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Survival Function Estimation based on Neutrosophic two-parameter XLindley distribution

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Abstract The probability distribution is of paramount importance in probability theory as it abounds in application across most disciplines comprising science. It is reported to be used preferentially for actuarial studies on insurance and finance, in the medical field, agriculture, demography and econometric analyses. The focus of the current research work is to introduce a novel extension termed as the neutrosophic two-parameter XLindley distribution (NTPXL). Many mathematical characteristics that model life survival have been developed and studied, involving survival and hazard functions, moment-generating functions and other tests of average, variance, and standard deviation, skewness and kurtosis. Monte Carlo method has been applied to investigate the effectiveness of the NTPXL distribution estimate. The results of the simulation conducted for this study show that the task of estimating with satisfactory accuracy is possible if the sample size is large enough. The actuality of premature infant staying time data has been used to explain the exact way through which the elaborated NTPXL distribution should be applied. On the application aspects, it has been shown in the subsequent sections that the NTPXL distribution is versatile because it can handle classical data as well as data that comprises uncertainties, ambiguity or imprecision.

Keywords Neutrosophic statistics, two-parameter XLindley distribution, survival analysis, premature infant

AMS 2010 subject classifications 62Exx, 03E72

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1. Introduction

Neutrosophy, philosophical and mathematical formation, was created by Florentin Smarandache [1]. As for the general themes, it refers to the interaction of opposites on the one hand and the issue of the study of indeterminacy on the other hand. Consequently, there appears a new set of approaches, neoclassical logic and set theory, which are the extensions of classical logic and set theory and aim at solving the problems of inconsistency, indeterminacy, and imperfect information.

The basis of neutrosophic statistics is that often in the data, there is information, which is vague and cannot be quantified in the classic sense and therefore cannot be properly processed in the framework of traditional statistical methods. Neutrosophic statistics is used as a way to handle and do more comprehensive analysis on such data. The use of fuzzy logic was expanded by [1] to create neutrosophy, which enables the depiction of uncertainty, ambiguity, and contradiction.

Traditional analysis often suggests that the deeper the data, the clearer it is, hence very often each of the observations gets a numerical value. However, as it has been observed, in most real life settings, information can be ambiguous or about which there is limited detail given. To get around these constraints, neosophic statistics offers ways of dealing with the unpredictable, scarce, and contradictory data [2, 3, 4].

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As already mentioned, neosophic statistics consider three measures that in some ways reflect the particularities of the evaluated propositions: truth membership, indeterminacy membership, and falsity membership. They all depict the extent of truth, openness, or falsehood that is correlated with a hypothesis or an observation. These degrees are represented in a manner similar to a fuzzy set by the membership functions [2, 3].

Neutrophic statistics are used in many different fields, including image processing, data mining, pattern recognition, and decision-making [4, 5, 6, 7]. It provides a flexible mathematical tool for the analysis and modeling of complex systems with a high level of imprecision and uncertainty.

The survival statistics are among the essential aspects of neutrosophic information that have to be examined. Basically, the idea of survival analysis, often termed as event-time analysis or time-to-event analysis, deals with the assessment of time to certain event of interest. It is commonly applied in social science, engineering, medical research and other fields where the time related results are issued. Most of the time when conducting research where the temporal order is not certain or where subjects may not have the same subsequent follow-ups survival analysis proves to be of great use. It could also be the case that occurrence of a particular event of interest; an event say of failure, a relapse or even a death, or any other event of interest [8].

Many statistical distributions are widely used in survival analysis to work on time-to-event data. This means that the features of the data and the assumptions made concerning the underlying survival process drives the choice of the distribution. These distributions are used for the assessment of time-to-event data in engineering, social sciences, and other medical disciplines. Different distributions may be chosen depending on the given characteristics of the data by the researchers as well as the hypotheses appropriate to the study. The literature review reveals that many articles address neutrosophic probability distribution [8, 20].

Applications for the two-parameter XLindley distribution can be found in many domains, including survival analysis. In this work, we extended the applications of the two-parameter XLindley distribution to include neutrosophical data in interval form with a degree of indeterminacy. Many qualities are investigated under the newly proposed distribution and their applications are described with the help of simulated and real data application.

2. Neutrosophic two-parameter XLindley distribution

Probability distributions help in the portrayal of uncertainty that is prevalent in the data set through depiction of the patterns of variation. In this regard distribution summaries, the observations into a mathematical form which contain a few unknown parameters and is the best possible understanding of the basic data generating mechanism. Survival time distribution which is the probability description of the behavior of length of life is to a certain extent depends on mode of succusses of the event under consideration. From the given data set, the selection of the right distribution depends with the extent of prior information regarding the physical characteristics of the process underlying the observed data [21, 22, 23].

