} I_2^{-1})^{'} D_1^{-1} l^{'} X \\ C_3 &= Y^{'} Z + \alpha^{'} I_R^{-1} - (Y^{'} l + \mu^{'} I_2^{-1})^{'} D_1^{-1} l^{'} Z - B_{21}^{'} D_2^{-1} (Z^{'} X - Z^{'} l D_1^{-1} l^{'} X) \\ N_0 &= Y^{'} Y + \gamma^{'} I_{P_1+P_2}^{-1} \gamma + \mu^{'} I_2^{-1} \mu + 2b - (Y^{'} l + \mu^{'} I_2^{-1})^{'} D_1^{-1} (Y^{'} l + \mu^{'} I_2^{-1}) - C_{'}^{21} D_2^{-1} C_{21} \\ N_1 &= N_0 + \alpha^{'} I_R^{-1} \alpha - C_3^{-1} + D_3^{-1} C_3 \end{aligned}$$

With the help of BF_{10}, posterior probability (PP) of H_1 is obtained for the given data which is,
$PP = P(Y|H_1) = [1 + BF_{10}^{-1}]^{-1}$
The Bayes factor makes it simple to decide whether a hypothesis should be accepted or rejected ([24], [25]). When the value of BF_{10}, is too much high, the null hypothesis (H_0), is rejected.

4. NUMERICAL ILLUSTRATION

We need to do some numerical study on the proposed model to evaluate its validity, applicability, and importance. For this, we applied two different data series on it. In the first section we generate a simulated series using the proposed model and in the second section, we illustrate the importance of the MM-AR model by an application on the ICICI bank data of India. By using both data series, we tested the proposed model by various estimating and testing methodologies.

4.1. MODEL VALIDATION

In this section, we have performed simulation analysis to assess the performance of various estimating approaches presented in previous sections. For this study first we, generated a series from proposed MM-AR model based on initial guess of the parameters by using the R programing and using the initial value of y_0 is 12. Using this simulated series we performed the estimation and testing procedures to check the model's adequacy and applicability. In this numerical illustration, the size of the generated series are $T = \{100, 200, 300\}$ and consider two known different time of merger say $T/4$, and $T/2$. These merger times are indicated by T_{m1} and T_{m2}. To obtain a more generalized idea of the model, we obtained the MSE and estimated the values of the parameters to compare different estimation methods under the classical and Bayesian approaches. For Bayesian computation used different loss functions. And also calculate the Bayes probabilities for the model selection. Figure 4 shows the performance of different estimation methods based on the MSE of the parameters. Whereas Table-(1) to Table-(3) shows that the estimated values of each parameter for different estimators under both the Classical and Bayesian approaches. Table-(4) represents the Bayes factors and posterior probability values at multiple merger time points for increasing series sizes, to the merger parameters of the series is affected or not.

Figure 4, shows that, as we increase the series size, the MSEs, tend to decrease for all the estimation methods. That's simply means is that model is good fit for the simulated series in the terms of MSE. Therefore, reduction in

