

Unraveling the Intuitionistic Octagonal Fuzzy Travelling Salesman Problem via Dhouib-Matrix-TSP1 Heuristic

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Abstract The Travelling Salesman Problem (TSP) is an NP-hard problem of optimization that its goal is to obtain the shortest cycle among all cities that should be visited only once by a salesperson. The main goal of a salesperson is to visit each city only once and to obtain the distance traveled as well as the travelling costs as low as possible. In real-life and due to the absence of information, variables coming from experts' collected data are usually uncertain and imprecise. In such cases, the decision maker cannot exactly expect the TSP cost. That's why in this paper, the TSP under the intuitionistic octagonal fuzzy environment is considered and solved by adapting the very recent greedy method namely Dhouib-Matrix-TSP1 (DM-TSP1). This heuristic is very simple and it is composed of four steps. DM-TSP1 uses the Sum metric and is enriched with a ranking function. This current research work presents the first resolution of the TSP under the intuitionistic octagonal fuzzy domain. For this reason, new case studies are generated in order to carry out the experimental results. Moreover, a step-by-step execution of DM-TSP1 is detailed in order to prove its effectiveness.

Keywords Artificial Intelligence, Travelling Salesman Problem, Optimization, Dhouib-Matrix-TSP1, Intuitionistic Octagonal Fuzzy number, Heuristic, Operation Research.

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1. Introduction

The Travelling Salesman Problem (TSP) is a combinatorial problem aiming to obtain the shortest cycle between all cities that should be visited by a salesperson. This latter has to keep the distance (the travel cost etc.) as minimal as possible and visit each city only once. Several approximation methods are used to optimize the TSP: The Ant Colony Optimization technique is applied in [1], the spider monkey method is used in [2], a hybrid method where the Max-Min Ant System and the Particle Swarm Optimization are combined in [3], the Particle Swarm Optimization and the Artificial Bee Colony metaheuristics are improved in [4], a novel local search method named Far-to-Near is designed to optimize the route of holes drilling in a printed circuit board where the holes represent the cities and the tool path signifies the salesman in [5], the Ant Colony Optimization metaheuristic is applied to optimize the problem of drilling of printed circuit board in [6], a modified metaheuristic is developed to optimize the TSP in [7] and the discrete Pigeon-Inspired optimization algorithm for a multiple TSP is applied in [8].

Moreover, a hybrid Genetic Algorithm is implemented to solve the truncated TSP in [9], a Linearithmic heuristic is proposed in [10], an enhanced Artificial Bee Colony metaheuristic is presented in [11], an integrated Genetic Algorithm with the Simulated Annealing is introduced in [12], an application of the TSP to generate a tool path without collision for the Controlled Numerical Computer Drilling is implemented in [13], a modified Particle Swarm Optimization metaheuristic with the 3-Opt and the Cauchy operator is designed in [14], a Genetic Algorithm

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with a local search method is developed in [15], several improved Genetic Algorithms with clusters are presented in [16], a multiagent learning heuristic hybridized with a Genetic Algorithm is introduced in [17], a new approach built on the Ant Colony Optimization and the Artificial Bee Colony metaheuristics is exploited in [18], a novel algorithm based on several iterated metaheuristics for the clustered TSP is invented in [19], a heuristic composed of two phases for solving the multiple TSP is developed in [20], a Harris Hawk Optimizer method is implemented in [21], a dynamic encoding as well as a graph model for the large-scale TSP is designed [22] and a modified Genetic Algorithm for the TSP is developed in [23]. Furthermore, three enriched formulations for the flying sidekick TSP are presented in [24], a local search heuristic to solve the flying sidekick TSP is exploited in [25], the Branch-and-Cut method for optimizing the TSP with drone is designed in [26], an iterative algorithm for the TSP with drone is introduced in [27] and the TSP with a drone Station is considered in [28]. Also, the Ant Colony Optimization algorithm for solving the multiple TSP is implemented in [29], the Coyote optimizer is enhanced in [30], a new Genetic Algorithm for solving the large scale TSP is exploited in [31], the Symbiotic Organisms Search algorithm is hybridized with the Simulated Annealing to unravel the TSP in [32], the Branch and Bound method is designed for the constrained TSP in [33], the generalized TSP is solved in [34, 35], the Harris Hawk Optimization algorithm with the Ant Colony Optimization metaheuristic is turned for the TSP in [36], a smart solution for the semantic TSP through a hybrid Genetic Algorithm with the Ant Colony Optimization method is proposed in [37], a Redesigned Tree-Seed Optimization algorithm to solve the symmetric TSP is used in [38].

In real-life and due to the absence of information, variables coming from experts' collected data are usually uncertain and imprecise. In such cases, the decision maker cannot exactly expect the TSP cost. That's why the uncertainty theory appears and is proved to be a successful method. More precisely, the Intuitionistic Fuzzy Set (IFS) was proposed by [39] to deal with these uncertainties. The concept of the IFS can be applied in various optimization problems like the Travelling Salesman Problem (TSP), the Transportation Problem (TP), the Assignment Problem (AP), the Linear Programming Problem (LPP), etc. A new programming method to solve the intuitionistic fuzzy LPP is designed in [40], the LPP under interval valued intuitionistic domain is optimized in [41], the intuitionistic LPP is solved by a ranking function in [42], a solution for the intuitionistic fuzzy LPP via a sensitivity analysis and a simplex algorithm is proposed in [43], an approach to solve the intuitionistic fuzzy AP is designed in [44], a novel approach for solving the solid AP under the intuitionistic fuzzy domain is exploited in [45], a novel PSK method is developed to optimize the intuitionistic fuzzy AP in [46], the intuitionistic fuzzy AP is solved by the means of score functions and similarity measures in [47]. The intuitionistic fuzzy TP using the revised distribution technique is optimized in [48], the TP in the interval valued intuitionistic domain is solved in [49], a mathematical model for the intuitionistic fuzzy TP is proposed in [50] and a novel solution method for the intuitionistic TP is invented in [51].

