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Improved Non-Parametric Double Homogenously Weighted Moving Average Control Chart for Monitoring Changes in Process Location

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Abstract Parametric control charts' statistical performance often raises concerns when dealing with processes that lack a predefined probability distribution. In such cases, non-parametric control charts emerge as a viable alternative, and the adoption of ranked set sampling, with its ability to reduce process parameter variability and enhance control chart performance, proves to be advantageous over traditional simple random sampling techniques. The study focusses on enhancing the sensitivity and robustness of the Non-Parametric Double Homogenously Weighted Moving (NPDHWMA) for detection of shift in process mean and make it more reliable across diverse applications. This study aims to improve the non-parametric double homogeneously weighted moving average control chart, employing the Wilcoxon signed rank (WSR) test and leveraging the ranked set sampling method, referred to as non-parametric improved robust DHWMA(NPIDHWMA-WSR) in this paper. To evaluate the efficiency of the proposed control chart, a comparison was conducted against non-parametric double exponentially weighted moving average using signed rank test (NPRDEWMA-SR) and non-parametric double homogeneously weighted moving average (NPDHWMA) control charts. The comparative analysis highlights the superior performance of the proposed NPIDHWMA-WSR control chart, in scenarios involving minimal to moderate changes in process location, as evidenced by metrics such as average run length (ARL), standard deviation run length (SDRL), and median deviation run length (MDRL). The study presents a practical application, providing practitioners with tangible evidence of the chart's effectiveness in maintaining both product and process quality. Moreover, the NPIRDHWMA-WSR control chart identified an out-of-control (OOC) condition by the 18th sample, whereas the competing NPDHWMA-RSS control chart didn't signal OOC until the 22nd sample.

Keywords Average run length, Control chart, Non-parametric, Signed-rank test, Wilcoxon signed rank

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1. Introduction

Industrial processes are prone to changes due to various factors such as equipment malfunction, material variations, and operator errors. Timely detection of these changes is crucial to ensure the quality and efficiency of production. Industrial settings frequently employ control charts to keep track of process stability and spot changes from intended behavior. However, traditional control charts based on normality assumptions and constant process parameters may not be suitable for all scenarios. Non-parametric methods and robust statistical techniques provide more flexible and robust alternatives for keeping track of changes in industrial processes. The literature extensively promotes the utilization of ranked set sampling (RSS) in statistical process monitoring (SPM) due to its significant benefits in increasing the efficiency of related control charts and reducing variability. For example, when visually

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assessing different levels of contamination at hazardous waste sites based on soil stains, employing traditional nonparametric (NP) statistics to locate hazardous compounds and evaluate their ecological impact would be impractical and costly [1]. In response to such challenges, numerous researchers have developed various NP control charts specifically designed to address these unique scenarios.

[2] presented a Wilcoxon signed rank (WSR) test and an NP exponentially weighted moving average control chart to effectively track changes in the process's location. [3] developed a location monitoring technique using the NP Shewhart-SR (Non-Parametric Shewhart- Signed Ranks) control chart, whereas [4] represented process location on a control chart using an EWMA coupled with the sign (SN) test. [5, 6] introduced an NP EWMA-SR control chart to monitor modifications in the location of the process, and [7, 8] improved NP EWMA SN control charts using RSS methods.

Furthermore, [9, 10] improved the detection capabilities of location shifts by introducing NP double generally weighted moving average (GWMA) SN statistics-based control charts and GWMA-SR for process percentages, respectively. These advancements in NP control charting techniques, incorporating RSS and various statistical tests, have significantly enhanced the ability to effectively monitor and detect process location shifts in specific situations, such as the assessment of contamination levels at hazardous waste sites based on soil stains.

Moreover, [11, 12] conducted a comprehensive analysis of recent advancements in NP control charts, focusing on both univariate and multivariate data. [13] further improved upon this concept by developing auxiliary homogenous weighted moving average (AHWMA) control charts for supplementary information, aiming to enhance the efficiency of process placement. Additionally, [14] proposed the control chart for homogeneously weighted moving averages (HWMA), which proved valuable, precisely determining areas where processes occur. In the realm of process monitoring, [15, 16] introduced parametric and nonparametric control plots using ranked set sampling (RSS). These control charts were designed to effectively track the process mean. Expanding on this research, [17, 18] proposed the auxiliary double homogenous weighted moving average (ADHWMA) control chart as an enhancement to the AHWMA, providing more accurate process location monitoring. [19] developed a DHWMA-SR control chart using ranked set sampling (NPDHWMA-RSS) to track shifts in process locations, leveraging the non-parametric DHWMA control chart combined with signed ranks (SR) and RSS methodologies. For the account of parametric and NP HWMA-type charts, readers are referred to the review article by [20].

[21] sensitized multivariate homogeneously weighted moving average (HWMA) charts, specifically designed for Phase II monitoring of a multivariate process mean, aim to improve the detection of small shifts in the process mean compared to traditional methods. NPDHWMA-RSS has shown effectiveness in monitoring process control, but it has some limitations that need improvement such as sensitivity to small shifts, robustness, and computational efficiency. According to the literature review, NPDHWMA-RSS has not been improved upon to this point. The motivation for the study is to enhance the sensitivity and robustness of NPDHWMA-RSS, making it more reliable across diverse applications and distribution. Extending its use to various non-parametric scenarios and improving computational efficiency for real-time monitoring. Thus, the study focuses on sensitivity to small shifts; robustness across different distributions and computational efficiency of the NPDHWMA-RSS. Because industries need precise control charts for quality and efficiency, enhancing NPDHWMA-RSS will in no doubt benefit the field of statistical process control, since better control charts improve process monitoring, reducing waste, and boosting productivity. Thus, there is a research gap in this area that needs to be investigated. Consequently, this study focus is on a non-parametric improved robust DHWMA (NPIRDHWMA-WSR) control chart for symmetric and continuous distribution under RSS for tracking shifts in process location.

In this study, we propose a nonparametric double homogenously weighted moving average, using a Wilcoxon signed-rank (NPIDHWMA-WSR) control chart. By avoiding distributional assumptions and incorporating robust estimators, the NPIDHWMA-WSR control chart can handle a wider range of process changes, including shifts in location, scale, and distribution shape.

