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A Redundancy Allocation Model for Uncertainty Random Water Supply System in Water Saving Management Contract Projects

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Abstract Water saving management contract (WSMC) projects provide advanced technology and management for a water supply system to achieve water conservation and set redundant components to ensure water supply reliability. Project managers focus on the reliability optimization problem and require redundancy allocation strategies of the above system. This paper presents an optimization method by dealing with the lifetime of the whole water supply system. Assuming the lifetimes of advanced components are uncertain variables and the old ones are random variables, a reliability optimization model of water supply systems is established based on chance theory, and the redundancy allocation solutions are obtained by an optimization toolkit. A WSMC case in Shenzhen, China is studied and the results show that the reliability of the water supply system has been in a high state based on the allocation strategy. This study provides theoretical support for improving water-saving safety and popularizing the WSMC service mechanism.

Keywords WSMC, Water Supply System, Redundancy Allocation Problem, Uncertain Random Variable

AMS 2010 subject classifications 28A99, 60E05

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1. Introduction

Water saving management contract (WSMC) is an innovative method for a water saving service company (WSSC) to generate profit by providing water users with services such as capital collection, advanced technology integration, and water-saving enhancement [15, 17]. The premise for promoting new projects is to ensure their safety and reliability, which requires the traditional system reliability assessment method [9, 13]. Reliability assessment occupies an important position in all industries and attracts the attention of many scholars [1, 2].

Due to the uncertainties of advanced water-saving technologies, component reliability and water supply safety are concerns for project managers [34]. System reliability is mainly determined by the lifetime of components, and increasing redundancy is one of the common methods to improve system reliability [6]. Generally, the more redundancy a component has, the longer the lifetime of that component, but also the higher the cost [3]. Therefore, managers need to trade-off between cost and reliability, which is called the redundancy allocation problem (RAP) [27]. At present, RAP has attracted many scholars to study, including different form systems and solving algorithms [4, 14]. Studying RAP requires understanding the lifetimes of individual components and their relationship to the lifetime of the entire system [29, 38].

Traditional research usually makes the component lifetimes constants or random variables, and calculates the lifetime of the whole system according to the connection structure among different components [7, 8, 39]. Some scholars have evaluated the life of the entire system based on different connection structures or component life distributions [12, 25, 33]. However, some scholars believe that the lifetime of advanced components is not suitable

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to be described by random variables, because these components have few statistical data [36]. They also argue that the lifetime of a system composed of advanced components is better characterized by uncertain variables [18, 37].

The water supply system in the WSMC project is composed of advanced components and traditional components [26]. The lifetime of such a system cannot be described by only uncertain variables or random variables. Fortunately, Liu proposed the chance theory, which is a theoretical system that characterizes the lifetime of a system where uncertain variables and random variables coexist [22] . Based on chance theory, some scholars have considered reliability optimization problems. Wen and Kang first studied the reliability analysis of uncertain random systems based on chance theory and they describe each component's reliability as an uncertain variable or a random variable, respectively, according to component data information [35]. Gao and Yao [10] proposed the concept of importance index to describe the importance of individual components and component groups in uncertain random systems where both uncertain variables and random variables exist. However, at present, no research has been found on the reliability optimization of water supply systems in WSMC projects, especially the coexistence of advanced components and traditional components.

Based on the traditional redundancy allocation method, this paper establishes a redundancy allocation model for the mixed components of the water supply system of WSMC project by using chance theory, and solves the above model through equivalence transformation and optimization toolbox. In this paper, the lifetime of advanced components in the water supply system is set as an uncertain variable, and the lifetime of traditional components is set as a random variable. Then an uncertain random redundancy allocation model (UR-RAM) is established based on chance theory. Finally, a solution tool is used to obtain the reliability optimization strategy of the water supply system in the WSMC project.

2. Preliminaries

This section briefly introduces the water supply system in WSMC projects, uncertainty theory, and chance theory.

2.1. The water supply system in WSMC projects

The WSMC project implements water-saving renovations on some of the core components of the water user's water supply system, while the remaining components remain in their original state. In this way, this water supply system is composed of advanced components (denoted by *New*) and traditional components (denoted by *Old*), as shown in Figure 1.

Water users implementing WSMC projects are usually highly water-dependent enterprises, such as car washes, hotels, etc [34]. In order to ensure the reliability of the water supply system after the implementation of the project, the administrator will set up redundancy for the core components of the system [11]. Managers take a cold redundancy approach, but instead of setting redundant components directly on the water supply system, they store these components in backup warehouses. When a component fails, maintenance personnel shall replace it in time. In the actual water supply process, the time consumed to replace the parts is negligible. Therefore, the lifetime of a component can be described as the sum of the lives of all its redundant components.