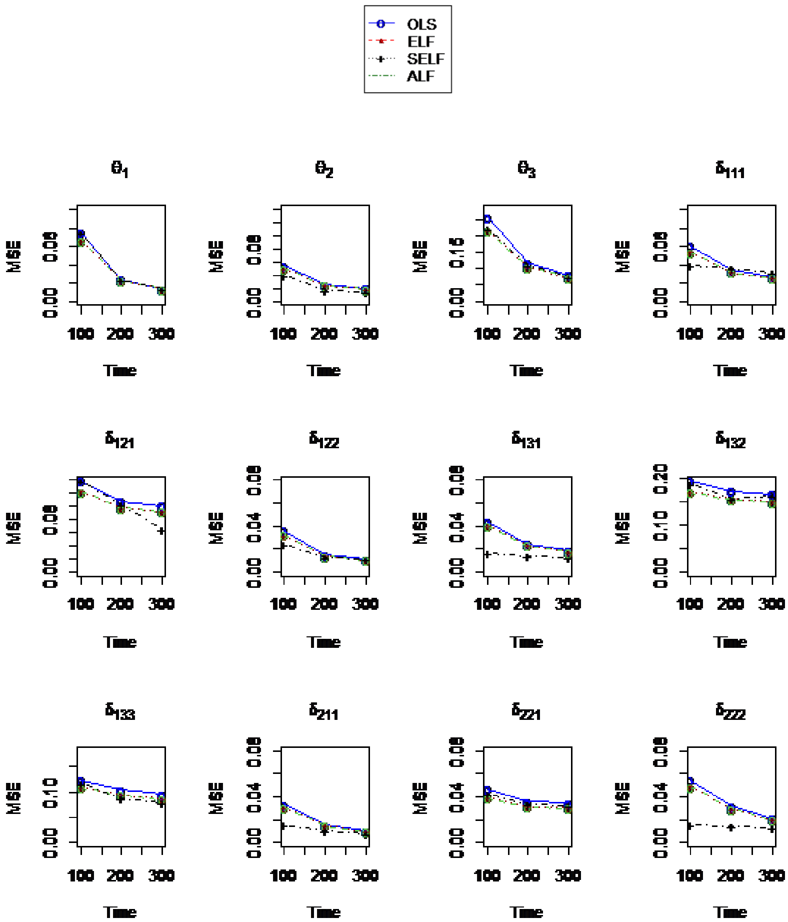
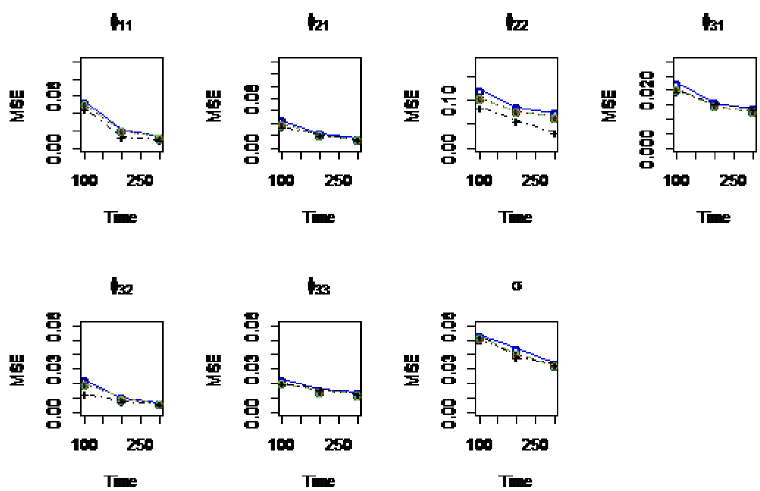


Figure 4. For Mean Squred Error



the MSE value as the series size increases is considered a positive indication of the model's accuracy and ability to make accurate predictions.

Figure 4 shows, as we increase the series size, the MSEs tends to decrease for all the estimation methods. Table 1-3, shows that, as we increase the series size, the AEs, tends to decrease for all the estimation methods. In overall comparison, Bayes estimates perform better as compare to OLS estimation method in terms of MSEs, ABs, and AEs. But when we make the comparison between the loss functions, then symmetric loss functions give better results in comparison of asymmetric loss function. Bayes factor for different series sizes are too much high i.e. merger is effective. So, we rejected the null hypothesis.

Table 1. Estimated values of the parameters ,$T = 100,\ T_{m1} = T/4,\ T_{m2} = T/2$