In the case of the TSP and by the use of the intuitionistic fuzzy set theory, the decision maker can choose between the level of acceptance and non-acceptance for the travelling cost. Few research papers were interested in solving the TSP under the intuitionistic fuzzy environment. A novel technique to optimize the TSP with intuitionistic triangular fuzzy numbers is designed in [52], an intuitionistic approach for the TSP is implemented in [53], the intuitionistic fuzzy TSP using the Genetic Algorithm is solved in [54], the time dependent TSP through the memetic algorithm and an intuitionistic fuzzy model is unraveled in [55], the TSP using a novel intuitionistic fuzzy technique is optimized in [56], the intuitionistic fuzzy TSP is modeled in [57], an intuitionistic model on the TSP is applied in [58], an intuitionistic fuzzy approach for solving the generalized trapezoidal TSP is developed in [59], an intuitionistic fuzzy method for the TSP is proposed in [60], a sensitivity analysis to solve the intuitionistic fuzzy TSP is presented in [61]. Also, a hybrid Particle Swarm Optimization method is used to unravel the intuitionistic fuzzy TSP in [62], a new model of the Hungarian approach for the intuitionistic fuzzy TSP is designed in [63], the Branch and Bound algorithm for solving the intuitionistic TSP is implemented in [64]. The dynamic programming technique is employed to optimize the intuitionistic TSP in [65]. The fuzzy Hungarian method is applied to solve the Intuitionistic Fuzzy TSP in [66]. A logarithmic function is used for unravelling the intuitionistic TSP in [67]. An intuitionistic method is implemented for optimizing the TSP in [68]. As far as we know, the TSP with the intuitionistic octagonal fuzzy numbers is not yet solved. For that, in this paper three new case studies are generated and solved by the very recent heuristic namely Dhouib-Matrix-TSP1 (DM-TSP1) which is characterized by its performance and its rapidity (just n iterations for generating a feasible solution).

The remaining of this manuscript is organized as follows. Section 2 is dedicated for detailing some intuitionistic fuzzy concepts. Section 3 describes the adaptation of the proposed approach DM-TSP1 for the case of the intuitionistic octagonal fuzzy TSP. Section 4 presents a step-by-step resolution of DM-TSP1 on three numerical examples. Section 5 concludes the paper and gives an idea about further research.

2. Preliminaries

2.1. The intuitionistic fuzzy number

The intuitionistic fuzzy number in X is denoted by $\tilde{F}^I = \{(x, \mu_{\tilde{F}^I}(x), \vartheta_{\tilde{F}^I}(x)) / x \in X\}$ that is characterized by two membership functions where $\mu_{\tilde{F}^I}(x) : X \rightarrow [0, 1]$ explains the membership mapping element of $x \in X$ to the set \tilde{F}^I and $\vartheta_{\tilde{F}^I}(x) : X \rightarrow [0, 1]$ presents the non-membership mapping element of $x \in X$ to the set \tilde{F}^I .

2.2. The intuitionistic octagonal fuzzy number

Let us define \tilde{F}^I as an intuitionistic octagonal fuzzy number denoted by $(f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8; f'_1, f'_2, f'_3, f'_4, f'_5, f'_6, f'_7, f'_8)$ with two membership functions:

$$\mu_{\tilde{F}^I}(x) = \begin{cases} 0 & \text{for } x \prec f_1 \\ k \left(\frac{x-f_1}{f_2-f_1} \right) & \text{if } f_1 \leq x \leq f_2 \\ k & \text{if } f_2 \leq x \leq f_3 \\ k + (1-k) \left(\frac{x-f_3}{f_4-f_3} \right) & \text{if } f_3 \leq x \leq f_4 \\ 1 & \text{if } f_4 \leq x \leq f_5 \\ k + (1-k) \left(\frac{f_6-x}{f_6-f_5} \right) & \text{if } f_5 \leq x \leq f_6 \\ k & \text{if } f_6 \leq x \leq f_7 \\ k \left(\frac{f_8-x}{f_8-f_7} \right) & \text{if } f_7 \leq x \leq f_8 \\ 0 & \text{for } x \succ f_8 \end{cases}$$

and

$$\vartheta_{\tilde{F}^I}(x) = \begin{cases} 0 & \text{for } x \prec f'_1 \\ k \left(\frac{x-f'_1}{f'_2-f'_1} \right) & \text{if } f'_1 \leq x \leq f'_2 \\ k & \text{if } f'_2 \leq x \leq f'_3 \\ k + (1-k) \left(\frac{x-f'_3}{f'_4-f'_3} \right) & \text{if } f'_3 \leq x \leq f'_4 \\ 1 & \text{if } f'_4 \leq x \leq f'_5 \\ k + (1-k) \left(\frac{f'_6-x}{f'_6-f'_5} \right) & \text{if } f'_5 \leq x \leq f'_6 \\ k & \text{if } f'_6 \leq x \leq f'_7 \\ k \left(\frac{f'_8-x}{f'_8-f'_7} \right) & \text{if } f'_7 \leq x \leq f'_8 \\ 0 & \text{for } x \succ f'_8 \end{cases}$$

Figure 1 illustrates the graphical representation of \tilde{F}^I .

