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Bitcoin Halving Cycles and Their Impact on the Gold Relationship

Mustapha KHALFOUNI[1,*], Rafia FRIJ[2],
Noaman LAKCHOUCH[1], Mohamed LAMARTI SEFIAN[1]

[1] *Sciences, Technology and Innovation (STI), Abdelmalek Essaadi University, Tetouan, Morocco*
[2] *National School of Commerce and Management of Tangier, Abdelmalek Essaadi University, Morocco*

Abstract This paper examines the interdependencies between Bitcoin and gold within the context of Bitcoin halving cycles. Using a comprehensive econometric approach, including cointegration tests, VAR and VECM models, DCC-GARCH modeling, and wavelet coherence analysis, we investigate short- and long-term dynamics linking these two assets. Our findings indicate that, over the long term, Bitcoin exhibits characteristics similar to gold as a safe haven despite its high volatility and sensitivity to short-term shocks. Moreover, the incorporation of macroeconomic variables, such as stock market indices and oil prices, highlights the significant influence of broader economic conditions on this relationship. These results suggest that while Bitcoin may serve as a complementary asset to gold in diversified portfolios, prudent management is essential to mitigate the risks associated with its speculative nature.

Keywords Bitcoin; Gold; Halving; Interdependencies; Volatility; Cointegration; Vector Error Correction Model (VECM), DCC-GARCH modeling

AMS 2010 subject classifications 91B62, 91B84, 91G70, 91G80, 62M10

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1. Introduction

The interplay between digital assets and precious metals, notably Bitcoin and gold, continues to draw scholarly and market attention. Bitcoin, frequently likened to gold for its perceived parallels, mirrors certain attributes of the traditional physical asset: finite availability, adoption as a means of preserving wealth, and utility in mitigating inflationary risks [1], [2]. Nonetheless, their market dynamics often diverge markedly under shifting economic or geopolitical conditions. Gold, for example, has long served as a stabilizing asset during periods of financial instability [3], whereas Bitcoin's reliability as a digital safeguard remains contested, with its appeal oscillating in response to regulatory shifts and investor sentiment [4].

Most research to date has focused on specific periods, such as the aftermath of financial crises or during extreme market volatility, often ignoring the stability of the relationship between Bitcoin and gold across different economic periods [5]. This gap in the literature highlights the need for studies that examine this relationship more continuously and comprehensively, particularly during significant Bitcoin-related events such as halvings [6].

This rigorous econometric analysis aims to uncover the dynamics and interdependencies that link these two valuable assets. By exploring their correlations and reactions to economic shocks, this study seeks to understand the nature of their relationship and specify this relationship between two halving dates. Thus, the central question of this analysis is: how does the relationship between Bitcoin and gold evolve during the intervals between two Bitcoin halvings, and what are the specific dynamics and interdependencies that link them as stores of value?

*Correspondence to: Mustapha KHALFOUNI (Email: mustapha.khalfouni@etu.uae.ac.ma). ENS-MARTIL, Abdelmalek Essaadi University, Tetouan, Morocco.

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2. Literature review

Gold and Bitcoin are two asset classes often compared, both being considered potential safe havens. However, there are fundamental differences between the two, particularly in terms of their nature, scarcity, and usage. This literature review explores the complex relationship between gold and Bitcoin, examining the perspectives of various researchers and analysts from a statistical and econometric standpoint.

The prices of gold and Bitcoin have been the subject of much debate and research. Some analysts have observed an increasing positive correlation between the two assets, suggesting that they may behave as alternative safe-havens [1]. Others argue that the relationship is more complex and can be influenced by various factors, such as the macroeconomic environment, investor sentiment, and government regulations [4].

Kyriazis [7], explores the possibility of Bitcoin playing the role of a haven, much like gold. His study reveals that Bitcoin is not as effective as gold in preserving its value during economic turmoil. While Bitcoin may offer some protection, its effect is weaker, and its correlation with the price of gold can even be negative at times. Kyriazis concludes that Bitcoin could still be an interesting complement to a portfolio that already contains gold, but currently, gold remains the most established option as a safe haven [8].

The article by Shehzad et al. (2021) [9] examines whether gold has proven to be a better investment than Bitcoin during the COVID-19 crisis. Using a mathematical approach called ”wavelets” to examine the relationship between gold and Bitcoin prices at different periods, the results suggest that investing in gold in most cases during the pandemic was more advantageous than investing in Bitcoin. Shehzad et al. found a stronger positive correlation between gold and certain stock indices (such as the CAC 40) at higher frequencies, indicating that gold could be a good diversification tool during the pandemic. In contrast, the relationship between Bitcoin and stock indices was weaker, or even negative at certain times [9].