Parameters	**OLS**	**ALF**	**SELF**	**ELF**
$\theta_1(0.3)$	0.3449	0.327	0.3329	0.5211
$\theta_2(0.2)$	0.2328	0.2232	0.2249	0.3142
$\theta_3(0.4)$	0.637	0.6134	0.6168	0.6326
$\phi_{11}(0.2)$	0.1821	0.1835	0.1823	0.3439
$\phi_{21}(0.3)$	0.3371	0.3244	0.326	0.3824
$\phi_{22}(0.1)$	0.0165	0.0267	0.0253	0.2103
$\phi_{31}(0.1)$	0.0566	0.0666	0.0657	0.1732
$\phi_{32}(0.3)$	0.2426	0.2448	0.2446	0.2753
$\phi_{33}(0.4)$	0.3578	0.354	0.3547	0.3745
$\delta_{111}(0.2)$	0.1775	0.1724	0.1733	0.2801
$\delta_{121}(0.4)$	0.3547	0.3835	0.3829	0.3425
$\delta_{122}(0.3)$	0.2834	0.2718	0.2737	0.34
$\delta_{131}(0.25)$	0.261	0.296	0.2968	0.2787
$\delta_{132}(0.01)$	0.0072	0.0086	0.0081	0.0094
$\delta_{133}(0.1)$	0.0177	0.0234	0.0221	0.2285
$\delta_{211}(0.3)$	0.2387	0.263	0.2638	0.2877
$\delta_{221}(0.5)$	0.4228	0.4578	0.4505	0.4586
$\delta_{222}(0.1)$	0.1563	0.1565	0.1458	0.1621
$\sigma(0.5)$	0.5436	0.4674	0.4626	0.4712

In table-4, we observe that there is strong evidence to support the presence of merger series because Bayes factor is so much high to reject the null hypothesis. The posterior under the alternative hypothesis is much closed to one. This shows that the simulated series is properly generated from the MM-AR model. Similarly results are obtained with an increase in the size of the series as well as a change in time of merger.

4.2. APPLICATION WITH BANKING DATA

The importance of MM-AR model is demonstrated with an application to quarterly data of ICICI Bank Ltd. For real application, we consider autoregressive data time series from the Indian banking system. As we know, ICICI Bank Ltd. acquired the Sangli Bank and The Bank of Rajasthan on April 10, 2006 and April 10, 2010 respectively. The reason for the bank's merger and acquisition is that the government expects it to be much easier for the government to pay and aid all of the aforementioned facilities to a single unified body, ICICI Bank Ltd., rather than paying to individual banks. We are using PCR (provisioning coverage ratio) data of ICICI Bank Ltd., PCR is the percentage of funds that a bank sets aside for bad debt losses. Banks can benefit from a high PCR to secure themselves against losses.

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Table 2. Estimated values of the parameters ;$T = 100,\ T_{m1} = T/4,\ T_{m2} = T/2$

Parameters	**OLS**	ALF	**SELF**	**ELF**
θ_1(0.3)	0.3058	0.2997	0.3008	0.3451
θ_2(0.2)	0.2123	0.2096	0.2099	0.2454
θ_3(0.4)	0.534	0.5246	0.526	0.5363
ϕ_{11}(0.2)	0.223	0.2232	0.223	0.2658
ϕ_{21}(0.3)	0.355	0.3485	0.3492	0.3689
ϕ_{22}(0.1)	0.0462	0.0503	0.0498	0.1512
ϕ_{31}(0.1)	0.0801	0.0844	0.0838	0.1378
ϕ_{32}(0.3)	0.2681	0.269	0.2692	0.2811
ϕ_{33}(0.4)	0.3782	0.3759	0.3764	0.385
δ_{111}(0.2)	0.1745	0.1718	0.1721	0.2148
δ_{121}(0.4)	0.3647	0.3875	0.3869	0.3658
δ_{122}(0.3)	0.3278	0.3221	0.3227	0.3407
δ_{131}(0.25)	0.2033	0.2018	0.2022	0.2327
δ_{132}(0.01)	0.0082	0.0089	0.0085	0.0096
δ_{133}(0.1)	0.0748	0.0793	0.076	0.7526
δ_{211}(0.3)	0.2457	0.2422	0.2428	0.2632
δ_{221}(0.5)	0.4441	0.4569	0.4637	0.47477
δ_{222}(0.1)	0.1319	0.1321	0.1221	0.1231
σ(0.5)	0.5425	0.4994	0.4985	0.5014

