

Supply Chain Networks Optimization under Uncertain Environment with Dhoub-Matrix-TP1 heuristic

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Abstract The transportation problem (TP) is a critical component of the supply chain network that involves determining the most efficient way to move goods from one location to another. TP is a generic name given to a whole class of problems in which diverse types of transportation modes are used to supply a product from sources to destinations. The TP is a common challenge in supply chain networks, where it aims to minimize the total cost of transportation to satisfy both supply and demand constraints. In this paper, the constructive heuristic Dhoub-Matrix-TP1 (DM-TP1) is adapted in order to solve the balanced and unbalanced TP with heptagonal fuzzy numbers. DM-TP1 needs a reduced number of iterations in order to generate a good initial basic feasible solution and uses a novel metric based on (Average-Min). Several numerical examples (balanced and unbalanced) are used to prove the performance of DM-TP1.

Keywords Supply chain performance, Fuzzy modelling, Transportation Decision, Artificial Intelligence, Optimization, Heuristic, Operations Research, Dhoub-Matrix.

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1. Introduction

Supply chain management and logistics are critical components of any business that relies on the movement of goods and services, and they are essential for ensuring that a business can meet customer demand while minimizing costs and/or maximizing profits. Supply chain networks involve the coordination of various activities, including transportation. By addressing the Transportation Problem (TP), companies can improve their supply chain network's overall efficiency, increase profits, and gain a competitive advantage. The TP was introduced by Hitchcock in [1]. It consists of supplying a product from a number of factories (supply origins or sources) to a number of cities (demand destinations) with the aim of minimizing the total shipping cost. The objective of the classical TP is to minimize the total cost of transportation while satisfying both supply and demand limits. Later, Dantzig in [2] developed a special form of the simplex method to solve the TP. Then, Charnes et al. in [3] developed a solution procedure from the simplex algorithm and a stepping stone method is developed to solve the TP in [4]. Besides, a new computational scheme for solving the TP is described in [5], a new computing procedure for the Hitchcock koopmans TP and the results are more efficient than the specialized form of the simplex method is proposed [6] and a primal-dual algorithm for the capacitated hitchcock problem is developed [7]. The TP can be stated as a linear programming problem in which there are m sources (suppliers) and n destinations (customers). Let c_{ij} the unit transportation cost from source i to destination j . Each sources has a supply of S_i units and each destination has a demand of D_j units. The objective function aims to minimize the total cost of transportation

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between various sources and destinations. The supply constraints ensure that the total units transported from the source i is less than or equal to its supply. The demand constraints ensure that the total units transported to the destination j is greater than or equal to its demand. Therefore, we have a set of $m * n$ decision variables and a set of $m + n$ constraints. The TP can be defined mathematically as follows: Primal model:

Subject to the constraints:

A balanced TP occurs if the sum of the supply S_i of all sources ($1 \leq i \leq m$) is equal to the sum of the demand D_j of all destinations ($1 \leq j \leq n$). Otherwise, the TP is unbalanced.

Dual model:

$$\text{Minimize } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

Subject to:

$$\sum_{j=1}^n x_{ij} = S_i, \quad i = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} = D_j, \quad j = 1, 2, \dots, n$$

$$x_{ij} \geq 0; i = 1, 2, \dots, m; j = 1, 2, \dots, n$$

A balanced TP occurs if the sum of the supply S_i of all sources ($1 \leq i \leq m$) is equal to the sum of the demand D_j of all destinations ($1 \leq j \leq n$). Otherwise, the TP is unbalanced.

Dual model:

$$\text{Maximize } R = \sum_{i=1}^m S_i Q_i + \sum_{j=1}^n D_j T_j$$

Subject to:

$$Q_i + T_j \leq c_{ij};$$

$$i = 1, 2, \dots, m; j = 1, 2, \dots, n$$

Q_i and T_j are the dual variables. Many other research works can be found in the literature since the TP is a linear programming problem that can be extended to worldwide applications such as plant location, production, scheduling and numerous other fields. For more details we refer to some noteworthy research contributors in [8], [9], [10]. In real-world applications, various transportation modes are used ship, plane, train, truck, etc. There are many different situations that provide different variants of TP such Maritime TP, Air TP, Rail TP, Space TP, Pipeline and Cable TP among others. A vast literature on different variant of TP is produced, we refer to [11] and the recent research proposed by [12]. The solution procedure for the TP consists of finding an initial basic feasible solution and then improve the current obtained solution until an optimal solution is obtained. Furthermore, various methods are developed for an initial solution or optimal solution. The well-known classical methods are North West Corner Rule (NWCR), Least Cost Method (LCM) and Vogel's Approximation Method (VAM) etc. The VAM method was introduced by [13] in order to find a good first feasible solution for the TP and it has since then been widely used. The VAM is a popular heuristic algorithm based on penalty calculation. Later, an improved Vogel's approximation method for an unbalanced TP is proposed in [14], an iterative algorithm for solving the TP when the shipping cost over each route is convex in [15] and a heuristic for obtaining an initial solution for the TP is developed in [16]. Recently, an advanced Vogel's approximation method (AVAM) and developed a new approach to determine penalty cost for better feasible solution of TP is proposed in [17] and an approach to find basic feasible solution for the problem based on logical development of Vogel's approximation method (LD-VAM) is proposed in [18].