The dynamics of Bitcoin and gold markets can also be analyzed through behavioral finance. According to Kahneman and Tversky (1979) [10], cognitive biases such as risk aversion, herd behavior, and overconfidence significantly influence investor decisions, especially in periods of uncertainty or volatility. In the context of Bitcoin halving events, investors may exhibit pronounced biases, amplifying price volatility due to speculative behavior and heightened emotional reactions [11].

This study focuses exclusively on the specific periods surrounding Bitcoin halving events to analyze the econometric relationship between Bitcoin and gold. The goal is to assess how Bitcoin's periodic supply shocks impact this relationship. This precise methodological approach allows for direct observation of potential structural changes caused by these supply reductions, providing insights into the stability and nature of the correlation between Bitcoin and gold.

3. Materials and Methods

This analysis examines the relationship between gold, a traditional safe haven, and Bitcoin, a modern cryptocurrency, particularly in the context of Bitcoin halvings. These halvings occur approximately every four years and involve reducing the rewards for Bitcoin miners. They have historically been crucial events, marking significant turning points in Bitcoin's price evolution and potentially impacting its correlation with gold.

Bitcoin halvings aim to control inflation by halving the reward for mining new blocks, thereby limiting the supply of new bitcoins. This mechanism, embedded in Bitcoin's protocol, has historically led to substantial increases in Bitcoin's price due to the reduced rate of new bitcoins being generated, increasing scarcity and increasing the price. The halving periods are defined as follows:

- The first halving period (Halving 1) is from November 8, 2012, to July 9, 2016;
- The second halving period (Halving 2) is from July 9, 2016, to May 11, 2020;
- The third halving period (Halving 3) is from May 11, 2020, to April 20, 2024.

3.1. Materials

This study relies heavily on archival market data. Daily Bitcoin price datasets will be sourced from established cryptocurrency market aggregators and financial repositories†. Similarly, gold price histories will be procured from trusted financial data providers‡. Data retrieval and subsequent analytical procedures will be executed programmatically using the R statistical environment.

To isolate Bitcoin's halving-driven market effects, the analysis restricts its scope to timeframes directly preceding and following these events, intentionally omitting alternative digital assets (e.g., Ethereum, Ripple) and non-halving-related periods. This exclusion minimizes noise from unrelated market mechanisms or external variables, enabling focused identification of halving-linked patterns—including price fluctuations and demand trajectories. Such methodological rigor enhances the validity of conclusions regarding Bitcoin's supply-driven market behavior.

Post-data acquisition, logarithmic returns and time-varying volatility for both Bitcoin and gold were derived using the standard formulations:

The daily returns R_t were calculated using the formula:

$$R_t = (\log(P_t) - \log(C_{P-1})) \times 100$$

where :

- R_t: is the daily return of day t
- P_t : is the closing price on day t
- P_{t-1}: is the closing price on day $(t-1)$
- log: is the natural logarithm function.

This methodology computes the logarithmic return by deriving the difference between the natural logarithms of the closing price on day t and day $t-1$. To analyze the broader price trajectory, the monthly return average was calculated, while the variance of these returns was used to evaluate historical price fluctuations, as defined by the formula below:

$$\overline{r} = \frac{1}{t-1}\sum_{i=1}^{t-1} r_i$$

$$\sigma = \sqrt{\frac{N_c}{t-2}\sum_{i=1}^{t-1}(r_i - \overline{r})^2}$$

Where:

- r_i : Log return on the ith day
- σ: Historical Volatility.
- t : Number of historical day used in the volatility estimate.
- N_c : The number of closing prices in a year.

3.2. Methodology

The first step of the study involves collecting historical daily data on bitcoin and gold closing prices over multiple Bitcoin halving cycles. The data will be cleaned and synchronized to align the time series. Daily returns will be calculated. Descriptive analysis will be conducted to understand the distribution and dispersion of prices. Cointegration tests and econometric models such as VECM and GARCH will be used to examine short-term and

†Bitcoin historical data source: https://coincodex.com/crypto/bitcoin/historical-data/

‡Gold futures (GC=F) historical data: https://finance.yahoo.com/quote/GC%3DF/history/?period1=1325376000&period2=1714521600

long-term relationships and price volatility. The analysis will compare price dynamics between different halving periods. The results will provide insights into the dynamics and interdependencies between Bitcoin and gold across different economic periods.

3.2.1. Dynamic Conditional Correlation (DCC) .

The Dynamic Conditional Correlation (DCC) model is a multivariate approach based on the GARCH methodology, designed to capture time-varying correlations between financial asset returns. Initially proposed by [13], this model differs from traditional constant-correlation approaches by allowing correlations to evolve dynamically over time. This flexibility enables it to better reflect the changing interactions between financial markets, particularly during periods of turbulence or crisis, The construction of the DCC model relies on two main steps:

Step 1: Volatility Modeling .

