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A Statistical Model for Analyzing the Impact of GDP Components on the Manufacturing Sector in Saudi Arabia

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Abstract This study examines the relationship between the Gross Domestic Product (GDP) of the manufacturing sector and various independent sectors in Saudi Arabia, utilizing a comprehensive dataset from 2010 to 2023 sourced from the General Authority for Statistics. A robust statistical model was developed to analyze these relationships, revealing significant insights into how changes in different sectors influence manufacturing GDP. Our findings indicate that increases in GDP from specific sectors, such as mining and construction, lead to growth in manufacturing GDP, while increases in other sectors, such as transport and community services, may negatively impact manufacturing performance. These insights are vital for policymakers, emphasizing the interconnectedness of economic sectors and the need for coordinated strategies. This research not only contributes to a deeper understanding of the dynamics shaping Saudi Arabia's economic landscape but also provides a valuable foundation for future studies and policy interventions aimed at strengthening the manufacturing sector and promoting sustainable economic development.

Keywords Regression analysis, Statistical Model, Regression Assumptions, Economic Sectors, Model Accuracy

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1. Introduction

The manufacturing sector is a critical indicator of Gross Domestic Product (GDP) and plays an integral role in the economic development of nations. It drives economic growth, generates employment opportunities, and fosters technological innovation [1]. By producing goods that meet both domestic and international demand, manufacturing generates substantial revenue, which directly impacts national income levels. Furthermore, a thriving manufacturing sector creates job opportunities, reducing unemployment and enhancing overall economic stability [2]. The sector also stimulates innovation and technological advancement, leading to increase productivity and enhanced competitiveness in the global market[3]. Understanding the dynamics of this sector is vital for policymakers, as effective strategies can enhance its contributions to national income and foster sustainable development. Therefore, the manufacturing sector is not just a driver of growth, but a fundamental pillar supporting the overall economic framework. In Saudi Arabia, the manufacturing sector is a cornerstone of the Vision 2030 initiative, which seeks to reduce the economy's dependence on oil dependency and promote sustainable development. This vision emphasizes enhancing the manufacturing industry to generate employment, promote economic advancement, and elevate the nation's global competitiveness[4]. The transition from a traditional economy, largely based on primary activities, to a modern economy where manufacturing with high productivity becomes central, is a fundamental aspect of economic development [5]. Understanding the dynamics of the manufacturing sector within this framework is crucial, especially in an increasingly interconnected global economy,

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where changes in one sector can have far-reaching implications. By examining these dynamics, we can better understand how various sectors interact and contribute to the overarching goals of Saudi Vision 2030.

Understanding the impact of other sectors on the manufacturing sector is crucial, as it contributes to the development of effective strategies that enhance the sector's competitiveness and support overall economic growth [6]. The interplay between sectors can directly affect manufacturing performance, making it essential to study these relationships to understand how to promote stability and growth in the economy as a whole.Moreover, these interactions can improve resource efficiency, leading to reduced waste and increased productivity. They can also foster collaboration among businesses across different sectors, facilitating the exchange of knowledge and modern technologies. Finally, understanding these relationships can assist policymakers in designing strategies that support innovation and address the evolving needs of the market.

We employed multiple linear regression, a robust statistical method for examining the relationship between one dependent variable and multiple independent variables. The significance of regression models lies in their capability to manage intricate relationships and generate predictions based on several factors [7]. This technique is extensively utilized across diverse fields, including economics, social sciences, and health research. Its application plays a critical role in supporting decision-making processes and informing the development of effective policy [8] [12].

To guarantee the validity and precision of our results, we carefully assessed the key assumptions of multiple linear regression [9]. Verifying these assumptions—normality of errors, constant variance of errors, and independence of errors—is essential, as they underpin the validity of the model [13] [10] [14]. When these conditions are satisfied, the least squares estimators are considered the best unbiased estimators with minimum variance [11]. However, failing to meet these assumptions can result in biased outcomes and unreliable predictions, affecting the integrity of hypothesis tests and the accuracy of parameter estimates [19].

In this study, we aim to explore the effect of different sectors on the manufacturing sector in terms of Saudi GDP. We will analyze how changes in the performance of different sectors can influence the dynamics of the manufacturing industry.By exploring these interrelationships, the research seeks to provide insights into the integration of the economy and the role that sectoral interactions play in shaping the manufacturing landscape. This statistical analysis will enhance our understanding of how developments in other sectors affect the performance and competitiveness of the manufacturing sector.

