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On Rainbow Vertex Antimagic Coloring of Related Prism Graphs and Its Operations

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Abstract Let $G = (V, E)$ be a simple, connected and un-directed graph, for $f : E(G) \to \{1, 2, \dots, |E(G)|\}$, the weight of a vertex $v \in V(G)$ under f is $w_f(v) = \Sigma_{e\in E(v)} f(e)$, where $E(v)$ is the set of vertices incident to v. The function f is called vertex antimagic edge labeling if every vertex has distinct weight. While, rainbow vertex coloring is a coloring of graph vertices where each vertex on the graph is connected by a path that all internal vertices on the $u - v$ path have different colors. The purpose of this research is to find the rainbow vertex antimagic coloring on prism graphs and it's operations. Rainbow vertex antimagic coloring is a combination of antimagic labeling and rainbow vertex coloring. The rainbow vertex antimagic connection number of G, denoted by $rvac(G)$, is the smallest number of colors taken over all rainbow colorings induced by rainbow vertex antimagic labelings of G. In this paper we aim to discover some new theorems regarding to $rvac(G)$.

Keywords Rainbow vertex antimagic coloring, Rainbow vertex antimagic connection number, Related prism graph

AMS 2010 subject classifications 05C15, 05C78

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1. Introduction

A graph G is a pair of sets $(V(G), E(G))$, where $V(G)$ is a non-empty finite set of elements called vertices, and $E(G)$ is a set (which can be empty) of unordered pairs (u, v) from the vertex (u, v) of the elements of $V(G)$, called edges [8],[13]. A graph is a finite set of vertices where the vertices are represented by node and edges are represented by lines [17]. A vertex without edges can be called a graph, while an edge without vertex cannot be called a graph [3]. A graph that only consists of one vertex without any edges is called a trivial graph [2]. The number of sets of vertex in a graph is called the cardinality of the vertex, generally denoted by $|V(G)|$. Meanwhile, the edge cardinality or the number of edge sets in a graph is denoted by $|E(G)|$ [7].

Antimagic Labeling was first introduced in 1990 by Hartsfield and Ringel with several graphs studied, namely path graphs, complete graphs and two conjectures about Antimagic Labeling. Antimagic Labeling is a bijective mapping of the vertex set or edge set such that the sum of each paired edge has a different value [1].

Graph coloring is a part of graph labeling. Vertex coloring in a graph is the assignment of colors to all vertex in a graph, provided that the two neighboring vertices must have different colors. The graph G is said to be rainbow connected if every two distinct vertices in G are connected by a rainbow path [5]. The minimum number of colors such that a graph is rainbow connected is called the rainbow connection number, denoted $rc(G)$.

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The *Rainbow Vertex Connection* is a development of the *Rainbow Connection* concept [4]. It is divided into two types namely *Rainbow Edge Connection* and *Rainbow Vertex Connection*. A rainbow vertex connection is the color of the vertex of a graph, where each vertex in the graph is connected by a path that has interior vertices of a different color [14]. The *Rainbow Vertex Connection Number* of a graph G is a number that expresses the number of minimal colors assigned to a graph G, the rainbow vertex connection number of a graph G is usually symbolized by rvc(G) [6].

Dafik, et al (2019) combined the concept of rainbow coloring and antimagic labeling [16]. Rainbow vertex antimagic coloring is a combination of rainbow vertex connection with antimagic labeling [10],[18]. For a bijective function mapping $f : E(G) \rightarrow \{1, 2, 3, ..., |E(G)|\}$ with $v \in V(G)$, a vertex weight function $w_f(v) = \Sigma_{e \in E(v)} f(e)$, where $E(v)$ is the set of edges adjacent to vertex v. The function f is called antimagic edge labeling, if each vertex has a different weight. A path P on graph G with edge labeling is called a rainbow path if for any two vertices u and v, all interior vertices on the path $u - v$ have different weights. If for any two vertices u and v of G, a rainbow path $u - v$ exists, then f is called the rainbow vertex antimagic labeling of G. When we assign each edge uv with a vertex weight color $wf(v)$, then we say the graph G is a rainbow vertex antimagic coloring. The rainbow vertex antimagic connection number of graph G denoted by $rvac(G)$, is the smallest number of colors drawn from all the rainbow colorings induced by rainbow vertex antimagic labeling of graph G.

Hereunder are the definitions and theorems employed in the present study. The graphs studied are prism graphs and it's operations. A prism graph is a connected graph which is the Cartesian product of a cycle graph C_n with n vertices and a path graph P_m with m vertices, where n and m are integer.

Definition 1.1. *A prism graph is a Cartesian product of a cycle graph C_n with $n = 3$ vertices and a path P_m with m vertices, with $m \in N$ can be donated by $Pr_{3,m}$.*

Definition 1.2. *A prism graph is a cartesian product of a cycle graph C_n with $n = 5$ vertices and a path P_m with m vertices, with $m \in N$ can be donated by $Pr_{5,m}$.*

Definition 1.3. *A prism graph is a Cartesian product of a cycle graph C_n with $n = 6$ vertices and a path P_m with m vertices, with $m \in N$ can be donated by $Pr_{6,m}$.*

Definition 1.4. *[9] A Shackle graph is a graph resulting from k copies of the graph G which is symbolized by $Shack(G, v, k)$ where $k \geq 2$ and k is a natural number.*

Definition 1.5. *[12] Let G_i be a finite collection of graphs and each G_i has a fixed vertex v_{oi} called a terminal. The amalgamation Amal$\{G_i, v_{oi}\}$ is formed by taking of all the G_i's and identifying their terminals.*

Theorem 1.6. *[15] Let G be a connected graph with $diam(G)$, then $rvc(G) \geq diam(G) - 1$.*

Theorem 1.7. *[11] Let G be a connected graph, then $rvac(G) \geq rvc(G)$*

2. Methodology

The research methods used in this study are (*pattern recognition*) and deductive axiomatic method. The research procedure carried out to determine *Rainbow Vertex Antimagic Coloring* is as follows:

1) Define a graph G;
   In this step, the researcher determines the research object. The graphs that used in this research are prism graphs and their operations namely $Pr_{3,m}$, $Pr_{5,m}$, $Pr_{6,m}$, *Shackle* ($Pr_{5,2}, v^k_{2,2}, k$), and $Amal(Pr_{5,3}, x_1, k)$.
2) Determine the cardinality of the graph G;
   In this step, researchers look for the cardinality of each graph namely $Pr_{3,m}$, $Pr_{5,m}$, $Pr_{6,m}$, *Shackle* ($Pr_{5,2}, v^k_{2,2}, t$), and $Amal(Pr_{5,3}, x_1, k)$.
3) Determine the label of *Rainbow Vertex Connection* on the graph. This is to obtain the *lower bound* which will later be used as the lower bound of the $rvac$ number of the graph;

4) Determine the coloring of *Rainbow Vertex Antimagic Coloring* on the graph. The coloring of *Rainbow Vertex Antimagic Coloring* on a graph starts by performing Antimagic labeling on the graph. We give a Label on each edge with an integer $k = \{1, 2, 3, ..., q\}$ so that the weight at each vertex is different. Then give color to each vertex. The color is obtained from the sum of the weights of each edge so that the weight at each vertex is different and the color is minimum ;
5) Determine the upper bound of $rvac(G)$ by constructing a bijective function, then showing that any two distinct vertices on graph G satisfy the requirement of *Rainbow Vertex Antimagic Coloring*. If the upper bound fulfill the definition of rainbow vertex antimagic coloring, where all the internal vertices on the graph have different colors and contain rainbow path, then the value of *Rainbow Vertex Antimagic Coloring* of graph G is obtained. It can be symbolized as $rvac(G)$.
6) A new theorem is obtained after proving the lower bound and upper bound of *rainbow vertex antimagic connection number* in graph G.;

3. Results and Discussion

This research formulates four theorems concerning rainbow vertex antimagic connection numbers in prism graphs and their operations, as elaborated below :

Theorem 3.1. *Rainbow Vertex Antimagic Connection Number of Prism Graph* $(Pr_{3,m})$ *for* $m \geq 3$ *is* $rvac(Pr_{3,m}) = m$.