Each asset's return series is modeled using a univariate GARCH process (e.g., GARCH(1,1)):

$$r_t = \mu_t + \epsilon_t, \quad \epsilon_t = H_t^{1/2} z_t$$

where:

- r_t: Asset returns at time t.
- μ_t: Conditional mean (often assumed to be zero).
- ϵ_t: Residuals.
- H_t: Conditional covariance matrix.
- z_t: Standardized residuals ($z_t \sim N(0, I)$).

The conditional variance $h_{i,t}$ for asset i is modeled as:

$$h_{i,t} = \omega_i + \alpha_i \epsilon_{i,t-1}^2 + \beta_i h_{i,t-1}$$

where ω_i, α_i, and β_i are GARCH parameters.

Step 2: Correlation Modeling .

The conditional covariance matrix H_t at time t is represented by:

$$H_t = D_t R_t D_t$$

where:

- $D_t = \text{diag}(h_{1,t}, \ldots, h_{N,t})$ is a diagonal matrix with conditional standard deviations $h_{i,t}$ derived from individual GARCH processes.
- R_t : Time-varying correlation matrix defined as: $R_t = \text{diag}(Q_t)^{-\frac{1}{2}}\, Q_t\, \text{diag}(Q_t)^{-\frac{1}{2}}$

The DCC model specifies the evolution of R_t as:

$$Q_t = (1 - \theta_1 - \theta_2)\bar{Q} + \theta_1(z_{t-1} z_{t-1}^t) + \theta_2 Q_{t-1}$$

where:

- Q_t: Intermediate correlation matrix.
- $\bar{Q}$: Unconditional covariance matrix of standardized residuals z_t defined as $z_t = D_t^{-1}\varepsilon_t$. Here, ε_t is the vector of standardized residuals.
- θ_1, θ_2: DCC parameters, with $\theta_1 > 0$, $\theta_2 > 0$ and $\theta_1 + \theta_2 < 1$ to ensure stationarity.

This model is popular because it accurately captures evolving market dynamics and remains computationally efficient. (Implementation in R in appendix).

3.2.2. wavelet coherence analysis .

Wavelet analysis extends classical spectral analysis by allowing the study of relationships between two time series across different time scales and at different points in time. Unlike the Fourier transform, it can capture local structures in time series.

The core tool of this analysis is the **continuous wavelet transform (CWT)**, defined as:

$$W_x(s,\tau) = \int_{-\infty}^{\infty} x(t)\psi^*\left(\frac{t-\tau}{s}\right) dt$$

where:

- $W_x(s,\tau)$ is the wavelet transform coefficient at scale s and time τ,
- $x(t)$ is the analyzed time series,
- $\psi^*(t)$ is the complex conjugate of the mother wavelet,
- s represents the scale parameter, linked to frequency.

The same transformation is applied to a second time series $y(t)$ (i.e., Bitcoin or gold returns).

Definition of Wavelet Coherence .

Wavelet coherence between two time series $x(t)$ (Bitcoin returns) and $y(t)$ (gold returns) is a normalized measure of their local correlation in the time-frequency domain. It is defined as:

$$R_x^2(s,\tau) = \frac{|S(s^{-1}W_x(s,\tau)W_y^*(s,\tau))|^2}{S(s^{-1}|W_x(s,\tau)|^2)\cdot S(s^{-1}|W_y(s,\tau)|^2)}$$

where:

- $R_x^2(s,\tau)$ is the wavelet coherence coefficient, ranging between 0 (no correlation) and 1 (perfect correlation),
- $W_x(s,\tau)$ and $W_y(s,\tau)$ are the wavelet transforms of $x(t)$ and $y(t)$,
- $S(\cdot)$ represents a smoothing operation to prevent over-interpretation of local fluctuations.

A high coherence level at a particular frequency suggests a stable relationship between Bitcoin and gold at that time scale. (Implementation in R in appendix)

3.2.3. Test of stationarity .

The widely used and popular test for non-stationarity is the unit root test proposed by Dickey and Fuller (ADF: Augmented Dickey-Fuller). The null hypothesis of the test is the presence of a unit root, indicating stochastic non-stationarity. The ADF test strategy and procedure are based on the following three models:

$$(M1): \Delta Y_t = \rho\cdot Y_{t-1} + \sum_{j=1}^{n}\phi_j\cdot\Delta Y_{t-j} + \epsilon_t$$

$$(M2): \Delta Y_t = \rho\cdot Y_{t-1} + c + \sum_{j=1}^{n}\phi_j\cdot\Delta Y_{t-j} + \epsilon_t$$

$$(M3): \Delta Y_t = \rho\cdot Y_{t-1} + c + \beta\cdot t + \sum_{j=1}^{n}\phi_j\cdot\Delta Y_{t-j} + \epsilon_t$$

The implementation of the ADF test follows a rigorous sequential approach (DICKEY et al.1979) [12]. It begins with determining the optimal lag order p using information criteria such as AIC or BIC. Once p is selected, the test systematically starts with the most comprehensive model (**M3**), which includes both a **constant term** (c) and a **deterministic time trend** (β_t). At this stage, critical values specific to M3 are used to assess the presence of a unit root. If the null hypothesis (non-stationarity) is rejected (ADF statistic below the critical threshold), supplementary analyses are conducted to verify the statistical significance of the c and β_t terms using hypothesis tests (e.g., F-test or t-test).