The two-parameter Xlindley distribution (TPXL), which was proposed by Ibrahim, Shah and Ahsan-ul-Haq [24], is one of the survival time distributions. Let X be continuous random variable follows a XLindley distribution, then its probability density function (pdf) is [25]:

$$f(x;\gamma) = \frac{\gamma^2(2+\gamma+x)}{(1+\gamma)^2} e^{-\gamma x} \quad , \qquad \gamma > 0; x > 0. \tag{1}$$

The pdf and the cumulative distribution function (cdf) of the TPXL distribution are, respectively, as [24]:

$$f(x) = \frac{\gamma}{1+\eta}(\eta + \frac{\gamma(1+x)}{1+\gamma}) e^{-\gamma x} \quad , \qquad \gamma > 0, \ \ \eta > 0 \, ; x > 0. \tag{2}$$

$$F(x) = P(X \leq x) = 1 - \left(1 + \frac{x\gamma}{(\eta+1)(1+\gamma)}\right) e^{-\gamma x} \tag{3}$$

The concept of neutrosophic probability as a function $NP :\rightarrow [0,\ 1]^3$was originally presented by [2], where $V$is a neutrosophic sample space and defined the probability mapping to take the form

$NP(\Omega) = (ch(\Omega), ch(neut\ \Omega), ch(anti\ \Omega)) = (\psi_1, \psi_2, \psi_3)$ with$0 \leq \psi_1, \psi_2, \psi_3 \leq 1$ *and* $0 \leq \psi_1 + \psi_2 + \psi_3 \leq 3$. The term Θ represents the set of sample space, R represents the set of real numbers, and ξ denotes a sample space event, X_N and Y_N denote neutrosophic random variable.

Definition 1 Consider $X$is the real-valued crisp random variable, which has the following definition: $X : \Theta \rightarrow R$

where Θ is the event space and X_N neutrosophic random variable as follows:

$X_N : \Theta \rightarrow R(I)$ and $X_N = X + I$, where I represents indeterminacy.

Theorem 1 Let $X_N = X + I$ be the neutrosophic random variable and the CDF and pdf of $X_N$are [13], respectively

$F_{X_N}(x) = F_X(x - I)$, and $f_{X_N}(x) = f_X(x - I)$,

Theorem 2 Let $X_N = X + I$ be the neutrosophic random variable, then the expected value and variance can be derived as follows: $E(X_N) = E(X) + I$and $V(X_N) = V(X)$ [13].

By supposing the neutrosophic variable could be expressed as: $x_N = x_L + x_U I_N$where $I_N \in \{I_L, I_U\}$ and x_L and $x_U I_N$ denote the determined and indeterminate parts, respectively, the neutrosophic random variable $x_N \in \{x_L, x_U\}$ which follows the TPXL distribution has neutrosophic parameters as: $\gamma_N \in \{\gamma_L, \gamma_U\}$ and $\eta_N \in \{\eta_L, \eta_U\}$ where the letters L and U are the lower values and the upper values, respectively.

Then, the neutrosophic CDF and pdf of neutrosophic TPXL (NTPXL) distribution is given by Eq. (4) and Eq. (5), respectively

$$f(x_N) = \frac{\gamma_N}{1+\eta_N}(\eta_N + \frac{\gamma_N(1+x_N)}{1+\gamma_N})e^{-\gamma_N x_N}, \qquad \gamma_N > 0,\ \eta_N > 0; x_N > 0. \tag{4}$$

$$F(x_N) = 1 - \left(1 + \frac{x_N \gamma_N}{(\eta_N + 1)(1+\gamma_N)}\right) e^{-\gamma_N x_N} \tag{5}$$

Figures 1 and 2 show the NTPXL distribution for different values of its parameters. Relating to Eq. (4) and Eq. (5), the neutrosophic survival and hazard functions of the NTPXL distribution are defined in Eq. (6) and Eq. (7), respectively,

$$S(x_N) = \left(1 + \frac{x_N \gamma_N}{(\eta_N + 1)(1+\gamma_N)}\right) e^{-\gamma_N x_N}, \quad x_N > 0. \tag{6}$$

$$h(x_N) = \frac{\gamma_N(\eta_N(1+\gamma_N) + \gamma_N(1+x_N))}{((\eta_N+1)(1+\gamma_N) + x_N\gamma_N)}, \qquad x_N > 0. \tag{7}$$

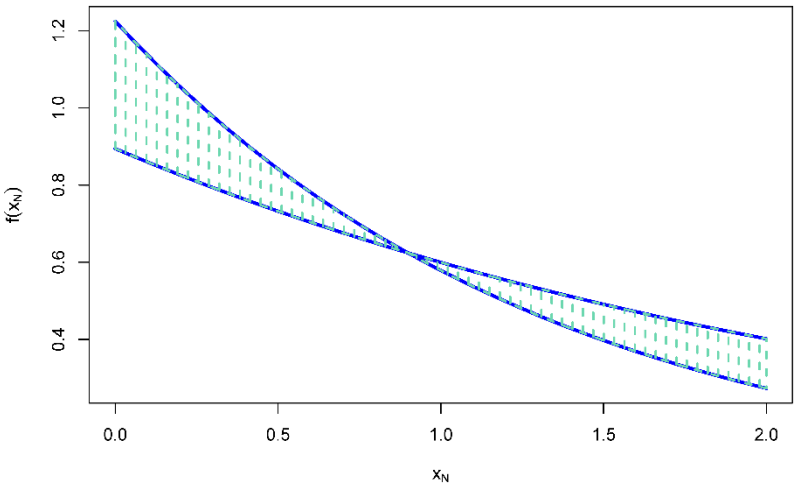

Figure 1. The pdf of NTPXL when $\gamma_N \in [0.4, 0.6]$ and $\beta_N \in [0.5, 0.7]$