2.3. The Intuitionistic arithmetic operations

The arithmetic operations on two intuitionistic octagonal fuzzy numbers \tilde{F}^I and \tilde{E}^I defined by $(f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8; f'_1, f'_2, f'_3, f'_4, f'_5, f'_6, f'_7, f'_8)$ and $(e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8; e'_1, e'_2, e'_3, e'_4, e'_5, e'_6, e'_7, e'_8)$ are the following:

The addition:

$$\tilde{F}^I \oplus \tilde{E}^I = \left((f_1+e_1, f_2+e_2, f_3+e_3, f_4+e_4, f_5+e_5, f_6+e_6, f_7+e_7, f_8+e_8); (f'_1+e'_1, f'_2+e'_2, f'_3+e'_3, f'_4+e'_4, f'_5+e'_5, f'_6+e'_6, f'_7+e'_7, f'_8+e'_8) \right)$$

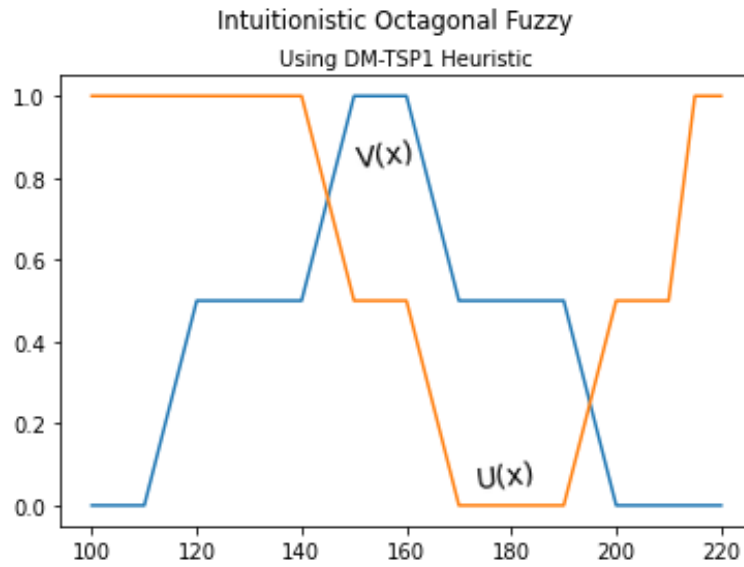


Figure 1. Graphical representation of an intuitionistic octagonal fuzzy number.

The subtraction:

$$\tilde{F}' \ominus \tilde{E}' = ((f_1 - e_1, f_2 - e_2, f_3 - e_3, f_4 - e_4, f_5 - e_5, f_6 - e_6, f_4 - e_1, f_3 - e_8), (f_1 - e_1, f_2 - e_2, f_3 - e_3, f_4 - e_4, f_5 - e_5, f_6 - e_6, f_7 - e_7, f_8 - e_8))$$

The multiplication:

$$\tilde{F}' \otimes \tilde{E}' = ((f_1 * e_1, f_2 * e_2, f_3 * e_3, f_4 * e_4, f_5 * e_5, f_6 * e_6, f_4 * e_1, f_3 * e_8), (f_1 * e_1, f_2 * e_2, f_3 * e_3, f_4 * e_4, f_5 * e_5, f_6 * e_6, f_7 * e_7, f_8 * e_8))$$

The division:

$$\tilde{F}' \tilde{E}' = ((f_1 / e_1, f_2 / e_2, f_3 / e_3, f_4 / e_4, f_5 / e_5, f_6 / e_6, f_4 / e_1, f_3 / e_8), (f_1 / e_1, f_2 / e_2, f_3 / e_3, f_4 / e_4, f_5 / e_5, f_6 / e_6, f_7 / e_7, f_8 / e_8))$$

2.4. Definition 4

In order to defuzzify an intuitionistic octagonal fuzzy number to a crisp one, the equation presented in [69] is used:

Where:

$$R[\tilde{F}^I] = \text{Max} [\text{Mag}_\mu(\tilde{F}^I), \text{Mag}_\vartheta(\tilde{F}^I)]$$

$$\text{Mag}_\mu(\tilde{F}^I) = \frac{2f_1 + 3f_2 + 4f_3 + 5f_4 + 5f_5 + 4f_6 + 3f_7 + 2f_8}{28}$$

$$\text{Mag}_\vartheta(\tilde{F}^I) = \frac{2f'_1 + 3f'_2 + 4f'_3 + 5f'_4 + 5f'_5 + 4f'_6 + 3f'_7 + 2f'_8}{28}$$

3. The proposed Dhouib-Matrix-TSP1 Method

The TSP deals with generating a minimal cycle between all cities. x_{ij} represents a binary decision variable and d_{ij} represents the distance between each two cities i and j . So, the TSP is mathematically defined as follows:

Minimize:

$$\sum_{i=1}^n \sum_{j=1}^n d_{ij} x_{ij}$$

Subject to:

$$\begin{aligned} \sum_{j=1}^n x_{ij} &= 1, \quad i = 1, \dots, n \\ \sum_{i=1}^n x_{ij} &= 1, \quad j = 1, \dots, n \\ x_{ij} &= 0 \text{ or } 1, \quad i = 1, \dots, n, j = 1, \dots, n \end{aligned}$$

In order to rapidly solve the TSP, a new greedy method named Dhouib-Matrix-TSP1 (DM-TSP1) which belongs to the Dhouib-Matrix concept is proposed in [70]. Next, a stochastic version of this method was presented in [71, 72]. Moreover, DM-TSP1 was tested on different fuzzy instances in [73, 74, 75]. Then, DM-TSP1 was applied on the TSP with neutrosophic numbers in [76, 77, 78, 79]. For other combinatorial problems, the second co-author Dhouib proposed several novel optimization methods: The Assignment Problems in [80, 81, 82], the Transportation Problems in [83, 84, 85], the Shortest Path Problems [86, 87] and the Minimum Spanning Tree Problem [88]. Moreover, for larger TSP instances three novel metaheuristics were designed: The Far-to-Near in [5], the Dhouib-Matrix-3 in [89, 90, 91, 92] and the Dhouib-Matrix-4 in [93, 94, 95, 96, 97].