Proof. Let $Pr_{3,m}$ be a Prism Graph $(Pr_{3,m})$ with a set of vertices $V(Pr_{3,m}) = \{v_{i,j}; 1 \leq i \leq 3, 1 \leq j \leq m\}$ and the edge set $E(Pr_{3,m}) = \{v_{i,j}v_{i,(j+1)}; 1 \leq i \leq 3, 1 \leq j \leq m-1\} \cup \{v_{i,j}v_{(i+1),j}; 1 \leq i \leq 2, 1 \leq j \leq m\} \cup \{v_{1,j}v_{3,j}; 1 \leq j \leq m\}$. First, we will show the lower bound of the *Rainbow Vertex Antimagic Coloring* of Prism Graph $(Pr_{3,m})$ by determining the diameter of the Prism Graph $(Pr_{3,m})$. The diameter of $Pr_{3,m}$ is $diam(Pr_{3,m}) = m+1$. Based on Theorem 1.6, the value of $rvc(Pr_{3,m}) \geq diam(Pr_{3,m}) - 1$. The identified diameter determines the value of $rvc(Pr_{3,m}) \geq m$.

$$rvc(Pr_{3,m}) \geq diam(Pr_{3,m}) - 1 = (m+1) - 1 = m$$

Based on Theorem 1.7, which states that $rvac(G) \geq rvc(G)$ we obtain the value of $rvac(Pr_{3,m}) \geq m$. Thus, we got the lower bound of Rainbow Vertex Antimagic Connection Number of Prism Graph $(Pr_{3,m})$ for $m \geq 3$ is $rvac(Pr_{3,m}) \geq m$. Second, we show the upper bound of *Rainbow Vertex Antimagic Coloring* on the Prism Graph $(Pr_{3,m})$, $rvac(Pr_{3,m}) \leq m$. The bijective function of the edge labels in the Prism graph $Pr_{3,m}$ is as follows:

$$f(v_{i,1}v_{(i+1),1}) = \begin{cases} 1,\ i = 1 \\ 2,\ i = 2 \end{cases}$$

$$f(v_{1,j}v_{3,j}) = 3,\ j = 1$$

$$f(v_{i,j}v_{(i+1),j}) = \begin{cases} 6j-3,\ i = 1, j = 0mod3, j \neq n \text{ or } i = 2, j = 1mod3, j \neq 1, n \\ 6j-4,\ i = 1j = 2mod3, j \neq n \text{ or } i = 2, j = 0mod3, j \neq n \\ 6j-5,\ i = 1j = 1mod3, j \neq 1, n \text{ or } i = 2, j = 2mod3, j \neq n \end{cases}$$

$$f(v_{i,j}v_{(i+1),j}) = \begin{cases} 6j-3,\ i = 1, j = 0mod3, j = n \text{ or } i = 2, j = 1mod3, j = n \\ 6j-4,\ i = 1j = 1mod3, j = n \text{ or } i = 2, j = 2mod3, j = n \\ 6j-5,\ i = 1j = 2mod3, j = n \text{ or } i = 2, j = 0mod3, j - n \end{cases}$$

$$f(v_{1,j}v_{3,j}) = \begin{cases} 6j-3,\ j = 2mod3, j \neq n \\ 6j-4,\ j = 1mod3, j \neq 1, n \\ 6j-5,\ j = 0mod3, j \neq n \end{cases}$$

$$f(v_{1,j}v_{3,j}) = \begin{cases} 6j-3,\ j = 2mod3, j = n \\ 6j-4,\ j = 0mod3, j = n \\ 6j-5,\ j = 1mod3, j = n \end{cases}$$

$$f(v_{i,j}v_{i,(j+1)}) = \begin{cases} 6j,\ i=1, j=0mod3 \text{ or } i=2, j=1mod3, \\ \quad\quad \text{or } i=3, j=2mod3 \\ 6j-1,\ i=1, j=1mod3 \text{ or } i=2, j=2mod3 \\ \quad\quad \text{or } i=3, j=0mod3 \\ 6j-2,\ i=1 j=2mod3 \text{ or } i=2, j=0mod3 \\ \quad\quad \text{or } i=3, j=1mod3 \end{cases}$$

Based on the edge labels above, we identify the vertex weight $w(v_{i,j}), 1 \le i \le 3, 1 \le j \le m$, as follows:
$w(v_{i,1}) = 9$
$w(v_{i,j}) = 24j - 16; 2 \le j \le m-1$
$w(v_{i,m}) = 18m - 5$

Next, we analyze the set of different edge weights at each vertex. Based on the vertex weight labels above, there are three different sets of vertex weights, namely $\{w(v_{i,1})\}, \{w(v_{i,j})\}$ and $\{w(v_{i,m})\}$. The following is a description of the different sets of vertex:
$|\{w(v_{i,1})\}| = 1$
$|\{w(v_{i,j})\}| = |\{32, 56, 80, 104, \ldots, 24m-40\}| = m-2$
$|\{w(v_{i,m})\}| = 1$

Based on the description of the set of vertex weights, the number of sets of vertex weights is derived as follows:

$$|W| = |\{w(v_{i,1})\}| + |\{w(v_{i,j})\}| + |\{w(v_{i,m})\}| = 1 + (m-2) + 1 = m$$

Based on the set of vertex weights, we determine the upper bound $rvac(Pr_{3,m}) = m$. Based on the lower bound and upper bound, the value obtained for the *Rainbow Vertex Antimagic Connetion Number*on the Prism graph $Pr_{3,m}$ is m, which can be symbolized by $rvac(Pr_{3,m}) = m$. The following Table 1 shows the rainbow path from the Prism graph $(Pr_{3,m})$.

Table 1. Rainbow Path vertex $u-v$ on the $Pr_{3,m}$

Case	u	v	*Rainbow Vertex Coloring* $u-v$
1	$v_{i,j}$	$v_{i,m}$	$v_{i,j}, v_{i,j+1}, v_{i,j+2}, \ldots, v_{i,m}$
2	$v_{i,j}$	$v_{i+1,m}$	$v_{i,j}, v_{i+1,j}, v_{i+1,j+1}, v_{i+1,j+2} \ldots, v_{i+1,m}$
3	$v_{1,j}$	$v_{3,m}$	$v_{i,j}, v_{3,j}, v_{3,j+1}, v_{3,j+2} \ldots, v_{3,m}$

Based on Table 1, each different vertex on the $Pr_{3,m}$ has interior vertex with different colors, also known as rainbow path, which complies with the definition of *Rainbow Vertex Antimagic Coloring*. Based on the lower and upper bound and Table 1, the value for the *Rainbow Vertex Antimagic Connetion Number* of the $Pr_{3,m}$ is m or can be denoted as $rvac(Pr_{3,m}) = m$. The following is an example of *Rainbow Vertex Antimagic Coloring* of $Pr_{3,5}$:

Theorem 3.2. *Rainbow Vertex Antimagic Connection Number of* $Pr_{5,m}$ *for* $m \ge 3$ *is* $rvac(Pr_{5,m}) = m$.

Proof. Let $Pr_{5,m}$ is a Prism Graph $(Pr_{5,m})$ with a vertex set $V(Pr_{5,m}) = \{v_{i,j}; 1 \le i \le 5, 1 \le j \le m\}$ and the edge set $E(Pr_{5,m}) = \{v_{i,j}v_{i,(j+1)}; 1 \le i \le 5, 1 \le j \le m-1\} \cup \{v_{i,j}v_{(i+1)j}; 1 \le i \le 4, 1 \le j \le m\} \cup \{v_{1,j}v_{(5,j}; 1 \le j \le m\}$. First, we will show the the *Rainbow Vertex Antimagic Coloring* of $Pr_{5,m}$ by determining the diameter of Prism Graph $Pr_{5,m}$. The diameter of the Prism Graph $Pr_{5,m}$ is $diam(Pr_{5,m}) = m+1$. Based on Theorem 1.6, the value of $rvc(Pr_{5,m}) \ge diam(Pr_{5,m}) - 1$. The diameter for the value of $rvc(Pr_{5,m}) \ge m$ has been obtained.

$$rvc(Pr_{5,m}) \ge diam(Pr_{5,m}) - 1 = (m+1) - 1 = m$$

Based on Theorem 1.7, which states that $rvac(G) \ge rvc(G)$ the value of $rvac(Pr_{5,m}) \ge m$ is obtained. Based on this condition, the lower bound of Rainbow Vertex Antimagic Connection Number in $Pr_{5,m}$ for $m \ge 3$ is $rvac(Pr_{5,m}) \ge m$. Second, the upper bound of *Rainbow Vertex Antimagic Coloring* on the Prism Graph $Pr_{5,m}$ is hereby shown. The bijective function of the edge labels on the Prism graph $Pr_{5,m}$ is as follows:

$$f(v_{i,1}v_{(i+1),1}) = \begin{cases} 1,\ i=1 \\ 4,\ i=2 \\ 2,\ i=3 \\ 5,\ i=4 \end{cases}$$