To assess the effectiveness of the proposed control chart, simulation studies are conducted under various scenarios. Furthermore, a real-world example is provided to demonstrate the practicality and the efficiency of the proposed control chart in monitoring changes in an industrial process. The Average Run length (ARL), Standard Deviation Run Length (SDRL), and Median Deviation Run Length (MDRL) were used to compare the performance of the proposed control chart with existing competing charts such as the non-parametric double exponentially weighted moving average, using Wilcoxon Signed-Rank (NPDEWMA-WSR), Non-Parametric Double Homogenous Weighted Moving Average, using Wilcoxon Signed-Rank (NPDHWMA-WSR), and double homogenous weighted moving average (DHWMA).

In summary, the study improved the robustness of the Non-Parametric Double Homogenous Weighted Moving Average control chart by using the Wilcoxon signed-rank test and the ranked set sampling, incorporates robust estimators to track various types of process changes (location, scale, and distribution shape), sample sizes, and shift levels, and compared the performance of the proposed control chart with competing control charts such as: Non-Parametric Double Exponentially Weighted Moving Average with Wilcoxon Signed-Rank (NPRDEWMA-SR); Non-Parametric Double Homogenous Weighted Moving Average (NPDHWMA) and Double Homogenous Weighted Moving Average (DHWMA), using various performance metrics such as Average Run Length (ARL), Standard Deviation Run Length (SDRL), and Median Deviation Run Length (MDRL).

The study highlights the practical necessity of precise control charts in industries, which can lead to better process monitoring, reduced waste, and increased productivity. The rest of the article is structured as follows. Section 2 describes the proposed and competing control charts, the performance measures and the simulation method. Section 3 presents the result. Section 4 discusses the result, while the article is summarized in Section 5.

2. Materials and Methods

2.1. Proposed and competing control charts

Various researchers utilized the Wilcoxon Signed-Rank Statistic based on RSS to track changes in the process location. This formula is employed in the RSS-based competing control charts and the proposed control chart statistic is:

$$\mathrm{WSR}_t = \sum_{j=1}^{n}\sum_{h=1}^{m} \mathrm{sign}(X_{tj(h)} - \theta_0) R^{+}_{tj(h)} \tag{1}$$

where, $\theta_0 =$ Process Median, $t =$ number of samples, $j =$ number of observations and $h =$ number of cycles.

According to [22], the statistic given in equation (1) has a mean and variance of $\mathbb{E}(\mathrm{WSR}_t) = 0$ and $\mathrm{Var}(\mathrm{WSR}_t) = -r(2r+1)(r+1)/6 \cdot \omega_0^2$, respectively. An essential quantity used to increase the sensitivity of the suggested chart is ω_0^2 or $-\xi_0^2 = 0.352$ which is obtained from [1] and corresponds to $n = 10$. In practical terms, using $-\xi_0^2 = 0.352$ allows quality control practitioners to establish control limits that are better aligned with the process's actual behavior. This enables them to detect process variations more accurately and make timely interventions when necessary to maintain product quality and customer satisfaction.

2.2. Non-Parametric Double Homogenous Weighted Moving Average with Ranked Set Sampling (NPDHWMA_RSS) Control Chart

The Non-Parametric Double Homogenous Weighted Moving Average with Ranked Set Sampling control chart statistic, according to [1], is given by

$$\mathrm{NPDH}_t = \lambda^2 \mathrm{WSR}_t + (1-\lambda^2)\overline{\mathrm{WSR}}_{t-1} \tag{2}$$

When $t = 0$, the value of NPDH $= 0$.

The NPDHWMA control chart's control limits are:

When $t = 1$

$$\left.\begin{aligned}
\mathrm{UCL}_{(\mathrm{NPDHWMA}_{\mathrm{RSS}})_t} &= \mu_0 + L\sqrt{\lambda^4\left(\frac{r(2r+1)(r+1)}{6}\right)\omega_0^2} \\
\mathrm{CL}_{(\mathrm{NPDHWMA}_{\mathrm{RSS}})_t} &= \mu_0 \\
\mathrm{LCL}_{(\mathrm{NPDHWMA}_{\mathrm{RSS}})_t} &= \mu_0 - L\sqrt{\lambda^4\left(\frac{r(2r+1)(r+1)}{6}\right)\omega_0^2}
\end{aligned}\right\} \tag{3}$$

When $t > 1$

$$\left.\begin{aligned} \mathrm{UCL}_{(\mathrm{NPDHWMA_{RSS}})_t} &= \mu_0 + L\sqrt{\lambda^4 + \left(\frac{(1-\lambda)^2(1+\lambda)^2}{t-1}\right)\left(\frac{r(r+1)(2r+1)}{6}\right)\omega_0^2} \\ \mathrm{CL}_{(\mathrm{NPDHWMA_{RSS}})_t} &= \mu_0 \\ \mathrm{LCL}_{(\mathrm{NPDHWMA_{RSS}})_t} &= \mu_0 - L\sqrt{\lambda^4 + \left(\frac{(1-\lambda)^2(1+\lambda)^2}{t-1}\right)\left(\frac{r(r+1)(2r+1)}{6}\right)\omega_0^2} \end{aligned}\right\} \tag{4}$$

A decision of out-of-control (OOC) is taken if

$$\mathrm{NPDH} < \mathrm{LCL}_{(\mathrm{NPDHWMA_{Bgss}})_t} \;\text{ or }\; \mathrm{NPDH} > \mathrm{UCL}_{(\mathrm{NPDHWMA_{Bgss}})_t}.$$

2.3. *Control Chart for Control Chart for Non-Parametric Double Exponentially Weighted Moving Average with signed ranks (NPRDEWMA-SR)*