2. Methods

Firstly, we built a statistical model to examine the relationship between the Gross Domestic Product (GDP) index for manufacturing and various other industries in Saudi Arabia. The dataset spans the years from 2010 to 2023 and was sourced from the General Authority for Statistics in Saudi Arabia.

The dataset includes the Gross Domestic Product (GDP) for manufacturing as the dependent variable, with nine independent variables that represent the contributions from other sectors of the economy.

The regression model:

$$Y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 + \beta_7 X_7 + \beta_8 X_8 + \beta_9 X_9 + \epsilon_i \tag{1}$$

Where Y_i = the Manufacturing GDP, X_1 = the Agriculture GDP, X_2 = the Mining GDP, X_3 = the Electricity Gas Water GDP, X_4 = the Construction GDP, X_5 = the Wholesale, Retail GDP, X_6 = the Transport, Storage Communication GDP, X_7 = the Finance, Insurance, Real, Estate GDP, X_8 = the Community, Social, Personal GDP, X_9 = the Government, Activities GDP,

and ϵ_i = the error term, independent $\epsilon_i \sim N(0, \sigma^2)$.

In matrix form, the multiple regression model can be expressed as:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \tag{2}$$

Where

$$\mathbf{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} 1 & X_{1,1} & X_{1,2} & \cdots & X_{1,9} \\ 1 & X_{2,1} & X_{2,2} & \cdots & X_{2,9} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & X_{n,1} & X_{n,2} & \cdots & X_{n,9} \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_9 \end{bmatrix}, \quad \boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix} \tag{3}$$

It is known that the least squares estimates $\hat{\boldsymbol{\beta}}$ are obtained by:

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top \mathbf{Y} \tag{4}$$

If the assumptions of the model are met, then $\hat{\beta} \sim \mathcal{N}\left(\beta, \sigma^2 (X^T X)^{-1}\right)$.

In other words:

- If the assumptions of the model are met, then the distribution of $\hat{\beta}$ will be as stated

- If the assumptions are violated, the stated distribution may no longer be accurate. In such cases, alternative methods or estimation techniques might be necessary to obtain reliable results.

By using R program, the estimated regression function is:

$$\hat{Y} = 2.09 \times 10^5 + 2.08X_1 + 0.13X_2 - 18.58X_3 + 1.95X_4 - 0.019 - 3.14X_5 - 0.95X_6 + 7.30X_8 + 1.07X_9 \tag{5}$$

Meaning of Regression Coefficients:

- β_0 stands for the intercept, which means the mean of Manufacturing GDP is 2.09×10^5 when all independent variables are held at zero.

- β_i $(i = 1, 2, ..., 9)$indicates the change in the mean Manufacturing GDP per unit increase in X_i when other variables are held constant.

3. Checking Assumptions of the Multiple Regression Model

To ensure the validity of the results obtained from the multiple regression analysis, it is essential to check the underlying assumptions of the model. These assumptions include linearity, independence, and normality of the residuals. Verifying these assumptions helps confirm that the model appropriately fits the data and that the inferences drawn from the analysis are reliable. Let's begin with the linearity assumption, which we will assess using the Residuals vs. Fitted Values plot. This visual representation will allow us to determine if the residuals are randomly dispersed around zero, indicating a linear relationship between the predictors and the dependent variable.