Several approaches are developed to determine the initial feasible solution we refer research works explored by [19, 20, 21, 22, 23, 24]. It is still challenging to develop a better method for finding initial feasible solution, an Inverse Coefficient of Variation Method (ICVM) was proposed in [25]. The method performs well and leads to the optimal solution for many problems. The TOCM-MEDM approach for finding an initial basic feasible solution of a balanced TP is introduced in [26]. The new algorithm is set up by applying Modified Extremum Difference Method (MEDM) on Total Opportunity Cost Matrix (TOCM). In addition, a new method called Bilqis Chastine Erma method (BCE) for finding the initial basic feasible solution is proposed in [27], a new method called Karagul-Sahin Approximation Method to find the initial solution to the TP in [28] and they conclude that the solutions are as good as those obtained with Vogel's approach and as fast as the Northwest Corner Method. Recently, a new method called Maximum Difference Extreme Difference Method (MDEDM) for finding the initial basic feasible solution of both cost minimization TP and profit maximization TP in [29] and a combination of Total Difference Method (TDM) and Karagul-Sahin Approximation Method (KSAM) algorithm to determine the initial feasible solution of TP in [30]. Researchers are working on TP, for more detailed bibliographies and survey we refer to [31, 32, 33, 34]. Various methods for finding an optimal solution for TP, have been developed we refer to some recent research contributors: A separation method (based on zero-point method) to find an optimal solution for integer TP is proposed in [35]. A new approach to solve the TP with the max min total opportunity cost method based on the Total Opportunity Cost (TOC) of a transportation table and maximum minimum penalty approach is developed in [36]. A new approach named Loop Product Difference for optimizing the initial basic feasible solution of a balanced TP is proposed in [37]. A new approach to find the optimum solution for the TP is performed in [38]. Recently, a Modified ASM method to produce optimal solutions directly is developed in [39] and the results shows that the proposed method solved successfully the problem of balanced and unbalanced transportation. Very recently, a new column-row heuristic, called Dhouib-Matrix-TP1 (DM-TP1), is designed in [40] to solve the TP with crisp parameters. In this study the novel heuristic DM-TP1 is adapted to solve TP under heptagonal fuzzy TP. The main contribution in this paper is to enhance the DM-TP1 with a new metric (Average-Min) and to solve the TP under uncertain domain with a step-by-step application of DM-TP1 for balanced and unbalanced TP. This paper is structured as follows: Section 2 starts with a review of relevant work reported in literature that highlights the TP under fuzzy environments. In section 3, a brief introduction to the fuzzy concept with details of the heptagonal fuzzy number is presented. In section 4, the novel heuristic DM-TP1 is then described. In section 5, example studies for balanced and unbalanced TP are then presented to prove the robustness of DM-TP1. Section 6 concludes the paper and gives some prospective future extensions.

2. Fuzzy Transportation Problem

In the real-world applications, due to some factors, the transportation costs, supply and demand quantities in a TP can be considered as fuzzy quantities. To deal with this imprecise information, the concept of fuzzy decision making is involved. The objective of the Fuzzy Transportation Problem (FTP) is to minimize the total fuzzy transportation cost while satisfying fuzzy supply and demand limits. Solving the TP is essential for the effective management of the supply chain network, as it helps reducing transportation costs, improving delivery times, and ensuring customer satisfaction. The fuzzy set concept is introduced in [41] where fuzzy numbers are considered to represent imprecise data. Besides, a decision making in a fuzzy environment is studied in [42] and a fuzzy linear programming with several objective functions is presented in [43]. Several researchers used the fuzzy set theory in many fields. Later, a comparative study on TP in fuzzy environment is introduced in [44] where a new algorithm called the fuzzy zero-point method for finding a fuzzy optimal solution of FTP in single stage with new multiplication operation is developed. Also, a time-cost minimization method to solve an FTP with fuzzy parameters is proposed in [45]. A fuzzy version of Vogels and MODI algorithms to find fuzzy basic feasible and fuzzy optimal solution of FTP without converting them to classical TP are developed in [46]. Later, a new approach to find a fuzzy optimal solution to Fully Fuzzy Transportation Problem (FFTP) using triangular fuzzy numbers is proposed in [47]. The proposed approach is an extension of NWC, LC and VAM. Many algorithms were developed to solve FTP using the ranking method to convert fuzzy numbers into crisp numbers: A method for solving FTP using a Robust's

