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# Discrimination between Quantile Regression Models for Bounded Data

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**Abstract** Most often when we use the term 'bounded', we mean a response variable that retains inherent upper and lower boundaries; for instance, it is a proportion or a strictly positive for example incomes. This constraint has implications for the type of model to be used since most traditional linear models may not respect these boundaries. Parametric quantile regression with bounded data thus comes with a framework for analysis and interpretation of how the predictor of interest influences the response variable over different quantiles while constrained by the bounds of the theoretically assumed distribution. In this paper, several parametric quantile regression models are explored and their performance is investigated under several conditions. Our Monte Carlo simulation results suggest that some of these parametric quantile regression models can bring significant improvement relative to other existing models under certain conditions.

**Keywords** Bounded data, beta distribution, quantile regression model, Monte Carlo simulation, Survival Analysis, SCSO

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## 1. Introduction

Regression analysis is the fit of a model with the aim of expressing it as a numerical vector, that is, it works to predict the value of a numerical variable using a single estimate, the mean, and sometimes the need to predict the full range or distribution of the target variable, however, statisticians have developed increasingly advanced methods of regression, and for this reason Quantitative regression techniques have been developed.

This interest is due to several natural and anthropogenic phenomena are measured as indexes, percentages, proportions, rates and ratios, which are bounded on a certain interval, usually the unit interval. "In many real-world applications, data is limited to a specific period of time, often within the range [0,1], and the limited nature of this data presents unique modeling challenges, prompting researchers to focus on addressing these issues, as The need to develop unit interval distributions is rapidly increasing because of their applications in engineering, economics, psychology, and biology. The unit interval or distribution bounded by the interval [0,1] is important for modeling data contained in the intervals between zero and one, such as ratios, rates, and percentages. For example, in psychology percentages and proportions are useful for judgment probabilities, which are the percentage of the mind section captured by a given region. In economics, the variable or data under study is generally limited to unit intervals, for example, market share, capital structure, and percentage of income spent on non-permanent uses. It is also noted that unit distributions have attractive bathtub-like hazard rate shapes.

Quantitative regression is one of the methods that has taken wide application in the past two decades, presented by [1], and has become attractive to researchers as it provides a framework for modeling the relationship between the response variable and the covariates using the quantitative function. The ordinary least squares regression is

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that the quantitative regression estimates are more robust against the extreme values in the response measurements, that is, it is considered one of the robust methods as it gives more details of the effect of the explanatory variables on the response variable , It also has a methodology for understanding the conditional distribution of the response variable by looking at the values of some common variables at different levels (quantities), thus providing users with a more complete picture [2] The simplest definition of quantitative regression is the value that divides a set of data into groups of equal size; Thus, the quantitative values define the boundaries between groups. Statistically speaking, quantities are values taken at regular intervals from the inverse of the cumulative distribution function (CDF) of a random variable. It is important to note that the usual quantitative regression is able to approximate the conditional quantities of a response variable in the unit period, by the methods of the equivalence principle.

A series of distributions with unit interval support has been studied in the trend literature on cumulative distribution function (CDF) transformations. From the component, we mention the log-Bilai [18], inverse Gaussian unit [21],Modeling Bounded Data under a Unit Birnbaum–Saunders Distribution with Applications in Medicine and Politics [24], Lindley unit [22], In order to evaluate the effect of one or more joint searches on the average coordination distribute on bounded by the unit interval, the following distributions can be used: beta rectangular [20], and log-Lindley [22]

In applied statistics, it is very common to deal with the uncertainty of a finite phenomenon. In many fields of knowledge, we often encounter variables such as the proportions of a particular property, the scores of some ability tests, and various indicators and rates, which are located on the period (0,1), as the distributions of units specified in the interval of units are applied to model the behavior of a random variable limited by commas Temporal (0,1) has found applications in areas such as health, biology, meteorology, hydrology, financial modeling and other sciences.

The beta distribution is the well-known statistical distribution for modeling data sets on the interval (0,1) and is a suitable and useful model in many areas of statistics, but in some cases its ability to model data is not sufficient to explain it.

The beta distribution has been widely used in statistical theory and practice for more than a hundred years, and the beta distribution is the most widely used for modeling unit outcomes, i.e. the time period is between (0, 1), and beta regression is useful for understanding the effect of covariates on the average response, as it can data as rates or ratios. On the other hand, Kumaraswamy argues that the beta distribution is not faithfully proportional to hydrological random variables like daily precipitation, daily stream flow, etc. also but it is much simpler to use especially in simulation studies where probability density function, cumulative distribution function and quantitative functions can be expressed In closed form [3]. Accordingly, there are alternative distributions to the beta distribution that have been defined and applied in the literature, such as:
The Johnson SB distribution, Kumaraswamy distribution , The Log-extended exponential-geometric distribution, Unit Generalized Half Normal distribution, The unit-Birnbaum-Saunders distribution, The unit-Burr-XII distribution, The unit-Chen distribution, The unit-Half-Normal -X distribution, The unit-Gompertz distribution, The unit-Weibull distribution, and The unit-Logistic distribution.