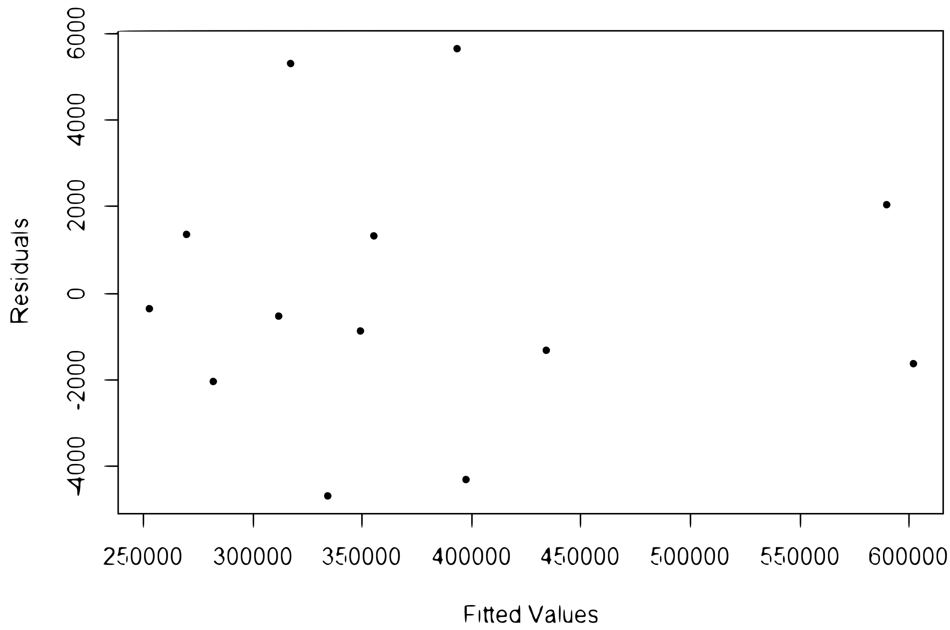

Figure 1. Residuals vs. Fitted Values plot

Linearity Assumption Check: In the Residuals vs. Fitted Values plot, the majority of the residuals are concentrated around zero, indicating that the model's predictions are generally accurate. However, the presence of a few outliers at the ends of the x-axis suggests that there are specific instances where the model underestimates or overestimates the dependent variable significantly. These outliers may warrant further investigation, as they could indicate influential data points that affect the model's performance. Despite these outliers, the overall pattern supports the assumption of linearity, suggesting that the model captures the primary relationships effectively.

Normality Assumption Check: To assess the normality of the residuals from the multiple regression analysis, we conducted the Shapiro-Wilk normality test. The null hypothesis for this test is that the residuals are normally distributed, which can be expressed mathematically as:

$$H_0 : \text{The residuals follow a normal distribution.}$$

$$H_a : \text{The residuals do not follow a normal distribution.}$$

In our analysis, we obtained a Shapiro-Wilk test statistic:

$$W = \frac{(\sum_{i=1}^{n} a_i X_{(i)})^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2} = 0.94192 \tag{6}$$

Hence, a Shapiro-Wilk test statistic $W = 0.94192$ with a corresponding p-value of 0.4823. Since the p-value exceeds the significance level of 0.05, we fail to reject the null hypothesis, indicating that the residuals are normally distributed. This suggests that the assumption of normality is met for the analysis.

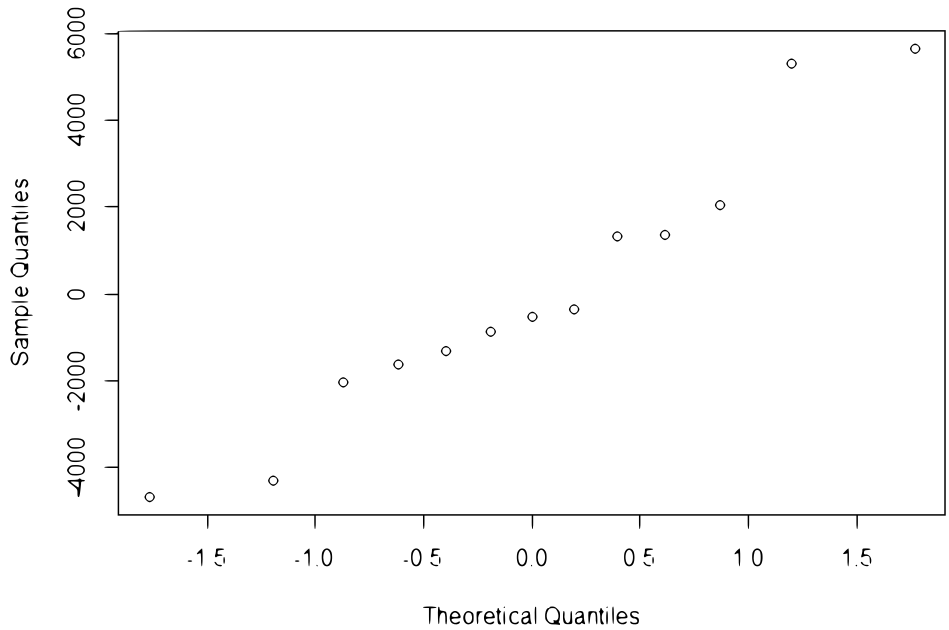

Figure 2. QQ plot of the residuals

To check the normality of the residuals, we also created a QQ plot. In the QQ plot, the points closely align with the straight line, indicating that the residuals follow a normal distribution.