ranking technique for the representative value of the fuzzy numbers is introduced in [48]. They used an allocation table method (ATM) to find an initial basic feasible solution for the problem. They also improved the basic feasible solution by modified distribution method (MODIM) to find the optimal solution. A FFTP where transportation costs, supply and demand are triangular fuzzy numbers is proposed in [49]. A new fuzzy transportation algorithm using a new total integral value ranking method with novel left, right and integral values resulting from inverse functions of fuzzy numbers is developed in [50]. The algorithm is described to find the fuzzy optimal solutions for a FTP. In addition, the Multi-Objective Fixed-Charge Solid Transportation Problem (MOFCSTP) is studied in [51]. They used a linear membership and non-membership functions to solve the problem. The authors used Fuzzy Programming (FP), Intuitionistic Fuzzy Programming (IFP) and Goal Programming (GP) in order to find best Pareto-optimal solutions. A new ranking function of interval-valued intuitionistic fuzzy sets (IVIFSs) is developed in [52]. The new function depends on both values of variable and interval-valued intuitionistic fuzzy degrees. For recent survey on single and multi-objective FTP the reader can refer to [53]. Several approaches have been developed in order to characterize the vague parameters that arise in real life problems. The trapezoidal fuzzy numbers have attracted research attention due to their vast applications, for example a two-stage method to solve FTP is proposed in [54] where a parametric approach is used to obtain a fuzzy solution in the form of a trapezoidal fuzzy number. A parametric method to solve an FTP based on transportation costs, supply and demand which are considered as trapezoidal fuzzy numbers and transformed into crisp quantities in [55]. Also, new methods to find an initial basic feasible solution and the fuzzy optimal solution of FTP where transportation cost, availability and demand of the product are represented by generalized trapezoidal fuzzy numbers in [56]. Furthermore, a new approach based on ranking function for solving FTP is developed in [57] using generalized trapezoidal fuzzy numbers. A fuzzy approach to solve a FTP without converting it into a crisp TP is considered in [58]. An improved method is developed in [59] in order reduced the computational complexity of the existing method. He developed a new approach for solving FTP where the values of transportation costs are represented by generalized trapezoidal fuzzy numbers and the values of supply and demand of products are represented by real numbers. In addition, a new algorithm is developed for solving FTP with trapezoidal fuzzy numbers in [60], an approach based on trapezoidal fuzzy numbers to optimize TP in fuzzy environment is introduced in [61] and a new algorithm for solving FTP with triangular fuzzy numbers is proposed in [62]. Later, a new distance ranking method to solve FTP where supply, demand and transportation costs are trapezoidal fuzzy numbers is developed in [63]. Recently, the heptagonal class of fuzzy numbers was frequently used in practical purposes. The heptagonal fuzzy numbers and defined its arithmetic operations is introduced in [64]. They studied a representation and ranking of fuzzy numbers with heptagonal membership function value and ambiguity index. Later, a general fuzzy transportation problem is discussed in [65]. He proposed a new ranking procedure to compare the heptagonal fuzzy numbers and the new ranking method converted the FTP to a crisp valued transportation problem. Besides, an unbalanced FTP where cost, requirements and availabilities are heptagonal fuzzy numbers is considered in [66]. They converted the FTP into a crisp valued TP using Robust Ranking method and presented a comparative study using Vogel's Approximation Method, New Method and Best Candidate Method. Also, a heptagonal FTP under budgetary constraint is solved in [67] where the ranking method is used to convert the problem into the corresponding crisp TP and the Goal Programming (GP) approach is applied to obtain the optimal solution. The crisp TP is converted into the FTP by using triangular, pentagonal, and heptagonal fuzzy numbers in [68]. They compared the minimum fuzzy transportation cost obtained from different methods and introduced the Lagrange's polynomial to determine the approximate fuzzy transportation cost for Nonagon and Hendecagonal. Besides, a range technique is used to convert fuzzy heptagonal numbers into crisp values and the MAX-MIN method is used to solve the problem in [69].

3. Preliminaries

In this section we review basic notions of fuzzy numbers and we defined three relevant classes of fuzzy numbers which are frequently used in practical purposes: triangular, trapezoidal and heptagonal fuzzy numbers.

3.1. Definition: Fuzzy Set

\tilde{F} is a fuzzy set on $R(\tilde{F} : R \rightarrow [0, 1])$. A fuzzy set can be described by a membership function of x denoted $\mu_{\tilde{F}}(x)$.

3.2. Definition: Fuzzy Number

A fuzzy number \tilde{F} is defined in an interval $[f_l, f_u]$, where f_l and f_u are respectively lower and upper boundaries of \tilde{F} .

3.3. Definition: Triangular Fuzzy Number

Triangular fuzzy number is a fuzzy number represented with three points (See Figure 1) and can be indicated as a triplet $\tilde{F} = (f_l, f_m, f_u)$ where f_l, f_m and f_u are real numbers and $f_l \leq f_m \leq f_u$.

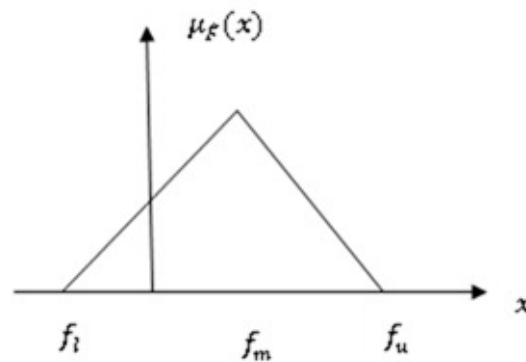


Figure 1. Graphical representation of triangular fuzzy number.

The membership function $\mu_{\tilde{F}}(x)$ for a triangular fuzzy number \tilde{F} is defined by:

$$\mu_{\tilde{F}}(x) = \begin{cases} \frac{x-f_l}{f_m-f_l} & \text{if } f_l \leq x \leq f_m \\ \frac{f_u-x}{f_u-f_m} & \text{if } f_m \leq x \leq f_u \\ 0 & \text{otherwise} \end{cases}$$

3.4. Definition: Trapezoidal Fuzzy Number

Trapezoidal fuzzy number can be indicated as a quartet $\tilde{F} = (f_l, f_h, f_m, f_u)$ where f_l, f_h, f_m and f_u are real numbers and $f_l \leq f_h \leq f_m \leq f_u$ (See Figure 2).