The components of a water system are usually connected in a grid structure [32]. The connection mode of the water supply system considered in this paper is in the form of parallel series. When characterizing the entire system lifetime, calculations are made based on the connection structure [30]. To simplify this calculation process, this paper assumes that each subsystem of the water supply system consists of the same advanced types of components (all *New* or all *Old*), as shown in Figure 1.

2.2. Uncertainty theory

While the assumption of random variables is generally accepted and reasonable in most cases, in broader applications such as the space shuttle system, this assumption does not apply because the lack of precision in the data does not allow us to estimate the probability distribution or density function of the component life [20]. Uncertainty theory is a branch of axiomatic mathematics for deals with the imprecise phenomena in the production and life of the people. As one of the main elements, the uncertain measure is defined based on *normality axiom*,

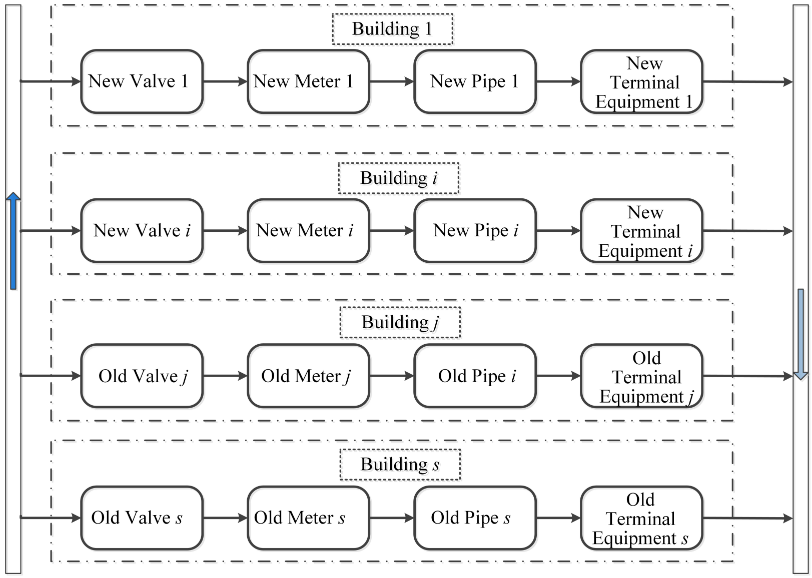

Figure 1. A hybrid water supply system coexist the *New* components and *Old* components

duality axiom, and *subadditivity axiom* to evaluate the possibility that the event happened in uncertainty theory. In particular, the product uncertain measure is introduced as *product axiom*, which is different to probability theory.

The concept of uncertainty distribution is introduced in order to describe the regularity of distribution of an uncertain variable, which implies the significant information for the studied uncertainty event. Some special uncertainty distributions, like normal, lognormal, and the properties of them are given. As a particular emphasis on the preceding properties, there is a method to obtain the uncertainty distribution of the function for several independent uncertain variables, which is very helpful for the subsequent researches. More details about the above concepts or properties can be explained in *Uncertainty Theory* [19].

2.3. Chance theory

Facing the phenomena that uncertainty and randomness coexist, chance theory seems an optional tool to deal with them. Chance space, defined as the product of uncertainty space and probability space, is a foundational concept in this theory. Important concepts such as chance measure, uncertain random variables, and chance distribution are all defined in chance space. In particular, for uncertain random variables that contain multiple uncertain variables and random variables, its chance distribution is fortunately obtained. For more details on these concepts or properties can be explained in the literature [19, 21].

3. Methodology

This section will establish a redundant allocation method for water supply systems for WSMC projects. First, the symbols and assumptions involved in this method are introduced. Secondly, based on chance theory, the life of mixed system of different types of components in WSMC project is described. Thirdly, with the maximum reliability of the system as the target and the constraints of cost and space, an optimal model of uncertain random redundancy allocation is established. Finally, the optimization model is solved by equivalent form transformation and MATLAB optimization toolbox.

3.1. Notations and assumptions

The mathematical notations used in the problem formulation are given in Table 1.

Table 1. Notations

s	number of subsystems
i	index of subsystems, $1 \le i \le s$
n_i	number of different components available for subsystem i, $1 \le i \le s$
x_{ij}	number of elements in jth component in subsystem i, $1 \le j \le n_i, 1 \le i \le s$
u_i	upper bounds on the number of redundant components in subsystem $i, 1 \le i \le s$
ξ_{ijk}	the uncertain lifetime of redundant element k in jth component in the subsystem i, $1 \le k \le x_{ij}, 1 \le j \le n_i, 1 \le i \le s$
η_{ijk}	the random lifetime of redundant element k in jth component in the subsystem i, $1 \le k \le x_{ij}, 1 \le j \le n_i, 1 \le i \le s$
T^0	preselected threshold system lifetime
$R_{T^0}(\mathbf{x})$	system reliability for a decision $\mathbf{x}$ at the threshold lifetime T^0
c_{ij}	the cost of each element in jth component in subsystem i, $1 \le j \le n_i, 1 \le i \le s$
c^0	the maximum capital available
N^*	set of positive integer