This result supports the Shapiro-Wilk normality test results, which indicated $W = 0.94192$ and a p-value $= 0.4823$. Since the p-value is greater than 0.05, we do not reject the null hypothesis of normality. Thus, both the QQ plot and the Shapiro-Wilk test suggest that the normality assumption is satisfied.

Independence of Errors Check: To assess the independence of errors in the regression model, we calculated the Durbin-Watson statistic, which is defined by the equation:

$$D_W = \frac{\sum_{t=1}^{n}(e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2}$$

In this analysis, the statistic yielded a value of $D_W = 2.7968$. This value, being close to 2, suggests that the residuals do not display any significant autocorrelation. Consequently, we conclude that the assumption of independence of errors is satisfied in this regression model.

We obtained the scatter plot using R to visualize the relationship between two variables and to gain a comprehensive overview of the relationships among all variables in the dataset. Using them together enhances our understanding of the data's relationships and trends, which aids in making data-driven decisions.

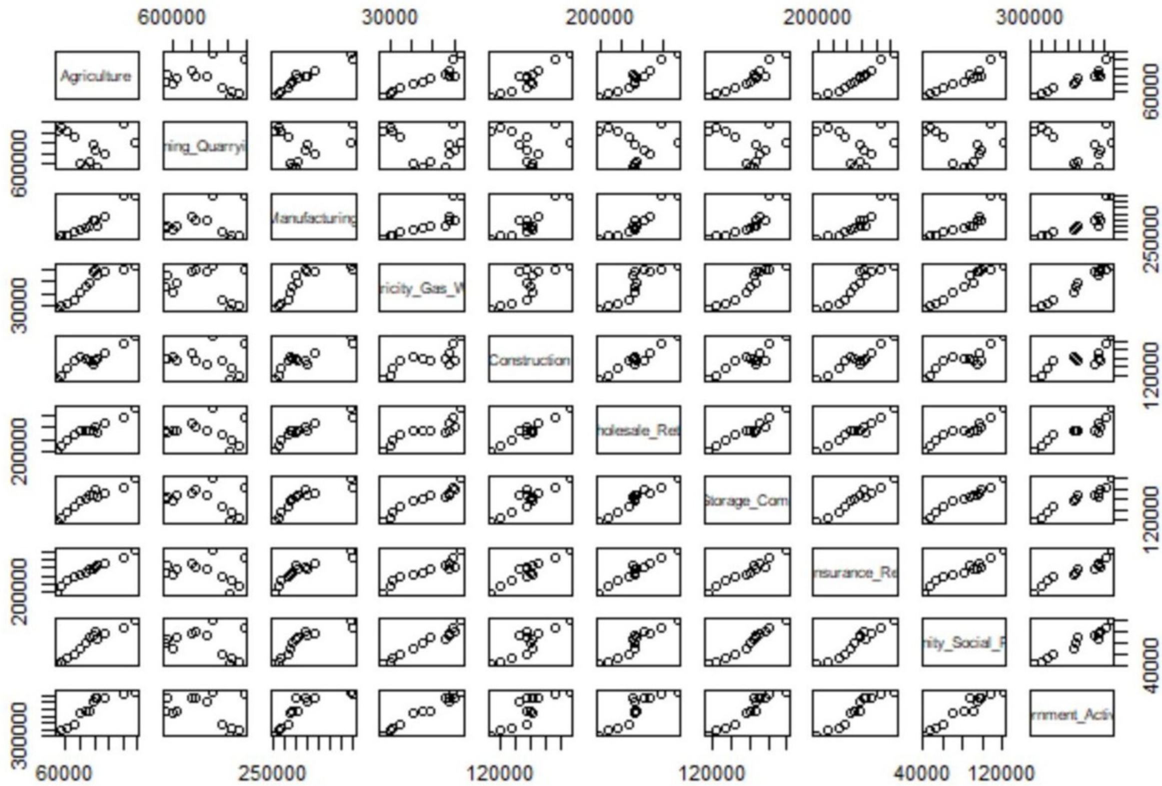

Figure 3. The scatter plot

The correlation matrix serves a similar purpose to a scatter plot by visually representing the relationships between different sectors in the economy. It reveals strong positive correlations, such as between agriculture and manufacturing (0.9618), indicating that growth in one sector often corresponds with growth in the other. Notably, the electricity, gas, and water sector also show high correlations with manufacturing (0.8159), suggesting that increased utility services are essential for manufacturing activities. Conversely, some sectors exhibit negative correlations, like mining with electricity, gas, and water (-0.3319), hinting at potential conflicts or resource allocation issues. Overall, the matrix highlights the interconnectedness of sectors, emphasizing the role of agricultural productivity in stimulating growth across various industries and providing valuable insights for policymakers to enhance economic synergy.