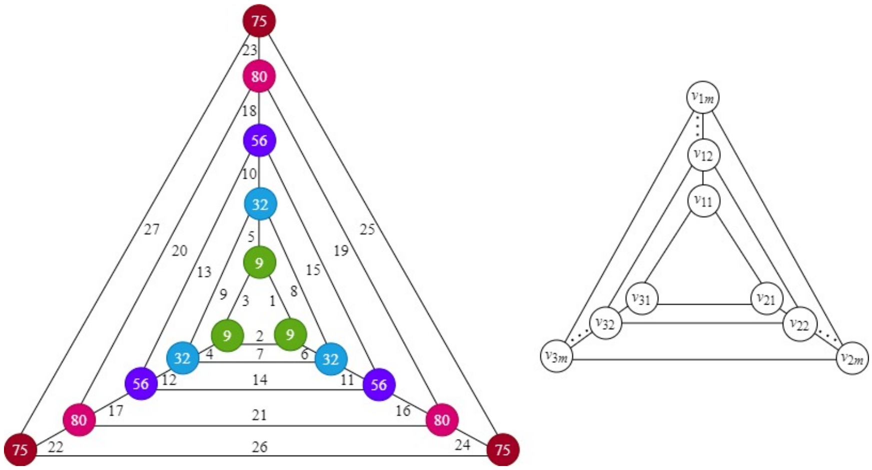

Figure 1. *Rainbow Vertex Antimagic Coloring* of $Pr_{3,5}$

$$f(v_{1,j}v_{5,j}) = 3,\ j = 1$$

$$f(v_{i,j}v_{(i+1),j}) = \begin{cases} 10j-5,\ i=1, j=3mod5, j\neq n \text{ or } i=2, j=4mod5, j\neq n, \\ \qquad \text{or } i=3, j=0mod5, j\neq n \text{ or } i=4, j=1mod5, j\neq 1,n \\ 10j-6,\ i=1 j=4mod5, j\neq n \text{ or } i=2, j=0mod5, j\neq n \\ \qquad \text{or } i=3, j=1mod5, j\neq 1,n \text{ or } i=4, j=2mod5, j\neq n \\ 10j-7,\ i=1 j=0mod5, j\neq n \text{ or } i=2, j=1mod5, j\neq 1,n \\ \qquad \text{or } i=3, j=2mod5, j\neq n \text{ or } i=4, j=3mod5, j\neq n \\ 10j-8,\ i=1 j=1mod5, j\neq 1,n \text{ or } i=2, j=2mod5, j\neq n \\ \qquad \text{or } i=3, j=3mod5, j\neq n \text{ or } i=4, j=4mod5, j\neq n \\ 10j-9,\ i=1 j=2mod5, j\neq n \text{ or } i=2, j=3mod5, j\neq n \\ \qquad \text{or } i=3, j=4mod5, j\neq n \text{ or } i=4, j=0mod5, j\neq n \end{cases}$$

$$f(v_{i,j}v_{(i+1),j}) = \begin{cases} 10j-5,\ i=1, j=4mod5, j=n \text{ or } i=2, j=0mod5, j=n, \\ \qquad \text{or } i=3, j=1mod5, j=n \text{ or } i=4, j=2mod5, j=n \\ 10j-6,\ i=1 j=1mod5, j=n \text{ or } i=2, j=2mod5, j=n \\ \qquad \text{or } i=3, j=3mod5, j=n \text{ or } i=4, j=4mod5, j=n \\ 10j-7,\ i=1 j=3mod5, j=n \text{ or } i=2, j=4mod5, j=5n \\ \qquad \text{or } i=3, j=0mod5, j=n \text{ or } i=4, j=1mod5, j=n \\ 10j-8,\ i=1 j=0mod5, j=n \text{ or } i=2, j=1mod5, j=n \\ \qquad \text{or } i=3, j=2mod5, j=n \text{ or } i=4, j=3mod5, j=n \\ 10j-9,\ i=1 j=2mod5, j=n \text{ or } i=2, j=3mod5, j=n \\ \qquad \text{or } i=3, j=4mod5, j=n \text{ or } i=4, j=0mod5, j=n \end{cases}$$

$$f(v_{1,j}v_{5,j}) = \begin{cases} 10j-5,\ j=2mod5, j\neq n \\ 10j-6,\ j=3mod5, j\neq n \\ 10j-7,\ j=4mod5, j\neq n \\ 10j-8,\ j=0mod5, j\neq n \\ 10j-9,\ j=1mod5, j\neq 1,n \end{cases}$$

$$f(v_{1,j}v_{5,j}) = \begin{cases} 10j-5,\ j=3mod5, j=n \\ 10j-6,\ j=0mod5, j=n \\ 10j-7,\ j=2mod5, j=n \\ 10j-8,\ j=4mod5, j=n \\ 10j-9,\ j=1mod5, j=n \end{cases}$$

$$f(v_{i,j}v_{i,(j+1)}) = \begin{cases} 10j,\ i=1, j=1mod5 \text{ or } i=2, j=2mod5, \text{ or } i=3, \\ \qquad j=3mod5 \text{ or } i=4, j=4mod5 \text{ or } i=5, j=0mod5 \\ 10j-1,\ i=1, j=0mod5 \text{ or } i=2, j=1mod5, \text{ or } i=3, \\ \qquad j=2mod5 \text{ or } i=4, j=3mod5 \text{ or } i=5, j=4mod5 \\ 10j-2,\ i=1, j=4mod5 \text{ or } i=2, j=0mod5, \text{ or } i=3, \\ \qquad j=1mod5 \text{ or } i=4, j=2mod5 \text{ or } i=5, j=3mod5 \\ 10j-3,\ i=1, j=3mod5 \text{ or } i=2, j=4mod5, \text{ or } i=3, \\ \qquad j=0mod5 \text{ or } i=4, j=1mod5 \text{ or } i=5, j=2mod5 \\ 10j-4,\ i=1, j=2mod5 \text{ or } i=2, j=3mod5, \text{ or } i=3, \\ \qquad j=4mod5 \text{ or } i=4, j=0mod5 \text{ or } i=5, j=1mod5 \end{cases}$$

Based on edge label above,we obtain the vertex weights of the $Pr_{5,m}$ $(w(v_{i,j})), 1 \leq i \leq 5, 1 \leq j \leq m$, as defined below:
$w(v_{i,1}) = 14$
$w(v_{i,j}) = 40j - 28; 2 \leq j \leq m-1$
$w(v_{i,m}) = 30m - 26$

Next, we analyze the set of different edge weights at each vertex. Based on the vertex weight labels above, three different sets of vertex weights are identified, namely $\{w(v_{i,1})\}, \{w(v_{i,j})\}$ and $\{w(v_{i,m})\}$. The following is a description of the different sets of edge weights in succession:
$|\{w(v_{i,1})\}| = 1$
$|\{w(v_{i,j})\}| = |\{52, 92, 132, 172, \ldots, 40m - 68\}| = m - 2$
$|\{w(v_{i,m})\}| = 1$

As such, based on the description of the set of vertex weights above, we obtain the number of sets of vertex weights as follows:

$$|W| = |\{w(v_{i,1})\}| + |\{w(v_{i,j})\}| + |\{w(v_{i,m})\}| = 1 + (m-2) + 1 = m$$

Based on the set of vertex weights, we determine the upper bound $rvac(Pr_{5,m}) = m$. Based on the lower bound and upper bound, the value for the *Rainbow Vertex Antimagic Connetion Number* on the Prism graph $Pr_{5,m}$ is m, which can be symbolized as follows $rvac(Pr_{5,m}) = m$. The (*rainbow path*) form the Prism Graph $Pr_{5,m}$ is displayed in Table 2.

Table 2. Rainbow Path Vertex $u - v$ on the Prism Graph $Pr_{5,m}$

Case	u	v	*Rainbow Vertex Coloring* $u - v$
1	$v_{i,j}$	$v_{i,m}$	$v_{i,j}, v_{i,j+1}, v_{i,j+2}, \cdots, v_{i,m}$
2	$v_{i,j}$	$v_{i+1,m}$	$v_{i,j}, v_{i+1,j}, v_{i+1,j+1}, v_{i+1,j+2} \cdots, v_{i+1,m}$
3	$v_{i,j}$	$v_{5,m}$	$v_{i,j}, v_{5,j}, v_{5,j+1}, v_{5,j+2} \cdots, v_{5,m}$
4	$v_{1,j}$	$v_{3,m}$	$v_{i,j}, v_{i+1,j}, v_{i+1,j+1}, v_{i+1,j+2} \cdots, v_{i+1,m}, v_{i+2,m}$
5	$v_{1,j}$	$v_{4,m}$	$v_{i,j}, v_{5,j}, v_{5,j+1}, v_{5,j+2} \cdots, v_{5,m}, v_{4,m}$

Table 2 demonstrates that each different vertex on the Prism graph $Pr_{5,m}$ has interior vertex with different colors, known as rainbow path. These vertex comply with the definition of *Rainbow Vertex Antimagic Coloring*. Based on lower bound and upper bound and Table 2. the value obtained for the *Rainbow Vertex Antimagic Connetion Number* of the Prism graph $Pr_{5,m}$ is m and can be denoted as $rvac(Pr_{5,m}) = m$.