## 2. Methodology

### *2.1. The Johnson SB distribution(johnsonsb)*

Havley and Schroeder (1977) first introduced a new class of distributions called Johnson SB in the forestry literature. It is widely used in forestry to represent the empirical distributions of forest tree variables to represent the empirical distributions of forest tree variables such as diameter, height, and volume, and has since been widely used. Wide in forest diameter and height distribution [4].
Density function ($f_{SB}$), Distribution function $F_{SB}$ , Quantile function $Q_{SB}$ for the The Johnson SB distribution reparametrized in terms of the $\tau$-th quantile, $\tau \in (0; 1)$.

$$f_{SB}\left(y;\alpha,\beta\right)=\frac{\beta}{\sqrt{2\pi}\,y\left(1-y\right)}exp\left\{-\frac{1}{2}\left[\alpha+\beta\,log\left(\frac{y}{1-y}\right)\right]^{2}\right\} \qquad \ldots \quad (1)$$

The probability density function (p.d.f.) corresponding to (**??**) is :

$$F_{SB}\left(y;\alpha,\beta\right)=\emptyset\left[\alpha+\beta\ log\emptyset^{-1}\left(\tau\right)\right] \quad , \quad y\in(0,1) \qquad \ldots \quad (2)$$

$$Q_{SB}\left(\tau;\alpha,\beta\right)=\frac{exp\left[\frac{\emptyset^{-1}(\tau)-\alpha}{\beta}\right]}{1+exp\left[\frac{\emptyset^{-1}(\tau)-\alpha}{\beta}\right]} \qquad \ldots \quad (3)$$

where $\alpha>0$ and $\beta>0$ are shape parameters and $y\in(0,1)$

### 2.2. *leeg The Log-extended exponential-geometric distribution*

It is a probability distribution derived from a two-factor expanded exponential geometric (EEG) distribution with finite support [7].

$$f_L\left(y;\alpha,\beta\right)=\frac{\alpha y^{\alpha-1}\left(1+\beta\right)^{\beta-1}}{\left(1+\beta y^{\alpha}\right)^{2}} \quad , \quad y\in(0,1) \ \ ,\alpha>0\ ,\ \beta>-1 \qquad \ldots \quad (7)$$

$$F_L\left(y;\alpha,\beta\right)=\frac{y^{\alpha}\ \left(1+\beta\right)}{1+\beta\ y^{\alpha}} \qquad \ldots \quad (8)$$

where $\alpha$ and $\beta$ are the model parameters. In particular, the case $\alpha>0$ and $\beta\in$ (-1, 0) corresponds to the exponential-geometric distribution .
In the sequel, the random variable defined by will be referred to as theLog-extended exponential-geometric (Log-extended exponential-geometric distribution). The leeg distribution presentsan advantage with respect to the beta distribution since it does not include special functions in its formulation.

$$Q_L\left(\tau;\alpha,\beta\right)=\left[\frac{\tau}{1+\alpha\left(1-\tau\right)}\right]^{\frac{1}{\beta}} \qquad \ldots \quad (9)$$

### 2.3. *Unit Generalized Half Normal distribution (ughn)*

Let the r.v. Y follow a ughn distribution with probability density function (pdf) and cumulative distribution function (cdf) [9] :

$$f_{\text{ughn}}\ \left(y;\alpha,\beta\right)=\sqrt{\frac{2}{\pi}}\,\frac{\beta}{y(-\log\left(y\right)\,)}\left(\frac{log(y)}{\alpha}\right)^{\beta}exp\left[-\frac{1}{2}(\frac{log(y)}{\alpha})\right]^{2\beta} \qquad \ldots\ (10)$$

The new pdf can be obtained with transformation of the $Y\ =\ e^{\,-y}$ rv, where $Y$ has GHN r.v. On the other word, a rv $Y$ is distributed unit GHN distribution on the interval (0,1) if its log transformation, $-logy$, is distributed GHN($\alpha$, $\beta$). We denote it with ughne($\alpha$, $\beta$). For $\alpha$ = 1, unit half normal distribution is obtained. The corresponding CDF is given by :

$$F_{ughn}\left(y;\alpha,\beta\right)=2\emptyset\left[-\left(-\frac{log(y)}{\alpha}\right)^{\beta}\right] \quad , \quad y\in(0,1) \qquad \ldots \quad (11)$$

respectively, where $\alpha,\ \beta\ >\ 0$, $\emptyset\left[\cdot\right]$ is the pdf of standard normal distribution and $\emptyset\left[\cdot\right]$ is the cdf of standard normal distribution

$$Q_{ughn}\left(\tau;\alpha\right)=\quad exp\ \left\{-\alpha\Big[-\emptyset^{-1}\left(\frac{\tau}{2}\right)\Big]^{\frac{1}{\beta}}\right\} \qquad \ldots \quad (12)$$

### *2.4. The unit-Birnbaum-Saunders distribution (ubs)*

Density function, distribution function, quantile function and random number generation function for the (ubs) distribution reparametrized in terms of the $\tau$-th quantile, $\tau \in (0;\ 1)$ [10].

$$f_{ubs}\left(y;\alpha,\beta\right)=\frac{1}{2y\alpha\beta\sqrt{2\pi}}\left[\left(-\frac{\alpha}{\log(y)}\right)^{\frac{1}{2}}+\left(-\frac{\alpha}{log(y)}\right)^{\frac{3}{2}}\right]exp\left[\frac{1}{2\beta^2}\left(2+\frac{log(y)}{\alpha}\right)+\frac{\alpha}{log(y)}\right] \quad \dots (17)$$

$$F_{ubs}\left(y;\alpha,\beta\right)=\frac{y^{\alpha}\ (1+\beta)}{1+\beta\ y^{\alpha}} \quad , \quad 0<x<1 \quad \dots \quad (13)$$

where , $\alpha > 0$ is the shape parameter

**Quantile function**

$$Q_{ubs}\left(\tau;\alpha,\beta\right)=1-\emptyset\left\{\frac{1}{\beta}\left[-\left(\frac{log(y)}{\alpha}\right)^{\frac{1}{2}}-\left(-\frac{\alpha}{log(y)}\right)^{\frac{1}{2}}\right]\right\} \quad \dots \quad (14)$$