Non-Parametric Double Exponentially Weighted Moving Average with signed ranks control chart utilizes the non-parametric Wilcoxon Rank tests that were signed to keep track of process shifts [14]. It incorporates a double exponentially (DE) weighted moving average approach to assign appropriate weights to historical observations. By leveraging the Wilcoxon Signed Rank test, this chart offers robustness against non-normality and provides effective detection of process changes. It performs better in terms of process location shift detection than the NPREWMA-SR control chart [14]. The chart statistics are given as follows:

$$E_{(WSR)t} = \lambda WSR_t + (1-\lambda)E_{(WSR)t-1} \tag{5}$$

$$DE_{(WSR)t} = \lambda E_{(WSR)t} + (1-\lambda)DE_{(WSR)t-1} \tag{6}$$

Where $\lambda \in (0, 1]$ is a smoothing constant.
The following equations were used to build the NPRDEWMA-SR control chart's control limits:

$$\left.\begin{aligned} \mathrm{UCL}_{(\text{NPRDEWMA-SR})t} &= +\mathrm{L}\sqrt{\left(\frac{r(2r+1)(r+1)}{6}\omega_0^2\right)\lambda^4\frac{1+\lambda^2-(t^2+2t+1)\lambda^{2t}+(2t^2+2t-1)\lambda^{2t+2}-t^2}{(1-\lambda^2)^3}} \\ \mathrm{CL}_{(\mathrm{NPDHWMA_{RSS}})t} &= \mu_0 \\ \mathrm{LCL}_{(\text{NPRDEWMA-SR})t} &= -\mathrm{L}\sqrt{\left(\frac{r(2r+1)(r+1)}{6}\omega_0^2\right)\lambda^4\frac{1+\lambda^2-(t^2+2t+1)\lambda^{2t}+(2t^2+2t-1)\lambda^{2t+2}-t^2}{(1-\lambda^2)^3}} \end{aligned}\right\} \tag{7}$$

2.4. *Proposed Improved Nonparametric Double Homogenously Weighted Moving Average, using a Wilcoxon signed-rank (NPIRDHWMA-WSR) Control Chart*

The statistical parameters governing the construction of the NPIRDEWMA-WSR control chart are delineated as follows:

$$\left.\begin{aligned} \mathrm{NPRDH}_i &= \lambda^2\mathrm{WSR}_t + (1-\lambda^2)\overline{\mathrm{WRS}}_{t-1} \\ \mathrm{NPRDH}_0 &= 0 \end{aligned}\right\} \tag{8}$$

$$E(\mathrm{WSR}_r) = 0, \quad \text{and}$$

$$\mathrm{Var}(\mathrm{WSR}_r) = \begin{cases} \lambda^5\left(\frac{r(2r+1)(r+1)}{6}\right)\xi_0^2, & t = 1 \\ \lambda^5 + \left(\frac{(1-\lambda)^2(1+\lambda)^2}{t-1}\right)\left(\frac{r(r+1)(2r+1)}{6}\right)\xi_0^2, & t > 1 \end{cases} \tag{9}$$

Where $\lambda \in [0, 1]$ is a smoothing constant.

To increase the sensitivity of the proposed chart, we increase the power of the smoothing constant. Thus, the introduction of λ^5 (smoothing constant) to the variance of in the existing model led to the robustness of the sensitivity of the proposed chart. Building on the NPDHWMA-RSS chart's foundation proposed by [19], the NPIRDHWMA-WSR chart introduces refined control limit calculations that enhance the chart's sensitivity to process shifts. Using the values of $E(WSR_t)$ and $Var(WSR_t)$, the following are the control limits for the proposed chart:

When $t = 1$

$$\left.\begin{aligned} \text{UCL}_{(\text{NPIRDHWMA-WSR})_t} &= \mu_0 + L\sqrt{\lambda^5\left(\frac{r(r+1)(2r+1)}{6}\right)\xi_0^2} \\ \text{CL}_{(\text{NPDHWMA}_{\text{RSS}})_t} &= \mu_0 \\ \text{LCL}_{(\text{NPIRDHWMA-WSR})_t} &= \mu_0 - L\sqrt{\lambda^5\left(\frac{r(r+1)(2r+1)}{6}\right)\xi_0^2} \end{aligned}\right\} \tag{10}$$

When $t > 1$

$$\left.\begin{aligned} \text{UCL}_{(\text{NPIRDHWMA-WSR})_t} &= \mu_0 + L\sqrt{\lambda^5 + \left(\frac{(1-\lambda)^2(1+\lambda)^2}{t-1}\right)\left(\frac{r(r+1)(2r+1)}{6}\right)\xi_0^2} \\ \text{CL}_{(\text{NPDHWMA}_{\text{RSS}})_t} &= \mu_0 \\ \text{LCL}_{(\text{NPIRDHWMA-WSR})_t} &= \mu_0 - L\sqrt{\lambda^5 + \left(\frac{(1-\lambda)^2(1+\lambda)^2}{t-1}\right)\left(\frac{r(r+1)(2r+1)}{6}\right)\xi_0^2} \end{aligned}\right\} \tag{11}$$

If $\text{NPIRDH}_t > \text{UCL}_{(\text{NPIRDHWMA-WSR})_t}$ or $\text{NPIRDH}_t < \text{LCL}_{(\text{NPIRDHWMA-WSR})_t}$, the underpinning process is OOC, otherwise, it is IC.

2.5. *Performance Assessment of proposed control chart*

This section explores the performance metrics comprising the In-Control (IC) and Out-of-Control (OOC) assessments of the effectiveness of the NPIDHWMA-WSR control chart in tracking shifts in process location. The assessment of the control chart's effectiveness is measured by the Average Run Length (ARL). The ARL signifies the anticipated amount of sample points necessary before the control chart signal an Out-of-Control (OOC) point. The In-Control ARL is denoted as ARL0 while the Out-of-Control ARL is denoted as ARL1.

In instances where a process is in a stable, in-control state, ARL_0 assumes paramount importance as it necessitates substantial values to mitigate undue false alarms. Conversely, the ARL_1 parameter should exhibit brevity to ensure swift detection of shifts. For optimal control chart performance, the ARL_1 value should be comparatively lower than that of other control charts while maintaining a fixed ARL0 value. In this study, ARL0 is set at 500, with a sample size (n) of 10. The sample size (n) employed for both Median Run Length and Standard Run Length control charts is set at 10. This combination of a fixed ARL_0 value and a sample size of 10 ensure-that the control charts are capable of quickly detecting shifts in the process mean or process variability while also maintaining a level of stability as indicated by the ARL_0 value.