If these terms are deemed insignificant, the model is considered overparameterized, and the test is repeated with the constrained model **M2** (including only the constant). Should M2 also prove inappropriate, the minimalist model **M1** (no constant or trend) is ultimately adopted. This hierarchical procedure ensures optimal model specification, avoiding biases from omitted variables or redundant terms. The final interpretation concludes stationarity if H_0 is rejected in the selected model or necessitates differencing if H_0 cannot be rejected. This method safeguards against erroneous conclusions arising from misspecification [14]. (Implementation in R in appendix)

3.2.4. Cointégration Test .

Cointegration is a statistical property of a set of non-stationary time series that share a long-term equilibrium relationship. Formally, if two or more *integrated* series (typically **I(1)**) can be linearly combined to form a **stationary** series (**I(0)**), they are said to be cointegrated. This implies deviations from their equilibrium relationship are temporary and mean-reverting.

Engle-Granger Two-Step Method .

Step 1: Estimate Long-Run Relationship

$$y_t = \beta_0 + \beta_1 x_t + \epsilon_t$$

where y_t and x_t are I(1) variables, and ϵ_t represents residuals.

Step 2: Test Residual Stationarity ADF test applied to residuals:

$$\Delta\epsilon_t = \alpha + \gamma\epsilon_{t-1} + \sum_{i=1}^{p} \phi_i \Delta\epsilon_{t-i} + u_t$$

- Null Hypothesis (H_0): $\gamma = 0$ (non-stationarity)
- Alternative Hypothesis (H_1): $\gamma < 0$ (stationarity)

If the null hypothesis of non-stationarity ($\gamma = 0$) is rejected, the two series are concluded to be cointegrated.

Johansen Test (Multivariate) .

The **Johansen cointegration test** is a statistical method used to determine whether multiple time series are cointegrated, meaning they share a long-term equilibrium relationship despite being potentially non-stationary. Based on a **Vector AutoRegressive (VAR) model**, the test follows Johansen's representation [19] to examine the presence of cointegration within a system of multiple variables. The test relies on the **Vector Error Correction Model (VECM)** equation:

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta X_{t-i} + \epsilon_t$$

where X_t is a vector of the time series studied (e.g., $X_t = [BTC_t, XAU_t]$ for Bitcoin and Gold), ΔX_t is the first difference of X_t, Π is a matrix containing cointegration information, Γ_i are short-term coefficient matrices, ϵ_t is a white noise error term, and p is the number of lags. The test examines the **rank of the matrix** Π: if rank$(\Pi) = 0$, there is **no cointegration**; if rank$(\Pi) = k$ (k represents the number of time series) , all series are stationary; if $0 <$ rank$(\Pi) < k$, some **cointegrating relationships exist**, meaning that linear combinations of the non-stationary variables are stationary.

The test evaluates the following statistical hypotheses: the **null hypothesis** H_0 states that there are at most r cointegration relations, while the **alternative hypothesis** H_1 suggests more than r cointegration relations. There are two test versions: the **trace test** (λ_{trace}), given by

$$\lambda_{\text{trace}}(r) = -T \sum_{i=r+1}^{k} \ln(1 - \lambda_i)$$

which tests the null that there are at most r cointegration relations, rejecting H_0 if λ_{trace} exceeds a critical value, and the **maximum eigenvalue test** (λ_{max}), given by

$$\lambda_{\text{max}}(r, r+1) = -T \ln(1 - \lambda_{r+1})$$

which tests the null that there are exactly r cointegration relations against $r+1$, rejecting H_0 if λ_{max} exceeds a critical value. The test conclusions are: if **no cointegration vector is found** ($r = 0$), the series are not cointegrated, and a **VAR model in differences** should be used; if $0 < r < k$, cointegration exists, meaning a **VECM model** is appropriate to capture long-term relationships; if $r = k$, all series are stationary, and a **standard VAR model** is suitable. This test is particularly useful for assessing long-term interactions between financial assets like Bitcoin and Gold, helping to evaluate their potential for diversification or their role as stores of value. (Implementation in R in appendix)

3.2.5. VAR Model .

A VAR (vector autoregressive) model is a statistical model that explains the dynamics of a set of variables based on their past values. The VAR model is based on the idea that variables are interconnected and their evolution is influenced by their own past and the past of other variables [14].