Table 3. Estimated values of the parameters ;$T = 300,\ T_{m1} = T/4,\ T_{m2} = T/2$

Parameters	**OLS**	**ALF**	**SELF**	**ELF**
θ_1(0.3)	0.2933	0.2903	0.2908	0.3116
θ_2(0.2)	0.2064	0.2046	0.2049	0.2257
θ_3(0.4)	0.4984	0.4927	0.4933	0.501
ϕ_{11}(0.2)	0.2436	0.2435	0.2434	0.2629
ϕ_{21}(0.3)	0.3497	0.3456	0.3458	0.3583
ϕ_{22}(0.1)	0.0597	0.0623	0.0617	0.1282
ϕ_{31}(0.1)	0.0857	0.0886	0.0885	0.1241
ϕ_{32}(0.3)	0.2791	0.2793	0.2793	0.2866
ϕ_{33}(0.4)	0.3828	0.3815	0.3815	0.3873
δ_{111}(0.2)	0.1752	0.1733	0.1737	0.1976
δ_{121}(0.4)	0.3747	0.3895	0.3889	0.3858
δ_{122}(0.3)	0.3463	0.3421	0.3427	0.3518
δ_{131}(0.25)	0.2075	0.2057	0.2061	0.2236
δ_{132}(0.01)	0.0092	0.0099	0.0095	0.0098
δ_{133}(0.1)	0.0764	0.0798	0.0776	0.7652
δ_{211}(0.3)	0.2536	0.2513	0.2515	0.2636
δ_{221}(0.5)	0.455	0.457	0.4694	0.4766
δ_{222}(0.1)	0.1002	0.1004	0.1003	0.1022
σ(0.5)	0.5371	0.5077	0.5078	0.5091

for the government to pay and aid all of the aforementioned facilities to a single unified body, ICICI Bank Ltd., rather than paying to individual banks. We are using PCR (provisioning coverage ratio) data of ICICI Bank Ltd., PCR is the percentage of funds that a bank sets aside for bad debt losses. Banks can benefit from a high PCR to

Table 4. BF and PP; $T_{m1} = T/4, T_{m2} = T/2$

Time	BF	PP
T=100	3.80E+02	0.99
T=200	1.45E+04	1
T=300	1.93E+14	1

secure themselves against losses. The values used in the following models are presented in the appendix .
Model for SELF

$$y_t = \begin{cases} 1.7222 + 0.4751y_{t-1} + 0.37084Z_{1,t-1} - 0.5027Z_{2,t-1} - 0.8349Z_{3,t-1} + \varepsilon_t & 0 < t \leq 21 \\ 1.3077 + 0.9094y_{t-1} + 0.06873Z_{t-1} + \varepsilon_t & 21 < t \leq 35 \\ 0.1165 + 0.9734y_{t-1} + \varepsilon_t & 21 < t \leq 35 \end{cases}$$

Model for ALF

$$y_t = \begin{cases} 1.8546 + 0.4777y_{t-1} + 0.36020Z_{1,t-1} - 0.51852Z_{2,t-1} - 0.0944Z_{3,t-1} + \varepsilon_t & 0 < t \leq 21 \\ 1.2735 + 0.8621y_{t-1} + 0.1177Z_{t-1} + \varepsilon_t & 21 < t \leq 35 \\ 0.1111 + 0.9740y_{t-1} + \varepsilon_t & 21 < t \leq 35 \end{cases}$$

Model for ELF

$$y_t = \begin{cases} 1.7939 + 0.5081y_{t-1} + 0.3915Z_{1,t-1} - 0.5290Z_{2,t-1} - 0.1155Z_{3,t-1} + \varepsilon_t & 0 < t \leq 21 \\ 1.3083 + 0.9640y_{t-1} + 0.3159Z_{t-1} + \varepsilon_t & 21 < t \leq 35 \\ 0.1276 + 0.9735y_{t-1} + \varepsilon_t & 21 < t \leq 35 \end{cases}$$