### *2.5. The unit-Burr-XII distribution (uburrxii)*

Let Y be a unit random variable having the UBXII distribution. The probability density function, cumulative distribution function, quantile function and random number generation function for the of Y are [11]

$$f_{\boldsymbol{uburrxii}}\left(y;\alpha,\beta\right)=\frac{\alpha\beta}{y}[-log(y)]^{\alpha-1}\ \ [1+(-\log(y))^{\alpha}]^{-\beta-1} \quad \dots (15)$$

$$F_{\boldsymbol{uburrxii}}\left(y;\alpha,\beta\right)=\ [1+(-log(y))^{\alpha}]^{-\beta} \quad \dots \quad (16)$$

respectively, where $\alpha > 0$ and $\beta > 0$ are shape parameters. The quantile function of $y$ $Y$ follows by inverting (**??**) , namely

$$Q_{\boldsymbol{uburrxii}}\left(y;\alpha,\beta\right)=exp\left[-\left(\tau^{-\frac{1}{\beta}}-1\right)\right]^{\frac{1}{\alpha}} \quad \dots \quad (17)$$

Henceforth, if $y$ is a random variable with pdf (**??**), we write $Y \sim \mathrm{uburrxii}\ (\alpha,\ \beta)$. For $\alpha = 1$, the $\mathrm{uburrxii}$ distribution reduces to the unit Lomax distribution [21]. By taking $\beta = 1$, it is a special case of the unit log-logistic distribution [22].

### *2.6. The unit-Chen distribution (uchen)*

This interest is due to the many natural and human phenomena being measured as indicators, percentages, proportions, rates and proportions, which are constrained to a certain period of time, usually a unit period. The need for modeling and analyzing limited data occurs in many real-life fields such as medicine, politics and psychology hence, to describe this type of data statistically, distributions with limited support are needed [12].

$$f_{uchen}\left(y;\alpha,\beta\right)=\frac{\alpha\beta}{y}(-log(y))^{\beta-1}exp\left\{[-\log(y)]^{\beta}exp\left\{\alpha\left\{1-exp[-log(y)]^{\beta}\right\}\right\}\right\} \quad \dots (18)$$

$$F_{uchen}\left(y;\alpha,\beta\right)=exp\left\{\alpha\left\{1-exp[-log(y)]^{\beta}\right\}\right\} \quad , \quad y\in(0,1)\ \ ,\alpha>0\ ,\ \beta>-1 \quad \dots \quad (19)$$

where , $\alpha$>0 is the shape parameter and $\Phi$ is the CDF of the standard normal distribution. It is noteworthy that $\gamma = \exp(-\beta)$ is a scale parameter and is also the median of the distribution of *X*, since F($\delta$;$\alpha$,$\beta$)=0.5. In addition, the *r*-th moment of *X* is given by

$$Q_{uchen}\left(\tau;\alpha,\beta\right)=exp\left\{-\left[log\left(1-\frac{\log(\tau)}{\alpha}\right)\right]^{\frac{1}{\beta}}\right\} \quad \dots \quad (20)$$

### *2.7. The unit-Half-Normal-X distribution (ughnx)*

Let the r.v. Y follow a ughnx distribution with probability density function (pdf) and cumulative distribution function (cdf) :

$$f_{\mathrm{ughnx}}\ (y;\alpha,\beta)=\sqrt{\frac{2}{\pi}}\,\frac{\beta}{y(1-y)}\left(\frac{y}{\alpha(1-y)}\right)^{\beta} exp\left[-\frac{1}{2}(\frac{y}{\alpha(1-y)})\right]^{2\beta} \quad \ldots (21)$$

The new pdf can be obtained with transformation of the $Y = e^{-y}$ rv, where $Y$ has GHN r.v. On the other word, a rv $Y$ is distributed unit GHN distribution on the interval (0,1) if its log transformation, $-logy$, is distributed GHN($\alpha$, $\beta$). We denote it with ughne($\alpha$, $\beta$). For $\alpha$ = 1, unit half normal distribution is obtained. The corresponding CDF is given by :

$$F_{ughnx}\,(y;\alpha,\beta)=2\emptyset\left[\frac{y}{\alpha(1-y)}\right]^{\beta}-1 \quad , \quad y\in(0,1) \quad \ldots \quad (22)$$

$$Q_{ughnx}\,(\tau;\alpha)= \frac{\alpha\left[\emptyset^{-1}\left(\frac{\tau+1}{2}\right)\right]^{\frac{1}{\alpha}}}{1+\alpha\left[\emptyset^{-1}\left(\frac{\tau+1}{2}\right)\right]^{\frac{1}{\alpha}}} \quad \ldots \quad (23)$$