A VAR model can be written in the form of a system of linear equations. Each equation in the system represents the evolution of a variable based on its own past values and the past values of other variables. For example, a VAR model with two variables, x and y, can be written in the form.

$$Y_{1(t)} = \beta_0 + \beta_1 Y_{1(t-1)} + \beta_2 Y_{2(t-1)} + \epsilon_1(t)$$
$$Y_{2(t)} = \alpha_0 + \alpha_1 Y_{1(t-1)} + \alpha_2 Y_{2(t-1)} + \epsilon_2(t)$$

Here is a breakdown of the equation:

- $Y_{1(t)}$ and $Y_{2(t)}$ are the values of the variables at time t.
- β_0 and α_0 are the intercept terms.
- β_1 and β_2, α_1, and α_2 are the coefficients on the lagged values of the variables.
- $\epsilon_1(t)$ and $\epsilon_2(t)$ are the error terms.

The coefficients of the VAR model can be estimated using a variety of methods, including ordinary least squares, generalized least squares, and weighted least squares, [15] . VAR models are used in many fields, including economics and finance. They are used to forecast the evolution of variables, analyze the relationships between variables, and test hypotheses about the dynamics of variables following a random shock using the impulse response function, [16]. (the R code to estimate VAR model in the appendix)

3.2.6. Vector Error Correction Model (VCEM) .

The **VECM (Vector Error Correction Model)** is an extension of the VAR (Vector Autoregression) model used to analyze multivariate time series that are non-stationary but **cointegrated**. It allows for the modeling of both long-term (equilibrium) relationships and short-term dynamics between variables.

Long-Term Relationship (Cointegration Equation) .

If we have k variables $Y_t = (y_{1t}, y_{2t}, \ldots, y_{kt})'$ that are cointegrated, there exists a long-term relationship of the form:

$$\beta' Y_t = \varepsilon_t$$

where:

- β is a vector of cointegration coefficients (describes the long-term relationship).
- ε_t is a stationary error term (represents deviations from the equilibrium).

General Form of the VECM .

The VECM model is written as:

$$\Delta Y_t = \Pi Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \varepsilon_t$$

where:

- $\Delta Y_t = Y_t - Y_{t-1}$ is the first difference of the variables (to make the series stationary).
- $\Pi = \alpha\beta'$ is the error correction matrix, where:
  - α is the matrix of adjustment speeds (measures how quickly variables return to the long-term equilibrium).
  - β is the matrix of cointegration coefficients (describes the long-term relationship).
- Γ_i are the matrices of coefficients for the lagged terms ΔY_{t-i} (capture short-term dynamics).
- ε_t is a vector of error terms (assumed to be white noise).

Interpretation of the Components .

- **Error Correction Term** (ΠY_{t-1}): Captures deviations from the long-term equilibrium and the speed at which variables return to this equilibrium.
- **Lagged Terms** ($\sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i}$): Model the short-term dynamics between the variables.

When to Use the VECM Model? .

The **VECM (Vector Error Correction Model)** is used in the following cases:

- **Stationarity tests (ADF-test)** show that the variables are non-stationary in levels but become stationary after differencing (i.e., they are $I(1)$).
- **Cointegration tests (such as the Johansen test)** indicate a long-term relationship between the variables. A linear combination of the variables is stationary.

Conclusion.

- If a cointegration relationship exists, using a VECM model is recommended, as it captures both short-term dynamics and long-term relationships between the variables.
- If no cointegration relationship is found, use a VAR model in first differences.

3.2.7. Macroeconomic Control Variables and Data Sources .

To better isolate the relationship between Bitcoin and gold, we integrate macroeconomic control variables into our econometric model. These variables include the S&P 500 returns (representing stock market dynamics), oil prices (capturing the impact of commodity fluctuations), and inflation rates (as an indicator of monetary conditions). The inclusion of these controls ensures that our analysis accounts for exogenous shocks that may simultaneously influence Bitcoin and gold.

We apply a VECM to analyze the relationship between Bitcoin and gold while incorporating these macroeconomic factors. The model is formulated as:

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \beta Z_t + \epsilon_t$$

where:

- X_t is a vector containing the Bitcoin and gold returns,
- Π is the cointegration matrix that captures the long-term relationships,
- Γ_i are short-term adjustment coefficients,
- Z_t is a vector of macroeconomic control variables including:
  - $SP500_t$: S&P 500 prices,
  - Oil_t: Oil prices.
- ϵ_t represents the error terms assumed to be white noise.