Model for OLS

$$y_t = \begin{cases} 1.9177 + 0.4783y_{t-1} + 0.5494Z_{1,t-1} - 0.5290Z_{2,t-1} - 0.1155Z_{3,t-1} + \varepsilon_t & 0 < t \leq 21 \\ 1.3145 + 0.8607y_{t-1} + 0.0818Z_{t-1} + \varepsilon_t & 21 < t \leq 35 \\ 0.1052 + 0.9798y_{t-1} + \varepsilon_t & 21 < t \leq 35 \end{cases}$$

Where, ε_t is i.i.d normally distributed with mean zero and variance σ^2. The above fitted models for different estimators' summaries the estimated values of all the parameters. By using these estimated values of the parameters one can calculate the standard error of the parameters and decide that which estimation method performs best for the developed model. Table (6) represents the standard error values of the estimation methods in the terms of the model parameters. Table-(6) shows the SE for different estimators under Bayesian and classical methods. Under Bayesian estimation SELF have the least SE for most of the parameters that means SELF gives the best performance in comparison to all estimation procedure.

Table 5. BF and PP $T_{m1} = 21,\ T_{m2} = 35$

Series size	BF	PP
78	108.465	0.99

The impact of merged series on ICICI Bank Ltd. is reported in the Table (5). It is concluded that the (MM-AR) model satisfies the merger situation because Bayes factor is sufficiently higher, i.e. 108.4651, to reject the null hypothesis. The coverage probability is 0.99. Overall, analysis demonstrates that the proposed MM-AR model is well adequate to understand the merger and acquisition.

Table 6. Standard Error (SE) ; $T = 78;\ T_{m1} = 21,\ T_{m2} = 35$

Parameters	**SELF**	**ALF**	**ELF**	**OLS**
θ_1	0.042	0.0442	0.0436	0.0457
θ_2	0.0803	0.0866	0.0814	0.0933
θ_3	0.0565	0.0594	0.0569	0.0585
ϕ_{11}	0.0121	0.0121	0.0106	0.0122
ϕ_{21}	0.0234	0.0248	0.0121	0.025
ϕ_{31}	0.016	0.0155	0.0108	0.0152
δ_{111}	0.0314	0.0262	0.0475	0.2102
δ_{121}	0.0255	0.0185	0.0561	0.2196
δ_{131}	0.0201	0.0199	0.0115	0.0318
δ_{212}	0.0208	0.0212	0.0121	0.0326
σ^2	0.037	0.0388	0.0392	0.0449

Table 7. MSE and RMSE of Model under different estimation methods

Comparison method	**ALF**	**SELF**	**ELF**
MSE	7.8456	6.4575	8.4623
RMSE	2.801	2.5411	2.9090

Table 7 shows the sensitive analysis of the model under the different loss functions using the MSE and RMSE of the model. Under the Bayesian analysis using the different loss functions, SELF perform better than the other loss functions. SELF penalizes larger errors more heavily, leading to improved precision in parameter estimation. Thus SELF are better suited to the nuances of MM-AR models, making them preferable choices for accurate estimation in the context of time series data.

Table 8. Model Comparison through AIC and BIC

Models	**AIC**	**BIC**
AR(1)	485.5723	494.2505
MM-AR(1,1,1,4)	338.4456	361.54663
AR(1,7)	396.8273	417.3547

Table 8 has the comparison of the different possible models with the study MM-AR model. First model is the simple AR(1) model value of AIC, BIC is 485.5723 and 494.2505 respectively. Second model is the MM-AR model in this model the order of the dependent series is one and the order of associate series is four. The AIC and BIC values of the MM-AR model is 338.4456 and 361.5466 respectively. This is the minimum in comparison to all possible models. Last model has the order of dependent series one and the number of associated series is seven. So, one can say that the MM-AR model is performing better than the other mentioned models in the Table 8.