A parallel-series system (S) composing of s subsystems is considered (see Figure 2). Each subsystem i consists of n_i components and the jth component consists of $x_{ij} - 1$ redundant elements in subsystem $i(j = 1, 2, \cdots, n_i, i = 1, 2, \cdots, s)$, respectively. System S includes two classes of the components, namely the *New* and the *Old*. Specifically, for a given $q(1 \le q \le s-1)$, the components in subsystem $i(1 \le i \le q)$ are wholly *New* while the components in subsystems $i(q+1 \le i \le s)$ are all *Old*. For each of the above components, there are several redundant elements in cold standby way. We assume that the lifetimes of standby elements in *New* components and *Old* components are uncertain variables and random variables, respectively. For each component, the lifetimes of each element are independent and identically distributed. Compared with component's lifetime, the time that workers change the elements is negligible.

3.2. System lifetime

The main problem in RAP is how to deal with the system lifetime when the numbers of redundant elements in all components are determined. In system S, let

$$\mathbf{x} = (x_{11}, x_{12}, \cdots, x_{1n_1}, x_{21}, \cdots, x_{2n_2}, \cdots, x_{sn_s}) \tag{1}$$

be the redundant elements allocation. Then lifetime of system S under the above allocation can be described by the following theorem.

Theorem 1
For the redundant element k in jth component in subsystem i of system S, let ξ_{ijk} be the uncertain lifetime and η_{ijk} is the random lifetime, respectively, $1 \le k \le x_{ij}, 1 \le j \le n_i, 1 \le i \le s$. Then the lifetime of system S under the allocation (1) can be described as an uncertain random variable shown by the follows.

$$T(\mathbf{x}) = \left[\bigvee_{i=1}^{q} \bigwedge_{j=1}^{n_i} \left(\sum_{k=1}^{x_{ij}} \xi_{ijk}\right)\right] \bigvee \left[\bigvee_{i=q+1}^{s} \bigwedge_{j=1}^{n_i} \left(\sum_{k=1}^{x_{ij}} \eta_{ijk}\right)\right]. \tag{2}$$

Proof
In subsystem $1 \le i \le q$, let ξ_{ijk} be the uncertain lifetime of the redundant element k in jth component, $1 \le k \le x_{ij}, 1 \le j \le n_i$, then the total lifetime of jth component is