### *2.8. The unit-Gompertz distribution (ugompertz)*

The Gompertz unit (UG) distribution shows a right-skewed (unimodal) density and an inverted J-shaped intensity while the hazard rate is constant and increasing and an inverted bathtub then the bathtub hazard rate. Benjamin Gompertz in 1825 introduced the Gompertz distribution in relation to human mortality and actuarial tables. Since then, it has received much attention from demographers and actuaries. This distribution is a generalization of the exponential distribution and has many real-world applications, especially in medical and actuarial studies. It has some good relationships with some well-known distributions such as the exponential, double exponential, Weibull, maximum value (Gumbel distribution) or the generalized logistic distribution.
we describe the new bounded distribution, which arises from a logarithmic transformation in the Gompertz distribution. This transformation is also considered in Grassia (1977) for the unit-Gamma distribution and Gómez-Déniz et al. (2014) for the log-Lindley distribution.
Let Y be a non negative random variable with Gompertz distribution, then its probability density function is given by [13]:

$$f_{\mathbf{ugompertz}}\ (y;\alpha,\beta)=\frac{\alpha\beta}{y}\,exp\left\{\alpha-\beta\log\left(y\right)\ -\alpha\ exp\left[-\beta\log(y)\right]\right\} \quad \ldots (24)$$

where $y\in(0,1)$ , $\beta>0$ and $\alpha>0$ are scale and shape parameters, respectivel . for x $\in$ (0, 1). The corresponding cumulative distribution function are given by :

$$\mathrm{F}_{\mathrm{ugompertz}}\,(y;\alpha,\beta)=exp\left[\alpha\left(1-y^{\beta}\right)\right] \quad \ldots \quad (25)$$

The quantile function $y=Q\left(\tau\right)=F^{-1}\left(\tau\right)\ ,\quad for\quad 0<\tau<1$ of the $\mathrm{ugompertz}$ distribution is obtained by inverting Equation (F) is given by

$$Q_{ugompertz}\,(\tau;\alpha,\beta)= \frac{\left[\alpha-\log(\tau)\right]^{-\frac{1}{\beta}}}{\alpha} \quad \ldots \quad (26)$$

### *2.9. The unit-Weibull distribution (uweibull)*

we obtain the Unit-Weibull (uweibull) distribution with p.d.f :

$$f_{\mathrm{uweibull}}\,(y;\alpha,\beta)=\frac{\alpha\beta}{y}\,\left[-\log(y)\right]^{\beta-1}exp\left\{-\alpha[-\log(y)]^{\beta}\right\} \quad \ldots (27)$$

and cumulative distribution function (c.d.f.) given by :

$$\mathrm{F}_{\text{uweibull}}\left(y;\alpha,\beta\right)=exp\left[\alpha\left(1-y^{\beta}\right)\right]\quad,\quad 0<y<1 \qquad \ldots \quad (28)$$

where $\alpha>0$ and $\beta>0$ are shape parameters. Special cases of the uweibull distributions include. the standard uniform distribution over the interval $(0,1)$ $(\alpha=\beta=1)$, the power function distribution $(\beta=1)$ and the unit-Rayleigh distribution ($\beta$=2). Therefore, the new distribution has connection with some well-known distributions, and hence, it can be very useful in many practical situations.
Since it is not possible to obtain a simple analytic expressions for $E(y)$, it is difficult to model the mean of $Y$ in the absence/presence of covariates. On the other hand, the quantile function of the uweibull distribution has a simple analytic expression given by [15] :

$$Q_{uweibull}\left(\tau;\alpha,\beta\right)=\ \exp\left\{-\left[-\frac{log\left(\tau\right)}{\alpha}\right]^{\frac{1}{\beta}}\right\} \qquad \ldots \quad (29)$$

### 2.10. The unit-Logistic distribution (ulogistic)

There are continuous distributions that still have limited support that needs further study, such as the L-Logistic distribution originally proposed by Tadikamala and Johnson (1982). By transforming the logistic criterion, this construction is similar to the SB system proposed by Johnson (1949).
We say that the random variable (r.v.) $y$ follows a ulogistic distribution, denoted by $y\sim$ ulogistic$(\alpha,\beta)$, if its probability density function (pdf) is given by [17] :

$$f_{\text{ulogistic}}\left(y;\alpha,\beta\right)=\frac{\beta\exp\left(\alpha\right)\left(\frac{y}{1-y}\right)^{\beta-1}}{\left[1+\exp(\alpha)\left(\frac{y}{1-y}\right)^{\beta}\right]^{2}}\quad,\quad 0<y<1 \qquad \ldots\ (30)$$

Where , $0<\beta<1$ and $\alpha>0$ , Depending on the parameters $\alpha$ and $\beta$, the ulogistic distribution takes on a variety of shapes .
The cumulative distribution function (cdf) of the ulogistic distribution is given by :

$$\mathrm{F}_{\text{ulogistic}}\left(y;\alpha,\beta\right)==\frac{exp\left(\alpha\right)\left(\frac{y}{1-y}\right)^{\beta}}{1+exp\left(\alpha\right)\left(\frac{y}{1-y}\right)^{\beta}}\quad,\quad 0<y<1 \qquad \ldots \quad (31)$$

function The quantile function of the ulogistic distribution has a simple analytic expression given by :

$$Q_{ulogistic}\left(\tau;\alpha,\beta\right)=\mathrm{F}^{-1}{}_{\text{ulogistic}}\left(y;\alpha,\beta\right)=\frac{exp\left(-\frac{\alpha}{\beta}\right)\left(\frac{\tau}{1-\tau}\right)^{\frac{1}{\beta}}}{1+exp\left(-\frac{\alpha}{\beta}\right)\left(\frac{\tau}{1-\tau}\right)^{\frac{1}{\beta}}}\qquad 0<\tau<1 \qquad \ldots (32)$$

This would readily enable a quantile-based analysis of this model. Note that if $\beta=1-\beta=0.5$, then$Q_{ulogistic}\left(\tau;\alpha,\beta\right)=\alpha$, which means that the parameter $\alpha$ is indeed the 50th percentile or the median of the ulogistic distribution.