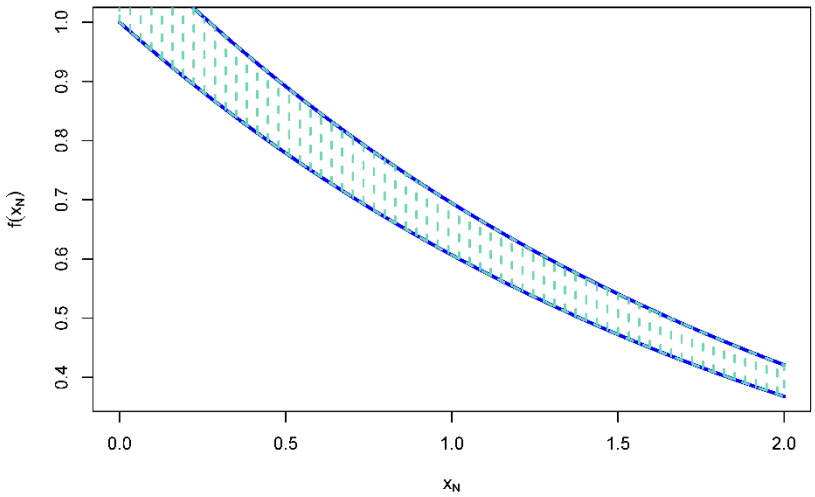

Figure 2. The pdf of NTPXL when $\gamma_N \in [0.3, 0.5]$ and $\beta_N \in [0.3, 0.7]$

3. Parameter Estimation of NTPXL Distribution

Five methods for estimating the NTPXL distribution parameters are described: (1) the maximum likelihood Method (MLE), (2) Anderson Darling method (AD), (3) Cramér-von Mises method (CVM), (4) weighted least-squares method (WLS), and (5) the maximum product spacing method (MPS).

3.1. MLE method

Assuming each of the random samples $x_1, x_2, \ldots, x_n$ follows NTPXL distribution, the log-likelihood function is given by

$$\ell(\gamma_N, \eta_N) = n \log \gamma_N + \sum_{i_N=1}^{n} \log\left(\eta_N + \frac{\gamma_N (1 + x_{i_N})}{1 + \gamma_N}\right) - \gamma_N \sum_{i=1}^{n} x_{i_N} - n \log(1 + \gamma_N). \tag{8}$$

The MLE of the parameters $\gamma_N$and η_N are the solutions of the following simultaneous equations:

$$\frac{\partial \ell}{\partial \gamma_N} = 0, \frac{\partial \ell}{\partial \eta_N} = 0$$

$$\frac{\partial \ell(\gamma_N, \eta_N)}{\partial \gamma_N} = \frac{n}{\gamma_N} + \sum_{i_N=1}^{n} \frac{(1 + x_{i_N})}{(1 + \gamma_N)^2 \left(\eta_N + \frac{\gamma_N (1 + x_{i_N})}{1 + \gamma_N}\right)} - \sum_{i_N=1}^{n} x_{i_N} \tag{9}$$

$$\frac{\partial \ell(\gamma_N, \eta_N)}{\partial \eta_N} = \sum_{i_N=1}^{n} \frac{1}{\left(\eta_N + \frac{\gamma_N (1 + x_{i_N})}{1 + \gamma_N}\right)} - \frac{n}{1 + \eta_N} = 0 \tag{10}$$

3.2. AD method

The AD estimates of the parameters $\gamma_N$and η_N are attained by minimizing the following equation with respect to the unknown parameters:

$$AD(\gamma_N, \eta_N) = -n - \sum_{i_N=1}^{n} \frac{(2i_N - 1)}{n} \left[\begin{array}{c} \log\left(F\left(X_{i:nN}; \gamma_N, \eta_N\right)\right) \\ + \log\left(1 - F\left(X_{i:nN}; \gamma_N, \eta_N\right)\right) \end{array} \right] \tag{11}$$

3.3. CVM method

The CVM estimates $\widehat{\gamma}_N$ and $\widehat{\eta}_N$ are derived by minimizing the following expression with respect to NTPXL parameters as:

$$CVM\left(\gamma_N,\eta_N\right)=\frac{1}{12n}+\sum_{i_N=1}^{n}\left(F\left(X_{i:nN};\gamma_N,\eta_N\right)-\frac{2i_N-1}{2n}\right)^2 \tag{12}$$

3.4. WLS method

The WLS estimates $\widehat{\gamma}_N$ and $\widehat{\eta}_N$ are derived by minimizing Eq. (13) with respect to NTPXL parameters as:

$$WLS\left(\gamma_N,\eta_N\right)=\sum_{i_N=1}^{n}\frac{(1+n)^2(2+n)}{i_N\left(n-i_N+1\right)}\left[F\left(X_{i:nN};\gamma_N,\eta_N\right)-\frac{i_N}{n+1}\right]^2. \tag{13}$$

3.5. MPS method

The estimation of NTPXL parameters using MPS method can be obtained by maximizing Eq. (14) as:

$$MPS\left(\gamma_N,\eta_N\right)=\frac{1}{n+1}\sum_{i_N=1}^{n+1}\log\left(F\left(X_{i:nN};\gamma_N,\eta_N\right)-F\left(X_{i-1:nN};\gamma_N,\eta_N\right)\right), \tag{14}$$

4. Simulation results

To investigate the efficacy of the NTPXL of the neutrosophic parameters $\gamma_N$and η_N of the suggested TPXL, simulation research is conducted in this section. A random sample of sizes, n = 30, 50, 150 and 250, is created from NTPXL using different amalgams of neutrosophic parameters for the simulation. Estimated MLEs, AD, CVM, WLS, and MPS of the neutrosophic parameters for 1000 replications at different sample sizes using simulated data. Thus, for all sample sizes, the neutrosophic mean square error (NMSE) and the neutrosophic average bias (NAB) are derived. The superior neutrosophic estimator's properties are evaluated using the estimations of NAB and NMSE [26, 27, 28].