2.6. *Simulation Study*

A series of Monte Carlo simulations were performed to determine the Run Length (RL) pattern obtained from 10,000 iterations conducted within the R programming environment. Various settings were used to investigate how the proposed NPIRDEWMA-WSR control chart performed $\lambda = (0.05, 0.10, 0.25, 0.50)$ and $\delta = (0.25, 0.05, 0.075, 0.10, 0.25, 0.50, 0.75, 1.00, 1.50, 2.00, 2.50, 3.00, 5.00)$ are used. The ensuing algorithm details the simulation process using the following steps:

Step 1: *Create samples from the chosen distributions, a finite loop is started.*

Step 2: *Specify process parameters: λ and L are given.*

Step 3: *From the distribution relevant to the investigation, a sample is taken.*

Step 4: *The statistic $NPIRDH_t$ is calculated using Equation (8).*

Step 5: *Determine the $LCL_{(NPIRDHWMA\text{-}WSR)_t}$ and $UCL_{(NPIRDHWMA\text{-}WSR)_t}$ using Equations (10) and (11) respectively.*

Step 6: *Determine the Run Length (RL).*

Step 7: *Steps (2) to (5) are iteratively executed 10,000 times and the accumulated RLs are recorded.*

For the computation of ARL_1 values, a perturbed sample is drawn based on the assumed distribution, and steps (2) through (6) are reiterated to assess the modified performance characteristics under such circumstances.

3. Results

The key elements explored in the simulation study are the robustness, behavior under circumstances of in-control (IC), and reaction to out-of-control (OOC) scenarios induced by shifts in process location of the proposed NPIRDHWMA-WSR control chart, Table 1 presents, the Run Length (RL) features when there are location shifts. Assessment is conducted across two categories of both normal and non-normal continuous symmetric distributions. The distribution under scrutiny encompasses standard normal distribution (N(0,1)) , the student's t distribution designated as t(4), and the contaminated normal (CN) distribution represented by a composite of N(0, σ_0^2) and N(0,σ_1^2) distributions. Notably, all distributions are re-parameterized using a unit variance and zero mean/median to allow for meaningful comparison.

The In-Control (IC) run length properties of all symmetric continuous distributions considered consistently retain their stability. To establish a meaningful context for comparison, identical parameters are employed. The Average Run Length (ARL) metrics are aptly deployed to facilitate a comparative analysis between the proposed NPIRDHWMA-WSR control chart and its competitive counterparts.

4. Discussion of Results

In this section, we discuss the results obtained in three sections. The first section discusses the performance of the proposed control chart with the Non parametric double exponential weighted moving average sign rank (NPDEWMA-SR) and the second section dwelt on the comparison of the proposed control chart with the NPDHWMA-RSS control chart. Finally, a summary of the result of the real-life application was also presented.

4.1. Comparison of Proposed Control Chart with NPDEWMA-SR Control Chart

The NPIRDHWMA-WSR controls graph was extensively assessed across various parameter combinations (λ and L) and a wide range of significance levels, all assuming a normal distribution. In specific scenarios, such as when λ = 0.05 and L = 1.064, the NPIRDHWMA-WSR chart demonstrated an Average Run Length (ARL) of approximately 234.20 at a significance level of 0.05. This indicates that, on average, it detected a process shift after observing 234.20 data points. Simultaneously, the SDRL, or Standard Deviation of Run Length, was around 121.09, suggesting a moderate level of variability in detection times and this result suggests that the control scheme with parameters (0.05, 1.064) has relatively high ARL values, meaning it takes a considerable number of observations, on average, to detect a change. However, once a change is detected, it tends to have consistent detection performance, indicated by the low SDRL values. Additionally, the MDRL values suggest that the scheme generally detects changes relatively quickly, though there are instances where it may have a longer time to detection. A value of 1.00 indicates that the control scheme signals a change immediately upon detecting it. (see Table 1)

Table 1. The proposed NPIRDHWMA-WSR control chart's RL characteristics for various distributions with nominal ARL_0 = 500 and n = 10.

(λ, L)							δ								
	Distrn.	Metrics	0.025	0.05	0.075	0.1	0.25	0.5	0.75	1	1.5	2	2.5	3	5
0.05, 1.064	Normal	ARL	234.23	212.32	200.36	184.48	113.23	46.57	16.41	5.07	1.12	1	1	1	1
		MDRL	287	270	255	240	170	1	1	1	1	1	1	1	1
		SDRL	121.73	120.07	113.37	110.16	88.26	56.01	30.86	14.47	2.04	0	0	0	0
	CN	ARL	15.88	14.18	12.44	11.57	6.18	2.34	1.32	1.05	1	1	1	1	1
		MDRL	1	1	1	1	1	1	1	1	1	1	1	1	1
		SDRL	32.77	30.66	28.46	27.05	18.13	8.41	3.75	1.29	0	0	0	0	0
	t(4)	ARL	83.18	78.16	75.06	70.85	48.31	24.82	10.27	4.14	1.11	1	1	1	1
		MDRL	132	127	123	118	1	1	1	1	1	1	1	1	1
		SDRL	71.61	69.83	68.05	66.15	55.04	37.89	22.65	12.23	1.89	0.27	0	0	0
0.1, 1.306	Normal	ARL	158.76	145.38	134.03	123.15	76.09	33.5	13.16	4.67	1.15	1	1	1	1
		MDRL	169	157	146	135	91	51	1	1	1	1	1	1	1
		SDRL	55.28	53.67	50.69	48.59	40.03	28.2	17.51	9.14	1.55	0.12	0	0	0
	CN	ARL	15.15	13.42	12.55	11.55	6.69	2.52	1.37	1.07	1	1	1	1	1
		MDRL	1	1	1	1	1	1	1	1	1	1	1	1	1
		SDRL	19.92	18.82	18.04	17.22	12.43	6.06	2.77	1.12	0	0	0	0	0
	t(4)	ARL	52.41	49.29	47.44	45.09	32.64	17.64	8.43	3.61	1.13	1.01	1	1	1
		MDRL	69	66	64	62	49	1	1	1	1	1	1	1	1
		SDRL	34.21	33.67	32.73	32.04	27.55	20.19	13.24	7.49	1.44	0.25	0	0	0
0.25, 2.113	Normal	ARL	180.76	163.29	147.19	132.98	72.94	31.25	15.03	7.19	1.56	1.01	1	1	1
		MDRL	177	160	143	130	71	32	18	10	1	1	1	1	1
		SDRL	53.02	47.91	43.55	39.67	21.96	10.88	8.21	6.11	1.88	0.22	0	0	0
	CN	ARL	19.79	18.9	18.04	17.09	12.14	6.26	2.92	1.51	1.01	1	1	1	1
		MDRL	21	21	20	19	15	1	1	1	1	1	1	1	1
		SDRL	7.46	7.43	7.37	7.31	7.1	5.79	3.74	1.9	0.23	0	0	0	0
	t(4)	ARL	46.45	43.79	41.33	39.3	29.27	17.68	10.21	5.57	1.53	1.02	1	1	1
		MDRL	46	44	41	39	30	20	13	1	1	1	1	1	1
		SDRL	16.02	15.26	14.64	14.1	11.3	8.8	7.21	5.38	1.85	0.32	0	0	0
0.5, 2.376	Normal	ARL	416.53	381.34	348.71	306.68	102.76	26.42	10.94	5.59	1.69	1.03	1	1	1
		MDRL	500	492	386	297	88	24	10	6	1	1	1	1	1
		SDRL	131.82	148.76	154.26	155.01	62.83	12.71	4.87	2.87	1.27	0.25	0.02	0	0
	CN	ARL	15.28	14.35	13.32	12.67	9	5.55	3.36	2.01	1.06	1	1	1	1
		MDRL	14	14	13	12	9	6	4	1	1	1	1	1	1
		SDRL	5.53	5.19	4.68	4.31	3.08	2.35	2.04	1.5	0.37	0.05	0	0	0
	t(4)	ARL	42.92	40.67	37.5	35.18	23.4	12.82	7.53	4.57	1.7	1.05	1	1	1
		MDRL	38	37	34	32	21	12	8	5	1	1	1	1	1
		SDRL	22.76	21.23	19.04	18.17	11.32	5.76	3.57	2.58	1.27	0.32	0.06	0	0