The proposed DM-TSP1 heuristic is composed of four simple steps (see Figure 2) and repeated n iterations (with n is the number of cities). Besides, DM-TSP1 can use any differential statistical metric (Min, Max, Average, Range, etc.) and in this paper the Sum function is used (for more clarification the next section will describe a step-by-step execution of DM-TSP1).

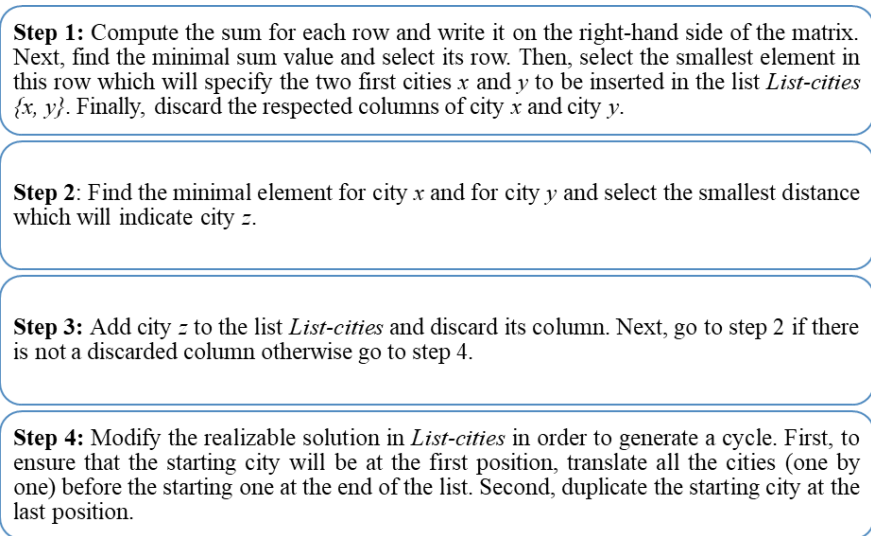


Figure 2. Flowchart of the proposed heuristic DM-TSP1.

4. Numerical Example

According to our knowledge, there is no paper in the literature that discussed a TSP with intuitionistic octagonal fuzzy numbers. For that, new instances are generated with a step-by-step resolution by the means of the novel proposed heuristic DM-TSP1 using the Sum function.

4.1. Case study 1

Let us consider a 5x5 distance matrix with intuitionistic octagonal fuzzy numbers described in Figure 3.

Where

$$\begin{aligned} d_{12} &= (7, 10, 17, 18, 23, 24, 27, 33; 5, 9, 15, 17, 22, 24, 25, 27), \\ d_{13} &= (6, 7, 12, 15, 19, 24, 25, 27; 3, 8, 11, 13, 15, 17, 18, 19), \\ d_{14} &= (0, 5, 9, 13, 15, 20, 22, 29; 2, 5, 10, 18, 20, 25, 29, 34), \\ d_{15} &= (3, 6, 12, 16, 20, 21, 22, 23; 4, 5, 10, 12, 19, 20, 24, 25), \end{aligned}$$

$$\begin{bmatrix} \infty & d_{12} & d_{13} & d_{14} & d_{15} \\ d_{21} & \infty & d_{23} & d_{24} & d_{25} \\ d_{31} & d_{32} & \infty & d_{34} & d_{35} \\ d_{41} & d_{42} & d_{43} & \infty & d_{45} \\ d_{51} & d_{52} & d_{53} & d_{54} & \infty \end{bmatrix}$$

Figure 3. The intuitionistic octagonal fuzzy set matrix.

$$\begin{aligned} d_{21} &= (6, 10, 20, 24, 29, 32, 35, 40; 4, 8, 16, 24, 26, 28, 29, 30), \\ d_{23} &= (8, 11, 22, 26, 33, 35, 36, 38; 10, 13, 16, 20, 24, 27, 28, 30), \\ d_{24} &= (13, 17, 21, 22, 23, 24, 26, 28; 15, 16, 18, 20, 22, 23, 25, 27), \\ d_{25} &= (3, 13, 16, 19, 22, 25, 28, 31; 5, 10, 21, 24, 27, 31, 33, 35), \\ d_{31} &= (4, 5, 10, 12, 19, 20, 24, 25; 2, 3, 6, 7, 17, 18, 19, 20), \\ d_{32} &= (10, 12, 14, 16, 18, 20, 22, 24; 11, 12, 17, 22, 26, 28, 30, 38), \\ d_{34} &= (4, 10, 17, 18, 20, 24, 26, 31; 6, 8, 15, 19, 20, 21, 22, 24), \\ d_{35} &= (12, 15, 20, 23, 25, 27, 29, 30; 8, 10, 15, 17, 19, 23, 24, 26), \\ d_{41} &= (15, 20, 25, 28, 35, 40, 43, 51; 15, 17, 20, 25, 35, 40, 43, 51), \\ d_{42} &= (6, 10, 20, 24, 29, 32, 35, 40; 4, 8, 16, 24, 26, 28, 29, 30), \\ d_{43} &= (4, 10, 17, 18, 20, 24, 26, 31; 6, 8, 15, 19, 20, 21, 22, 24), \\ d_{45} &= (6, 10, 20, 24, 29, 32, 35, 40; 4, 8, 16, 24, 26, 28, 29, 30), \\ d_{51} &= (3, 6, 12, 16, 20, 21, 22, 23; 4, 5, 10, 12, 19, 20, 24, 25), \\ d_{52} &= (1, 5, 8, 9, 11, 12, 13, 14; 5, 6, 8, 10, 11, 12, 13, 14), \\ d_{53} &= (7, 10, 17, 18, 23, 24, 27, 33; 5, 9, 15, 17, 22, 24, 25, 27), \\ d_{54} &= (6, 10, 20, 24, 29, 32, 35, 40; 4, 8, 16, 24, 26, 28, 29, 30). \end{aligned}$$