$$\sum_{k=1}^{x_{ij}} \xi_{ijk}, 1 \le j \le n_i. \tag{3}$$

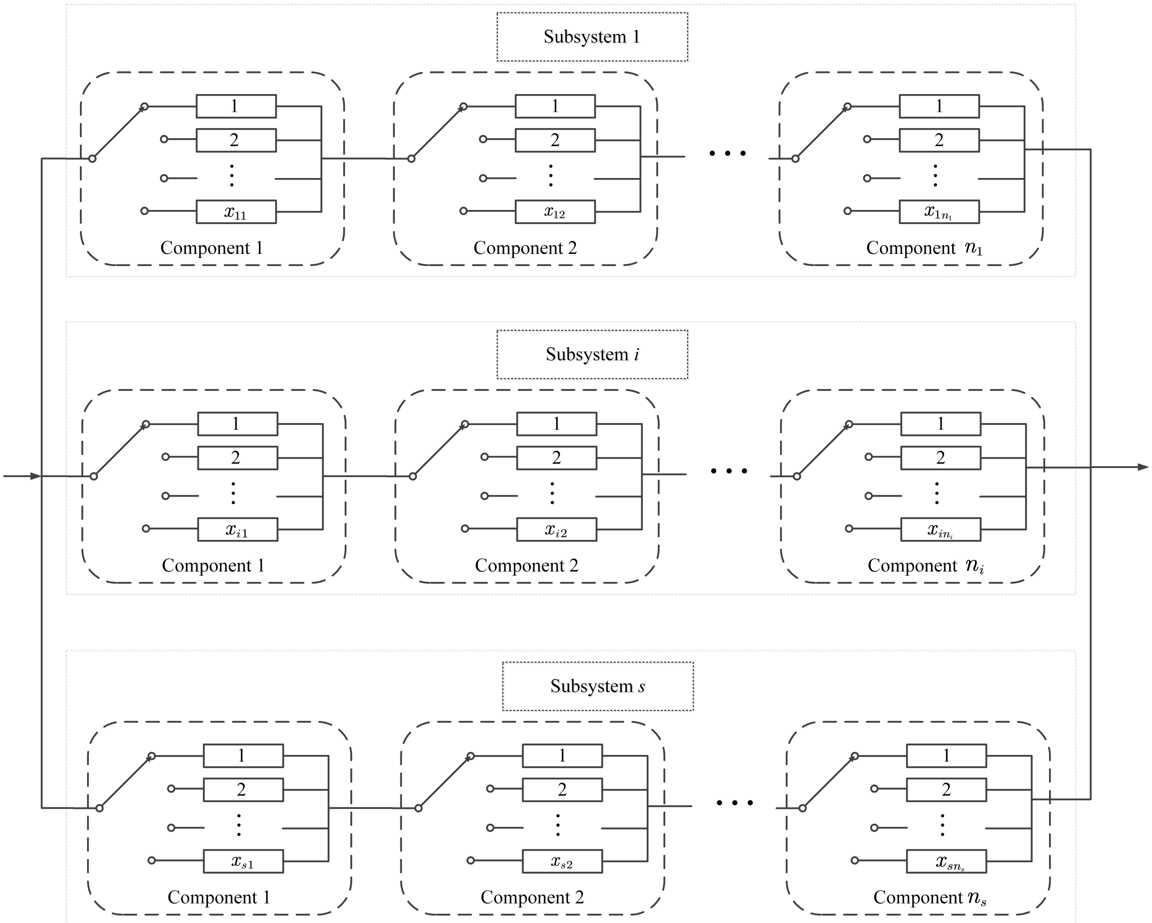

Figure 2. A s-stage parallel-series system S

Thus the lifetime of subsystem i is

$$\bigwedge_{j=1}^{n_i}\left(\sum_{k=1}^{x_{ij}}\xi_{ijk}\right). \tag{4}$$

Therefore, the maximal lifetime of the subsystems 1 to q is

$$\bigvee_{i=1}^{q}\bigwedge_{j=1}^{n_i}\left(\sum_{k=1}^{x_{ij}}\xi_{ijk}\right). \tag{5}$$

Similarly, the maximal lifetime of the subsystem $q+1$ to s is

$$\bigvee_{i=q+1}^{s}\bigwedge_{j=1}^{n_i}\left(\sum_{k=1}^{x_{ij}}\eta_{ijk}\right) \tag{6}$$

where η_{ijk} is the random lifetime of the redundant element k in jth component in the subsystem $i, 1 \le k \le x_{ij}, 1 \le j \le n_i, q+1 \le i \le s$. Therefore for given allocation $\mathbf{x}$, the lifetime of system S can be described as

$$T(\mathbf{x}) = \left[\bigvee_{i=1}^{q}\bigwedge_{j=1}^{n_i}\left(\sum_{k=1}^{x_{ij}}\xi_{ijk}\right)\right]\bigvee\left[\bigvee_{i=q+1}^{s}\bigwedge_{j=1}^{n_i}\left(\sum_{k=1}^{x_{ij}}\eta_{ijk}\right)\right]. \tag{7}$$

where $\mathbf{x}$ is defined by Equation (1). By Definition 1 in [22], $T(\mathbf{x})$ is an uncertain random variable.

Then it is necessary to determine the chance distribution of $T(\mathbf{x})$ for the further calculation, and the following theorem presents its chance distribution.

Theorem 2
Let $T(\mathbf{x})$ be an uncertain random variable which given by Equation (7). Then the chance distribution of $T(\mathbf{x})$ is

$$\begin{aligned}\Upsilon(x) &= Ch\{T(\mathbf{x}) \le x\}\\ &= \left(\bigwedge_{i=1}^{q}\Phi^i(x)\right)\cdot\left(\prod_{i=q+1}^{s}F^i(x)\right),\end{aligned} \tag{8}$$

where $\Phi^i(x)$ and $F^i(x)$ are the uncertain distribution and probability distribution determined by elements' lifetime distribution, respectively.

Proof
On the one hand, in subsystems $i(1 \le i \le q)$, for any component $j(1 \le j \le n_i)$, we assume that the lifetimes $\xi_{ijk}(1 \le k \le x_{ij})$ are independent uncertain variables with the same uncertainty distribution $\Phi_{ij}(x)(1 \le j \le n_i, 1 \le i \le q)$. By Theorem 2.1 in [19], we have that

$$\sum_{k=1}^{x_{ij}}\xi_{ijk}, 1 \le j \le n_i, 1 \le i \le q \tag{9}$$

and

$$\bigwedge_{j=1}^{n_i}\left(\sum_{k=1}^{x_{ij}}\xi_{ijk}\right), 1 \le i \le q \tag{10}$$