To assess whether Manufactoring GDP is significantly related to the other sectors, we will perform an F-test for the regression model.

We formulate the hypotheses as follows:

- $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8 = 0 = \beta_9 = \beta_1 0$
- H_1 : Not all β_i are equal to zero, $i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$

We use the test statistic:

$$F^* = \frac{\text{MSR}}{\text{MSE}} = \frac{18,367,387,550}{39,230,000} = 467.52$$

$$F_{9,3,0.95} = 8.8123$$

Decision

Table 1. Correlation Matrix

$$\begin{bmatrix} 1.0000 & -0.1203 & 0.9618 & 0.9056 & 0.9348 & 0.9667 & 0.9449 & 0.9780 & 0.9557 & 0.9194 \\ -0.1203 & 1.0000 & 0.1211 & -0.3319 & -0.2076 & -0.1692 & -0.3353 & -0.2478 & -0.2853 & -0.3670 \\ 0.9618 & 0.1211 & 1.0000 & 0.8159 & 0.8859 & 0.9270 & 0.8608 & 0.9054 & 0.8851 & 0.8235 \\ 0.9056 & -0.3319 & 0.8159 & 1.0000 & 0.7856 & 0.8669 & 0.9482 & 0.9311 & 0.9746 & 0.9907 \\ 0.9348 & -0.2076 & 0.8859 & 0.7856 & 1.0000 & 0.9734 & 0.9145 & 0.9475 & 0.8691 & 0.8352 \\ 0.9667 & -0.1692 & 0.9270 & 0.8669 & 0.9734 & 1.0000 & 0.9634 & 0.9750 & 0.9386 & 0.8903 \\ 0.9449 & -0.3353 & 0.8608 & 0.9482 & 0.9145 & 0.9634 & 1.0000 & 0.9807 & 0.9842 & 0.9583 \\ 0.9780 & -0.2478 & 0.9054 & 0.9311 & 0.9475 & 0.9750 & 0.9807 & 1.0000 & 0.9703 & 0.9497 \\ 0.9557 & -0.2853 & 0.8851 & 0.9746 & 0.8691 & 0.9386 & 0.9842 & 0.9703 & 1.0000 & 0.9690 \\ 0.9194 & -0.3670 & 0.8235 & 0.9907 & 0.8352 & 0.8903 & 0.9583 & 0.9497 & 0.9690 & 1.0000 \end{bmatrix}$$

Source	Df	Sum Sq	Mean Sq
Regression	8	146,939,100,400	18,367,387,550
Residuals	3	117,690,000	39,230,000

Table 2. ANOVA Table

Since $F^* = 467.52 > 8.8123 = F_{9,3,0.95}$, we reject H_0.
Conclusion : At the 5% significance level, there is a relationship between manufacturing GDP and other industries.

We will now concentrate on the Coefficient of Multiple Determination, typically referred to as R^2 . This statistic plays a crucial role in our research, as it indicates how much of the variation in the dependent variable is explained by the independent variables in our regression analysis.
The calculation of the Coefficient of Multiple Determination, R^2 is as follows:

$$R^2 = \frac{SSR}{SSTO} = \frac{146,939,100,400}{147,056,790,400} \approx 0.9992$$

Thus, 99% of the variation in construction is explained by the multiple relationship with independent variable .

We want to test whether there is a industry that can be dropped from the model or not.
We want to test the hypothesis for the variable X_1 as follows:

- Null Hypothesis: $H_0 : \beta_1 = 0$
- Alternative Hypothesis: $H_1 : \beta_1 \neq 0$

The full model is:

$$Y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 + \beta_7 X_7 + \beta_8 X_8 + \beta_9 X_9 + \epsilon_i \tag{7}$$

The reduced model is:

$$Y_i = \beta_0 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 + \beta_7 X_7 + \beta_8 X_8 + \beta_9 X_9 + \epsilon_i \tag{8}$$

Test Statistics:

$$F^* = \frac{\text{MSR}(X_1 \mid X_2, ..., X_9)}{\text{MSE}(X_1, X_2, ..., X_9)} = 2.95$$

$$F_{,8,0.95} = 6.60$$

Decision: Since $F^* = 2.95 < 6.60 = F_{0.95,1,8}$, we fail to reject H_0.