## 3. Bayesian Information Criterion (BIC)

We obtain this formula from the formula of the AIC standard by multiplying the degree of freedom df by the amount Ln(n) instead of the value 2, and the general formula of the (BIC) standard is as follows [14, 26, 27, 28] :

$$BIC=-2Ln\left(Lik\right)+\mathrm{Ln}\left(n\right)df \qquad \ldots \quad (33)$$

Table 1. The best fitting model when n=30 for 70% $>$0.5 and 30% $>$0.5

| Tau | P | Case: 70% $>$0.5 +30% $<$0.5 | Case: 30% $>$0.5 +70% $<$0.5 |
|---|---|---|---|
| 0.2 | 5 | Ughnx | Uchen |
| | | Johnsonsb | Ulogistic |
| | | Uweibull | Johnsonsb |
| | | Ulogistic | Ugompertz |
| | | Uburrxii | Leeg |
| | 10 | Ughnx | Uchen |
| | | Ulogistic | Ugompertz |
| | | Leeg | Leeg |
| | | Johnsonsb | Johnsonsb |
| | | Ugompertz | Uweibull |
| 0.5 | 5 | Ughnx | Uchen |
| | | Johnsonsb | Ulogistic |
| | | Uweibull | Johnsonsb |
| | | Ubs | Ugompertz |
| | | Ulogistic | Leeg |
| | 10 | Ubs | Uchen |
| | | Ulogistic | Ugompertz |
| | | Johnsonsb | Johnsonsb |
| | | Leeg | Uweibull |
| | | Ugompertz | Ughne |
| 0.7 | 5 | Ughnx | Uchen |
| | | Johnsonsb | Ulogistic |
| | | Uweibull | Johnsonsb |
| | | Ubs | Ugompertz |
| | | Ulogistic | Leeg |
| | 10 | Ughne | Uchen |
| | | Ugompertz | Ugompertz |
| | | Ulogistic | Johnsonsb |
| | | Johnsonsb | Uweibull |
| | | Leeg | Ughne |
| 0.9 | 5 | Johnsonsb | Uchen |
| | | Uweibull | Ulogistic |
| | | Ubs | Johnsonsb |
| | | Uburrxii | Ugompertz |
| | | Ugompertz | Leeg |
| | 10 | Ughne | Uchen |
| | | Ugompertz | Ugompertz |
| | | Ulogistic | Johnsonsb |
| | | Johnsonsb | Uweibull |
| | | Uweibull | Ughne |

## 4. Simulation and Results

In this section, we explain how to apply the ideas and concepts mentioned in theoretical consideration and what are the results obtained. In order to achieve the goal of this research, we relied on Monte Carlo simulation in the R programming language to examine the performance of distributions and compare them using the Bayesian Information Criterion (BIC).

Many scenarios were applied for the simulation experiments. Different samples (n=30,50,200) were taken with parameters ($\tau$ =0.2, 0.5, 0.7, 0.9) and concentrations (70%, 80%, 90%) for each group.

Explanatory variables (X = 5, 10, 15) were generated, and the response variable Y was deduced through the model used in the simulation experiments, using regression functions in terms of the explanatory variables generated above, plus random error.

Depending on the BIC criterion , the best model are highlited . Tables (1-9) summarize the best selected model . Several observations can be concluded .

1- For (n=30) , regardless the value of $\tau$ , the Ughnx regression model came in the first rank for fitting the data in most cases when $(p = 5)$ and when 70% of the data $> 0.5$ , the the other hand , when 30% of the data $> 0.5$ , the best model is Uchen regression model . Regarding the $(p = 10)$, Uchen regression model be the best .

Table 2. The best fitting model when n=30 for 80% $>$0.5 and 20% $>$0.5

| **Tau** | **P** | **Case: 80% $>$0.5 +20% $<$0.5** | **Case: 20% $>$0.5 +80% $<$0.5** |
|---|---|---|---|
| 0.2 | 5 | Ughnx | Uchen |
| | | Johnsonsb | Ulogistic |
| | | Uweibull | Johnsonsb |
| | | Ulogistic | Ugompertz |
| | | Uburrxii | Leeg |
| | 10 | Ughnx | Uchen |
| | | Ulogistic | Ugompertz |
| | | Leeg | Leeg |
| | | Johnsonsb | Johnsonsb |
| | | Ugompertz | Uweibull |
| 0.5 | 5 | Ughnx | Uchen |
| | | Johnsonsb | Ulogistic |
| | | Uweibull | Johnsonsb |
| | | Ubs | Ugompertz |
| | | Ulogistic | Leeg |
| | 10 | Ubs | Uchen |
| | | Ulogistic | Ugompertz |
| | | Johnsonsb | Johnsonsb |
| | | Leeg | Uweibull |
| | | Ugompertz | Ughne |
| 0.7 | 5 | Ughnx | Uchen |
| | | Johnsonsb | Ulogistic |
| | | Uweibull | Johnsonsb |
| | | Ubs | Ugompertz |
| | | Ulogistic | Leeg |
| | 10 | Ughne | Uchen |
| | | Ugompertz | Ugompertz |
| | | Ulogistic | Johnsonsb |
| | | Johnsonsb | Uweibull |
| | | Leeg | Ughne |
| 0.9 | 5 | Johnsonsb | Uchen |
| | | Uweibull | Ulogistic |
| | | Ubs | Johnsonsb |
| | | Uburrxii | Ugompertz |
| | | Ugompertz | Leeg |
| | 10 | Ughne | Uchen |
| | | Ugompertz | Ugompertz |
| | | Ulogistic | Johnsonsb |
| | | Johnsonsb | Uweibull |
| | | Uweibull | Ughne |