Table 2. RL features of the NPRDEWMA-SR control chart for various distributions with nominal ARL_0 = 500 and n = 10.

(λ, L)							δ								
	Distrn.	Metrics	0.025	0.05	0.075	0.1	0.25	0.5	0.75	1	1.5	2	2.5	3	5
0.05, 2.975	Normal	ARL	435.33	429.54	419.76	414.77	364.27	211.05	5.06	1.74	1.02	1	1	1	1
		SDRL	167.61	173.78	183.32	187.81	222.07	244.49	9.95	2.24	0.26	0.03	0	0	0
	CN	ARL	10.18	6.5	4.75	3.87	2.29	1.25	1.05	1	1	1	1	1	1
		SDRL	48.01	26.46	10.09	6.18	3.53	1.28	0.47	0.12	0	0	0	0	0
	t(4)	ARL	314.23	314.97	309.88	300.35	197.54	11.41	2.87	1.52	1.02	1	1	1	1
		SDRL	241.09	240.86	242.06	244.06	240.99	48	4	1.76	0.24	0	0	0	0
	t(8)	ARL	377.75	373.75	367.87	355.69	303.24	72	3.63	1.65	1.02	1	1	1	1
		SDRL	214.64	216.95	220.19	226.26	243.74	165.52	5.07	2.05	0.26	0	0	0	0

Table 3. RL characteristics of the NPDHWMARSS control chart for various distributions with nominal ARL_0 = 500 and n = 10.

(λ, L)							δ								
	Distrn.	Metrics	0.025	0.05	0.075	0.1	0.25	0.5	0.75	1	1.5	2	2.5	3	5
0.05, 1.064	Normal	ARL	295.08	277.9	262.42	247.26	178.97	105.84	60.761	28.927	3.629	1.066	1	1	1
		SDRL	25.56	27.339	24.625	26.486	24.77	31.121	31.837	27.564	9.075	1.297	0	0	0
0.1, 1.306	Normal	ARL	176.155	162.942	151.094	140.642	95.584	55.072	32.787	17.773	3.167	1.092	1	1,00	1
		SDRL	26.316	23.468	21.541	19.448	13.506	12.057	13.005	12.501	5.459	1.048	0	0	0
0.25, 2.113	Normal	ARL	276.989	239.33	206.466	180.93	86.132	35.052	19.149	11.932	3.862	1.223	1.003	1	1
		SDRL	96.794	82.95	70.371	62.108	26.364	8.266	4.288	3.803	3.371	1.04	0.107	0	0
0.5, 2.376	Normal	ARL	496.814	494.429	490.99	485.375	349.09	47.705	14.651	7.127	2.554	1.17	1.002	1	1
		SDRL	30.397	39.345	50.875	64.17	158.66	28.316	6.559	2.658	1.535	0.57	0.057	0	0

What is noteworthy is that the NPIRDHWMA-WSR chart displayed consistency in performance across different parameter combinations. Unlike its competitor, it exhibited minimal variation in both ARL and SDRL, regardless of the chosen parameter values.

In contrast, the NPRDEWMA-SR control chart, evaluated under similar conditions (λ = 0.05, L = 2.975), achieved an ARL of approximately 435.33 and an SDRL of around 167.61. This suggests that, in this