Conclusion: At the 5% significance level,X_1 (Agriculture) can be dropped from the regression model that already contains Mining , Electricity , Construction , Wholesale, Transport, Finance, Community, and the Government GDP. We can present the remaining variables in the following table:

Industry	F^* **Test**	$F_{0.95}$	**Decision**	**Conclusion**
Mining	93.48	5.98	Reject H_0	Significant
Electricity Gas Water	3.85	5.98	Fail to Reject H_0	Not Significant
Construction	23.54	5.59	Reject H_0	Significant
Wholesale Retail	3.42	4.53	Fail to Reject H_0	Not Significant
Transport Storage Communication	22.20	5.11	Reject H_0	Significant
Finance Insurance Real Estate	9.02	5.11	Reject H_0	Significant
Community Social Personal	59.70	5.11	Reject H_0	Significant
Government Activities	4.63	5.11	Fail to Reject H_0	Not Significant

The model can be summarized to focus only on Mining , Electricity Gas Water, Construction, Transport Storage Communication, Finance Insurance Real Estate and Community Social Personal, excluding Agriculture, Wholesale Retail and Government Activities. After removing the non-significant variable, the estimated regression equation generated by the R program is:

$$Y_i = \beta_0 + \beta_2 X_2 + \beta_4 X_4 + \beta_6 X_6 + \beta_7 X_7 + \beta_8 X_8 + \epsilon_i$$

We examined the impact of the remaining variables on manufacturing Gross Domestic Product (GDP)and checked the assumptions of the new model. After testing and comparing the coefficients and p-values, we observed the following:

- The coefficient for **Finance/Insurance/Real Estate** was -0.6798 with a p-value of 0.115.
- In contrast, other significant variables had p-values less than 0.001 (e.g., Mining/Quarrying: $p < 0.001$, Construction: $p < 0.001$).

Thus, given that the p-value for Finance/Insurance/Real Estate is 0.115, which exceeds the significance threshold of 0.05, we determined that this variable does not have a statistically significant impact on the model. As a result, we concluded that it is appropriate to remove Finance/Insurance/Real Estate from the analysis, thereby enhancing the model's clarity and focus on the more impactful variables.

Final Revised Model Equation: The final revised model includes only the significant independent variables that have a measurable impact on manufacturing GDP, our dependent variable. Specifically, the model is expressed as:

$$Y_i = \beta_0 + \beta_2 \text{MiningGDP} + \beta_4 \text{ConstructionGDP} + \beta_6 \text{TransportGDP} + \beta_8 \text{CommunityGDP} + \epsilon_i \tag{9}$$

The estimated model coefficients were calculated using R, yielding the following results. Each coefficient represents the predicted change in manufacturing GDP for a unit increase in the respective independent variable.

$$\hat{Y} = \ 1.17 \times 10^5 + 0.115 X_2 + 2.87 X_4 - 5.08 X_6 - 6.46 X_8 \tag{10}$$

The estimated model reveals significant insights into the relationships between various GDP sectors and the GDP of the manufacturing industry. The positive coefficient for mining GDP (0.115) indicates that increases in this sector contribute to the growth of manufacturing GDP, highlighting its supportive role. Construction GDP shows an even stronger positive impact (2.87), emphasizing its critical importance in driving manufacturing growth. Conversely, the negative coefficients for transport (-5.08) and community GDP (-6.46) suggest that expansions in these sectors could detract from manufacturing GDP, potentially due to inefficiencies or resource reallocation. These findings underscore the importance of the study, as they clarify the dynamics influencing manufacturing GDP and provide actionable insights for policymakers to enhance collaboration among GDP sectors and address challenges in transport and community services to foster overall economic growth.

4. Conclusion

This study has elucidated the relationships between manufacturing GDP and various other sectors of GDP in Saudi Arabia. Our analysis revealed that increases in GDP from the mining and construction sectors contribute to the growth of manufacturing GDP. This positive relationship underscores the critical importance of investing in these sectors as a strategy to enhance manufacturing performance. Conversely, our findings indicate that the transport and community sectors may negatively impact manufacturing growth, suggesting that inefficiencies in these areas could hinder overall economic development. These insights are essential for policymakers, as they highlight the interconnected nature of economic sectors. By understanding these dynamics, effective strategies can be developed to promote collaboration and address existing challenges. Ultimately, this research provides a valuable foundation for future studies and policy interventions aimed at strengthening the manufacturing sector and the broader economic landscape of Saudi Arabia. The study has the following limitations:

- The study relies on national-level data, which may not fully capture regional disparities in economic activity within Saudi Arabia. The lack of disaggregated data at the regional or provincial level means that the unique characteristics and variations of different areas of the country are not addressed, potentially overlooking regional differences in how GDP components influence the manufacturing sector.