Three cases of the NTPXL neutrosophic parameters are determined: Case 1: $\gamma_N\in[0.5,0.8]$, $\eta_N\in[0.6,1]$, Case 2: $\gamma_N\in[0.8,1]$, $\eta_N\in[1,1.2]$, and Case 3: $\gamma_N\in[1.2,1.6]$, $\eta_N\in[1.2,1.7]$. The results are given in Tables 1 – 6.

From Tables 1, 3, and 5, in terms of NAB, it is seen that, as predicted, the NAB for $\hat{\gamma}_N$ and $\hat{\eta}_N$ decrease as sample sizes rise. It can also be deduced from Tables 1, 3, and 5 that the NAB values of the five estimators are varying as expected. The NAB values of $\hat{\gamma}_N$ and $\hat{\eta}_N$ for MPS, WLS, and CVM are higher than MLE and AD estimators. For example, from Table 1 when n=30, the NAB values of $\hat{\gamma}_N$ for MPS, WLS, and CVM is higher than MLE estimator by 71.27%, 65.05%, and 55.37% for the lower bound of $\hat{\gamma}_N$. While NAB values were higher than MLE by 71.01%, 65.21%, and 55.15% for the upper bound of $\hat{\gamma}_N$.

Concerning the NMSE values, and for all sample sizes, the MLE estimator of $\hat{\gamma}_N$and $\hat{\eta}_N$ has the smallest values comparing with AD, CVM, WLS, and MPS estimators. Further, it is noticed from Tables 2, 4, and 6 that when the $\hat{\gamma}_N$ and $\hat{\eta}_N$ increase regardless the values of the n, the NMSE are decreasing.

5. Real Application

From our study, we have used premature infant staying time data that we gathered from Kirkuk hospital, Iraq for about six months to apply our proposed NTPXL distribution. The time corresponds to the number of days that the premature infant is alive after discharge from the hospital. The subject population in the study is 120 premature infants. However, premature infant times are not recorded accurately, the member countries need to develop clear and accurate time definitions. Therefore, defining the number of days from the birth of a premature infant with

Table 1. Average values of NAB for case 1

n		30	50	150	250
MLE	$\hat{\gamma}_N$	[0.1146,0.1161]	[0.1122,0.1137]	[0.1098,0.1113]	[0.1074,0.1089]
	$\hat{\eta}_N$	[0.1247,0.1273]	[0.1223,0.1249]	[0.1199,0.1225]	[0.1175,0.1201]
AD	$\hat{\gamma}_N$	[0.18547,0.1872]	[0.1833,0.1848]	[0.1809,0.1824]	[0.1785,0.1801]
	$\hat{\eta}_N$	[0.1958,0.1984]	[0.1934,0.196]	[0.191,0.1936]	[0.1886,0.1912]
CVM	$\hat{\gamma}_N$	[0.2568,0.2583]	[0.2544,0.2559]	[0.252,0.2535]	[0.2496,0.2511]
	$\hat{\eta}_N$	[0.2669,0.2695]	[0.2645,0.2671]	[0.2621,0.2647]	[0.2597,0.2623]
WLS	$\hat{\gamma}_N$	[0.3279,0.3294]	[0.3255,0.3271]	[0.3231,0.3246]	[0.3207,0.3222]
	$\hat{\eta}_N$	[0.338,0.3406]	[03356,0.3382]	[0.3332,0.3355]	[0.3308,0.3334]
MPS	$\hat{\gamma}_N$	[0.399,0.4005]	[0.3966,0.3981]	[0.3942,0.3957]	[0.3918,0.3933]
	$\hat{\eta}_N$	[0.4091,0.4117]	[0.4067,0.4093]	[0.4043,0.4069]	[0.4019,0.4045]

Table 2. Average values of NMSE for case 1

n		30	50	150	250
MLE	$\hat{\gamma}_N$	[0.3203,0.3218]	[0.3179,0.3194]	[0.3155,0.317]	[0.3131,0.3146]
	$\hat{\eta}_N$	[0.3304,0.333]	[0.328,0.3306]	[0.3256,0.3282]	[0.3232,0.3258]
AD	$\hat{\gamma}_N$	[0.3914,0.3929]	[0.389,0.3905]	[0.3866,0.3881]	[0.3842,0.3857]
	$\hat{\eta}_N$	[0.4015,0.4041]	[0.3991,0.4017]	[0.3967,0.3993]	[0.3943,0.3969]
CVM	$\hat{\gamma}_N$	[0.4625,0.464]	[0.4601,0.4616]	[0.4577,0.4592]	[0.4553,0.4568]
	$\hat{\eta}_N$	[0.4726,0.4752]	[0.4702,0.4728]	[0.4678,0.4704]	[0.4654,0.468]
WLS	$\hat{\gamma}_N$	[0.5336,0.5351]	[0.5312,0.5327]	[0.5288,0.5303]	[0.5264,0.5279]
	$\hat{\eta}_N$	[0.5437,0.5463]	[0.5413,0.5439]	[0.5389,0.5415]	[0.5365,0.5391]
MPS	$\hat{\gamma}_N$	[0.6047,0.6062]	[0.6023,0.6038]	[0.5999,0.6014]	[0.5975,0.599]
	$\hat{\eta}_N$	[0.6148,0.6174]	[0.6124,0.615]	[0.61,0.6126]	[0.6076,0.6102]