The following ranking function is used to defuzzify the intuitionistic octagonal fuzzy cost matrix to a crisp cost matrix (see Figure 4). This function is taken from [69].

$$\begin{bmatrix} \infty & 20 & 17 & 18 & 16 \\ 25 & \infty & 27 & 22 & 24 \\ 15 & 23 & \infty & 19 & 23 \\ 32 & 25 & 19 & \infty & 25 \\ 16 & 10 & 20 & 25 & \infty \end{bmatrix}$$

Figure 4. The crisp cost matrix.

The first step consists in computing the sum of each row (see Figure 5), identifying the smallest value (at the first row which is equal to 71) then selecting the smallest element in the identified row (which is equal to 16 at position d_{15}).

The element d_{15} is selected then city 1 and city 5 are inserted in $List_cities = \{1-5\}$ and their respective columns are discarded. Next, search the smallest element between row 1 and row 5 (1 and 5 represent the extremity points in the list) which is the element d_{52} with the value of 10 (see Figure 6).

Besides, insert city 2 after city 5 in $List_cities = \{1-5-2\}$ and discard its column (see Figure 7). Now, select the minimal value in row 1 and row 2 which is equal to 17 at position d_{13} .

∞	20	17	18	16	71
25	∞	27	22	24	98
15	23	∞	19	23	80
32	25	19	∞	25	101
16	10	20	25	∞	71

Figure 5. Computing the Sum of each row.

∞	20	17	18	16
25	∞	27	22	24
15	23	∞	19	23
32	25	19	∞	25
16	10	20	25	∞

Figure 6. Discarding columns 1 and 5 and selecting element d_{52} .

∞	20	17	18	16
25	∞	27	22	24
15	23	∞	19	23
32	25	19	∞	25
16	10	20	25	∞

Figure 7. Discarding column 2 and selecting element d_{13} .

Furthermore, add city 3 before city 1 (because city 3 is joined from city 1) in $List_cities = \{3-1-5-2\}$ and discard its column (see Figure 8). Now, select the minimal value between row 3 and row 2 which is equal to 19 at position d_{34} .

∞	20	17	18	16
25	∞	27	22	24
15	23	∞	19	23
32	25	19	∞	25
16	10	20	25	∞

Figure 8. Discarding column 3 and selecting element d_{34} .

Hence, insert city 4 before city 3 (because city 4 is joined with city 3) in $List_cities = \{4-3-1-5-2\}$ and discard its column (see Figure 9).

∞	20	17	18	16
25	∞	27	22	24
15	23	∞	19	23
32	25	19	∞	25
16	10	20	25	∞

Figure 9. Discarding column 4.

All columns are discarded, then Step 4 is launched to generate the cycle by translating every city (which are 4 and 3) before the starting city (1) at the end of the list and also by duplicating the starting city (1) at the end: so, from 4-3-1-5-2, the cycle 1-5-2-4-3-1 is generated with a total crisp cost equal to $82 = 16+10+22+19+15$. The corresponding total intuitionistic octagonal fuzzy cost is $\tilde{F}^I = (25, 43, 68, 77, 93, 101, 111, 121; 19, 25, 45, 55, 84, 89, 97, 103)$ and it is graphically illustrated in Figure 10.

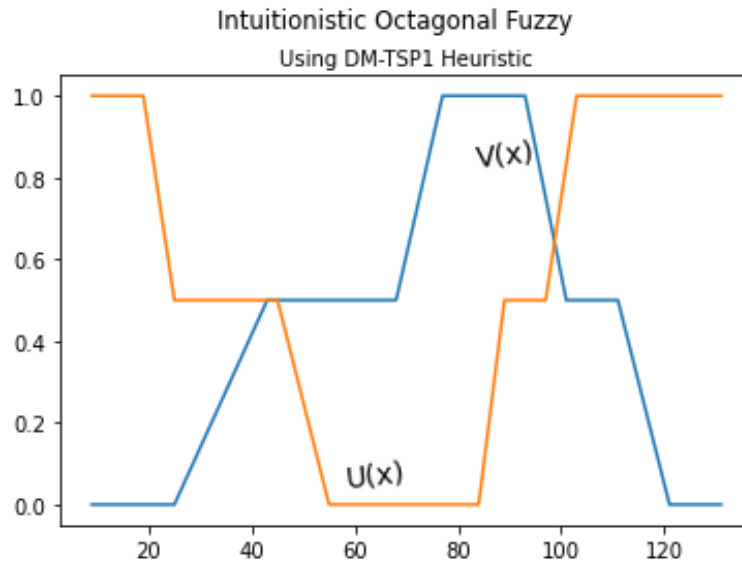


Figure 10. Graphical representation of the generated intuitionistic octagonal fuzzy solution.