The membership function $\mu_{\tilde{F}}(x)$ for a trapezoidal fuzzy number \tilde{F} is defined by:

$$\mu_{\tilde{F}}(x) = \begin{cases} \frac{x-f_l}{f_h-f_l} & \text{if } f_l \leq x \leq f_h \\ 1 & \text{if } f_h \leq x \leq f_m \\ \frac{f_u-x}{f_u-f_m} & \text{if } f_m \leq x \leq f_u \\ 0 & \text{otherwise} \end{cases}$$

3.5. Definition: Heptagonal Fuzzy Number

Heptagonal fuzzy number can be indicated by 7 tuples (See Figure 3), $\tilde{F} = (f_l, f_h, f_m, f_n, f_k, f_p, f_u)$ where $f_l, f_h, f_m, f_n, f_k, f_p$ and f_u are real numbers and $f_l \leq f_h \leq f_m \leq f_n \leq f_k \leq f_p \leq f_u$.

The membership function $\mu_{\tilde{F}}(x)$ for an heptagonal fuzzy number \tilde{F} is defined by:

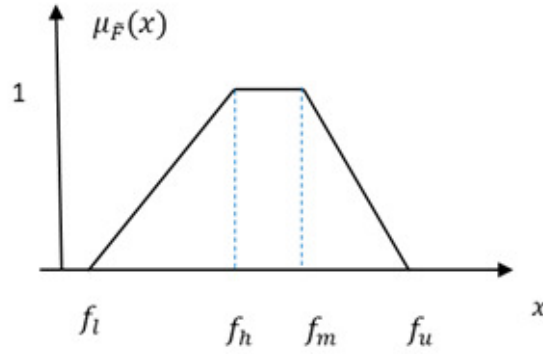


Figure 2. Graphical representation of trapezoidal fuzzy number.

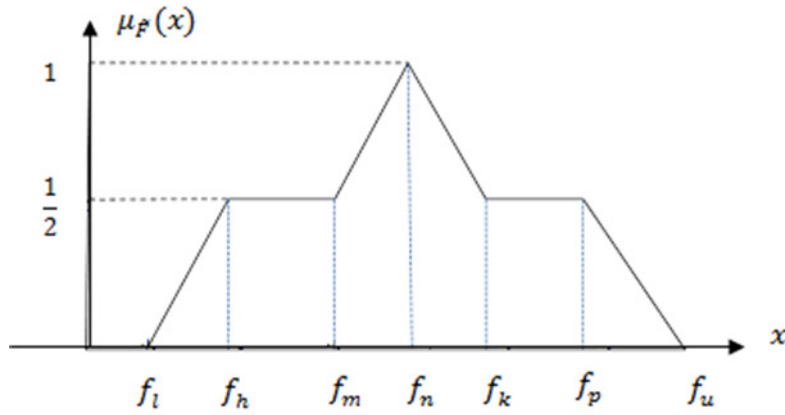


Figure 3. Graphical representation of heptagonal fuzzy number.

$$\mu_{\bar{F}}(x) = \begin{cases} 0 & \text{if } x < f_l \\ \frac{1}{2} \left(\frac{x-f_l}{f_h-f_l} \right) & \text{if } f_l \leq x \leq f_h \\ \frac{1}{2} & \text{if } f_h \leq x \leq f_m \\ \frac{1}{2} + \frac{1}{2} \left(\frac{x-f_m}{f_n-f_m} \right) & \text{if } f_m \leq x \leq f_n \\ \frac{1}{2} + \frac{1}{2} \left(\frac{f_k-x}{f_k-f_n} \right) & \text{if } f_n \leq x \leq f_k \\ \frac{1}{2} & \text{if } f_k \leq x \leq f_p \\ \frac{1}{2} \left(\frac{f_u-x}{f_u-f_p} \right) & \text{if } f_p \leq x \leq f_u \\ 0 & \text{if } x > f_u \end{cases}$$

4. The proposed Dhouib-Matrix-TSP1 Method

Recently, a new column-row heuristic called Dhouib-Matrix-TP1 (DM-TP1) is developed to solve the TP in [40]. Besides, the DM-TP1 method is enhanced to solve the trapezoidal fuzzy TP in [70] where a robust ranking function is used and a new operation to select nodes is added. Next, the first adaptation of the Dhouib-Matrix-TP1 heuristic to solve the TP in single-valued trapezoidal neutrosophic environment is introduced in [71]. The author exploited a defuzzification function in order to convert the single-valued trapezoidal neutrosophic numbers to crisp numbers

and proposed a metric function (Average-Min) to perform the nodes selection process. DM-TP1 is subdivided to nine simple steps (See Figure 4) and it is characterized by its rapidity (just $n + m$ iterations) to generate an initial basic feasible solution.