are uncertain variables, and their uncertainty distributions are

$$\begin{aligned}\Phi_j(z) &= \left\{\sum_{k=1}^{x_{ij}}\xi_{ijk} \le z\right\}\\ &= \sup_{z_1+z_2+\cdots+z_{x_{ij}}}\Phi_{ij}(z_1)\wedge\Phi_{ij}(z_2)\wedge\cdots\wedge\Phi_{ij}(z_{x_{ij}}), 1 \le j \le n_i, 1 \le i \le q\end{aligned} \tag{11}$$

and

$$\begin{aligned}\Phi^i(z) &= \left\{\bigwedge_{j=1}^{n_i}\left(\sum_{k=1}^{x_{ij}}\xi_{ijk}\right) \le z\right\}\\ &= \Phi_1(z)\vee\Phi_2(z)\vee\cdots\vee\Phi_{n_i}(z), 1 \le i \le q,\end{aligned} \tag{12}$$

respectively, by Theorem 2.16 in [19].

On the other hand, in the subsystems $i(q+1 \le i \le s)$, for any component $j(1 \le j \le n_i)$, let $\eta_{ijk}(1 \le k \le x_{ij})$ be independent random variables with the same probability distributions $F_{ij}(x)(1 \le j \le n_i, q+1 \le i \le s)$. By the definition of random variable, we obtain that

$$\sum_{k=1}^{x_{ij}} \eta_{ijk}, \quad 1 \leq j \leq n_i, q+1 \leq i \leq s \tag{13}$$

and

$$\bigwedge_{j=1}^{n_i} \left(\sum_{k=1}^{x_{ij}} \eta_{ijk} \right), \quad q+1 \leq i \leq s \tag{14}$$

are random variables, and their probability distributions are

$$\begin{aligned} F_j(y) &= Pr\left\{ \sum_{k=1}^{x_{ij}} \eta_{ijk} \leq y \right\} \\ &= \int_{y_1+y_2+\cdots+y_{x_{ij}}} d(F_{ij}(y_1))d(F_{ij}(y_2))\cdots d(F_{ij}(y_{x_{ij}})), \quad 1 \leq j \leq n_i, q+1 \leq i \leq s \end{aligned} \tag{15}$$

and

$$\begin{aligned} F^i(y) &= Pr\left\{ \bigwedge_{j=1}^{n_i} \left(\sum_{k=1}^{x_{ij}} \eta_{ijk} \right) \leq y \right\} \\ &= 1-(1-F_1(y))(1-F_2(y))\cdots(F_{n_i}(y)), \quad q+1 \leq i \leq s, \end{aligned} \tag{16}$$

respectively. Therefore by Theorem 1 in [22], yields

$$\begin{aligned} \Upsilon(x) &= Ch\{T(\mathbf{x}) \leq x\} \\ &= \left(\bigwedge_{i=1}^{q} \Phi^i(x) \right) \cdot \left(\prod_{i=q+1}^{s} F^i(x) \right) \end{aligned} \tag{17}$$

is the chance distribution of $T(\mathbf{x})$.

3.3. Uncertain random redundancy allocation model (UR-RAM)

In this paper, the reliability of a system is defined as the possibility that the lifetime of this system goes beyond a certain threshold. Since the lifetime of system S is an uncertain random variable, the reliability of it is, by chance theory, described as

$$R_{T^0}(\mathbf{x}) = Ch\{T(\mathbf{x}) \geq T^0\} \tag{18}$$

where T^0 is specifiable threshold lifetime. Considering the cost of a system, we have the following constraint

$$\sum_{i=1}^{s} \sum_{j=1}^{n_i} c_{ij} x_{ij} \leq c^0 \tag{19}$$

where c_{ij} is the cost of each element in jth component in subsystem i, $1 \leq j \leq n_i, 1 \leq i \leq s$ and c^0 is the maximum capital available. Redundancy level indicates total quantities of redundant components in each subsystem, which means the change count of each component. Thus we have

$$\sum_{j=1}^{n_i} x_{ij} \leq u_i \tag{20}$$

where u_i is upper bounds on the number of redundant components in subsystem $i, 1 \le i \le s$. Until now we have formed the redundancy allocation model by maximizing the reliability of system S, i.e.

[UR-RAM]

$$\begin{array}{rl} max & R_{T^0}(\mathbf{x}) = Ch\{T(\mathbf{x}) \ge T^0\} \\ subject\ \ to & \\ & \displaystyle\sum_{i=1}^{s}\sum_{j=1}^{n_i} c_{ij}x_{ij} \le c^0, \\ & \displaystyle\sum_{j=1}^{n_i} x_{ij} \le u_i, \qquad 1 \le i \le s, \\ & x_{ij} \in N^*, \qquad j = 1, \ldots, n_i, 1 \le i \le s, \end{array} \tag{21}$$

where N^* is set of positive integer.