4.2. Case study 2

A second example is considered with a 6x6 distance matrix using intuitionistic octagonal fuzzy numbers (see Figure 11). Where

- $d_{12} = (6, 7, 12, 15, 19, 24, 25, 27; 3, 8, 11, 13, 15, 17, 18, 28),$
- $d_{13} = (6, 10, 20, 24, 29, 32, 35, 40; 4, 8, 16, 24, 26, 28, 29, 30),$
- $d_{14} = (13, 17, 21, 22, 23, 24, 26, 28; 15, 16, 18, 20, 22, 23, 25, 27),$
- $d_{15} = (1, 5, 8, 9, 11, 12, 13, 14; 5, 6, 8, 10, 11, 12, 13, 14),$
- $d_{16} = (0, 3, 6, 9, 12, 15, 20, 25; 0, 3, 4, 5, 6, 8, 16, 18),$
- $d_{21} = (3, 6, 12, 16, 20, 21, 22, 23; 4, 5, 10, 12, 19, 20, 24, 25),$
- $d_{23} = (1, 5, 8, 9, 11, 12, 13, 14; 5, 6, 8, 10, 11, 12, 13, 14),$
- $d_{24} = (7, 10, 17, 18, 23, 24, 27, 33; 5, 9, 15, 17, 22, 24, 25, 27),$

$$\begin{bmatrix} \infty & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & \infty & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & \infty & d_{34} & d_{35} & d_{36} \\ d_{41} & d_{42} & d_{43} & \infty & d_{45} & d_{46} \\ d_{51} & d_{52} & d_{53} & d_{54} & \infty & d_{56} \\ d_{61} & d_{62} & d_{63} & d_{64} & d_{65} & \infty \end{bmatrix}$$

Figure 11. The intuitionistic octagonal fuzzy set 6x6 matrix.

$$\begin{aligned} d_{25} &= (6, 10, 20, 24, 29, 32, 35, 40; 4, 8, 16, 24, 26, 28, 29, 30), \\ d_{26} &= (4, 5, 10, 12, 19, 20, 24, 25; 2, 3, 6, 7, 17, 18, 19, 20), \\ d_{31} &= (13, 17, 21, 22, 23, 24, 26, 28; 15, 16, 18, 20, 22, 23, 25, 27), \\ d_{32} &= (10, 12, 14, 16, 18, 20, 22, 24; 11, 12, 17, 22, 26, 28, 30, 38), \\ d_{34} &= (4, 10, 17, 18, 20, 24, 26, 31; 6, 8, 15, 19, 20, 21, 22, 24), \\ d_{35} &= (12, 15, 20, 23, 25, 27, 29, 30; 8, 10, 15, 17, 19, 23, 24, 26), \\ d_{36} &= (0, 5, 9, 13, 15, 20, 22, 29; 2, 5, 10, 18, 20, 25, 29, 34), \\ d_{41} &= (15, 20, 25, 28, 35, 40, 43, 51; 15, 17, 20, 25, 35, 40, 43, 51), \\ d_{42} &= (6, 10, 20, 24, 29, 32, 35, 40; 4, 8, 16, 24, 26, 28, 29, 30), \\ d_{43} &= (4, 10, 17, 18, 20, 24, 26, 31; 6, 8, 15, 19, 20, 21, 22, 24), \\ d_{45} &= (1, 5, 8, 9, 11, 12, 13, 14; 5, 6, 8, 10, 11, 12, 13, 14), \\ d_{46} &= (7, 10, 17, 18, 23, 24, 27, 33; 5, 9, 15, 17, 22, 24, 25, 27), \\ d_{51} &= (0, 3, 4, 5, 6, 8, 16, 18; 1, 4, 8, 11, 15, 18, 22, 25), \\ d_{52} &= (8, 11, 22, 26, 33, 35, 36, 38; 10, 13, 16, 20, 24, 27, 28, 30), \\ d_{53} &= (13, 17, 21, 22, 23, 24, 26, 28; 15, 16, 18, 20, 22, 23, 25, 27), \\ d_{54} &= (3, 13, 16, 19, 22, 25, 28, 31; 5, 10, 21, 24, 27, 31, 33, 35), \\ d_{56} &= (15, 20, 25, 28, 35, 40, 43, 51; 15, 17, 20, 25, 35, 40, 43, 51), \\ d_{61} &= (0, 3, 6, 9, 12, 15, 20, 25; 0, 3, 4, 5, 6, 8, 16, 18), \\ d_{62} &= (1, 5, 8, 9, 11, 12, 13, 14; 5, 6, 8, 10, 11, 12, 13, 13), \\ d_{63} &= (0, 5, 9, 13, 15, 20, 22, 29; 2, 5, 10, 18, 20, 25, 29, 34), \\ d_{64} &= (3, 6, 12, 16, 20, 21, 22, 23; 4, 5, 10, 12, 19, 20, 24, 25), \\ d_{65} &= (0, 3, 6, 9, 12, 15, 20, 25; 1, 4, 8, 11, 15, 18, 22, 25), \end{aligned}$$

DM-TSP1 needs only 6 simple iterations ($n=6$) to solve this problem. Figure 12 illustrates the result generated for each step (Also, in this example the ranking function proposed by [69] is used for defuzzification).

4.3. Case study 3

A third example is generated with a 6x6 distance matrix using intuitionistic octagonal fuzzy numbers defined in Figure 13.

Where

$$\begin{aligned} d_{12} &= (10, 13, 15, 18, 19, 21, 23, 29; 5, 9, 12, 16, 18, 20, 21, 23), \\ d_{13} &= (3, 5, 13, 20, 23, 28, 31, 34; 11, 13, 17, 20, 22, 25, 28, 33), \\ d_{14} &= (5, 8, 13, 16, 21, 23, 25, 31; 6, 9, 13, 16, 19, 21, 24, 28), \\ d_{15} &= (3, 4, 7, 10, 13, 15, 17, 19; 10, 13, 15, 17, 19, 21, 23, 26), \\ d_{16} &= (2, 5, 8, 11, 13, 16, 18, 21; 4, 7, 9, 11, 14, 16, 19, 23), \\ d_{21} &= (10, 13, 15, 18, 19, 21, 23, 29; 5, 9, 12, 16, 18, 20, 21, 23), \\ d_{23} &= (0, 3, 9, 12, 15, 18, 21, 24; 2, 5, 8, 15, 17, 22, 24, 27), \\ d_{24} &= (3, 6, 7, 9, 13, 16, 19, 24; 1, 6, 9, 12, 15, 18, 20, 22), \\ d_{25} &= (1, 5, 7, 8, 13, 15, 17, 22; 2, 5, 8, 11, 14, 16, 19, 24), \end{aligned}$$