Hereunder is the example of *Rainbow Vertex Antimagic Coloring* on Prism Graph $Pr_{5,6}$ (See Figure 2):

Theorem 3.3. *Rainbow Vertex Antimagic Connection Number on Prism Graph $Pr_{6,m}$ for $m \geq 3$ is $rvac(Pr_{6,m}) = m + 2$.*

Proof. Let $Pr_{6,m}$ is Prism Graph $Pr_{6,m}$ with vertex set $V(Pr_{6,m}) = \{v_{i,j}; 1 \leq i \leq 6, 1 \leq j \leq m\}$ and edge set $E(Pr_{6,m}) = \{v_{i,j}v_{i,(j+1)}; 1 \leq i \leq 6, 1 \leq j \leq m-1\} \cup \{v_{i,j}v_{(i+1)j}; 1 \leq i \leq 5, 1 \leq j \leq m\} \cup \{v_{1,j}v_{(6,j}; 1 \leq j \leq m\}$. First, we will show the lower bound of the *Rainbow Vertex Antimagic Coloring* on the Prism Graph $Pr_{6,m}$ Graph by determining the diameter of the Prism Graph $Pr_{6,m}$. The diameter of the Prism Graph $Pr_{6,m}$ is $diam(Pr_{6,m}) = m + 3$. Therefore, based on Theorem 1.6, the value of $rvc(Pr_{6,m}) \geq diam(Pr_{6,m}) - 1$. As

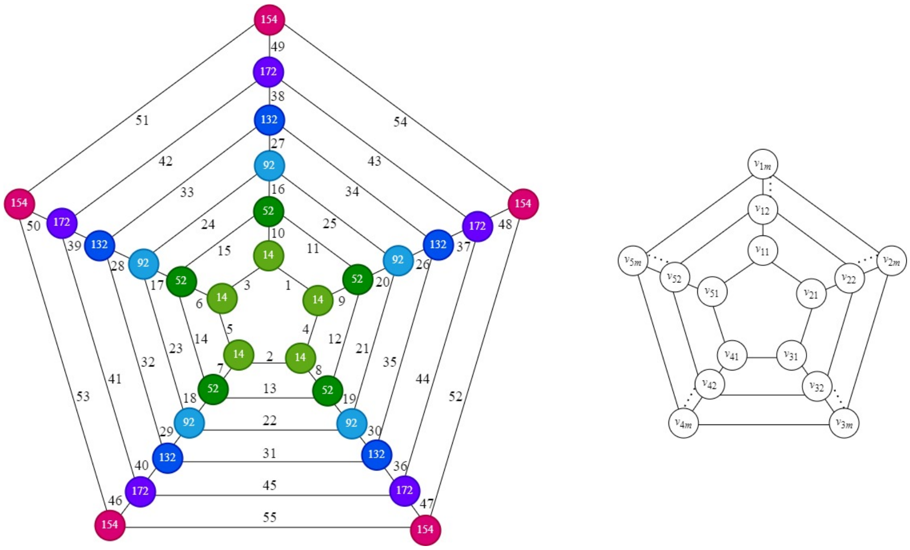

Figure 2. *Rainbow Vertex Antimagic Coloring* on Prism Graph $Pr_{5,6}$

such, from the obtained diameter, we determine the value of $rvc(Pr_{6,m}) \geq m+2$. Following Theorem 1.7. which states that $rvac(G) \geq rvc(G)$ we affirm the value of $rvac(Pr_{6,m}) \geq m+2$. Under this condition, the value of $rvac(Pr_{6,m}) \geq m+2$ is as follows.

$$rvc(Pr_{6,m}) \geq diam(Pr_{6,m}) - 1 = (m+3) - 1 = m+2$$

Second, we show the upper bound of *Rainbow Vertex Antimagic Coloring* of Prism Graph $rvac(Pr_{6,m}) \leq m+2$. The bijective function of the edge labels on the Prism Graph $Pr_{6,m}$ is as follows:

$$f(v_{i,1}v_{(i+1),1}) = \begin{cases} 4, & i=1 \\ 1, & i=2 \\ 5, & i=3 \\ 2, & i=4 \\ 6, & i=5 \end{cases}$$

$$f(v_{1,j}v_{(6,j)}) = 3, \ j=1$$

$$f(v_{i,j}v_{(i+1),j}) = \begin{cases} 12j-6, & i=1, j=2mod6, j\neq n \text{ or } i=2, j=3mod6, j\neq n, \\ & \text{or } i=3, j=4mod6, j\neq n \text{ or } i=4, j=5mod6, j\neq n \\ & \text{or } i=5, j=0mod6, j\neq n \\ 12j-7, & i=1 j=3mod6, j\neq n \text{ or } i=2, j=4mod6, j\neq n \\ & \text{or } i=3, j=5mod6, j\neq n \text{ or } i=4, j=0mod6, j\neq n \\ & \text{or } i=5, j=1mod6, j\neq 1,n \\ 12j-8, & i=1 j=4mod6, j\neq n \text{ or } i=2, j=5mod6, j\neq n \\ & \text{or } i=3, j=0mod6, j\neq n \text{ or } i=4, j=1mod6, j\neq 1,n \\ & \text{or } i=5, j=2mod6, j\neq n \\ 12j-9, & i=1 j=5mod6, j\neq n \text{ or } i=2, j=0mod6, j\neq n \\ & \text{or } i=3, j=1mod6, j\neq 1,n \text{ or } i=4, j=2mod6, j\neq n \\ & \text{or } i=5, j=3mod6, j\neq n \end{cases}$$

$$f(v_{i,j}v_{(i+1),j}) = \begin{cases} 12j-10, & i=1 j=0mod6, j\neq n \text{ or } i=2, j=1mod6, j\neq 1,n \\ & \text{or } i=3, j=2mod6, j\neq n \text{ or } i=4, j=3mod6, j\neq n \\ & \text{or } i=5, j=4mod6, j\neq n \\ 12j-11, & i=1 j=1mod6, j\neq 1,n \text{ or } i=2, j=2mod6, j\neq n \\ & \text{or } i=3, j=3mod6, j\neq n \text{ or } i=4, j=4mod6, j\neq n \\ & \text{or } i=5, j=5mod6, j\neq n \end{cases}$$

$$f(v_{i,j}v_{(i+1),j}) = \begin{cases} 12j-6, & i=1, j=0mod6, j=n \text{ or } i=2, j=1mod6, j=n, \\ & \text{or } i=3, j=2mod6, j=n \text{ or } i=4, j=3mod6, j=n \\ & \text{or } i=5, j=4mod6, j=n \\ 12j-7, & i=1 j=2mod6, j=n \text{ or } i=2, j=3mod6, j=n \\ & \text{or } i=3, j=4mod6, j=n \text{ or } i=4, j=5mod6, j=n \\ & \text{or } i=5, j=0mod6, j=n \\ 12j-8, & i=1 j=4mod6, j=n \text{ or } i=2, j=5mod6, j=n \\ & \text{or } i=3, j=0mod6, j=n \text{ or } i=4, j=1mod6, j=n \\ & \text{or } i=5, j=2mod6, j=n \\ 12j-9, & i=1 j=5mod6, j=n \text{ or } i=2, j=0mod6, j=n \\ & \text{or } i=3, j=1mod6, j=n \text{ or } i=4, j=2mod6, j=n \\ & \text{or } i=5, j=3mod6, j=n \\ 12j-10, & i=1 j=1mod0, j=n \text{ or } i=2, j=2mod6, j=n \\ & \text{or } i=3, j=3mod6, j=n \text{ or } i=4, j=4mod6, j=n \\ & \text{or } i=5, j=5mod6, j=n \\ 12j-11, & i=1 j=3mod6, j=n \text{ or } i=2, j=4mod6, j=n \\ & \text{or } i=3, j=5mod6, j=n \text{ or } i=4, j=0mod6, j=n \\ & \text{or } i=5, j=1mod6, j=n \end{cases}$$