The Johansen cointegration test is used to determine the number of potential cointegration relationships between the variables, When a relationship is identified, the error correction model (VECM) allows to analyze both the short-term dynamics and the long-term equilibrium adjustments between Bitcoin, gold, and the inserted macroeconomic variables. This methodology allows to understand more the Bitcoin-gold relationship by isolating the intrinsic interdependencies from general economic trends, thus ensuring that the results do not reflect solely the global market conditions.

The macroeconomic control variables used in this section, notably the S&P 500 index and OIL [§].

4. Analyse and Results

In this section, we will start by providing descriptive statistics for each period between two consecutive halving dates. After that, we will briefly analyze the dynamic interaction between gold and Bitcoin from 2012 to 2024. Next, we'll delve into the interaction between gold and Bitcoin for each halving period, focusing primarily on cointegration to study both short-term and long-term relationships.

4.1. Descriptive Statistic

Table 1. Descriptive Statistics of Gold and Bitcoin Across Halving Periods.

	Halving. 1		Halving. 2		Halving. 3	
Statistics	Gold	BTC.	Gold	BTC.	Gold	BTC.
mean	1359.8	275.5	1330.1	5754.6	1873.6	33048.8
sd.	204.6	243.2	125.6	3693.6	126.9	15802.9
Min.	1050.7	4.3	1110.0	513.4	1621.8	8813.9
Max.	1792.9	1237.6	1756.7	18972.3	2388.6	73066.7
Skewness	0.64	0.89	1.24	0.33	1.11	0.43
Kortosis	−0.96	0.57	0.97	−0.28	2.37	−0.63

The Table n° 1 presents descriptive statistics for Gold and Bitcoin (BTC) across three halving periods. For Gold, the mean price has shown a steady increase from 1359.8 in Halving 1 to 1873.6 in Halving 3, with relatively stable standard deviations indicating consistent variability. Skewness is positive across all periods, suggesting a longer right tail in the distribution, while kurtosis shifts from platykurtic in Halving 1 to leptokurtic in Halving 3, indicating an increase in outliers over time. For Bitcoin, the mean price has increased dramatically from 275.5 in Halving 1 to 33048.8 in Halving 3, with extremely high standard deviations reflecting significant volatility. Skewness is positive but decreases slightly across periods, and kurtosis values indicate a mixture of leptokurtic and platykurtic distributions, reflecting varying degrees of outliers. These statistics highlight the differing nature

[§]The historical data of the S&P 500 and OIL are publicly available on Yahoo Finance: https://finance.yahoo.com of these two assets, with Bitcoin exhibiting more speculative characteristics and higher risk compared to the more stable and traditional safe-haven asset, Gold.