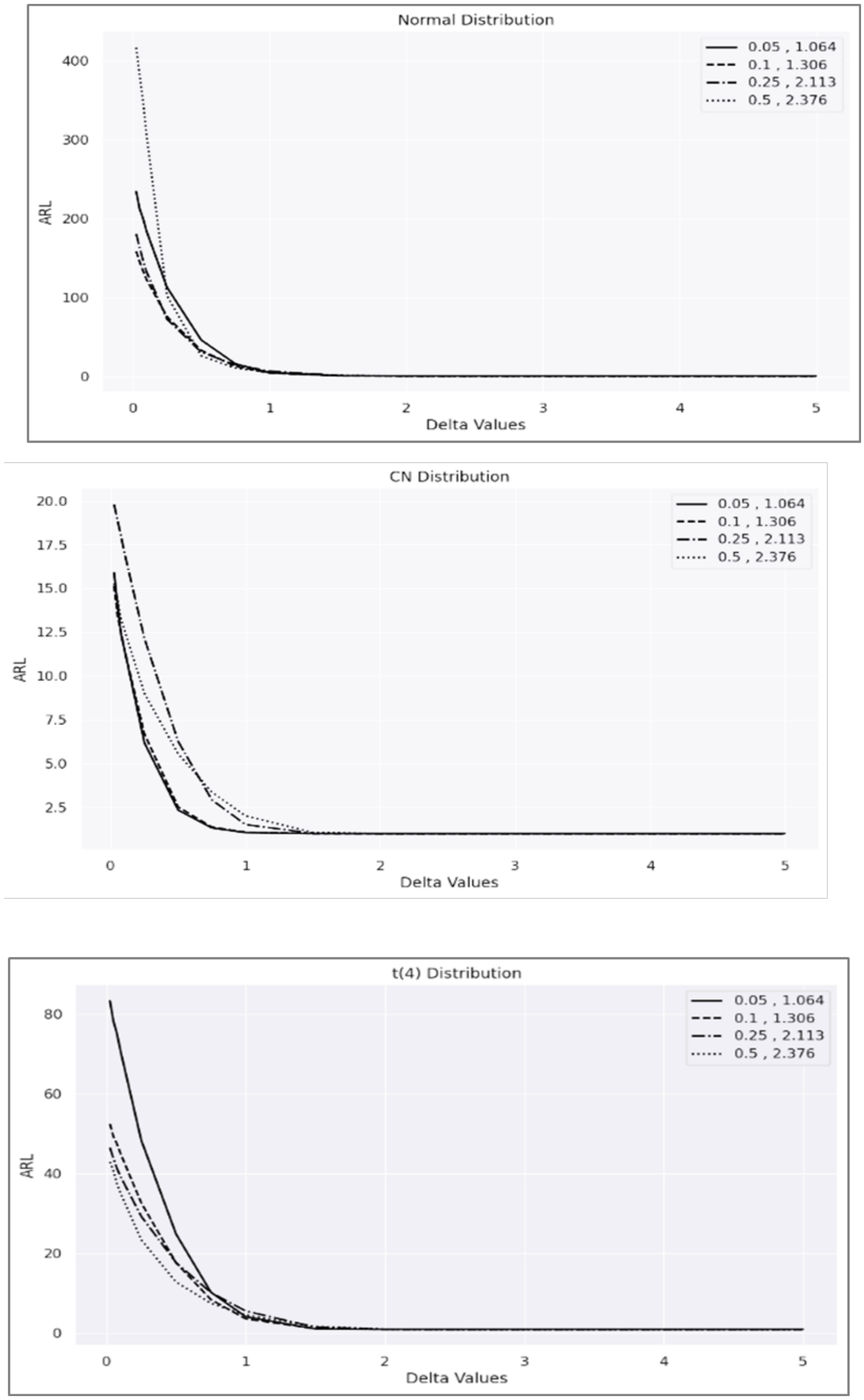

Figure 1. The proposed NPIRDHWMA-WSR control chart's ARL features for various values of, L when n = 10 and ARL_0 = 500.

specific scenario, the NPRDEWMA-SR chart had a longer ARL but a slightly higher SDRL compared to the NPIRDHWMA-WSR chart (see Table 2).