5. Conclusion

This paper introduces a multiple-merger time series model capable of accommodating the merging of multiple series at various time points within a single observed series. Employing Bayesian estimation, the model effectively captures merged series dynamics, yielding favorable results in terms of Mean Squared Error (MSE), Absolute Bias (AB), and Absolute Error (AE), with the SELF estimator demonstrating superior performance. By rigorously testing null and alternative Model using Bayes factor analysis, we ascertain the effectiveness of series mergers, showing positive impacts in both simulated and real-world data analyses. Our testing procedure facilitates the identification of merged series within observed data, underpinning the beneficial effects of mergers on model dynamics. Utilizing PCR banking data from ICICI Banks Ltd., we validate the model's applicability and relevance in empirical settings, while suggesting potential extensions to panel multiple-merger models, thereby enriching our understanding of merger dynamics within financial contexts. In general, this research offers a novel approach to comprehending merged time series and provides empirical evidence supporting the positive outcomes of mergers.

Appendix A

Table 9. MSE, $T = 100,\ T_{m1} = T/4,\ T_{m2} = T/2$

Parameters	**OLS**	**ALF**	**SELF**	**ELF**
$\theta_1(0.3)$	0.0745	0.0663	0.0662	0.0742
$\theta_2(0.2)$	0.0545	0.0499	0.0503	0.0390
$\theta_3(0.4)$	0.2562	0.2136	0.2166	0.2211
$\phi_{11}(0.2)$	0.0531	0.0508	0.0496	0.0449
$\phi_{21}(0.3)$	0.0452	0.0389	0.0395	0.0335
$\phi_{22}(0.1)$	0.1211	0.1036	0.1048	0.0853
$\phi_{31}(0.1)$	0.0867	0.074	0.0746	0.022
$\phi_{32}(0.3)$	0.0221	0.0199	0.0201	0.0123
$\phi_{33}(0.4)$	0.0226	0.0203	0.0205	0.0197
$\delta_{111}(0.2)$	0.0603	0.0537	0.0544	0.0392
$\delta_{121}(0.4)$	0.1392	0.1225	0.1231	0.1382
$\delta_{122}(0.3)$	0.0358	0.032	0.0319	0.0247
$\delta_{131}(0.25)$	0.0440	0.0400	0.0404	0.0166
$\delta_{132}(0.01)$	0.1950	0.1691	0.1706	0.1870
$\delta_{133}(0.1)$	0.1228	0.1076	0.1087	0.1166
$\delta_{211}(0.3)$	0.0332	0.0306	0.0307	0.0159
$\delta_{221}(0.5)$	0.0469	0.0389	0.0399	0.0422
$\delta_{222}(0.1)$	0.0542	0.0486	0.0492	0.0163
$\sigma(0.5)$	0.0535	0.0522	0.0506	0.0529

Table 10. MSE, $T = 200,\ T_{m1} = T/4,\ T_{m2} = T/2$

Parameters	**OLS**	**ALF**	**ELF**	**SELF**
θ_1(0.3)	0.0241	0.0227	0.0227	0.0232
θ_2(0.2)	0.0254	0.0248	0.0247	0.0166
θ_3(0.4)	0.1179	0.1046	0.1051	0.1082
ϕ_{11}(0.2)	0.0207	0.0204	0.0201	0.0123
ϕ_{21}(0.3)	0.0228	0.0208	0.021	0.0209
ϕ_{22}(0.1)	0.0856	0.0764	0.0767	0.0559
ϕ_{31}(0.1)	0.0631	0.0559	0.0562	0.0302
ϕ_{32}(0.3)	0.0098	0.0093	0.0094	0.0077
ϕ_{33}(0.4)	0.0158	0.0146	0.0146	0.0154
δ_{111}(0.2)	0.0339	0.032	0.0319	0.0362
δ_{121}(0.4)	0.1079	0.0975	0.0979	0.1026
δ_{122}(0.3)	0.0152	0.014	0.0142	0.0132
δ_{131}(0.25)	0.0246	0.0231	0.0231	0.0138
δ_{132}(0.01)	0.1726	0.1542	0.1548	0.1560
δ_{133}(0.1)	0.1033	0.0932	0.0934	0.0885
δ_{211}(0.3)	0.0157	0.0151	0.0152	0.0106
δ_{221}(0.5)	0.0360	0.0313	0.0317	0.0337
δ_{222}(0.1)	0.0318	0.0295	0.0295	0.0151
σ(0.5)	0.0441	0.0408	0.0401	0.0385