3.4. Model solving

Because the optimization model UR-RAM contains uncertain random variables, it cannot be solved directly, so it needs to be transformed into an equivalent deterministic form first. Then the following theorem illustrates that the above UR-RAM's equivalent model.

Theorem 3
The UR-RAM (21) is equivalent to the following model.

$$\begin{array}{rl} min & \Upsilon(\mathbf{x}, T^0) \\ subject\ \ to & \\ & \displaystyle\sum_{i=1}^{s}\sum_{j=1}^{n_i} c_{ij}x_{ij} \le c^0, \\ & \displaystyle\sum_{j=1}^{n_i} x_{ij} \le u_i, \qquad 1 \le i \le s, \\ & x_{ij} \in N^*, \qquad j = 1, \ldots, n_i, 1 \le i \le s, \end{array} \tag{22}$$

where $\Upsilon(x)$ is obtained by Equation (8), which is the chance distribution of the uncertain random variable $T(\mathbf{x})$ and N^* is set of positive integer.

Proof
By Equation (17), the equivalency is obviously proved.

The equivalent UR-RAM is clearly a crisp integer programming model and we can solve it by the integer programming software MATLAB optimization toolbox.

Step 1: Program the objective function of the model (22) into the MATLAB function.
Step 2: Import the constraint data of the model (22) into the MATLAB optimization toolbox.
Step 3: Set the decision variable and its value range.
Step 4: Run the optimization toolbox to get the optimal solution of redundancy allocation.

4. Case study

In this section, a portion of Shenzhen PT university WSMC project is taken as an example to illustrate the feasibility of UR-RAM. To be more general, we assume the whole water supply system is divided into 2 subsystems.

4.1. Study area and data

Shenzhen PT university has 5 campuses, of which LXD and XLH are the primary educational and teaching bases, with about 15,000 full-time boarding students. Before the school implemented the WSMC, there were problems such as burst and leakage of water supply pipes, inaccurate measurement of old mechanical water meters, outdated water-using appliances, and low water efficiency. Shenzhen PT university signed WSMC contract with Shenzhen KXJY company in 2017.More detailed information about this WSMC project can be accessed to the web site as follows (https://htjs.waterconserving.cn/Article/detail/726). The contract period is 10 years. LXD campus is one of the campuses carried out WSMC of Shenzhen PT university. The dormitory area in LXD has been improved by changing some new type water supply components like remote water meter and new pedal valve. Meanwhile, the office area in LXD remains the original water supply components like traditional mechanical water meter and traditional pressing valve.

Based on contract, the water saving maintenance cost is 350000 yuan every year. The replacement cost of components converted to LXD campus is total 13000 yuan during the contract period. The university managers wish to set aside an appropriate component purchase fee to ensure the water supply reliability. Since the lifetimes and costs of different water supply components are not the same, the above problem is a RAP.

The buildings are simplified as two areas, dormitory area and office area. Each area represents one subsystem, respectively. In dormitory area, new remote water meter and new pedal valve are considered which are in series. In office area, equipped traditional mechanical water meter and traditional pressing valve are in series. Due to the special location of the buildings in LXD, the managers require that only one of the two areas has a normal water supply to meet the requirements of teachers and students. Hence, there are two subsystems which are in parallel for water supply system (see Figure 3).