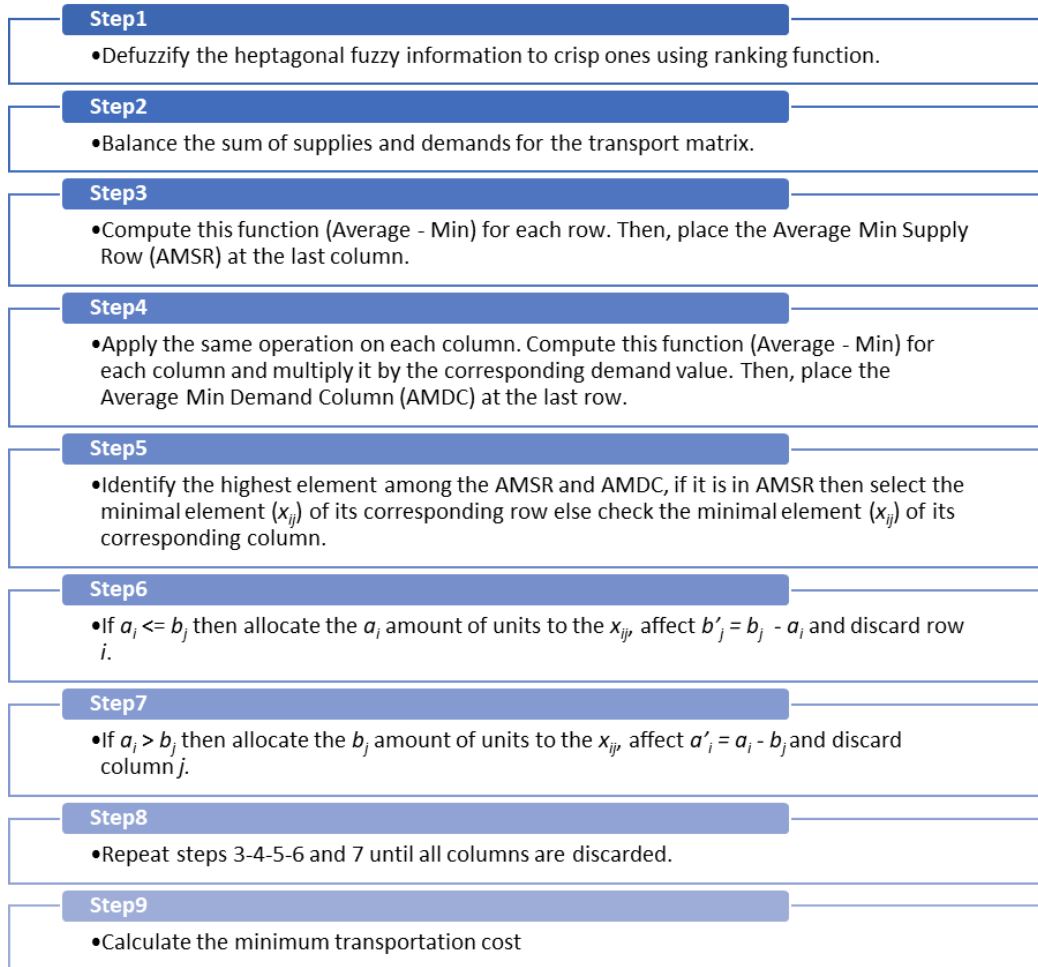


Figure 4. The nine steps of the DM-TP1.

Actually, DM-TP1 belong to the novel concept of Dhouib-Matrix (DM) where several heuristics, metaheuristics and optimal method are developed such as Dhouib-Matrix-TSP1 (DM-TSP1), Dhouib-Matrix-API (DM-API), Far-to-Near (FtN), Dhouib-Matrix3 (DM3), Dhouib-Matrix-4 (DM4) and Dhouib-Matrix-SPP (DM-SPP). In fact, the DM-TSP1 heuristic is introduced by [72, 73, 74] and tested under uncertain domain by [75, 76, 77, 78, 79, 80, 81, 82]. Whereas, the DM-API heuristic is presented in [83, 84, 85] and the FtN local search method is described in [86]. Furthermore, two metaheuristics are inspired from the novel heuristic DM-TSP1: The DM3 is introduced in [87, 88, 89]; and the DM4 is presented in [90, 91, 92, 93, 94, 95]. Moreover, the optimal method DM-SPP is designed in [96] to unravel the shortest path problem and advanced for mobile robot shortest path problem in [97, 98]. In addition, an original method namely DM-MSTP is invented in [99, 100].

5. Computational results

Two numerical examples (balanced and unbalanced) with step-by-step applications are used to show the performance of the novel heuristic DM-TP1.

5.1. Balanced numerical example

Let us consider this balanced TP example taken from [65] where all data are represented as heptagonal fuzzy number. Figure 5 summarized this example with three sources (denoted by rows S_1, S_2 and S_3) and four destinations (indicated by columns D_1, D_2, D_3 and D_4).

	D_1	D_2	D_3	D_4	Supply
S_1	(-1, 0, 1, 2, 4, 5, 6)	(0, 1, 2, 3, 5, 6, 7)	(8, 9, 10, 11, 13, 14, 15)	(4, 5, 6, 7, 9, 10, 11)	(1, 3, 5, 6, 8, 10, 12)
S_2	(-2, -1, 0, 1, 3, 4, 5)	(-3, -2, -1, 1, 2, 3, 4)	(2, 4, 5, 6, 8, 9, 11)	(-3, -1, 0, 1, 4, 5, 6)	(-2, -1, 0, 1, 3, 4, 5)
S_3	(2, 3, 4, 5, 7, 8, 9)	(3, 6, 7, 8, 10, 12, 13)	(11, 12, 14, 15, 17, 18, 21)	(5, 6, 8, 9, 11, 12, 15)	(5, 6, 8, 10, 13, 15, 17)
Demand	(4, 5, 6, 7, 9, 10, 11)	(1, 2, 3, 5, 7, 8, 10)	(0, 1, 2, 3, 5, 6, 7)	(-1, 0, 1, 2, 4, 5, 6)	

Figure 5. Transportation Problem with heptagonal fuzzy number.