Table 11. MSE, $T = 300,\ T_{m1} = T/4,\ T_{m2} = T/2$

Parameters	**OLS**	**ALF**	**ELF**	**SELF**
θ_1(0.3)	0.0136	0.0135	0.0134	0.0131
θ_2(0.2)	0.0196	0.019	0.0189	0.014
θ_3(0.4)	0.0795	0.0723	0.0722	0.0747
ϕ_{11}(0.2)	0.0124	0.0122	0.0121	0.0089
ϕ_{21}(0.3)	0.015	0.0139	0.014	0.0143
ϕ_{22}(0.1)	0.0726	0.0653	0.0656	0.0322
ϕ_{31}(0.1)	0.0571	0.051	0.0511	0.0341
ϕ_{32}(0.3)	0.0062	0.006	0.006	0.0054
ϕ_{33}(0.4)	0.0137	0.0126	0.0126	0.0133
δ_{111}(0.2)	0.0272	0.0256	0.0255	0.0311
δ_{121}(0.4)	0.1013	0.0921	0.0922	0.0654
δ_{122}(0.3)	0.0115	0.0106	0.0106	0.0109
δ_{131}(0.25)	0.0187	0.0177	0.0177	0.0128
δ_{132}(0.01)	0.1662	0.1499	0.1501	0.1633
δ_{133}(0.1)	0.0944	0.0856	0.0857	0.0791
δ_{211}(0.3)	0.0102	0.01	0.0100	0.0080
δ_{221}(0.5)	0.0347	0.0304	0.0307	0.0322
δ_{222}(0.1)	0.0215	0.0202	0.0202	0.0136
σ(0.5)	0.0341	0.0324	0.0321	0.0332

Table 12. AB, $T = 100,\ T_{m1} = T/4,\ T_{m2} = T/2$

Parameters	OLS	ALF	ELF	SELF
θ_1(0.3)	0.1997	0.1972	0.1985	0.1734
θ_2(0.2)	0.1779	0.1756	0.1779	0.1385
θ_3(0.4)	0.3588	0.3397	0.3419	0.3555
ϕ_{11}(0.2)	0.1739	0.1782	0.1761	0.1421
ϕ_{21}(0.3)	0.1592	0.1559	0.1571	0.1406
ϕ_{22}(0.1)	0.2895	0.2802	0.2817	0.2697
ϕ_{31}(0.1)	0.2485	0.2392	0.2402	0.2354
ϕ_{32}(0.3)	0.1122	0.1119	0.1122	0.0919
ϕ_{33}(0.4)	0.1271	0.1158	0.1164	0.1149
δ_{111}(0.2)	0.1873	0.1858	0.187	0.1099
δ_{121}(0.4)	0.3071	0.3031	0.3036	0.28217
δ_{122}(0.3)	0.1424	0.1414	0.1412	0.1203
δ_{131}(0.25)	0.1591	0.1592	0.16	0.1046
δ_{132}(0.01)	0.3800	0.3704	0.379	0.2119
δ_{133}(0.1)	0.2909	0.2853	0.2866	0.2153
δ_{211}(0.3)	0.1544	0.1498	0.1505	0.1566
δ_{221}(0.5)	0.1680	0.1593	0.1613	0.1657
δ_{222}(0.1)	0.1784	0.1773	0.1785	0.1057
σ(0.5)	0.2436	0.23777	0.2278	0.22915