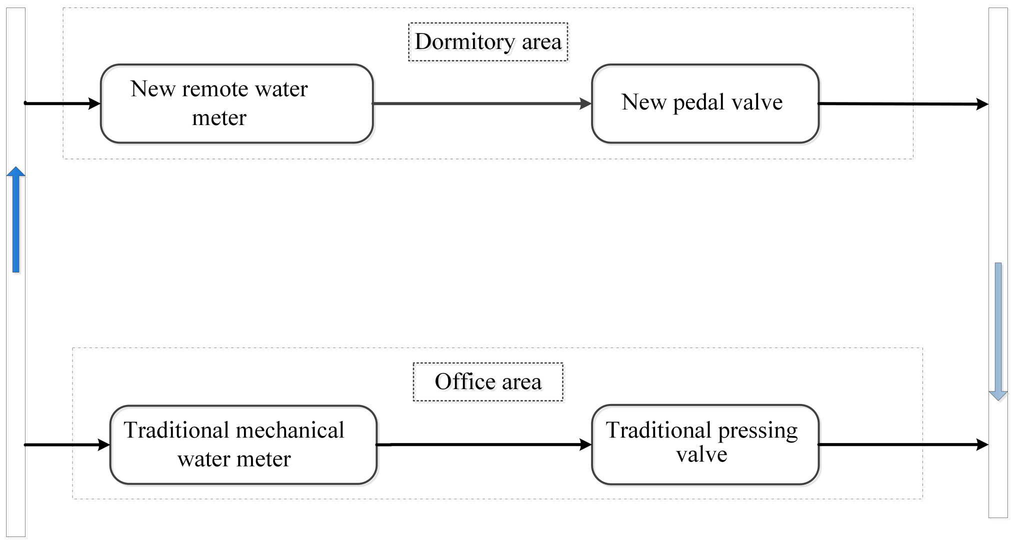

Figure 3. Portion of Shenzhen PT university WSMC project

The lifetimes of new remote water meter and new pedal valve are assumed as uncertain variables meanwhile the lifetimes of traditional mechanical water meter and traditional pressing valve are assumed as random variables. According to research results and experts inquiry [23, 24], the detailed database are shown in Table 2, where the third line shows that the element lifetimes in subsystem 1 are all *lognormal* uncertain variables (see Definition 2.10 in [19]) meanwhile the element lifetimes in subsystem 2 are all *normal* random variables, and the last line tells us the element costs which contain the service cost.

According to the water saving maintenance cost, the maximum expenditure is limited in 130 $\times 10^2$yuan each period for the above mentioned 4 components. The total count of cut off water supply caused by changing component is limited in 10 for subsystem 1 and in 5 for subsystem 2. The above different upper bounds on the

Table 2. The database of the example

Subsystem	Dormitory area		Office area	
Component	New meter	New valve	Old meter	Old valve
Element lifetime (years)	LOGN(2,0.7)	LOGN(1.6,0.6)	N(8,0.4)	N(6,0.5)
Element cost (10^2 RMB)	12	8	5	3

count of changing implies that WSCO places particular emphasis on the advanced components in allocation due to their instability. Finally, the threshold lifetime of water supply system is contract term plus extra 10 years, i.e. $T^0 = 20$.

4.2. *UR-RAM for Shenzhen PT university WSMC project*

In order to test the reliability of water supply for an entire water-saving project under extreme conditions, we established the UR-RAM for Shenzhen PT university based on the assumption that the two subsystems are in parallel in the water supply system and the whole system lifetime is determined by the maximum lifetime of these subsystems. This assumption seems different from the actual situation, but it may reflect the lifetime difference between the advanced components and traditional one. Let decision variables $x_{ij}, (i = 1, 2; j = 1, 2)$ represent the redundant allocation of *New* meter, *New* valve, *Old* meter and *Old* valve, respectively. According to the above data, we can obtain the following UR-RAM.

$$
\begin{aligned}
& min \quad \Upsilon(\mathbf{x}, 20) \\
subject \quad & to \\
& \quad 12x_{11} + 8x_{12} + 5x_{21} + 3x_{22} \leq 130, \\
& \quad x_{11} + x_{12} \leq 10, \\
& \quad x_{21} + x_{22} \leq 5, \\
& \quad x_{ij} \in N^*, \qquad j = 1, 2, i = 1, 2,
\end{aligned}
$$

where

$$
\begin{aligned}
\Upsilon(\mathbf{x}, 20) = & \left\{ \left[1 + \exp\left(\frac{\pi\left(2 - \ln\frac{20}{x_{11}}\right)}{\sqrt{3}\cdot 0.6} \right) \right]^{-1} \vee \left[1 + \exp\left(\frac{\pi\left(1.6 - \ln\frac{20}{x_{12}}\right)}{\sqrt{3}\cdot 0.6} \right) \right]^{-1} \right\} \cdot \\
& \left\{ 1 - \left(1 - \int_0^{20} \frac{1}{\sqrt{2\pi x_{21}}\cdot 0.4} \exp\left(-\frac{(t - 8x_{21})^2}{2x_{21}\cdot 0.4^2} \right) dt \right) \right\} \cdot \\
& \left\{ 1 - \int_0^{110} \frac{1}{\sqrt{2\pi x_{22}}\cdot 0.5} \exp\left(-\frac{(t - 6x_{22})^2}{2x_{22}\cdot 0.5^2} \right) dt \right\}.
\end{aligned}
$$

4.3. *Result and discussion*

A run of MATLAB optimization toolbox shows that the optimal solution is $x = (4, 6, 2, 3)$ and the corresponding maximum reliability is 0.7334, which tells us that in order to maximize the water supply system reliability, we should take 3 redundant element of *New* meter and 5 redundant elements of *New* valve in subsystem Dormitory area, meanwhile 1 redundant elements of *Old* meter and 2 redundant elements of *Old* valve in the subsystem Office area. Hence according to the above result, a water user can obtain the enough water supply with guarantee 73.34 % in WSMC projects. In general situations, the above water supply reliability could basically meet the water users' demand as for the university conditions [5].