- The quality of the data used in the study is a concern. Certain economic indicators may be subject to measurement errors, inconsistencies, or missing values. In particular, data related to some sectors, like non-oil industries or the informal sector, might not be as robust, which could introduce bias into the findings and limit the generalizability of the results.

- This study is based on cross-sectional data, which captures information at a single point in time. While this allows for an analysis of relationships between GDP components and the manufacturing sector, it is not possible to discern any causal effects or track the evolution of these relationships over time. As a result, this analysis is limited in its ability to explore the long-term dynamics of the economic factors under investigation.

- The study assumes linear relationships between the economic variables, but this may not always be accurate. Real-world economic relationships are often nonlinear and may involve more complex interactions between sectors, which a linear model cannot fully capture. Therefore, the model assumptions could limit the ability to accurately represent the true nature of the relationships.

- The study may not account for all relevant variables that could influence the manufacturing sector in Saudi Arabia. Factors such as technological advancements, changes in global oil prices, foreign investments, or geopolitical influences might significantly affect manufacturing but are not included in the analysis. The omission of such factors may introduce omitted variable bias, skewing the results.

- Measurement errors are a common challenge in economic studies, particularly in sectors where data collection can be inconsistent. For instance, certain sectors may be underreported or inaccurately quantified in official statistics, leading to distortions in the analysis. Such measurement errors could undermine the validity and reliability of the results.

- The study does not fully address potential endogeneity issues, such as reverse causality or simultaneity. For example, while we assume that changes in GDP components influence the manufacturing sector, it is possible that the performance of the manufacturing sector could also affect the GDP components. This bidirectional relationship could lead to biased estimates if not properly controlled for.
- The model does not account for external shocks, such as global economic crises, changes in oil prices, or unexpected geopolitical events (e.g., conflicts or pandemics), which could have a significant impact on the manufacturing sector. These factors could cause sudden and substantial changes in economic activity that are not reflected in the model.
- The analysis is limited by the time frame of the data available. The study may not capture long-term trends or shifts in the Saudi economy, particularly as the country undergoes significant economic diversification efforts. Short-term variations or recent changes in economic conditions may not be fully reflected, limiting the depth of the analysis.
- Some sectors in the study may overlap or influence each other in ways that are difficult to disentangle. For example, the services sector, which includes transportation and community services, might have indirect effects on manufacturing that are not fully captured in the analysis. The interactions between sectors could distort the estimated effects of individual GDP components on manufacturing output.

The following points are proposed for future research

I. Future research could benefit from a more detailed regional or provincial analysis, which would allow for a better understanding of the geographical variations in the impact of GDP components on manufacturing. Saudi Arabia's regional diversity may lead to different economic dynamics in areas like Riyadh, Jeddah, and the Eastern Province, and exploring these differences could provide valuable insights for more targeted policy recommendations.

II. To address potential issues of reverse causality and endogeneity, future studies could incorporate more advanced econometric techniques, such as instrumental variables (IV), difference-in-differences (DiD), or structural equation modeling (SEM). These methods would help establish stronger causal links between GDP components and the manufacturing sector, improving the credibility of the findings.

III. Future research could incorporate dynamic models such as vector autoregression (VAR) or error correction models (ECM) to examine both short-term fluctuations and long-term trends in the relationship between economic sectors and manufacturing. These models would allow for a deeper understanding of how the effects of GDP components evolve over time and under different economic conditions.

IV. Conducting a comparative analysis with other countries, especially those within the Gulf Cooperation Council (GCC) or emerging economies, would help contextualize the findings and allow for cross-country comparisons. This would highlight whether the findings from Saudi Arabia are consistent with broader regional or global trends and identify any unique characteristics of the Saudi economy.

V. Future research could explore the interdependencies between different economic sectors, especially focusing on the manufacturing sector's interactions with services and infrastructure. For example, how do transportation and communication services influence the growth of manufacturing industries? Understanding these interdependencies could lead to a more comprehensive policy framework that addresses multiple sectors simultaneously.