Table 13. AB, $T = 200,\ T_{m1} = T/4,\ T_{m2} = T/2$

Parameters	OLS	ALF	ELF	SELF
θ_1(0.3)	0.1203	0.1182	0.1183	0.1122
θ_2(0.2)	0.1301	0.1255	0.1298	0.1068
θ_3(0.4)	0.2515	0.2454	0.2462	0.2488
ϕ_{11}(0.2)	0.1088	0.1131	0.1123	0.0902
ϕ_{21}(0.3)	0.1141	0.1144	0.1147	0.1144
ϕ_{22}(0.1)	0.2547	0.2509	0.2515	0.2311
ϕ_{31}(0.1)	0.2209	0.2168	0.2174	0.2138
ϕ_{32}(0.3)	0.0752	0.0768	0.077	0.0708
ϕ_{33}(0.4)	0.1099	0.1002	0.1003	0.1036
δ_{111}(0.2)	0.1449	0.1475	0.1475	0.1094
δ_{121}(0.4)	0.2891	0.2876	0.2881	0.2075
δ_{122}(0.3)	0.0939	0.094	0.0947	0.0919
δ_{131}(0.25)	0.1219	0.1239	0.1239	0.0991
δ_{132}(0.01)	0.3779	0.3735	0.3741	0.1845
δ_{133}(0.1)	0.2848	0.2827	0.2831	0.2059
δ_{211}(0.3)	0.1377	0.1388	0.1393	0.1034
δ_{221}(0.5)	0.1544	0.1498	0.1505	0.1566
δ_{222}(0.1)	0.1385	0.1396	0.1396	0.1047
σ(0.5)	0.1925	0.1794	0.1685	0.1614

Table 14. AB, $T = 300,\ T_{m1} = T/4,\ T_{m2} = T/2$

Parameters	OLS	ALF	ELF	SELF
θ_1(0.3)	0.0983	0.0921	0.0918	0.0907
θ_2(0.2)	0.1301	0.1255	0.1298	0.1068
θ_3(0.4)	0.2135	0.2086	0.2087	0.2095
ϕ_{11}(0.2)	0.0840	0.0875	0.0873	0.0763
ϕ_{21}(0.3)	0.0926	0.0939	0.0941	0.0952
ϕ_{22}(0.1)	0.2104	0.2079	0.2085	0.1721
ϕ_{31}(0.1)	0.2144	0.2116	0.2117	0.2076
ϕ_{32}(0.3)	0.0630	0.0620	0.0619	0.0591
ϕ_{33}(0.4)	0.0945	0.0943	0.0944	0.0976
δ_{111}(0.2)	0.1335	0.1357	0.1353	0.1134
δ_{121}(0.4)	0.2877	0.2869	0.2870	0.2024
δ_{122}(0.3)	0.08227	0.0826	0.0827	0.0838
δ_{131}(0.25)	0.10783	0.1100	0.1101	0.0952
δ_{132}(0.01)	0.3769	0.3500	0.3475	0.1659
δ_{133}(0.1)	0.2781	0.2769	0.2771	0.1854
δ_{211}(0.3)	0.0102	0.0101	0.0102	0.0080
δ_{221}(0.5)	0.1576	0.1536	0.1545	0.1593
δ_{222}(0.1)	0.1151	0.1169	0.1170	0.0983
σ(0.5)	0.1371	0.1236	0.1227	0.1235

Table 15. $T = 78;$ Estimates; $T_{m1} = 21,\ T_{m2} = 35$

Parameters	SELF	ALF	ELF	OLS
θ_1	1.7222	1.8546	1.7939	1.9177
θ_2	1.3077	1.2735	1.3083	1.3145
θ_3	0.1165	0.1111	0.1276	0.1052
ϕ_{11}	0.4751	0.4777	0.5081	0.4783
ϕ_{21}	0.9094	0.8621	0.9640	0.8607
ϕ_{31}	0.9734	0.9740	0.9735	0.9798
δ_{111}	0.3708	0.3602	0.3915	0.5494
δ_{121}	-0.5027	-0.5185	0.5290	-0.5388
δ_{131}	-0.0834	-0.0944	0.1155	-0.1708
δ_{212}	-0.0687	-0.1177	0.3159	-0.0818
σ^2	0.0833	0.0864	0.0863	0.0355

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