Table 3. Average values of NAB for case 2

n		30	50	150	250
MLE	$\hat{\gamma}_N$	[0.1099,0.1114]	[0.1075,0.109]	[0.1051,0.1066]	[0.1027,0.1042]
	$\hat{\eta}_N$	[0.12,0.1226]	[0.1176,0.1202]	[0.1152,0.1178]	[0.1128,0.1154]
AD	$\hat{\gamma}_N$	[0.181,0.1825]	[0.1786,0.1801]	[0.1762,0.1777]	[0.1738,0.1753]
	$\hat{\eta}_N$	[0.1911,0.1937]	[0.1887,0.1913]	[0.1863,0.1889]	[0.1839,0.1865]
CVM	$\hat{\gamma}_N$	[0.2521,0.2536]	[0.2497,0.2512]	[0.2473,0.2488]	[0.2449,0.2464]
	$\hat{\eta}_N$	[0.2622,0.2648]	[0.2598,0.2624]	[0.2574,0.26]	[0.255,0.2576]
WLS	$\hat{\gamma}_N$	[0.3232,0.3247]	[0.3208,0.3223]	[0.3184,0.3199]	[0.316,0.3175]
	$\hat{\eta}_N$	[0.3333,0.3359]	[0.3309,0.3335]	[0.3258,0.3311]	[0.3261,0.3287]
MPS	$\hat{\gamma}_N$	[0.3943,0.3958]	[0.3919,0.3934]	[0.3895,0.391]	[0.3871,0.3886]
	$\hat{\eta}_N$	[0.4044,0.407]	[0.402,0.4046]	[0.3996,0.4022]	[0.3972,0.3998]

alive discharge without having explicit information is always problematic supervising unpredictable, insufficient, and incongruent data. The data related to exploratory factor analysis is given in Table 7.

Table 4. Average values of NMSE for case 2

n		30	50	150	250
MLE	$\hat{\gamma}_N$	[0.2837,0.2852]	[0.2813,0.2828]	[0.2789,0.2804]	[0.2765,0.278]
	$\hat{\eta}_N$	[0.2938,0.2964]	[0.2914,0.294]	[0.289,0.2916]	[0.2866,0.2892]
AD	$\hat{\gamma}_N$	[0.3548,0.3563]	[0.3524,0.3539]	[0.35,0.3515]	[0.3476,0.3491]
	$\hat{\eta}_N$	[0.3649,0.3675]	[0.3625,0.3651]	[0.3601,0.3627]	[0.3577,0.3603]
CVM	$\hat{\gamma}_N$	[0.4259,0.4274]	[0.4235,0.425]	[0.4211,0.4226]	[0.4187,0.4202]
	$\hat{\eta}_N$	[0.436,0.4386]	[0.4336,0.4362]	[0.4312,0.4338]	[0.4288,0.4314]
WLS	$\hat{\gamma}_N$	[0.497,0.4985]	[0.4946,0.4961]	[0.4922,0.4937]	[0.4898,0.4913]
	$\hat{\eta}_N$	[0.5071,0.5097]	[0.5047,0.5073]	[0.5023,0.5049]	[0.4999,0.5025]
MPS	$\hat{\gamma}_N$	[0.5681,0.5696]	[0.5657,0.5672]	[0.5633,0.5648]	[0.5609,0.5624]
	$\hat{\eta}_N$	[0.5782,0.5808]	[0.5758,0.5784]	[0.5734,0.576]	[0.571,0.5736]

Table 5. Average values of NAB for case 3

n		30	50	150	250
MLE	$\hat{\gamma}_N$	[0.0788,0.0803]	[0.0764,0.0779]	[0.074,0.0755]	[0.0716,0.0731]
	$\hat{\eta}_N$	[0.0889,0.0915]	[0.0865,0.0891]	[0.0841,0.0867]	[0.0817,0.0843]
AD	$\hat{\gamma}_N$	[0.1499,0.1514]	[0.1475,0.149]	[0.1451,0.1466]	[0.1427,0.1442]
	$\hat{\eta}_N$	[0.16,0.1626]	[0.1576,0.1602]	[0.1552,0.1578]	[0.1528,0.1554]
CVM	$\hat{\gamma}_N$	[0.221,0.2225]	[0.2186,0.2201]	[0.2162,0.2177]	[0.2138,0.2153]
	$\hat{\eta}_N$	[0.2311,0.2337]	[0.2287,0.2313]	[0.2263,0.2289]	[0.2239,0.2265]
WLS	$\hat{\gamma}_N$	[0.2921,0.2936]	[0.2897,0.2912]	[0.2873,0.2888]	[0.2849,0.2864]
	$\hat{\eta}_N$	[0.3022,0.3048]	[0.2998,0.3024]	[0.2974,0.3]	[0.295,0.2976]
MPS	$\hat{\gamma}_N$	[0.3632,0.3647]	[0.3608,0.3623]	[0.3584,0.3599]	[0.356,0.3575]
	$\hat{\eta}_N$	[0.3733,0.3759]	[0.3709,0.3735]	[0.3685,0.3711]	[0.3661,0.3687]