4.2. Price Evolution

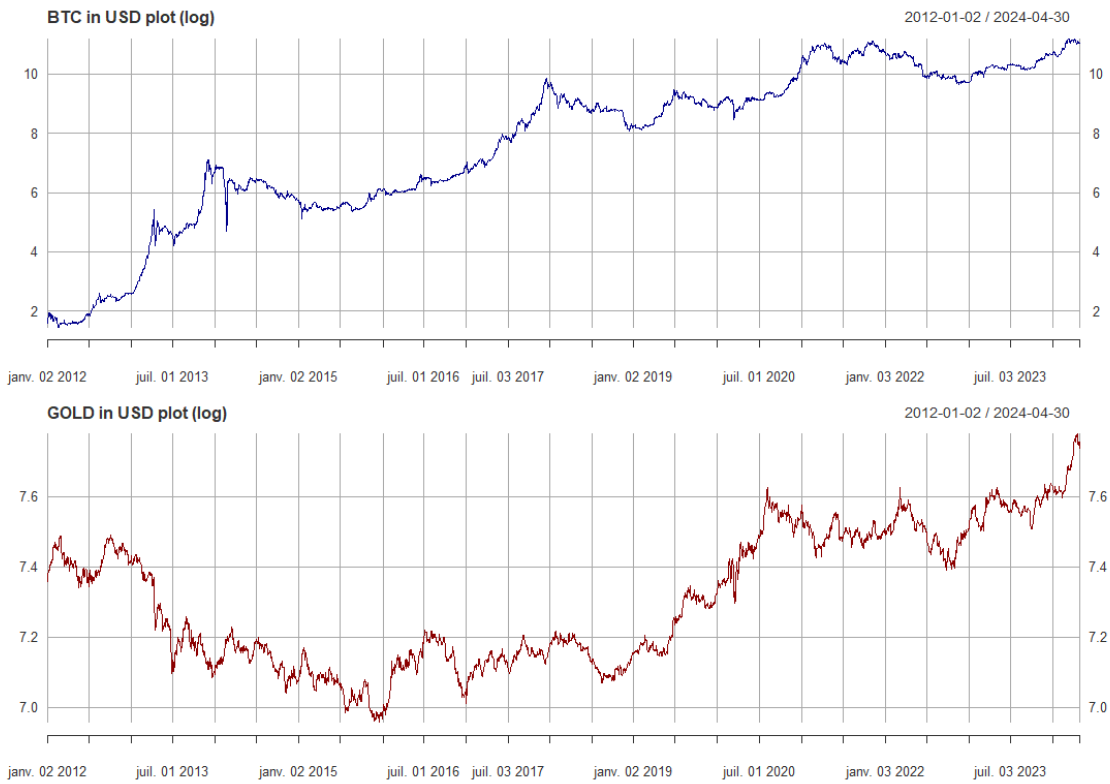

Figure 1. Evolution of BTC and Gold prices in logarithm between 2012 and 2024

The Figure N°: 1 presents two linear graphs comparing the prices of Bitcoin (BTC) and gold (GLD) in USD from January 2012 to April 2024. Bitcoin exhibits significant volatility, with initially stable prices followed by sharp increases and decreases, particularly with peaks at the end of 2017 and 2021. In contrast, gold shows a more stable trend, with a steady increase starting from 2019 after an initial decline until 2015. Overall, Bitcoin is highly volatile with speculative peaks, while gold maintains a stable and gradually increasing value, reflecting its role as a safe haven asset.

4.3. Evolution of volatility

The Figure N°: 2 illustrates the evolution of monthly volatility of Bitcoin (BTC) and gold (GLD) from January 2012 to May 2024. It can be observed that the volatility of Bitcoin is very high, with significant peaks in 2014 and 2020, surpassing 30%. On the other hand, gold exhibits much lower volatility, rarely exceeding 2.5%, with peaks around 2013 and 2020. Overall, Bitcoin is much more volatile than gold, which remains relatively.

4.4. Dynamic Conditional Correlation (DCC)

Figure n°: 3 illustrates the dynamic conditional correlation (DCC) between Bitcoin returns (BTC.Return) and gold returns (XAU.Return). This correlation fluctuates modestly, generally ranging between -0.1 and 0.3, indicating a moderate and unstable relationship between these two assets. Correlation peaks, notably between 0.2 and 0.3,

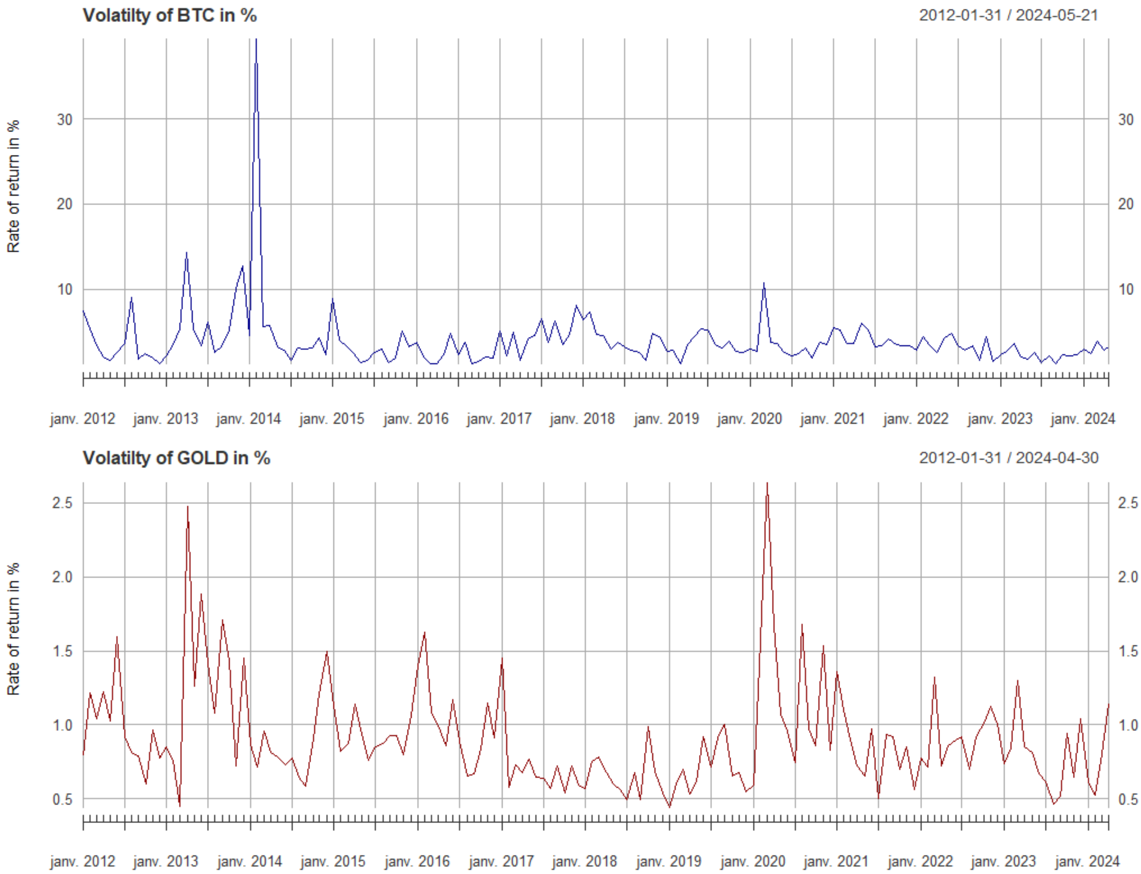

Figure 2. Comparison of Bitcoin and Gold Volatility (2012-2024)