$$f(v_{1,j}v_{6,j}) = \begin{cases} 12j-6, & j=1mod6, j\neq 1,n \\ 12j-7, & j=2mod6, j\neq n \\ 12j-8, & j=3mod6, j\neq n \\ 12j-9, & j=4mod6, j\neq n \\ 12j-10, & j=5mod6, j\neq n \\ 12j-11, & j=0mod6, j\neq n \end{cases}$$

$$f(v_{1,j}v_{6,j}) = \begin{cases} 12j-6, & j=5mod6, j=n \\ 12j-7, & j=1mod6, j=n \\ 12j-8, & j=3mod6, j=n \\ 12j-9, & j=4mod6, j=n \\ 12j-10, & j=0mod6, j=n \\ 12j-11, & j=2mod6, j=n \end{cases}$$

$$f(v_{i,j}v_{i,(j+1)}) = \begin{cases} 12j, \ i=1, j=0mod6 \text{ or } i=2, j=1mod6, \text{ or } i=3, \\ \qquad j=2mod6 \text{ or } i=4, j=3mod6 \text{ or } i=5, j=4mod6 \\ \qquad \text{ or } i=6, j=5mod6 \\ 12j-1, \ i=1, j=5mod6 \text{ or } i=2, j=0mod6, \text{ or } i=3, \\ \qquad j=1mod6 \text{ or } i=4, j=2mod6 \text{ or } i=5, j=3mod6 \\ \qquad \text{ or } i=6, j=4mod6 \\ 12j-2, \ i=1, j=4mod6 \text{ or } i=2, j=5mod6, \text{ or } i=3, \\ \qquad j=0mod6 \text{ or } i=4, j=1mod6 \text{ or } i=5, j=2mod6 \\ \qquad \text{ or } i=6, j=3mod6 \\ 12j-3, \ i=1, j=3mod6 \text{ or } i=2, j=4mod6, \text{ or } i=3, \\ \qquad j=5mod6 \text{ or } i=4, j=0mod6 \text{ or } i=5, j=1mod6 \\ \qquad \text{ or } i=6, j=2mod6 \\ 12j-4, \ i=1, j=2mod6 \text{ or } i=2, j=3mod6, \text{ or } i=3, \\ \qquad j=4mod6 \text{ or } i=4, j=5mod6 \text{ or } i=5, j=0mod6 \\ \qquad \text{ or } i=6, j=1mod6 \end{cases}$$

$$f(v_{i,j}v_{i,(j+1)}) = \begin{cases} 12j-5, \ i=1, j=1mod6 \text{ or } i=2, j=2mod6, \text{ or } i=3, \\ \qquad j=3mod6 \text{ or } i=4, j=4mod6 \text{ or } i=5, j=5mod6 \\ \qquad \text{ or } i=6, j=0mod6 \end{cases}$$

Based on the edge label above, we derive the vertex weights of the Prism Graph $(w(v_{i,j})), 1 \leq i \leq 6, 1 \leq j \leq m$, as follows:
$w(v_{1,1}) = 14$
$w(v_{i,1}) = 17; 2 \leq i \leq 6$
$w(v_{i,j}) = 48j - 34; 2 \leq j \leq m-1$

$$w(v_{i,m}) = 36m - 35 \begin{cases} 1 \leq i \leq 2, j = 3mod6 \\ 2 \leq i \leq 3, j = 4mod6 \\ 3 \leq i \leq 4, j = 5mod6 \\ 4 \leq i \leq 5, j = 0mod6 \\ 5 \leq i \leq 6, j = 1mod6 \\ i = 6, 1, j = 2mod6 \end{cases}$$

$$w(v_{i,m}) = 36m - 32; \begin{cases} i = 1, j = 4mod6 \\ i = 2, j = 5mod6 \\ i = 3, j = 0mod6 \\ i = 4, j = 1mod6 \\ i = 5, j = 2mod6 \\ i = 6, j = 3mod6 \end{cases}$$

$$w(v_{i,m}) = 36m - 29; \begin{cases} 3 \leq i \leq 5, j = 3mod6 \\ 4 \leq i \leq 6, j = 4mod6 \\ i = 1, 5, 6, j = 5mod6 \\ i = 1, 2, 6, j = 0mod6 \\ 1 \leq 3 \leq 6, j = 1mod6 \\ 2 \leq i \leq 4, j = 2mod6 \end{cases}$$

Next, we analyze the set of different edge weights at each vertex. The following is a description of the different sets of vertex weights in succession:
$|\{w(v_{1,1})\}| = 1$
$|\{w(v_{i,1}\}) = 1$
$|\{w(v_{i,j})\}| = \{62, 110, 158, 206, \ldots, 48m - 82\} = m - 2$
$|\{w(v_{i,m})\}| = 1$
$|\{w(v_{i,m})\}| = 1$
Based on the description of the set of vertex weights above, the number of vertex weight sets is determined as

follows:
$|W| = |\{w(v_{1,1})\}| + |\{w(v_{i,1})\}| + |\{w(v_{i,j})\}| + |\{w(v_{i,m})\}| + |\{w(v_{i,m})\}| = 1 + 1 + (m-2) + 1 + 1 = m + 2$

Based on the set of vertex weights, the upper bound $rvac(Pr_{6,m}) = m + 2$ is obtained. Based on lower bound and upper bound, the value for the *Rainbow Vertex Antimagic Connetion Number* on the Prism Graph $Pr_{6,m}$ is $m + 2$, which can be symbolized as follows $rvac(Pr_{6,m}) = m + 2$. The (*rainbow path*) of the Prism Graph $Pr_{6,m}$ which can be seen in Table 3 and Table 4.

Table 3. Rainbow Trajectory vertex $u - v$ on the Prism Graph$Pr_{6,m}$

Case	u	v	*Rainbow Vertex Coloring* $u - v$
1	$v_{i,j}$	$v_{i,m}$	$v_{i,j}, v_{i,j+1}, v_{i,j+2}, \dots, v_{i,m}$
2	$v_{i,j}$	$v_{i+1,m}$	$v_{i,j}, v_{i+1,j}, v_{i+1,j+1}, v_{i+1,j+2} \cdots, v_{i+1,m}$
3	$v_{i,j}$	$v_{6,m}$	$v_{i,j}, v_{6,j}, v_{6,j+1}, v_{6,j+2} \cdots, v_{6,m}$
4	$v_{i,j}$	$v_{i+2,m}$	$v_{i,j}, v_{i+1,j}, v_{i+1,j+1}, v_{i+1,j+2} \cdots, v_{i+1,m}, v_{i+2,m}$
5	$v_{1,j}$	$v_{5,m}$	$v_{1,j}, v_{6,j}, v_{6,j+1}, v_{6,j+2} \cdots, v_{6,m}, v_{5,m}$
6	$v_{2,j}$	$v_{6,m}$	$v_{2,j}, v_{1,j}, v_{1,j+1}, v_{1,j+2} \cdots, v_{1,m}, v_{6,m}$

Table 4. Rainbow Trajectory $u - v$ on The Prism Graph $Pr_{6,m}$

Case	u	v	Condition	*Rainbow Vertex Coloring* $u - v$
1	$v_{i,j}$	$v_{i+3,j}$	$i = 1; 3mod6$	$v_{i,j}, v_{i+1,j}, v_{i+1,j+1}, \dots, v_{i+2,m}, v_{i+3,m}$
	$v_{i,j}$	$v_{i+3,j}$	$i = 2; 3mod6$	$v_{i,j}, v_{i-1,j}, v_{5,j+1}, v_{5,j+2}, \dots, v_{5,m}, v_{i+3,m}$
	$v_{i,j}$	$v_{i+3,j}$	$i = 3; 3mod6$	$v_{i,j}, v_{i-1,j}, v_{i-2,j+1}, v_{i-2,j+2}, \dots, v_{i-2,m}, v_{i+3,m}$
2	$v_{i,j}$	$v_{i+3,j}$	$i = 1; 4mod6; 5mod6$	$v_{i,j}, v_{i+1,j}, v_{i+1,j+1}, \dots, v_{i+2,m}, v_{i+3,m}$
	$v_{i,j}$	$v_{i+3,j}$	$i = 2; 4mod6; 5mod6$	$v_{i,j}, v_{i+1,j+1}, v_{i+1,j+2}, \dots, v_{i+2,m}, v_{i+3,m}$
	$v_{i,j}$	$v_{i+3,j}$	$i = 3; 4mod6; 5mod6$	$v_{i,j}, v_{i+1,j+1}, v_{i+1,j+2}, \dots, v_{i+2,m}, v_{i+3,m}$
3	$v_{i,j}$	$v_{i+3,j}$	$i = 1; 0mod6; 1mod6$	$v_{i,j}, v_{i+5,j+1}, v_{i+5,j+2}, \dots, v_{i+4,m}, v_{i+3,m}$
	$v_{i,j}$	$v_{i+3,j}$	$i = 2; 0mod6; 1mod6$	$v_{i,j}, v_{i-1,j}, v_{i+4,j+2}, \dots, v_{i+4,m}, v_{i+3,m}$
	$v_{i,j}$	$v_{i+3,j}$	$i = 3; 0mod6; 1mod6$	$v_{i,j}, v_{i-1,j}, v_{i-2,j+2}, \dots, v_{i-2,m}, v_{i+3,m}$
	$v_{i,j}$	$v_{i+3,j}$	$i = 1; 2mod6$	$v_{i,j}, v_{i+5,j+1}, v_{i+5,j+2}, \dots, v_{i+4,m}, v_{i+3,m}$
	$v_{i,j}$	$v_{i+3,j}$	$i = 2; 2mod6$	$v_{i,j}, v_{i-1,j}, v_{i+4,j+2}, \dots, v_{i+4,m}, v_{i+3,m}$
	$v_{i,j}$	$v_{i+3,j}$	$i = 3; 2mod6$	$v_{i,j}, v_{i-1,j}, v_{i-2,j+2}, \dots, v_{i-2,m}, v_{i+3,m}$