Table 6. Average values of NMSE for case 3

n		30	50	150	250
MLE	$\hat{\gamma}_N$	[0.2616,0.2631]	[0.2592,0.2607]	[0.2568,0.2583]	[0.2544,0.2559]
	$\hat{\eta}_N$	[0.2717,0.2743]	[0.2693,0.2719]	[0.2669,0.2695]	[0.2645,0.2671]
AD	$\hat{\gamma}_N$	[0.3327,0.3342]	[0.3303,0.3318]	[0.3279,0.3294]	[0.3255,0.327]
	$\hat{\eta}_N$	[0.3428,0.3454]	[0.3404,0.343]	[0.338,0.3406]	[0.3356,0.3382]
CVM	$\hat{\gamma}_N$	[0.4038,0.4053]	[0.4014,0.4029]	[0.399,0.4005]	[0.3966,0.3981]
	$\hat{\eta}_N$	[0.4139,0.4165]	[0.4115,0.4141]	[0.4091,0.4117]	[0.4067,0.4093]
WLS	$\hat{\gamma}_N$	[0.4749,0.4764]	[0.4725,0.474]	[0.4701,0.4716]	[0.4677,0.4692]
	$\hat{\eta}_N$	[0.485,0.4876]	[0.4826,0.4852]	[0.4802,0.4828]	[0.4778,0.4804]
MPS	$\hat{\gamma}_N$	[0.546,0.5475]	[0.5436,0.5451]	[0.5412,0.5427]	[0.5388,0.5403]
	$\hat{\eta}_N$	[0.5561,0.5587]	[0.5537,0.5563]	[0.5513,0.5539]	[0.5489,0.5515]

An informal graphical technique has been utilized to show that the TPXL distribution is one of the plausible models for explaining the premature infant staying time data. Figure 3 displays a visual fit of the TPXL distribution. Further, the χ^2 test for the goodness of fit shows that the premature infant staying time data follows TPXL distribution with p-value=0.671. A descriptive assessment of the premature infant staying time data using NTPXL

Table 7. The staying time data

1.24,1.22, 1.23, 1.56, 1.54, 1.67, 1.49, [1.14,1.21], 1.11, 1.29, 2.03, 1.33, 1.65, 1.66, 2.81, 1.26, [1.85,2.01], 1.31,1.38, 1.13, 3.05, 1.48, 3.07, 1.27, 1.19, 1.31, [1.75,1.81], 1.62, 1.46, 1.81, [1.62,1.75], 1.62, 1.64, 1.29,1.19, 1.82, [1.66,1.74], 2.37,2.42, 1.84, 1.48,1.25, 1.48, 1.86, 2.22, [2.18,2.22], 1.96, 1.27, 1.14, 1.52, 1.64, 1.18, 2.37, 1.24, 2.23, 1.21, 1.72,1.90, 1.87, 1.74, 1.20, 2.03, 1.67, 2.03, 1.24, 1.11, 1.38, [1.33,1.41], 1.98, 2.28,1.21,2.21,1.65,1.45,1.73,1.87,1.57,1.44,2.46,1.14,1.23,1.77,1.81,1.19,1.51,1.70,1.44, 2.77,1.74,1.37,1.23,1.36,1.18,1.12,1.37, [1.83,1.91], 2.42, 1.46, 1.57,1.16,1.68,1.41, 1.47,2.45, [1.75,1.83], 1.80,1.24, [1.27,1.31],2.09, 2.01, 1.8, 1.34, 1.28,1.54, 1.56, 1.29,1.87, 1.12, 1.79,2.01.

is shown in Table 8. Table 8 makes it abundantly evident that uncertainties taken into account in the observed sample are the cause of discrepancies in a number of the critical numerical statistics of the failure times data. Further, it is more clearly shown from Table 8 that there are high varies among the estimation methods in estimating the NTPXL distribution parameters $\hat{\gamma}_N$and $\hat{\eta}_N$.

In terms of survival probability, Figure 4 displays the survival curve for the five estimation methods. It can be observed that the neutrosophic survival curve using MLE methods shows higher probability than the others. This suggests that the neutrosophic MLE is better than the other four methods. Based on this observation, Figure 5 depicts the margin of the survival function between lower and upper the premature infant staying time data.

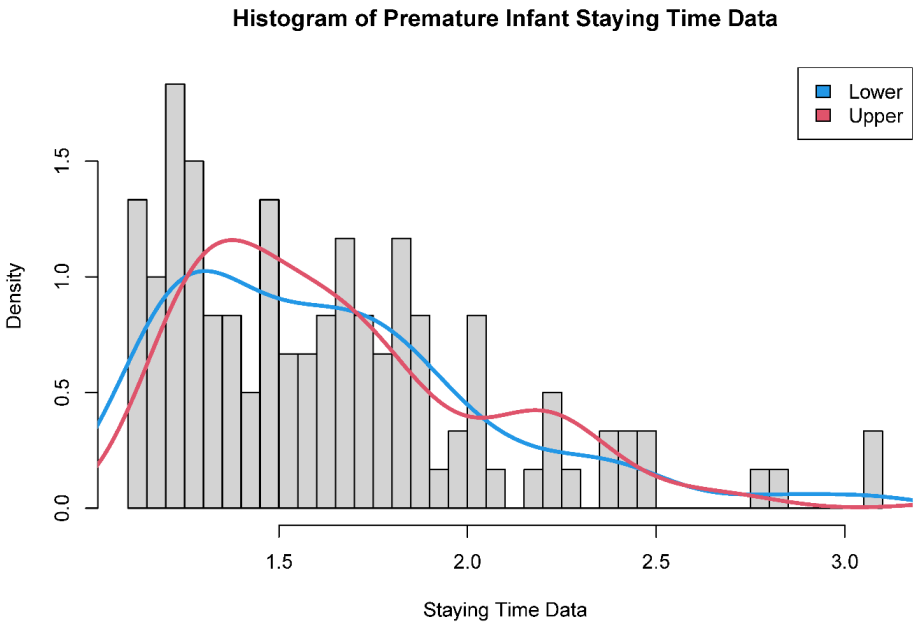

Figure 3. Fitting of TPXL distribution of staying time data

6. Conclusion

This paper presents an interesting extension known as the neutrosophic two-parameter Xlindley distribution. The concepts of neutrosophic calculus serve as the foundation for this new extension. The neutrosophic paradigm has been used to investigate a number of estimation methods. The study's numerical examples showed that NTPXL