2- When the percentage of the data values $> 0.5$ is 80% , fixing (n=30) regardless of $\tau$ values, the best model is Uburrxii regression model when $(p = 5)$ . In addition , when $(p = 10)$ the Uweibull regression model is the best in all cases . On the other hand , when 20% of the data values $> 0.5$ , the Johnsonsb regression model is the best for both $(p = 5)$ and $(p = 10)$
3- When 90% of data values $> 0.5$ and (n=30) , the best models is Ughnx for the most when $(p = 5)$ and $(p = 10)$ , while when 10% of the data values $> 0.5$ , the Kum and Johnsonsb regression model is the best .
4- When n is varying , the best model is Varies regardless the percentage value of the data $> 0.5$ .
5- When $\tau$ is varying , in most cases , there is no effect in changing the best model .

## 5. Real Application

This application is based on an empirical dataset that comes from a longitudinal study on the quality of life in patients with epilepsy in Iran from March 2014 to December 2015. The Bayesian Information Criterion (BIC) are presented in Table 10. We note that the BIC value for the Ugompertz model is smaller than the BIC value for

Table 3. The best fitting model when n=30 for 10% >0.5 and 90% >0.5

| Tau | P | Case: 90% >0.5 +10% <0.5 | Case: 10% >0.5 +90% <0.5 |
|---|---|---|---|
| 0.2 | 5 | Ughnx | Ughnx |
| | | Johnsonsb | Johnsonsb |
| | | Ulogistic | Leeg |
| | | Uchen | Ubs |
| | | Ughnx | Uweibull |
| | 10 | Ughnx | Kum |
| | | Leeg | Johnsonsb |
| | | Ubs | Leeg |
| | | Johnsonsb | Uchen |
| | | Uburrxii | Ubs |
| 0.5 | 5 | Ughnx | Johnsonsb |
| | | Johnsonsb | Leeg |
| | | Leeg | Ubs |
| | | Ubs | Ulogistic |
| | | Uweibull | Uweibull |
| | 10 | Ughnx | Kum |
| | | Ubs | Johnsonsb |
| | | Leeg | Leeg |
| | | Uburrxii | Uchen |
| | | Johnsonsb | Ubs |
| 0.7 | 5 | Uchen | Johnsonsb |
| | | Ughnx | Leeg |
| | | Johnsonsb | Ubs |
| | | Ulogistic | Ulogistic |
| | | Ubs | Uweibull |
| | 10 | Ughnx | Kum |
| | | Ubs | Johnsonsb |
| | | Leeg | Leeg |
| | | Uburrxii | Uchen |
| | | Johnsonsb | Ubs |
| 0.9 | 5 | Ughnx | Johnsonsb |
| | | Johnsonsb | Leeg |
| | | Leeg | Ubs |
| | | Ubs | Ulogistic |
| | | Uweibull | Uweibull |
| | 10 | Ughnx | Kum |
| | | Ubs | Johnsonsb |
| | | Leeg | Leeg |
| | | Uburrxii | Uchen |
| | | Johnsonsb | Ubs |

others". So, based on the BIC criterion, the Ugompertz distribution is preferable in fitting these data than the Beta distribution for the bounded data.

## 6. Conclusion

This methodology of using quantile regression is thus a way of expressing the dependence of a response variable on the explanatory variable and covariates in terms of conditional quantiles. This methodology not only provides a stronger procedure to substitute the measure of the center of location of the response but also enables a better understanding of the variation of the distribution of the response as a function of the predictor values especially at different quantiles. In this paper, the author carried out a comparative analysis of the various parametric models of quantile regression under several conditions. Consequently, it was shown with the help of the Monte Carlo simulation that some of the above outlined parametric quantile regression models can provide a distinct improvement in comparison to other existing models under certain circumstance.

Table 4. The best fitting model when n=50 for 70% >0.5 and 30% >0.5

| **Tau** | P | **Case: 70% >0.5 +30% <0.5** | **Case: 30% >0.5 +70% <0.5** |
|---|---|---|---|
| 0.2 | 5 | Ughnx | Ugompertz |
| | | Johnsonsb | Ughne |
| | | Ugompertz | Uchen |
| | | Uburrxii | Johnsonsb |
| | | Uweibull | Ulogistic |
| | 10 | Ugompertz | Uchen |
| | | Ughnx | Johnsonsb |
| | | Uweibull | Ulogistic |
| | | Leeg | Kum |
| | | Johnsonsb | Leeg |
| 0.5 | 5 | Johnsonsb | Ughne |
| | | Ugompertz | Uchen |
| | | Ubs | Johnsonsb |
| | | Uburrxii | Ulogistic |
| | | Uweibull | Kum |
| | 10 | Ugompertz | Uchen |
| | | Ughnx | Johnsonsb |
| | | Uweibull | Ulogistic |
| | | Johnsonsb | Kum |
| | | Leeg | Leeg |
| 0.7 | 5 | Johnsonsb | Ugompertz |
| | | Ubs | Ughne |
| | | Ugompertz | Uchen |
| | | Uburrxii | Johnsonsb |
| | | Uweibull | Ulogistic |
| | 10 | Ughnx | Johnsonsb |
| | | Uweibull | Ulogistic |
| | | Ugompertz | Kum |
| | | Leeg | Leeg |
| | | Ubs | Uchen |
| 0.9 | 5 | Johnsonsb | Ughne |
| | | Ugompertz | Uchen |
| | | Ubs | Johnsonsb |
| | | Uburrxii | Ulogistic |
| | | Uweibull | Kum |
| | 10 | Ugompertz | Uchen |
| | | Uweibull | Johnsonsb |
| | | Leeg | Ulogistic |
| | | Ubs | Kum |
| | | Ulogistic | Leeg |