From the optimal solution, the parameters in advanced components play more important roles in the reliability than ones in traditional components. That result implies the lifetimes of *New* components are more instability. In fact, there exist many components with similar characters in practical situations. For example, airplanes are often equipped with primary electronic gyroscopes and secondary mechanical gyroscopes. Apparently, the above mechanical gyroscopes are quite different from the electronic gyroscopes in both physical structure and reliability[31]. Hence in order to ensure the system reliability, managers usually intend to increase the redundant elements of these advanced components. In summary, UR-RAMs can be applied to solve the RAPs in WSMC projects, and to obtain the quantities of each component with optimal solution [28].

The sensitivity analysis is organized to analysis model validity. The sensitivity analysis of the main parameters of the model in this paper is shown in Table 3. Results show that element lifetime of *New* meter, total count and threshold lifetime are more sensitive parameters than others.

Table 3. Sensitivity analysis

Parameters	Value	Allocation results	Reliability
Element lifetime of *New* meter	LOGN(2,0.7)	4,6,2,3	0.7334
	LOGN(3,0.7)	2,8,2,3	0.8590
	LOGN(1,0.7)	6,4,2,3	0.3709
Element lifetime of *New* valve	LOGN(1.6,0.6)	4,6,2,3	0.7334
	LOGN(1.7,0.6)	4,6,2,3	0.7334
	LOGN(1.5,0.6)	4,6,2,3	0.7099
Element lifetime of *Old* meter	N(8,0.4)	4,6,2,3	0.7334
	N(9,0.4)	4,6,2,3	0.7334
	N(7,0.4)	4,6,2,2	0.7334
Element lifetime of *Old* valve	N(6,0.5)	4,6,2,3	0.7334
	N(7,0.5)	4,6,2,2	0.7334
	N(5,0.5)	4,6,3,2	0.7334
Element cost of *New* meter and *New* valve	12,8	4,6,2,3	0.7334
	13,9	4,6,2,3	0.7334
	11,7	4,6,2,3	0.7334
Element cost of *Old* meter and *Old* valve	5,3	4,6,2,3	0.7334
	6,4	4,6,2,3	0.7334
	4,2	4,6,2,3	0.7334
Maximum expenditure	130	4,6,2,3	0.7334
	140	4,6,2,3	0.7334
	120	4,6,2,3	0.7334
Total count of subsystem 1	10	4,6,2,3	0.7334
	11	5,6,2,3	0.7680
	9	4,5,2,3	0.6561
Total count of subsystem 2	5	4,6,2,3	0.7334
	6	4,6,3,3	0.7362
	4	4,6,2,2	0.7334
Threshold lifetime	20	4,6,2,3	0.7334
	21	4,6,2,2	0.7080
	19	4,6,2,3	0.7586

Model applicability and expansion. At present, the model established in this paper is suitable for the case where traditional parts and advanced parts are connected separately, and the system is facing parallel series. When the chance theory is used to characterize the lifetime of the above system, it is necessary for each component to be independent of each other. Compared with the more general scenario, the model built in this paper is mainly applicable to the water supply system of public institutions in the WSMC project. The type of commercial equipment for water supply systems in this field is usually small, the frequency of product replacement is not high, and the storage and application of redundant parts of the project can usually last the entire project contract period. On the other hand, the form of water supply system connection of public institutions is relatively simple, mostly in basic ways such as series connection. In chance theory, the theoretical research of describing the operation of random variables and uncertain variables has just started, and more algorithms need to be studied. Therefore, due to the limitation of theoretical methods, the lifetime characterization of hybrid connections between traditional parts and advanced parts still needs to be further studied. At the same time, the extension of parallel series system to series parallel and hybrid connection, and the characterization of system life and reliability in this case need additional research work.

In actual WSMC projects, managers generally replace water-saving equipment with new equipment before it fails to ensure the reliability and water-saving effect of the water-saving system. Therefore, managers usually need to be conservative based on the redundant optimization results to ensure that the WSMC project water supply system operates at a guaranteed level. Usually, water-saving service companies set a water-saving income guarantee value to encourage water users to participate, which requires the water-saving system's reliability and cost. Therefore, the RAP of the WSMC project still needs further research to consider the participants' enthusiasm. Some scholars [16] have studied the relationship between the cost of water-saving equipment and other factors in the WSMC project and the enthusiasm of participants, and the future research can be further promoted on the existing basis.

5. Conclusion

An urgent problem for WSMC project is the description of the water supply system reliability. By characterizing the advanced components lifetimes as uncertain variables and the traditional type ones as random variables, this paper firstly considered the water supply reliability optimization under WSMC based on RAP. An UR-RAM established in this paper, based on chance theory, dealt with the hybrid system reliability in which the components with lots of statistical data and the other with few statistical data coexist. This paper only considers the most classical parallel-series water supply system and assumes that the advanced components and traditional components will not be installed in one same subsystem. In future studies, more practical water supply system structures and components connection situations can be investigated.

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