Based on Table 3 and Table 4 each different vertex on the Prism graph $Pr_{6,m}$ has interior vertex with different colors, which satisfies the definition *Rainbow Vertex Antimagic Coloring*. As a corollary, the value obtained for the *Rainbow Vertex Antimagic Connetion Number* of the Prism Graph $Pr_{6,m}$ is $m + 2$ or denoted as $rvac(Pr_(6, m) = m + 2$.
The following is an example of *Rainbow Vertex Antimagic Coloring* on the Prism Graph $Pr_{6,6}$:

Theorem 3.4. *Rainbow Vertex Antimagic Connection Number in the Shackle of Prism Graph, Shackle* $(Pr_{5,2}, v^k_{2,2}, t)$*, for* $t \geq 3$ *and* $1 \leq k \leq t - 1$ *is* $rvac(Shackle(Pr_{5,2}, v^k_{2,2}, t)) = 3t - 1$.

Proof. Let *Shackle* $(Pr_{5,2}, v^k_{2,2}, t)$ is *Shackle* of Prism Graph with the vertex set $V(Shackle(Pr_{5,2}, v^k_{2,2}, t)) = \{v^k_{i,j}; 1 \leq i \leq 5, j = 1, 1 \leq k \leq t\} \cup \{v^k_{i,(j+1)}; 1 \leq i \leq 4, 1 \leq j \leq 2, 1 \leq k \leq t\} \cup \{v^k_{5,(j+1)}; j = 1, k = 1\}$ and edge set $E(Shackle(Pr_{5,2}, v^k_{2,2}, k)) = \{v^k_{i,j} v_{i,(j+1)^k}; 1 \leq i \leq 5, j = 1, 1 \leq k \leq t\} \cup \{v^k_{i,j} v^k_{(i+1),j}; 1 \leq i \leq 4, 1 \leq j \leq 2, 1 \leq k \leq t\} \cup \{v^k_{1,j} v^k_{5,j}; 1 \leq j \leq 2, 1 \leq k \leq t\}$. First, we will show the lower bound of the *Rainbow Vertex Antimagic Coloring* on the *Shackle* $(Pr_{5,2}, v^k_{2,2}, t)$ by finding the diameter of the *Shackle* Prism Graph . The diameter of *Shackle* Prism Graph, *Shackle* $(Pr_{5,2}, v^k_{2,2}, t)$, is $diam(Shackle(Pr_{5,2}, v^k_{2,2}, t)) = 2t + 1$. So Based on Theorem 1.6., value of $rvc(Shackle(Pr_{5,2}, v^k_{2,2}, t) \geq diam(Shackle(Pr_{5,2}, v^k_{2,2}, t) - 1$. So from the diameter that has been obtained the value $rvc(Shackle(Pr_{5,2}, v^k_{2,2}, t) \geq 2t$.

$$rvc(Shackle(Pr_{5,2}, v^k_{2,2}, t) \geq diam(Shackle(Pr_{5,2}, v^k_{2,2}, t) - 1 = (2t + 1) - 1 = 2t$$

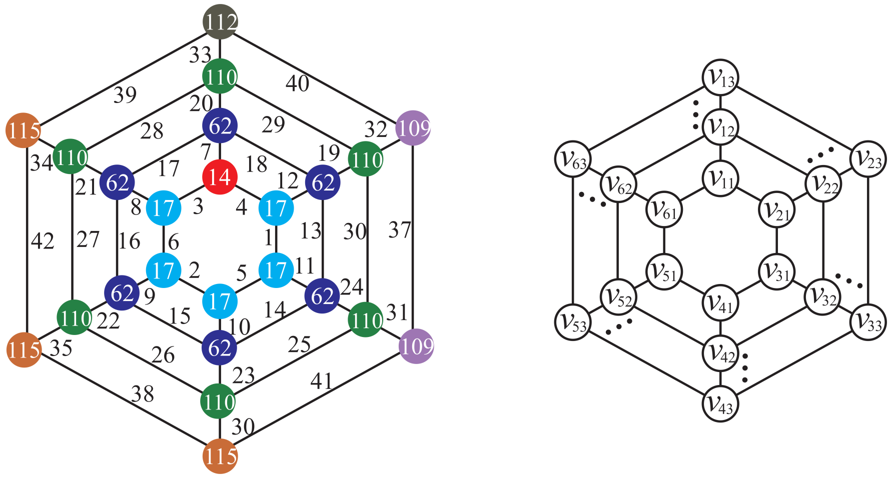

Figure 3. *Rainbow Vertex Antimagic Coloring* on the Prism Graph $Pr_{6,4}$

Based on this condition, we identify the value of $rvc(\mathit{Shackle}(Pr_{5,2}, v^k_{2,2}, t) \geq 2t$. Because the shackle operation connection vertex between the prism graphs t and $t+1$ is connected by a vertex, it must have a different color. Based on this condition, we obtain $rvc(\mathit{Shackle}(Pr_{5,2}, v^k_{2,2}, t) \geq 3t$. Each path from $v^1_{i,1} - v^k_{i,1}$ does not pass through the inner prism graph, so based on this condition we obtain $rvc(\mathit{Shackle}(Pr_{5,2}, v^k_{2,2}, t) \geq 3t - 1$. Based on Theorem 1.7 which states that $rvac(G) \geq rvc(G)$ then The value obtained is $rvac(\mathit{Shackle}(Pr_{5,2}, v^k_{2,2}, t) \geq 3t - 1$.Based on this condition, we determine the value $rvac(\mathit{Shackle}(Pr_{5,2}, v^k_{2,2}, t) \geq 3t - 1$.

Second, the upper bound of the *Rainbow Vertex Antimagic Coloring* on the *Shackle* of Prism Graph. The bijective function of the edge labels in the *Shackle* of Prism Graph, *Shackle* $(Pr_{5,2}, v^k_{2,2}, t)$, is presented below:

$$f(v^k_{i,1}v^k_{(i+1),1}) = \begin{cases} 15k - 14, & i = 1; 1 \leq k \leq t \\ 15k - 11, & i = 2; 1 \leq k \leq t \\ 15k - 13, & i = 3; 1 \leq k \leq t \\ 15k - 10, & i = 4; 1 \leq k \leq t \end{cases}$$

$$f(v^k_{i,2}v^k_{(i+1),2}) = \begin{cases} 15k - 4, & i = 1; 1 \leq k \leq t \\ 15k - 1, & i = 2; 1 \leq k \leq t \\ 15k - 3, & i = 3; 1 \leq k \leq t \\ 15k, & i = 4; 1 \leq k \leq t \end{cases}$$

$$f(v^k_{1,j}v^k_{5,j}) = 15k - 12, \ j = 1; 1 \leq k \leq t$$
$$f(v^k_{1,j}v^k_{5,j}) = 15k - 2, \ j = 2; 1 \leq k \leq t$$

$$f(v^k_{i,1}v^k_{i,2}) = \begin{cases} 15k - 5, & i = 1, 1 \leq k \leq t, \\ 15k - 6, & i = 2, 1 \leq k \leq t, \\ 15k - 7, & i = 3, 1 \leq k \leq t, \\ 15k - 8, & i = 4, 1 \leq k \leq t, \\ 15k - 9, & i = 5, 1 \leq k \leq t \end{cases}$$