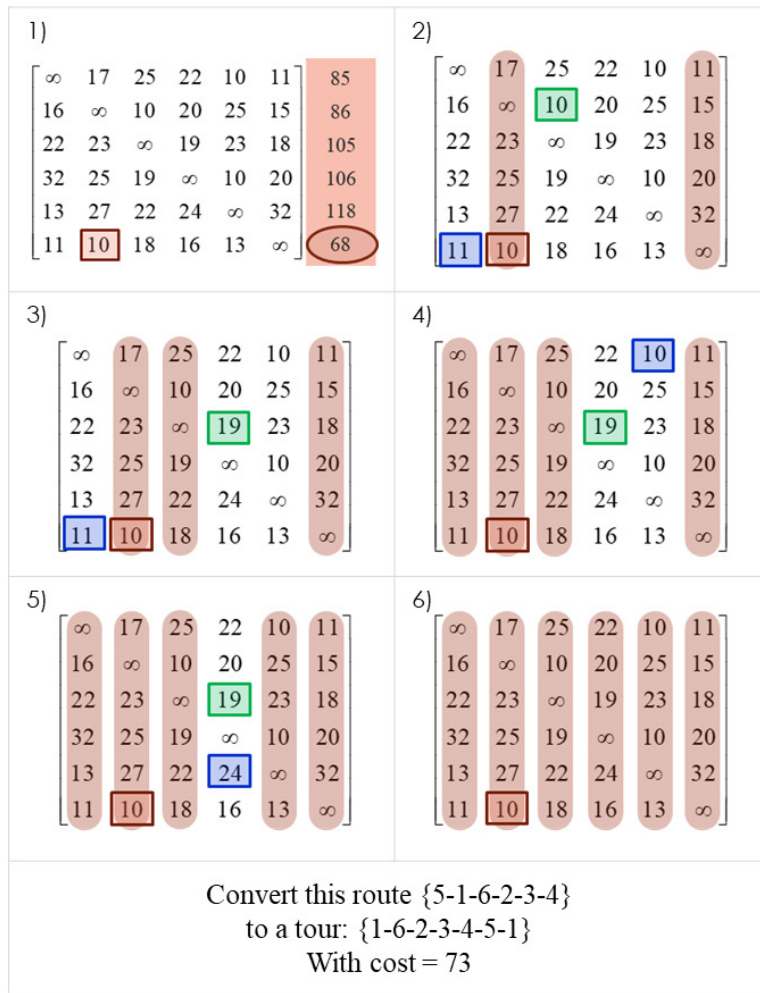


Figure 12. The step-by-step application of DM-TSP1 for 6 cities.

$$\begin{bmatrix}
 \infty & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\
 d_{21} & \infty & d_{23} & d_{24} & d_{25} & d_{26} \\
 d_{31} & d_{32} & \infty & d_{34} & d_{35} & d_{36} \\
 d_{41} & d_{42} & d_{43} & \infty & d_{45} & d_{46} \\
 d_{51} & d_{52} & d_{53} & d_{54} & \infty & d_{56} \\
 d_{61} & d_{62} & d_{63} & d_{64} & d_{65} & \infty
 \end{bmatrix}$$

Figure 13. The intuitionistic octagonal fuzzy set 6x6 matrix.

$$\begin{aligned}
 d_{26} &= (1, 2, 5, 7, 9, 12, 16, 19; 2, 3, 6, 7, 17, 18, 19, 20), \\
 d_{31} &= (3, 5, 13, 20, 23, 28, 31, 34; 11, 13, 17, 20, 22, 25, 28, 33), \\
 d_{32} &= (0, 3, 9, 12, 15, 18, 21, 24; 2, 5, 8, 15, 17, 22, 24, 27), \\
 d_{34} &= (0, 3, 6, 9, 12, 17, 19, 21; 2, 5, 7, 9, 13, 15, 17, 19), \\
 d_{35} &= (2, 4, 6, 8, 12, 14, 16, 18; 6, 8, 10, 12, 14, 16, 18, 23), \\
 d_{36} &= (4, 7, 8, 11, 13, 15, 17, 19; 4, 5, 6, 7, 11, 13, 14, 16),
 \end{aligned}$$

$$\begin{aligned}
d_{41} &= (5, 8, 13, 16, 21, 23, 25, 31; 6, 9, 13, 16, 19, 21, 24, 28), \\
d_{42} &= (3, 6, 7, 9, 13, 16, 19, 24; 1, 6, 9, 12, 15, 18, 20, 22), \\
d_{43} &= (0, 3, 6, 9, 12, 17, 19, 21; 2, 5, 7, 9, 13, 15, 17, 19), \\
d_{45} &= (4, 6, 7, 11, 14, 17, 18, 21; 5, 6, 8, 10, 11, 12, 13, 14), \\
d_{46} &= (2, 5, 7, 8, 12, 14, 17, 19; 3, 6, 8, 10, 13, 15, 17, 19), \\
d_{51} &= (3, 4, 7, 10, 13, 15, 17, 19; 10, 13, 15, 17, 19, 21, 23, 26), \\
d_{52} &= (1, 5, 7, 8, 13, 15, 17, 22; 2, 5, 8, 11, 14, 16, 19, 24), \\
d_{53} &= (2, 4, 6, 8, 12, 14, 16, 18; 6, 8, 10, 12, 14, 16, 18, 23), \\
d_{54} &= (4, 6, 7, 11, 14, 17, 18, 21; 5, 6, 8, 10, 11, 12, 13, 14), \\
d_{56} &= (3, 8, 13, 15, 18, 21, 26, 28; 6, 8, 13, 14, 16, 31, 35, 37), \\
d_{61} &= (2, 5, 8, 11, 13, 16, 18, 21; 4, 7, 9, 11, 14, 16, 19, 23), \\
d_{62} &= (1, 2, 5, 7, 9, 12, 16, 19; 2, 3, 6, 7, 17, 18, 19, 20), \\
d_{63} &= (4, 7, 8, 11, 13, 15, 17, 19; 4, 5, 6, 7, 11, 13, 14, 16), \\
d_{64} &= (2, 5, 7, 8, 12, 14, 17, 19; 3, 6, 8, 10, 13, 15, 17, 19), \\
d_{65} &= (3, 8, 13, 15, 18, 21, 26, 28; 6, 8, 13, 14, 16, 31, 35, 37).
\end{aligned}$$