The DM-TP1 heuristic starts by the defuzzification process: convert the heptagonal fuzzy number to crisp number using equation below described by [65].

$$R(N_h) = \frac{3n_1 + 6n_2 + 4n_3 + 10n_4 + 4n_5 + 6n_6 + 3n_7}{36}$$

Figure 6 describes the generated matrix after the defuzzification process. This matrix is balanced, AMSR and

	D_1	D_2	D_3	D_4	Supply
S_1	2.361	3.361	11.361	7.361	6.361
S_2	1.361	0.638	6.361	1.638	1.361
S_3	5.361	8.444	15.277	9.277	10.444
Demand	7.361	5.083	3.360	2.361	

Figure 6. The crisp matrix.

AMDC are computed using the (Average – Min) function (See Figure 7). The highest value (4.64) between the AMSR and the AMDC elements is at the third element of AMDC. Thus, the third column is selected, its smallest element is selected (6.361) at position d_{23} , the smallest value between supply and demand (1.36) is allocated and row 2 is discarded.

Now, compute AMSR and AMDC and choose their highest value (4.23). Then, the third row is selected, its smallest element is selected (5.361) at position d_{31} (See Figure 8), the smallest value between supply and demand (7.36) is allocated and column 1 is discarded.

	D_1	D_2	D_3	D_4	Supply	AMSR
S_1	2.361	3.361	11.361	7.361	6.36	3.75
S_2	1.361	0.638	6.361	1.638	1.36	1.86
S_3	5.361	8.444	15.277	9.277	10.44	4.23
Demand	7.36	5.08	3.36	2.36		
AMDC	1.66	3.51	4.64	4.46		

Figure 7. The element d_{23} is selected.

	D_1	D_2	D_3	D_4	Supply	AMSR
S_1	2.361	3.361	11.361	7.361	6.36	3.75
S_2						
S_3	5.361	8.444	15.277	9.277	10.44	4.23
Demand	7.36	5.08	2.00	2.36		
AMDC	1.50	2.54	1.96	0.96		

Figure 8. The element d_{31} is selected.

Again, compute AMSR and AMDC and choose their highest value (4.00). Then, the first row is selected, its smallest element is selected (3.361) at position d_{12} (See Figure 9), the smallest value between supply and demand (2.54) is allocated and column 2 is discarded.

	D_1	D_2	D_3	D_4	Supply	AMSR
S_1		3.361	11.361	7.361	6.36	4.00
S_2						
S_3		8.444	15.277	9.277	3.08	2.56
Demand		5.083	2.00	2.36		
AMDC		2.54	1.96	0.96		

Figure 9. The element d_{12} is selected.

Hence, compute AMSR and AMDC and choose their highest value (3.00). Then, the third row is selected, its smallest element is selected (9.277) at position d_{34} (See Figure 10), the smallest value between supply and demand (2.36) is allocated and column 4 is discarded.

Next, compute AMSR and AMDC and choose their highest value (2.00). Then, the first row is selected, its smallest element is selected (11.361) at position d_{13} (See Figure 11), the smallest value between supply and demand (1.28) is allocated and row 1 is discarded.

Finally, there is only one element in the matrix at position d_{33} (See Figure 12), the smallest value between supply and demand (0.72) is allocated and row 3 is discarded.

Therefore, the total transportation cost using the novel heuristic DM-TP1 is calculated as follows:

$$(x_{23} * 1.361) + (x_{31} * 7.361) + (x_{34} * 5.083) + (x_{34} * 2.361) + (x_{13} * 1.278) + (x_{33} * 0.722) = 112.655$$

$$(6.361 * 1.361) + (5.361 * 7.361) + (3.361 * 5.083) + (9.277 * 2.361) + (11.361 * 1.278) + (15.277 * 0.722) =$$

	D_1	D_2	D_3	D_4	Supply	AMSR
S_1			11.361	7.361	1.28	2.00
S_2						
S_3			15.277	9.277	3.08	3.00
Demand			2.00	2.36		
AMDC			1.96	0.96		

Figure 10. The element d_{34} is selected.

	D_1	D_2	D_3	D_4	Supply	AMSR
S_1			11.361		1.28	0.00
S_2						
S_3			15.277		0.72	0.00
Demand			2.00			
AMDC			1.96			

Figure 11. The element d_{13} is selected.

	D_1	D_2	D_3	D_4	Supply	AMSR
S_1						
S_2						
S_3			15.277		0.72	0.00
Demand			0.72			
AMDC			0.00			

Figure 12. The element d_{33} is selected.

112.655

Obviously, the proposed heuristic DM-TP1 can easily and rapidly (just after 6 iterations) generate the optimal solution. Figure 13 depicts the graphical representation of the transportation plan solution.

DM-TP1 is compared to other methods presented in [65]: the Vogel Approximation Method (VAM) and the MODified Distribution Method (MODI). Table 1 depicts the generated solution by VAM, MODI and DM-TP1 where DM-TP1 found directly the optimal solution with a deviation of (02.58%) to VAM.