Table 8. The estimated parameters of NTPXL distribution

Method	Estimated values	
	$\hat{\gamma}_N$	$\hat{\eta}_N$
MLE	[0.114,0.123]	[0.104,0.111]
AD	[0.128,0.134]	[0.113,0.117]
CVM	[0.133,0.141]	[0.120,0.129]
WLS	[0.144,0.152]	[0.131,0.135]
MPS	[0.151,0.160]	[0.134,0.138]

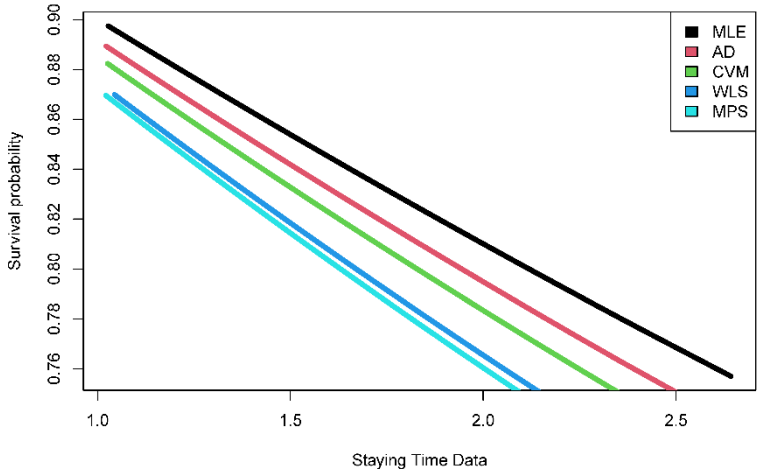

Figure 4. The survival curve plot for the NTPXL distribution under several estimation methods

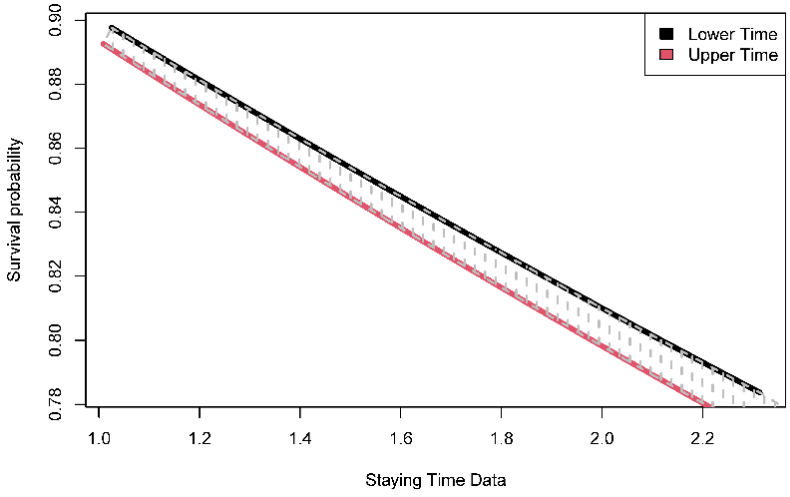

Figure 5. The survival curve plot for the NTPXL distribution under MLE method

distribution's theoretical conclusions are flexible and applicable to a wide range of data. The simulation study's findings suggest that a large sample size can yield accurate estimations. The premature infant staying time data have been employed to explicate the practical implementation of the suggested NTPXL distribution. The application

section has demonstrated that the NTPXL distribution is capable of analyzing both classical datasets and real-world data that contains uncertainties, ambiguity, or imprecision.