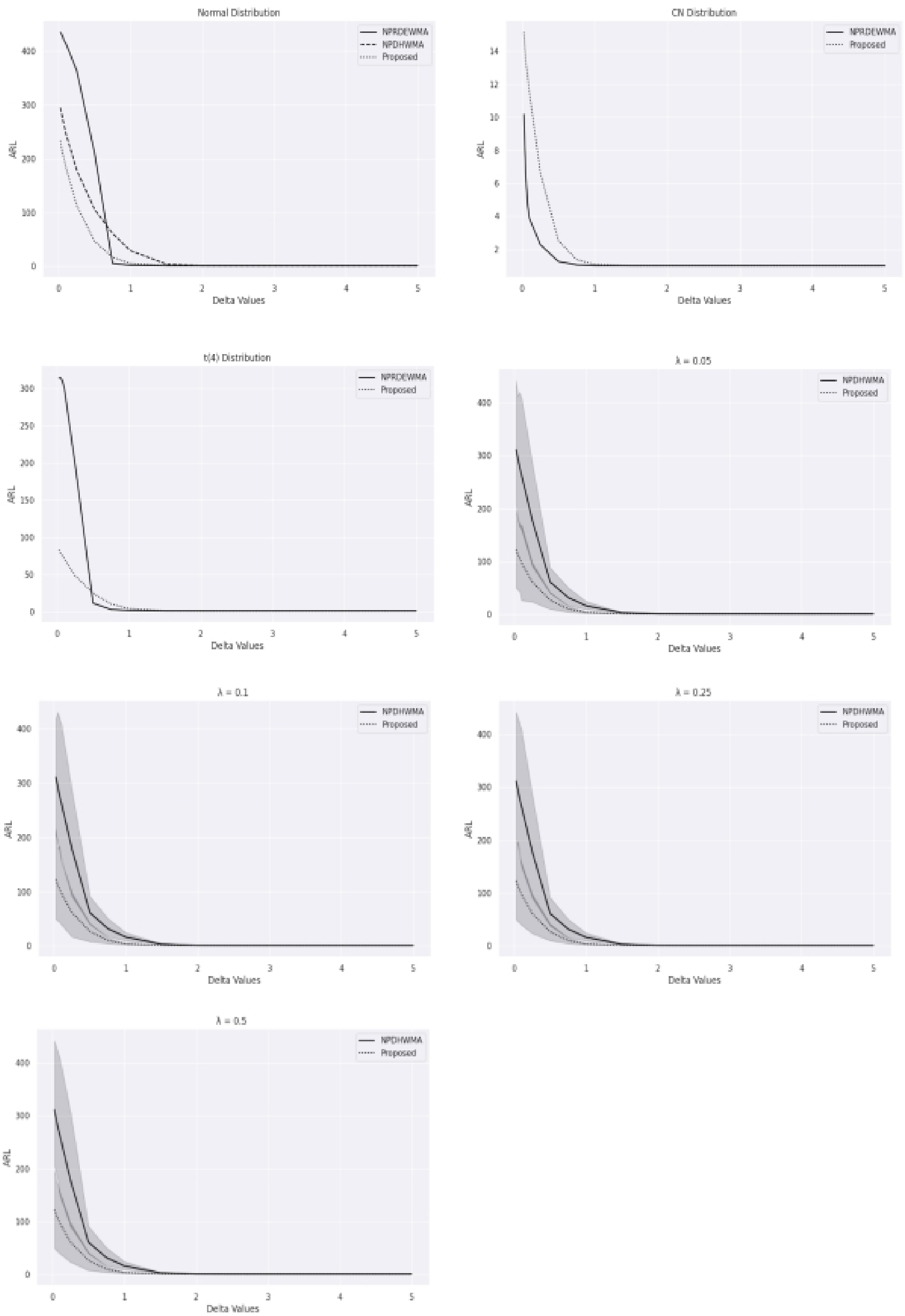

Figure 2. The ARL features of the proposed NPIRDHWMA-WSR control chart and competing charts for various distributions when n = 10 and $ARL_0 = 500$

Furthermore, the NPRDEWMA-SR chart displayed notable sensitivity to parameter choices. The selection of λ and L had a significant impact on ARL and SDRL, leading to marked fluctuations in detection times. For instance,

Table 4. Application of the proposed control charts in comparison to NPDHWMARSS.

Sample	Proposed			NPDHWMARSS		
	Statistic	UCL	LCL	Statistic	UCL	LCL
1	0	0.06	-0.06	0	0.25	-0.25
2	0.24	17.69	-17.69	0.3	24.4	-24.4
3	0.64	12.51	-12.51	0.49	17.25	-17.25
4	0.89	10.21	-10.21	0.8	14.09	-14.09
5	1.19	8.85	-8.85	1.19	12.2	-12.2
6	1.49	7.91	-7.91	1.63	10.91	-10.91
7	1.81	7.22	-7.22	1.88	9.96	-9.96
8	2.07	6.69	-6.69	2.15	9.23	-9.23
9	2.24	6.26	-6.26	2.45	8.63	-8.63
10	2.39	5.9	-5.9	2.75	8.14	-8.14
11	2.6	5.6	-5.6	2.99	7.72	-7.72
12	2.8	5.33	-5.33	3.2	7.36	-7.36
13	3.18	5.11	-5.11	3.44	7.05	-7.05
14	3.45	4.91	-4.91	3.62	6.77	-6.77
15	3.57	4.73	-4.73	3.87	6.53	-6.53
16	4.04	4.57	-4.57	4.04	6.3	-6.3
17	4.39	4.42	-4.42	4.3	6.11	-6.11
18	4.53	4.29	-4.29	4.37	5.92	-5.92
19	4.56	4.17	-4.17	4.6	5.76	-5.76
20	4.86	4.06	-4.06	4.88	5.6	-5.6
21	5.25	3.96	-3.96	5.2	5.46	-5.46
22	5.63	3.86	-3.86	5.48	5.33	-5.33
23	5.97	3.77	-3.77	5.78	5.21	-5.21
24	6.27	3.69	-3.69	5.82	5.09	-5.09
25	6.39	3.61	-3.61	5.97	4.99	-4.99
26	6.56	3.54	-3.54	6.13	4.89	-4.89
27	6.54	3.47	-3.47	6.42	4.79	-4.79
28	6.69	3.41	-3.41	6.76	4.7	-4.7
29	7.03	3.34	-3.34	6.98	4.62	-4.62
30	7.39	3.29	-3.29	7.21	4.54	-4.54
31	7.62	3.23	-3.23	7.46	4.46	-4.46
32	7.81	3.18	-3.18	7.7	4.39	-4.39
33	8.0	3.13	-3.13	7.95	4.32	-4.32
34	8.4	3.08	-3.08	8.11	4.25	-4.25
35	8.5	3.03	-3.03	8.39	4.19	-4.19
36	8.61	2.99	-2.99	8.82	4.13	-4.13
37	8.78	2.95	-2.95	8.89	4.07	-4.07
38	8.87	2.91	-2.91	9.09	4.02	-4.02
39	8.94	2.87	-2.87	9.27	3.97	-3.97
40	9.04	2.83	-2.83	9.44	3.92	-3.92

under the same significance level and distribution type, the NPDEWMA-SR chart's ARL ranged from 10.18 to 435.33, and SDRL varied from 0.00 to 244.49 across different parameter combinations (see Figure 1).

In summary, a direct comparison of specific instances and values reveals that the NPIRDHWMA-WSR chart generally demonstrated competitive or superior performance compared to the NPRDEWMA-SR chart in terms of ARL and SDRL. The NPIRDHWMA-WSR chart exhibited more consistent and predictable behavior across different scenarios, whereas the NPDEWMA-SR chart's performance varied significantly with parameter choices. Considering the paramount importance of timely detection of process shifts in quality control, the NPIRDHWMA-WSR chart's ability to provide more reliable and consistent performance positions it as the preferred choice for this purpose (see Figure 1, Tables 1 and 2).

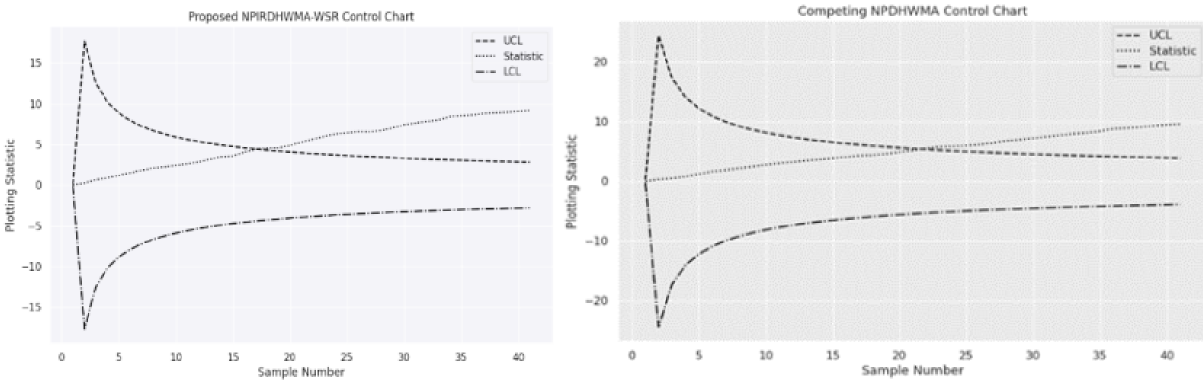

Figure 3. NPIRDHWMA-WSR and NPDHWMARSS control charts for real-life Data.