Table 1 Comparing DM-TP1 to other heuristics

Methods	VAM	MODI	DM-TP1
Best solution	115.564	112.655	112.655

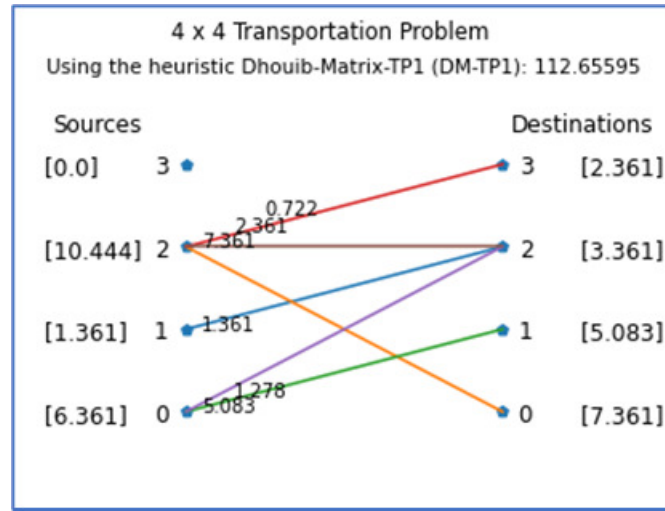


Figure 13. The transportation plan generated by DM-TP1.

5.2. Unbalanced numerical example

Let us consider this unbalanced TP example developed by [101] where three sources will cover the demand of three destinations. All data are indicated by heptagonal fuzzy number (see Figure 14.a) with three sources (denoted by rows S_1, S_2 and S_3) and three sources (represented by rows D_1, D_2 and D_3).

a)					b)				
	D_1	D_2	D_3	Supply		D_1	D_2	D_3	Supply
S_1	(3,6,2,1,5,0,4)	(2,3,1,4,3,6,5)	(2,4,3,1,6,5,2)	(2,2,1,2,1,1,0)	S_1	6	6.75	7.25	2.5
S_2	(2,7,7,6,3,2,1)	(1,3,5,7,9,11,13)	(0,1,2,4,6,0,5)	(3,2,1,4,5,0,1)	S_2	8.25	14.5	7	10.5
S_3	(3,6,3,2,1,8,7)	(3,4,3,2,1,1,0)	(2,4,6,8,10,12,14)	(2,4,3,1,6,5,2)	S_3	7.75	16.5	10.5	14.5
Demand	(0,1,2,4,6,0,5)	(0,4,6,4,6,2,0)	(2,7,7,6,3,2,1)		Demand	6	7	7.5	

Figure 14. TP matrix.

Before starting DM-TP1, the heptagonal fuzzy data are converted to crisp ones (see Figure 14.b) using equation below.

$$R(N_h) = \int_0^1 0.5 (f_\alpha^l, f_\alpha^u) d\alpha$$

$$R(N_h) = \int_0^1 0.5 \{ (n_2 - n_1)\alpha + n_1, n_4 - (n_4 - n_3)\alpha, (n_6 - n_5)\alpha + n_5, n_7 - (n_7 - n_5)\alpha \} d\alpha$$

Besides, this TP matrix is not balanced for that dummy row and column are added (See Figure 15).

Next, the row (AMSR) and column (AMDC) are computed and only six iterations are needed to solve this problem (See Figure 16).

Therefore, the total transportation cost using the novel heuristic DM-TP1 is:

$$(x_{12} * 2.5) + (x_{23} * 7.5) + (x_{31} * 6) + (x_{22} * 3) + (x_{32} * 1.5) + (x_{34} * 7) = 184.12$$

$$(6.75 * 2.5) + (7 * 7.5) + (7.75 * 6) + (14.5 * 3) + (16.5 * 1.5) + (0 * 7) = 184.12$$

Figure 17 depicts the best solution generated by DM-TP1.

	D_1	D_2	D_3	D_4	Supply
S_1	6	6.75	7.25	10000	2.5
S_2	8.25	14.5	7	10000	10.5
S_3	7.75	16.5	10.5	10000	14.5
S_4	10000	10000	10000	10000	0
Demand	6	7	7.5	7	

Figure 15. Balanced crisp matrix.