Figure 14 represents the generated crisp cost matrix after defuzzification.

$$\begin{bmatrix}
\infty & 18.39 & 21.04 & 17.86 & 18.00 & 12.75 \\
18.39 & \infty & 15.18 & 13.11 & 12.32 & 11.64 \\
21.04 & 15.18 & \infty & 10.93 & 13.21 & 11.79 \\
17.86 & 13.11 & 10.93 & \infty & 12.25 & 11.43 \\
18.00 & 12.32 & 13.21 & 12.25 & \infty & 19.32 \\
12.75 & 11.64 & 11.79 & 11.43 & 19.32 & \infty
\end{bmatrix}$$

Figure 14. The crisp cost matrix.

For this case study, there are six nodes ($n=6$) and DM-TSP1 needs just six iterations to solve this problem. Figure 15 illustrates the result generated for each step.

5. Conclusion

In the literature, many research works were interested in solving the Travelling Salesmen Problem (TSP) which is widely studied in the Operation Research field. In this paper, a first resolution of the intuitionistic octagonal fuzzy TSP is proposed. It is carried out thanks to the application of the very recent heuristic namely Dhouib-Matrix-TSP1 (DM-TSP1) using the Sum function as a statistical metric. DM-TSP1 is a technique based on four simple steps. The empirical results on new generated case studies prove the performance as well as the rapidity of the proposed approach (just n iterations). In this current research work, comparison with other existing optimization methods is not fulfilled viewing that this is the inaugural paper that discusses the resolution of the TSP under the intuitionistic octagonal domain. In fact, there is no paper in the literature that considers the optimization of the intuitionistic octagonal TSP. Further research work will deal about applying DM-TSP1 on the octagonal neutrosophic TSP.

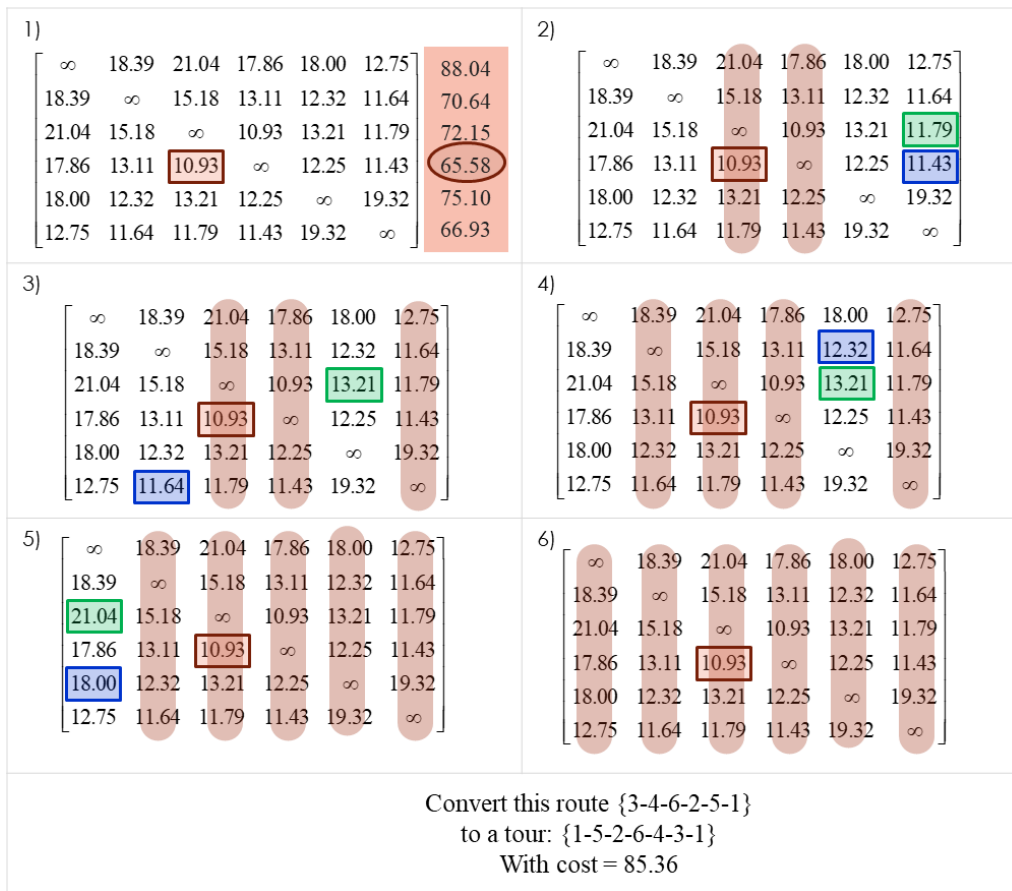


Figure 15. The step-by-step application of DM-TSP1 for the third case study.

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