Table 5. The best fitting model when n=50 for 80% >0.5 and 20% >0.5

| **Tau** | P | **Case: 80% >0.5 +20% <0.5** | **Case: 20% >0.5 +80% <0.5** |
|---|---|---|---|
| 0.2 | 5 | Uburrxii | Johnsonsb |
| | | Ugompertz | Kum |
| | | Ughnx | Leeg |
| | | Leeg | Ughne |
| | | Uchen | Uchen |
| | 10 | Johnsonsb | Johnsonsb |
| | | Ughnx | Ugompertz |
| | | Ugompertz | Leeg |
| | | Ulogistic | Ulogistic |
| | | Uweibull | Uchen |
| 0.5 | 5 | Uburrxii | Johnsonsb |
| | | Ugompertz | Kum |
| | | Ughnx | Leeg |
| | | Leeg | Ughne |
| | | Uchen | Uchen |
| | 10 | Johnsonsb | Johnsonsb |
| | | Ughnx | Ugompertz |
| | | Ugompertz | Leeg |
| | | Ulogistic | Ulogistic |
| | | Uweibull | Uchen |
| 0.7 | 5 | Uburrxii | Johnsonsb |
| | | Ugompertz | Kum |
| | | Ughnx | Leeg |
| | | Leeg | Ughne |
| | | Uchen | Uchen |
| | 10 | Johnsonsb | Johnsonsb |
| | | Ughnx | Ugompertz |
| | | Ugompertz | Leeg |
| | | Ulogistic | Ulogistic |
| | | Uweibull | Uchen |
| 0.9 | 5 | Uburrxii | Johnsonsb |
| | | Ugompertz | Kum |
| | | Ughnx | Leeg |
| | | Leeg | Ughne |
| | | Uchen | Uchen |
| | 10 | Johnsonsb | Johnsonsb |
| | | Ughnx | Ugompertz |
| | | Ugompertz | Leeg |
| | | Ulogistic | Ulogistic |
| | | Uweibull | Uchen |

Table 6. The best fitting model when n=50 for 10% $>$0.5 and 90% $>$0.5

| **Tau** | P | **Case: 90% $>$0.5 +10% $<$0.5** | **Case: 10% $>$0.5 +90% $<$0.5** |
|---|---|---|---|
| 0.2 | 5 | Uchen | Uburrxii |
| | | Ubs | Kum |
| | | Johnsonsb | Johnsonsb |
| | | Ulogistic | Leeg |
| | | Kum | Uchen |
| | 10 | Uchen | Ughnx |
| | | Ubs | Kum |
| | | Johnsonsb | Johnsonsb |
| | | Ulogistic | Ughne |
| | | Kum | Leeg |
| 0.5 | 5 | Uchen | Uburrxii |
| | | Ubs | Kum |
| | | Johnsonsb | Johnsonsb |
| | | Ulogistic | Leeg |
| | | Ughnx | Uchen |
| | 10 | Ughnx | Ughnx |
| | | Uweibull | Kum |
| | | Johnsonsb | Johnsonsb |
| | | Ughne | Ughne |
| | | Ubs | Leeg |
| 0.7 | 5 | Uchen | Uburrxii |
| | | Ubs | Kum |
| | | Johnsonsb | Johnsonsb |
| | | Ulogistic | Leeg |
| | | Ughnx | Uchen |
| | 10 | Ughnx | Ughnx |
| | | Kum | Kum |
| | | Ubs | Johnsonsb |
| | | Uweibull | Ughne |
| | | Johnsonsb | Leeg |
| 0.9 | 5 | Ubs | Uburrxii |
| | | Johnsonsb | Kum |
| | | Ulogistic | Johnsonsb |
| | | Ughnx | Leeg |
| | | Kum | Uchen |
| | 10 | Kum | Ughnx |
| | | Ubs | Kum |
| | | Ughnx | Johnsonsb |
| | | Uweibull | Ughne |
| | | Johnsonsb | Leeg |