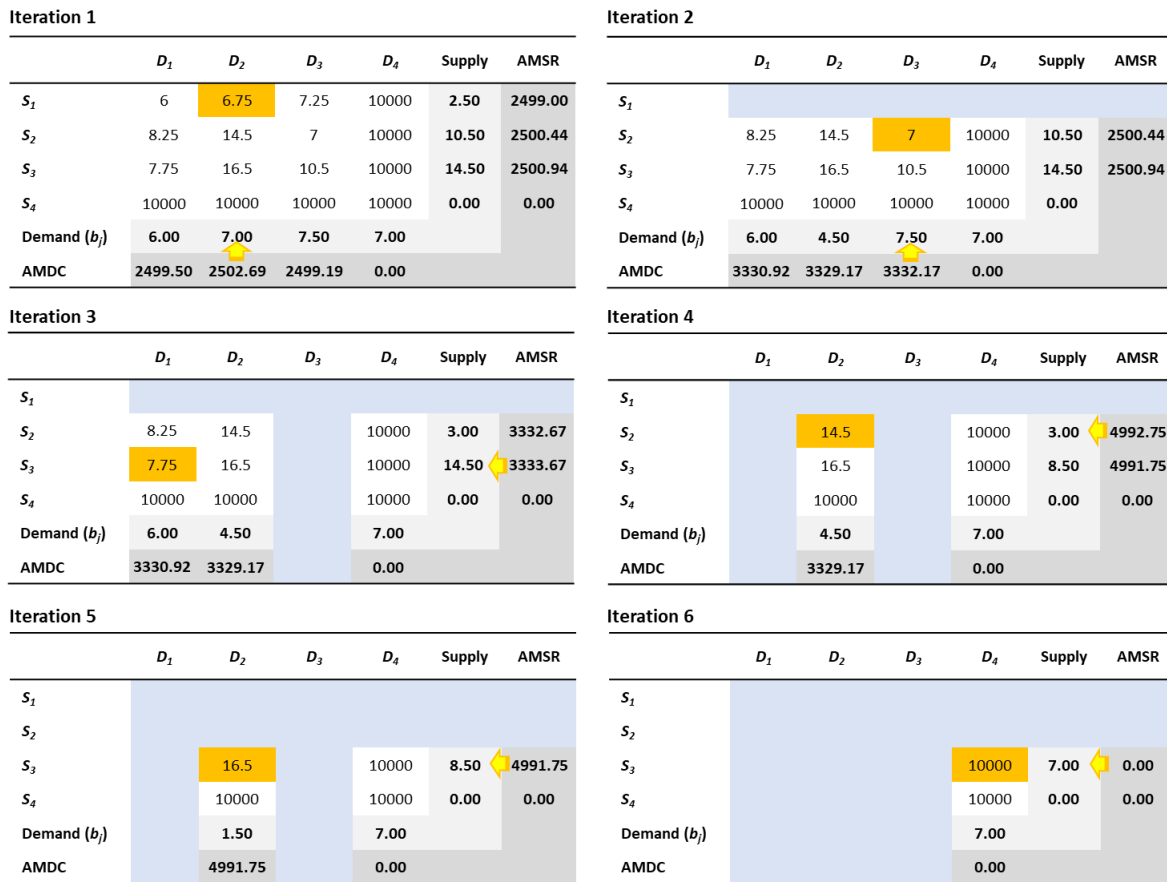


Figure 16. DM-TP1 needs only six simple iterations to solve the TP.

Table 2: comparing DM-TP1 to other methods

	Russel's Method	North West Corner Method	Least Cost Method	DM-TP1
Best solution	280.375	224.125	219.750	184.120
Deviation to DM-TP1	52.278	21.728	19.352	0.000

Figure 18 shows that the proposed heuristic DM-TP1 outperforms all the other methods (Russel's Method, North West Corner Method and Least Cost Method with corresponding deviations to DM-TP1 52.278

	D_1	D_2	D_3	D_4	Supply
S_1	6	6.75	7.25	10000	2.5
S_2	8.25	14.5	7	10000	10.5
S_3	7.75	16.5	10.5	10000	14.5
S_4	10000	10000	10000	10000	0
Demand	6	7	7.5	7	

Figure 17. The solution generated by DM-TP1.

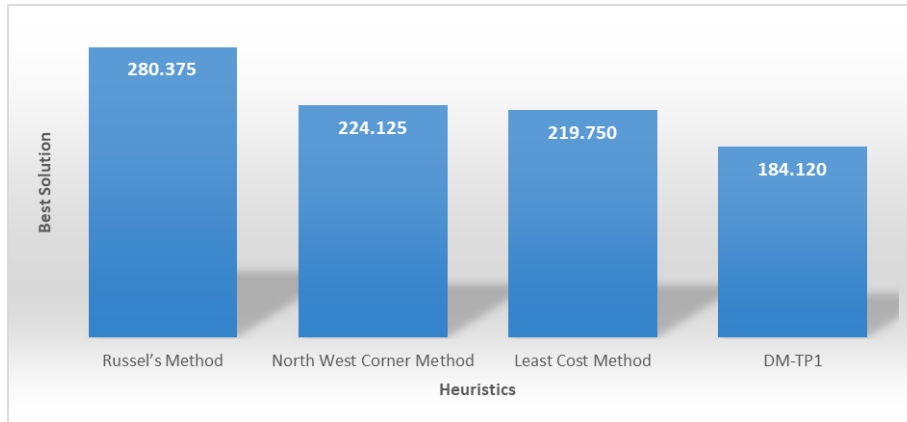


Figure 18. Graphical representation of method solutions.

6. Conclusion

The transportation problem (TP) is an integral part of the supply chain network and plays a crucial role because it directly affects the cost, efficiency and reliability of the overall system. The TP involves supplying products from several sources to different demand destinations with the aim of minimizing the total shipping cost of transportation while satisfying both supply and demand limits. In this paper, the TP is considered under heptagonal fuzzy number. To achieve efficient total shipping cost, the novel heuristic Dhouib-Matrix-TP1 (DM-TP1) is enhanced. Numerical simulations of balanced and unbalanced heptagonal fuzzy transportation problems show the performance of the novel heuristic DM-TP1. Further extensions of this research may include the application of DM-TP1 to solve the transportation problem under multi-objective or dynamic environments. In addition, exploring the DM-TP1 heuristic with Fermatean Neutrosophic number in transportation problem could further enhance the heuristic's capabilities.

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