4.2. Comparison of Proposed Control Chart with NPDHWMA-RSS

The NPIRDHWMA-WSR control chart consistently demonstrated competitive and stable performance across a range of scenarios. In cases where λ was set to 0.05 and L to 1.064 under a normal distribution, it achieved an Average Run Length (ARL) of approximately 234.20 at a significance level of 0.05, indicating efficient detection of process shifts. The SDRL, was 121.09, signifying moderate variability in detection times. This pattern of stability and efficiency was consistent across various parameter combinations, highlighting the chart's reliability (see Table 1).

In contrast, the NPDHWMA–RSS control chart displayed reliable but relatively longer detection times. For instance, when λ was set to 0.05 and L to 1.064 under a normal distribution, the ARL was approximately 295.081 at a significance level of 0.05, slightly higher than that of the NPIRDHWMA-WSR chart. The SDRL for this scenario was around 25.56, indicating stability but shorter compared to the NPIRDHWMA–WSR chart. This trend continued across different parameter combinations, suggesting stable but relatively slower detection times (see Table 3).

The obtained results revealed that the NPIRDHWMA-WSR chart is the preferred choice for detecting process shifts than NPDHWMARSS chart. It consistently demonstrated competitive or superior performance in terms of ARL and SDRL across various scenarios, emphasizing its efficiency and reliability in detecting process shifts, which is crucial in quality control applications (see Figure 2 and Tables 1 and 3).

Conversely, the NPDHWMARSS chart, while reliable, generally required more observations on average to detect process shifts. Therefore, the NPIRDHWMA-WSR chart is the more suitable option for timely and effective process shift detection, making it the preferred choice in quality control applications.

4.3. Real-life Data Result

The data used in this project is based on two datasets with a total of 1,143 observations. The Ranked Set Sampling (RSS) technique was utilized to create 40 sets of paired observations. As depicted in Figure 3, the proposed NPIRDHWMA-WSR promptly identified an OOC by the 18th sample. In contrast, the competing NPDHWMA-RSS control chart signal OOC at the 22nd sample. Specifically, the proposed NPIRDHWMA-WSR control chart detected a total of 22 OOC points, while the competing NPDHWMA-RSS control chart identified only 18 OOC points. This is an indication, that the proposed control chart is robust in OOC detection than the competing control charts discussed in this study.

5. Conclusion

We explored the development and application of non-parametric control charts, with a particular focus on the NPIRDHWMA-WSR control chart. Through extensive Monte Carlo simulations and real-life dataset applications, we evaluated the chart's performance in detecting out-of-control (OOC) processes. Our findings revealed that the NPIRDHWMA-WSR control chart demonstrated promising sensitivity in OOC detection, often outperforming competing charts. Moreover, its adaptability to diverse domains, as demonstrated in real-life applications, highlights its potential for broader quality assurance and process monitoring applications.

Drawing from the obtained results and the comprehensive analyses for sensitivity conducted, consequently, the following can be inferred:

i. The ARL and SDRL reported for each combination reflected that the choice of "λ" and "L" has a notable impact on both ARL and SDRL. Lower values of "λ" tend to lead to longer ARL, while higher values lead to shorter ARL. The impact of "L" is also evident, where larger values generally result in shorter ARL.
ii. Across all distributions, the ARL0 consistently hovers around the nominal value of 500, indicating that the control chart is appropriately sensitive under in-control conditions.
iii. ARL_1 values decrease as the significance level increases, indicating faster detection of process shifts for higher significance levels.
iv. Table 1 shows that ARL_1 when the process is out of control decreases as δ increases. For example, ARL_1 is 234.23 for δ = 0.025 and drops significantly to 1.00 at δ = 5. Similarly, ARL_1 decreases with increasing δ. For (λ = 0.25 and L = 2.113), ARL_1 decreases from 180.76 at (δ = 0.025 to 1.00 at (δ = 5.
v. As λ increases (from 0.05 to 0.5), the ARL_1 values do not consistently decrease. Specifically, for the Normal distribution, the ARL_1 values are 234.23, 158.76, 180.76, and 416.53 for λ values of 0.05, 0.1, 0.25, and 0.5, respectively. This indicates that while the ARL_1 decreases when λ increases from 0.05 to 0.1, it increases for higher λ values, showing that the control chart's responsiveness to process changes varies non-linearly with λ. This pattern is also observed for other distributions considered in this study. Thus, the relationship between the degree of shift and ARL_1 is a non-linear function which is an implication that higher λ does not uniformly improve the control chart's sensitivity to shifts.
vi. Generally, the control chart performs differently across the various distributions considered in this study, with the CN distribution showing earlier detection compared to the Normal and t(4) distributions.
vii. As L increases (from 1.064 to 2.376), ARL_1 values also tend to decrease, indicating improved sensitivity to shifts.
viii. The SDRL values tend to decrease as well with higher λ and L, indicating more consistent effectiveness of the control chart.
ix. The selection of significance level (alpha) affects the ARL_1 values. Smaller alpha values result in longer ARL_1 values, meaning that the control chart is less likely to signal a false alarm.

Based on the above, it is clear that the NPIRDHWMA-WSR control chart enhanced sensitivity in identifying process shifts. The ability to detect OOC conditions with fewer samples positions the proposed chart as a powerful tool for quality assurance. Future research in this domain should focus on refining and fine-tuning the NPIRDHWMA-WSR control chart's parameters to maximize its performance in various industries. Additionally, comparative studies with other non-parametric and parametric control charts can provide deeper insights into its relative strengths. Moreover, exploring applications in emerging fields, such as artificial intelligence and IoT-based quality monitoring, can extend its reach. Collaborations with industry partners and continuous validation in real-world settings are essential steps toward establishing the NPIRDHWMA-WSR control chart as a reliable and practical tool for quality assurance and process control.

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