REFERENCES

1. Smarandache, Florentin, *A unifying field in Logics: Neutrosophic Logic.*, American Research Press, 1–141, 1999.
2. Salama, AA and Smarandache, Florentin and Eisa, Mohamed, *Introduction to image processing via neutrosophic techniques* , Infinite study, 2014.
3. Smarandache, Florentin, *Neutrosophic Statistics is an extension of Interval Statistics, while Plithogenic Statistics is the most general form of statistics (second version)* , Infinite Study, vol. 2, 2022.
4. Guan, Hongjun and Dai, Zongli and Guan, Shuang and Zhao, Aiwu, *A neutrosophic forecasting model for time series based on first-order state and information entropy of high-order fluctuation*, Entropy, vol. 21, no. 5, pp. 455, 2019.
5. Mao, Xinna and Guoxi, Zhao and Fallah, Mohammad and Edalatpanah, SA, *A Neutrosophic-Based Approach in Data Envelopment Analysis with Undesirable Outputs*, Mathematical problems in engineering, vol. 2020, no. 1, pp. 7626102, 2020.
6. Aslam, Muhammad, *A new failure-censored reliability test using neutrosophic statistical interval method*, International Journal of Fuzzy Systems, vol. 21, pp. 1214–1220, 2019.
7. Taş, Ferhat and Topal, Selçuk and Smarandache, Florentin, *Clustering neutrosophic data sets and neutrosophic valued metric spaces*, Symmetry, vol. 10, no. 10, pp. 430, 2018.
8. Bibani, Alan Adham and Algamal, Zakariya Yahya and others, *Survival function estimation for fuzzy Gompertz distribution with neutrosophic data*, Full Length Article, vol. 21, no. 3, pp. 137–37, 2023.
9. Ahsan-ul-Haq, MJNSS, *Neutrosophic Kumaraswamy distribution with engineering application*, Neutrosophic Sets and Systems, vol. 49, no. 1, pp. 269–276, 2021.
10. Albassam, Mohammed and Ahsan-ul-Haq, Muhammad and Aslam, Muhammad, *Weibull distribution under indeterminacy with applications*, AIMS Mathematics, vol. 8, no. 5, pp. 10745–10757, 2023.
11. Alsoboh, Abdullah and Amourah, Ala and Darus, Maslina and Sharefeen, Rami Issa Al, *Applications of neutrosophic q-Poisson distribution series for subclass of analytic functions and bi-univalent functions*, Mathematics, vol. 11, no. 4, pp. 868, 2023.
12. Duan, Wen-Qi and Khan, Zahid and Gulistan, Muhammad and Khurshid, Adnan, *Neutrosophic exponential distribution: modeling and applications for complex data analysis*, Complexity, vol. 2021, no. 112, pp. 5970613, 2021.
13. Granados, Carlos, *Some discrete neutrosophic distributions with neutrosophic parameters based on neutrosophic random variables*, Hacettepe Journal of Mathematics and Statistics, vol. 51, no. 5, pp. 1442–1457, 2022.
14. Hassan, Marwa KH and Aslam, Muhammad, *DUS-neutrosophic multivariate inverse Weibull distribution: properties and applications*, Complex & Intelligent Systems, vol. 9, no. 5, pp. 5679–5691, 2023.
15. Khan, Kashif Ali and Jamil, Faizan and Ali, Javaid and Khan, Ilyas and Ahmed, Nauman and Andualem, Mulugeta and Rafiq, Muhammad, *Analytical simulation of heat and mass transmission in casson fluid flow across a stretching surface*, Mathematical Problems in Engineering, vol. 2022, no. 1, pp. 5576194, 2022.
16. Khan, Zahid and Amin, Adnan and Khan, Sajjad Ahmad and Gulistan, Muhammad, *Statistical development of the neutrosophic lognormal model with application to environmental data*, Neutrosophic Sets and Systems, vol. 47, no. 1, pp. 1, 2021.
17. Rao, Gadde Srinivasa, *Neutrosophic log-logistic distribution model in complex alloy metal melting point applications*, International Journal of Computational Intelligence Systems, vol. 16, no. 1, pp. 48, 2023.
18. Shah, Faisal and Aslam, Muhammad and Khan, Zahid and Almazah, Mohammed MA and Alduais, Fuad S, *[Retracted] On Neutrosophic Extension of the Maxwell Model: Properties and Applications*, Journal of Function Spaces, vol. 2022, no. 1, pp. 4536260, 2022.
19. Khan, Zahid and Gulistan, Muhammad and Kausar, Nasreen and Park, Choonkil, *Neutrosophic Rayleigh model with some basic characteristics and engineering applications*, IEEE Access, vol. 9, pp. 71277–71283, 2021.
20. Aslam, Muhammad Ahtisham, *Neutrosophic Rayleigh distribution with some basic properties and application*, Neutrosophic Sets in Decision Analysis and Operations Research, pp. 119–128, 2020.
21. Eliwa, MS and Altun, Emrah and Alhussain, Ziyad Ali and Ahmed, Essam A and Salah, Mukhtar M and Ahmed, Hanan Haj and El-Morshedy, M, *A new one-parameter lifetime distribution and its regression model with applications*, Plos one, vol. 16, no. 2, pp. e0246969, 2021.
22. Ijaz, Muhammad and Mashwani, Wali Khan and Belhaouari, Samir Brahim, *A novel family of lifetime distribution with applications to real and simulated data*, Plos one, vol. 15, no. 10, pp. e0238746, 2020.
23. Kharazmi, Omid and Paghale, Fatemeh Jamali and Nik, Ali Saadati and Dey, Sanku and Alizadeh, Morad, *A new two-sided class of lifetime distributions: Applications to complete and right censored data*, Statistics, Optimization & Information Computing, vol. 11, no. 3, pp. 595–614, 2023.
24. Ibrahim, Muhammad and Shah, Muhammad Kashif Ali and Ahsan-ul-Haq, Muhammad, *New two-parameter XLindley distribution with statistical properties, simulation and applications on lifetime data*, International Journal of Modelling and Simulation, pp. 1–14, 2023.
25. Chouia, Sarra and Zeghdoudi, Halim, *The xlindley distribution: Properties and application*, Journal of Statistical Theory and Applications, vol. 20, no. 2, pp. 318–327, 2021.
26. Mustafa, Marwah Yahya and Algamal, Zakariya Yahya and others, *Neutrosophic inverse power Lindley distribution: A modeling and application for bladder cancer patients*, Full Length Article, vol. 21, no. 2, pp. 216–16, 2023.
27. Alanaz, Mazin M and Algamal, Zakariya Yahya, *Neutrosophic exponentiated inverse Rayleigh distribution: Properties and Applications*, International Journal of Neutrosophic Science, vol. 21, no. 4, pp. 36–43, 23.
28. Alanaz, Mazin M and Mustafa, Marwah Yahya and Algamal, Zakariya Yahya and others, *Neutrosophic Lindley distribution with application for alloying metal melting point*, Full Length Article, vol. 21, no. 4, pp. 65–5, 2023.