Based on the edge label above, the vertex weight of *Shackle* $(Pr_{5,2}, v^k_{2,2}, t)$ is $(w(v^k_{i,j})), 1 \leq i \leq 5, 1 \leq j \leq 2; 1 \leq k \leq t$, as follows:

$$w(v^k_{i,j}) = 45k - 31$$

$$w(v^k_{i,(j+1)}) = \begin{cases} 45k - 11, & i = 1, 3, 4, t; 1 \leq k \leq t \\ 90k + 23, & i = 2; 1 \leq k \leq t - 1 \end{cases}$$

Next, we analyze the set of different edge weights at each vertex. What follows is the description of the different sets of vertex weights in succession:

$|\{w(v_{i,j}^k)\}| = |\{14, 59, 104, 149, \ldots, 45k - 31\}| = t$
$|\{w(v_{i,(j+1)}^k)\}| = |\{34, 79, 124, 169, \ldots, 45k - 11\}| = t$ for $i = 1, 3, 4, t; 1 \le k \le t$
$|\{w(v_{i,(j+1)}^k)\}| = |\{113, 203, 239, \ldots, 90k - 67\}| = t - 1$ for $i = 2; 1 \le k \le t - 1$

So, based on the description of the set of vertex weights above, we obtain the number of sets of vertex weights as follows:

$$|W| = |\{w(v_{i,j}^k)\}| + |\{w(v_{i,(j+1)}^k)\}| + |\{w(v_{i,(j+1)}^k)\}| = t + t + (t - 1) = 3t - 1$$

Based on the set of vertex weights, we determine the upper bound of $rvac$(*Shackle*$(Pr_{5,2}, v_{2,2}^k, t)$)= $3t - 1$. Based on lower bound and upper bound, the value of the *Rainbow Vertex Antimagic Connetion Number* on the Prism Graph *Shackle* $(Pr_{5,2}, v_{2,2}^k, t)$ is $3t - 1$, can be symbolized as $rvac(Pr_{5,2}^k) = 3t - 1$. The following rainbow path of the of the *Shackle* $(Pr_{5,2}, v_{2,2}^k, t)$ is shown in Table 5.

Table 5. Rainbow Path vertex $u - v$ on the Prism Graph *Shackle* $(Pr_{5,2}, v_{2,2}^k, t)$

Case	u	v	*Rainbow Vertex Coloring* $u - v$
1	$v_{i,1}^k$	$v_{i+2,1}^k$	$v_{i,1}^k, v_{i+1,1}^k, v_{i+2,1}^k$
2	$v_{1,1}^k$	$v_{4,1}^k$	$v_{1,1}^k, v_{5,1}^k, v_{4,1}^k$
3	$v_{i,1}^k$	$v_{i,2}^t$	$v_{i,1}^k, v_{i,2}^k, v_{2,2}^k, v_{1,2}^{k+1}, v_{1,2}^{k+2}, \ldots, v_{i,2}^t$
4	$v_{i,1}^k$	$v_{i,1}^t$	$v_{i,1}^k, v_{i,2}^k, v_{2,2}^k, v_{1,2}^{k+1}, v_{1,2}^{k+2}, \ldots, v_{1,2}^t, v_{i,1}^t$
5	$v_{i,2}^k$	$v_{i,2}^t$	$v_{i,2}^k, v_{2,2}^k, v_{1,2}^{k+1}, v_{2,2}^{k+1}, v_{1,2}^{k+2}, \ldots, v_{i,2}^t$
6	$v_{i,2}^k$	$v_{i,1}^t$	$v_{i,2}^k, v_{2,2}^k, v_{1,2}^{k+1}, v_{2,2}^{k+1}, v_{1,2}^{k+2}, \ldots, v_{1,2}^t, v_{i,1}^t$

Based on Table 5, we define that each different vertex on the *Shackle* $(Pr_{5,2}, v_{2,2}^k, t)$ has interior vertex with different colors, which satisfies the definition *Rainbow Vertex Antimagic Coloring*. Based on the lower and upper bound as well as Table 5, the *Rainbow Vertex Antimagic Connetion Number* value from the *Shackle* $(Pr_{5,2}, v_{2,2}^k, t)$ is $3t - 1$, which can be denoted as $rvac$(*Shackle*$(Pr_{5,2}, v_{2,2}^k, t)) = 3t - 1$.

Hereunder is an example of *Rainbow Vertex Antimagic Coloring* on the *Shackle* $(Pr_{5,2}, v_{2,2}^k, t)$:

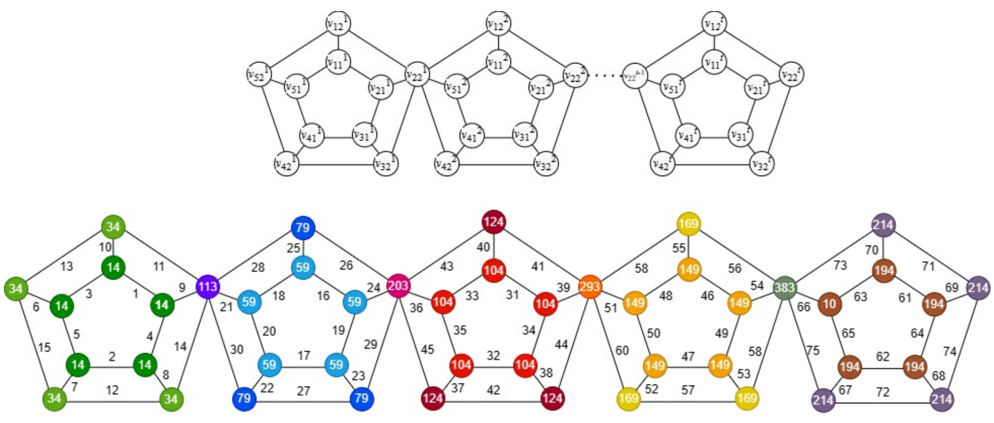

Figure 4. *Rainbow Vertex Antimagic Coloring* on the *Shackle* $(Pr_{5,2}, v_{2,2}^k, t)$

Theorem 3.5. *Rainbow Vertex Antimagic Connection Number on* $Amal(Pr_{5,3}, x_1, k)$ *for* $k \ge 2$ *is* $rvac(Amal(Pr_{5,3}, x_1, k)) = 3k + 1$.

Proof. Let *Amalgamation* $(Pr_{5,3}, x_1, k)$ is *Amalgamation* of Prism Graph with the vertex set $V(Amal(Pr_{5,3}, x_1, k)) = \{v_{i,j}^k; 1 \le i \le 5, 1 \le j \le 2, 1 \le k \le t\} \cup \{v_{i,3}^k; 2 \le i \le 5, j = 1, 1 \le k \le t\} \cup \{x_1\}$

and edge set $E(Amal(Pr_{5,3},x_1,k)) = \{v^k_{i,j}v_{i,(j+1)^k}; 1 \leq i \leq 5, 1 \leq j \leq 2, 1 \leq k \leq t\} \cup \{v^k_{i,j}v^k_{(i+1)j}; 1 \leq i \leq 4, 1 \leq j \leq 3, 1 \leq k \leq t\} \cup \{v^k_{1,j}v^k_{(5,j}; 1 \leq j \leq 3, 1 \leq t \leq k\}$. First, we will show the lower bound of the ***Rainbow Vertex Antimagic Coloring*** on the $Amal(Pr_{5,3},x_1,k)$ by finding the diameter of the $Amal(Pr_{5,3},x_1,k)$. The diameter of $Amal(Pr_{5,3},x_1,k)$ is $diam(Amal(Pr_{5,3},x_1,k)) = 3k+2$. So Based on Theorem 1.6., value of $rvc(Amal(Pr_{5,3},x_1,k))) \geq diam(Amal(Pr_{5,3},x_1,k)) - 1$. So from the diameter that has been obtained the value $rvc(Amal(Pr_{5,3},x_1,k)) \geq 3k+1$. is thus formulated as follow:

$$rvc(Amal(Pr_{5,3},x_1,k)) \geq diam(Pr^k_{5,3}) - 1 = (3k+2) - 1 = 3k+1$$

Second, we show the upper bound of *Rainbow Vertex Antimagic Coloring* on the $Amal(Pr_{5,3},x_1,k) \leq 3k+1$. The bijective function of the edge labels in the $Amal(Pr_{5,3},x_1,k)$ is defined as follows:

$$f(v^k_{i,j}v^k_{i,(j+1)}) = \begin{cases} 25k-15, \ i=1; j=1; 1 \leq k \leq t \\ 25k-16, \ i=2; j=1; 1 \leq k \leq t \\ 25k-17, \ i=3; j=1; 1 \leq k \leq t \\ 25k-18, \ i=4; j=1; 1 \leq k \leq t \\ 25k-19, \ i=5; j=1; 1 \leq k \leq t \end{cases}$$