Table 7. The best fitting model when n=200 for 70% >0.5 and 30% >0.5

| **Tau** | P | **Case: 70% >0.5 +30% <0.5** | **Case: 30% >0.5 +70% <0.5** |
|---|---|---|---|
| 0.2 | 5 | Uweibull | Johnsonsb |
| | | Ugompertz | Uchen |
| | | Leeg | Kum |
| | | Johnsonsb | Ulogistic |
| | | Uburrxii | Ughnx |
| | 10 | Ughnx | Johnsonsb |
| | | Uweibull | Ulogistic |
| | | Johnsonsb | Leeg |
| | | Ugompertz | Uchen |
| | | Ulogistic | Uweibull |
| 0.5 | 5 | Ubs | Johnsonsb |
| | | Uweibull | Uchen |
| | | Ugompertz | Kum |
| | | Leeg | Ulogistic |
| | | Johnsonsb | Ughnx |
| | 10 | Ughnx | Johnsonsb |
| | | Ubs | Ulogistic |
| | | Uweibull | Leeg |
| | | Johnsonsb | Uchen |
| | | Ugompertz | Uweibull |
| 0.7 | 5 | Ugompertz | Johnsonsb |
| | | Ughnx | Uchen |
| | | Ubs | Kum |
| | | Uweibull | Ulogistic |
| | | Leeg | Ughnx |
| | 10 | Ughnx | Johnsonsb |
| | | Ubs | Ulogistic |
| | | Uweibull | Leeg |
| | | Johnsonsb | Uchen |
| | | Ugompertz | Uweibull |
| 0.9 | 5 | Ugompertz | Johnsonsb |
| | | Ughnx | Uchen |
| | | Ubs | Kum |
| | | Uweibull | Ulogistic |
| | | Leeg | Ughnx |
| | 10 | Ughnx | Johnsonsb |
| | | Ubs | Ulogistic |
| | | Uweibull | Leeg |
| | | Johnsonsb | Uchen |
| | | Ugompertz | Uweibull |

Table 8. The best fitting model when n=200 for 80% >0.5 and 20%

| **Tau** | P | **Case: 80% >0.5 +20% <0.5** | **Case: 20% >0.5 +80% <0.5** |
|---|---|---|---|
| 0.2 | 5 | Ugompertz | Uchen |
| | | Uweibull | Johnsonsb |
| | | Leeg | Ulogistic |
| | | Uchen | Ubs |
| | | Uburrxii | Ugompertz |
| | 10 | Johnsonsb | Uchen |
| | | Uburrxii | Johnsonsb |
| | | Ulogistic | Leeg |
| | | Kum | Ulogistic |
| | | Leeg | Ubs |
| 0.5 | 5 | Ugompertz | Uchen |
| | | Uweibull | Johnsonsb |
| | | Leeg | Ulogistic |
| | | Uchen | Ubs |
| | | Uburrxii | Ugompertz |
| | 10 | Johnsonsb | Uchen |
| | | Uburrxii | Johnsonsb |
| | | Ulogistic | Leeg |
| | | Kum | Ulogistic |
| | | Leeg | Ubs |
| 0.7 | 5 | Ugompertz | Uchen |
| | | Uweibull | Johnsonsb |
| | | Leeg | Ulogistic |
| | | Uchen | Ubs |
| | | Uburrxii | Ugompertz |
| | 10 | Johnsonsb | Uchen |
| | | Uburrxii | Johnsonsb |
| | | Leeg | Leeg |
| | | Ulogistic | Ulogistic |
| | | Kum | Ubs |
| 0.9 | 5 | Ugompertz | Uchen |
| | | Uweibull | Johnsonsb |
| | | Leeg | Ulogistic |
| | | Uchen | Ubs |
| | | Uburrxii | Ugompertz |
| | 10 | Johnsonsb | Uchen |
| | | Uburrxii | Johnsonsb |
| | | Ulogistic | Leeg |
| | | Kum | Ulogistic |
| | | Leeg | Ubs |

Table 9. The best fitting model when n=200 for 10% >0.5 and 90%

| **Tau** | P | **Case: 90% >0.5 +10% <0.5** | **Case: 10% >0.5 +90% <0.5** |
|---|---|---|---|
| 0.2 | 5 | Kum | Uchen |
| | | Ughnx | Ugompertz |
| | | Johnsonsb | Ulogistic |
| | | Ubs | Uburrxii |
| | | Ugompertz | Uweibull |
| | 10 | Ughnx | Ugompertz |
| | | Uchen | Kum |
| | | Kum | Ughne |
| | | Ubs | Uchen |
| | | Johnsonsb | Ulogistic |
| 0.5 | 5 | Kum | Uchen |
| | | Ughnx | Ugompertz |
| | | Johnsonsb | Ulogistic |
| | | Ubs | Uburrxii |
| | | Ugompertz | Uweibull |
| | 10 | Ughnx | Kum |
| | | Uchen | Ugompertz |
| | | Kum | Ulogistic |
| | | Ubs | Uchen |
| | | Johnsonsb | Leeg |
| 0.7 | 5 | Kum | Uchen |
| | | Ughnx | Ugompertz |
| | | Johnsonsb | Ulogistic |
| | | Ubs | Uburrxii |
| | | Ugompertz | Uweibull |
| | 10 | Ughnx | Ugompertz |
| | | Uchen | Kum |
| | | Kum | Uchen |
| | | Ubs | Ughne |
| | | Johnsonsb | Ulogistic |
| 0.9 | 5 | Kum | Uchen |
| | | Ughnx | Ugompertz |
| | | Johnsonsb | Ulogistic |
| | | Ubs | Uburrxii |
| | | Ugompertz | Uweibull |
| | 10 | Ughnx | Ugompertz |
| | | Uchen | Kum |
| | | Kum | Ulogistic |
| | | Ubs | Uchen |
| | | Johnsonsb | Ughne |

Table 10. The best fitting model for the application data

| Model | BIC value |
|---|---|
| **Ugompertz** | **-1833.281** |
| Uweibull | -1821.399 |
| Leeg | -1813.187 |
| Uchen | -1802.668 |
| Uburrxii | -1791.237 |

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