VI. Including policy variables, such as government spending on industrial development, trade policies, or foreign direct investment, would enrich the analysis. Future research could explore how specific policies have shaped the manufacturing sector's growth and how they interact with broader economic components like GDP growth, oil prices, and global economic trends.

VII. Expanding the analysis to include a longitudinal dataset could provide a better understanding of long-term trends and dynamics. This would allow researchers to track how shifts in economic policies, global market conditions, or technological advancements impact the manufacturing sector over time and assess the sustainability of observed relationships.

VIII. To strengthen the validity of the findings, future research should conduct robustness checks by testing alternative model specifications, such as including interaction terms, employing logarithmic transformations, or considering different lag structures. Sensitivity analysis would help ensure that the results are not driven by specific assumptions or model choices and increase confidence in the study's conclusions.

IX. Investigating the role of globalization, including trade liberalization, foreign direct investment, and global supply chain integration, could provide important insights into how these external factors affect Saudi manufacturing. Given Saudi Arabia's ongoing efforts to diversify its economy, exploring these global dynamics could help understand the future trajectory of the manufacturing sector.

X. The role of technological innovation, automation, and digital transformation in shaping the manufacturing sector's performance could be further explored. Research could investigate how technological advancements influence productivity and efficiency within manufacturing industries in Saudi Arabia and how they are linked to broader economic factors such as GDP components, labor market conditions, and global market trends.

REFERENCES

1. W. Naudé and A. Szirmai, *The importance of manufacturing in economic development: Past, present and future perspectives*, UNU-MERIT, 2012.
2. J. Konings, *Job creation and job destruction in the UK manufacturing sector*, Oxford Bulletin of Economics & Statistics, vol. 57, no. 1, pp. 1–18, 1995.
3. N. Becheikh, R. Landry, and N. Amara, *Lessons from innovation empirical studies in the manufacturing sector: A systematic review of the literature from 1993–2003*, Technovation, vol. 26, no. 5-6, pp. 644–664, 2006.
4. M. B. Khan and S. Iqbal, *Vision 2030 and the national transformation program*, In *Research, Innovation and Entrepreneurship in Saudi Arabia*, pp. 146–166, Routledge, 2020.
5. A. Faudot, *Saudi Arabia and the rentier regime trap: A critical assessment of the plan Vision 2030*, Resources Policy, vol. 62, pp. 94–101, 2019.
6. N. Haraguchi, C. F. C. Cheng, and E. Smeets, *The importance of manufacturing in economic development: has this changed?*, World Development, vol. 93, pp. 293–315, 2017.
7. J. Fox, *Applied regression analysis and generalized linear models*, Sage Publications, 2015.
8. R. B. Darlington and A. F. Hayes, *Regression analysis and linear models: Concepts, applications, and implementation*, Guilford Publications, 2016.
9. S. Chatterjee and A. S. Hadi, *Regression analysis by example*, John Wiley & Sons, 2015.
10. D. C. Montgomery, E. A. Peck, and G. G. Vining, *Introduction to linear regression analysis*, John Wiley & Sons, 2021.
11. J. O. Rawlings, S. G. Pantula, and D. A. Dickey, *Applied regression analysis: A research tool*, Springer, 1998.
12. D. C. Hoaglin, F. Mosteller, and J. W. Tukey, *Understanding robust and exploratory data analysis*, John Wiley & Sons, vol. 76, 2000.
13. M. Tranmer and M. Elliot, *Multiple linear regression*, The Cathie Marsh Centre for Census and Survey Research (CCSR), vol. 5, no. 5, pp. 1–5, 2008.
14. T. Lumley, P. Diehr, S. Emerson, and L. Chen, *The importance of the normality assumption in large public health data sets*, Annual Review of Public Health, vol. 23, no. 1, pp. 151–169, 2002.
15. S. Weisberg, *Applied linear regression*, Wiley, 2005.
16. W. D. Berry, *Understanding regression assumptions*, Sage, vol. 92, 1993.
17. R. J. Casson and L. D. M. Farmer, *Understanding and checking the assumptions of linear regression: A primer for medical researchers*, Clinical & Experimental Ophthalmology, vol. 42, no. 6, pp. 590–596, 2014.
18. D. A. Belsley, E. Kuh, and R. E. Welsch, *Regression diagnostics: Identifying influential data and sources of collinearity*, John Wiley & Sons, 2005.
19. M. H. Kutner, C. J. Nachtsheim, J. Neter, and W. Li, *Applied linear statistical models*, McGraw-Hill, 2005.