$$f(v^k_{i,j}v^k_{i,(j+1)}) = \begin{cases} 25k-5, \ i=2; j=2; 1 \leq k \leq t \\ 25k-6, \ i=3; j=2; 1 \leq k \leq t \\ 25k-7, \ i=4; j=2; 1 \leq k \leq t \\ 25k-8, \ i=5; j=2; 1 \leq k \leq t \\ 25k-9, \ i=1; j=2; 1 \leq k \leq t \end{cases}$$

$$f(v^k_{i,j}v^k_{(i+1),j}) = \begin{cases} 25k-20, \ i=4; j=1; 1 \leq k \leq t \\ 25k-21, \ i=2; j=1; 1 \leq k \leq t \\ 25k-23, \ i=3; j=1; 1 \leq k \leq t \\ 25k-24, \ i=1; j=1; 1 \leq k \leq t \end{cases}$$

$$f(v^k_{i,j}v^k_{(i+1),j}) = \begin{cases} 25k-11, \ i=4; j=2; 1 \leq k \leq t \\ 25k-12, \ i=3; j=2; 1 \leq k \leq t \\ 25k-13, \ i=2; j=2; 1 \leq k \leq t \\ 25k-14, \ i=1; j=2; 1 \leq k \leq t \end{cases}$$

$$f(v^k_{i,j}v^k_{(i+1),j}) = \begin{cases} 25k-1, \ i=3; j=3; 1 \leq k \leq t \\ 25k-2, \ i=1; j=3; 1 \leq k \leq t \\ 25k-3, \ i=4; j=3; 1 \leq k \leq t \\ 25k-4, \ i=2; j=3; 1 \leq k \leq t \end{cases}$$

$$f(v^k_{1,j}v^k_{5,j}) = \begin{cases} 25k-22, \ j=1; 1 \leq k \leq t \\ 25k-10, \ j=2; 1 \leq k \leq t \\ 25k, \ j=3; 1 \leq k \leq t \end{cases}$$

Based on the edge label above, the vertex weight of $Amal(Pr_{5,3},x_1,k)$ is $\{w(v^k_{i,j}), 1 \leq i \leq 5, 1 \leq j \leq 2; 1 \leq k \leq t\}, \{w(v^k_{i,3}), 2 \leq i \leq 5, j=1; 1 \leq k \leq t\}$, and $\{w(x_1)\}$ as follows:

$$w(v^k_{i,j}) = \begin{cases} 75k-61, 1 \leq i \leq 5; j=1; 1 \leq k \leq t \\ 100k-48, 1 \leq i \leq 5; j=2; 1 \leq k \leq t \end{cases}$$

$w(v^k_{i,3}) = 75n-11, 2 \leq i \leq 5; 1 \leq k \leq t$

$w(x_1) = \frac{1}{2}(75n^2 + 53n)$

Next, we analyze the set of different edge weights at each vertex. Based on the vertex weight labels above, we identify nine different sets of vertex weights, namely $\{w(v^k_{i,j})\}, \{w(v^k_{i,3})\}$, and $\{w(x_1)\}$ Next, we analyze the set of different edge weights at each vertex. Based on the vertex weight labels above, we identify nine different sets of vertex weights, namely

$|\{w(v^k_{i,j})\}| = \{14, 89, 164, 239, \ldots, 75k-61\} = k$

$|\{w(v^k_{i,(j+1)})\}| = \{52, 152, 252, 352, \ldots, 100k-48\} = k$

$|\{w(v_{i,3}^k)\}| = \{64, 139, 214, 289, \ldots, 75k - 11\} = k$
$|\{w(x_1)\}| = \{64, 203, 417, 706 \ldots, \frac{1}{2}(75n^2 + 53n)\} = 1$

Based on the description of the set of vertex weights above, the number of sets of vertex weights is formulated as follows:

$$|W| = |\{w(v_{i,j}^k)\}| + |\{w(v_{i,(j+1)}^k)\}| + |\{w(v_{i,3}^k)\}| + |\{w(x_1)\}| = k + k + k + 1 = 3k + 1$$

Based on the set of vertex weights, we determine the upper bound $rvac(Amal(Pr_{5,3}, x_1, k)) = 3k + 1$. Based on lower bound and upper bound, the value of *Rainbow Vertex Antimagic Connetion Number* on the $Amal(Pr_{5,3}, x_1, k)$ is $3k + 1$, can be symbolized as $rvac(Amal(Pr_{5,3}, x_1, k)) = 3k + 1$. The rainbow path of the $Amal(Pr_{5,3}, x_1, k)$ is presented in Table 6.

Table 6. Rainbow Trajectory vertex $u - v$ on the $Amal(Pr_{5,3}, x_1, k)$

Case	u	v	*Rainbow Vertex Coloring* $u - v$
1	$v_{i,1}^k$	$v_{i+2,1}^k$	$v_{i,1}^k, v_{i+1,1}^k, v_{i+2,1}^k$
2	$v_{1,1}^k$	$v_{4,1}^k$	$v_{1,1}^k, v_{5,1}^k, v_{4,1}^k$
3	$v_{i,1}^k$	$v_{i,2}^t$	$v_{i,1}^k, v_{i,2}^k, v_{2,2}^k, v_{1,2}^{k+1}, v_{1,2}^{k+2}, \ldots, v_{i,2}^t$
4	$v_{i,1}^k$	$v_{i,1}^t$	$v_{i,1}^k, v_{i,2}^k, v_{2,2}^k, v_{1,2}^{k+1}, v_{1,2}^{k+2}, \ldots, v_{1,2}^t, v_{i,1}^t$
5	$v_{i,2}^k$	$v_{i,2}^t$	$v_{i,2}^k, v_{2,2}^k, v_{1,2}^{k+1}, v_{2,2}^{k+1}, v_{1,2}^{k+2}, \ldots, v_{i,2}^t$
6	$v_{i,2}^k$	$v_{i,1}^t$	$v_{i,2}^k, v_{2,2}^k, v_{1,2}^{k+1}, v_{2,2}^{k+1}, v_{1,2}^{k+2}, \ldots, v_{1,2}^t, v_{i,1}^t$

Table 6 shows that each different vertex on the $Amal(Pr_{5,3}, x_1, k)$ has interior vertex with different colors, which also complies with the definition *Rainbow Vertex Antimagic Coloring*. Based on the lower and upper bound and the details in Table 6, the value for the *Rainbow Vertex Antimagic Connetion Number* of the $Amal(Pr_{5,3}, x_1, k)$ is $3k + 1$ which can be denoted as $rvac(Amal(Pr_{5,3}, x_1, k)) = 3k + 1$.
Hereunder is an example of *Rainbow Vertex Antimagic Coloring* on the $Amal(Pr_{5,3}, x_1, k)$:

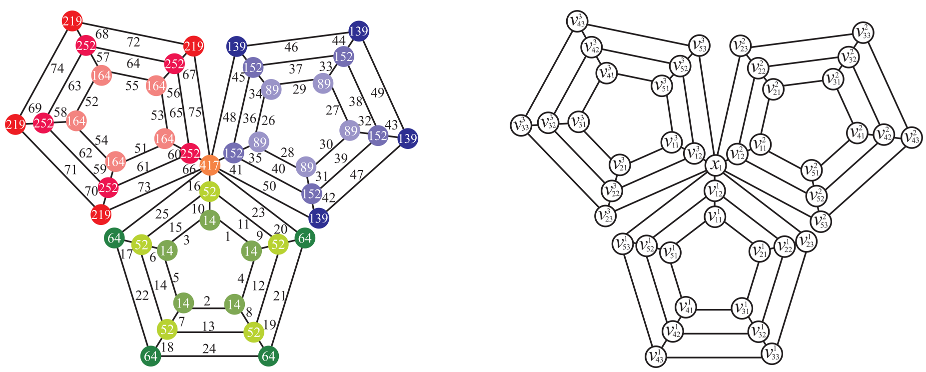

Figure 5. *Rainbow Vertex Antimagic Coloring* on the $Amal(Pr_{5,3}, x_1, 3)$

4. Conclusion

In this research, we have examined the accurate value of *rainbow vertex antimagic connection number* on a prism graph and its operations, namely $Pr_{3,m}$, $Pr_{5,m}$, $Pr_{6,m}$, *Shackle* ($Pr_{5,2}, v_{2,2}^k, t$), and $Amal(Pr_{5,3}, x_1, k)$. The research results have concluded four theorems regarding the *rainbow vertex antimagic connection number* in prism graphs and their operations.

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