coincide with significant macroeconomic events or periods of heightened market volatility. These peaks suggest temporary alignment in the behavior of Bitcoin and gold during specific market conditions. The low and irregular amplitude of the correlation underscores the distinct underlying factors influencing each asset, emphasizing that Bitcoin, as an emerging and speculative asset, and gold, traditionally viewed as a stable store of value, do not consistently move in tandem, although temporary convergence can occur during particular market events.

Graph n° 4 illustrates the evolution of the returns of an equally weighted (EW) portfolio with 1% Value at Risk (VaR) limits, based on a conditional weighted density. The blue line represents the portfolio returns, while the green and red lines mark the upper and lower 1% VaR boundaries, respectively. Extreme spikes, particularly around 2015 and 2020, indicate periods of high volatility and financial stress. Outside these episodes, the returns remain largely contained within these thresholds, suggesting relative portfolio stability under normal market conditions.

4.5. Wavelet coherence analysis

The wavelet coherence analysis reveals a notable duality in the relationship between Bitcoin and gold , Figure N° 5. Over long-term periods (exceeding 64 intervals), a strong coherence—highlighted by red and yellow zones—indicates a stable correlation between the two assets, suggesting a persistent interdependence. This dynamic likely arises from their shared perception as safe-haven assets during times of economic uncertainty. In contrast, on short-term scales, blue and green zones dominate, showing weak or unstable correlations. Arrows illustrating phase relationships depict frequent shifts in directional influence, indicating whether Bitcoin influences

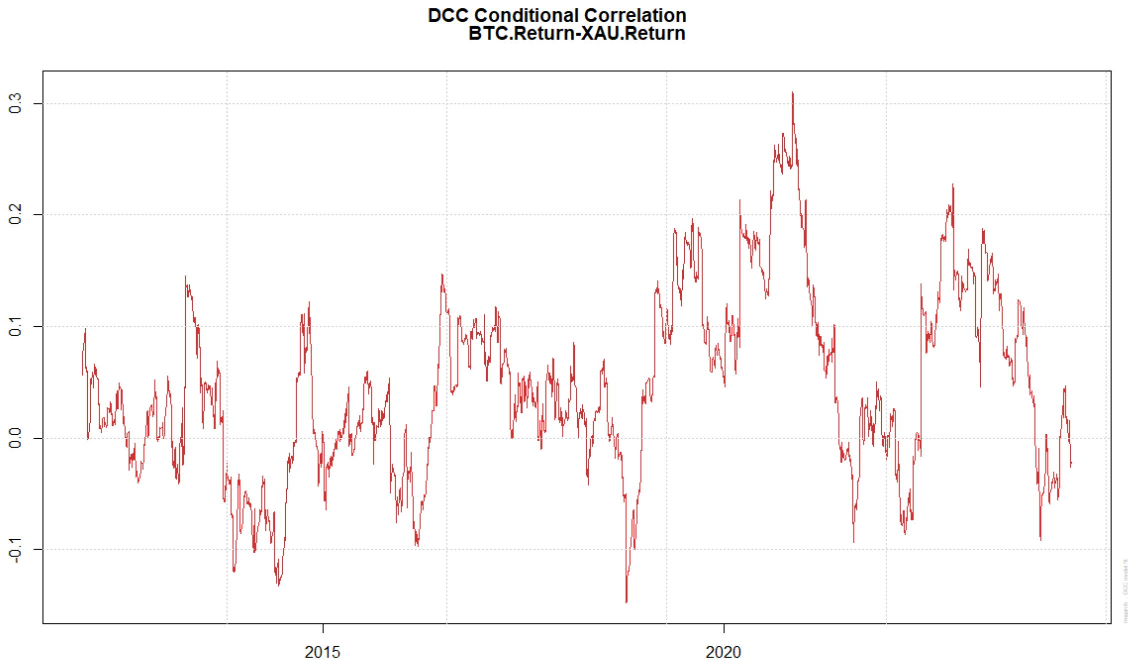

Figure 3. Dynamic Conditional Correlation (DCC) Between Bitcoin and Gold Returns

Figure 4. Evolution of an Equally Weighted Portfolio with 1% Value at Risk (VaR) Limits

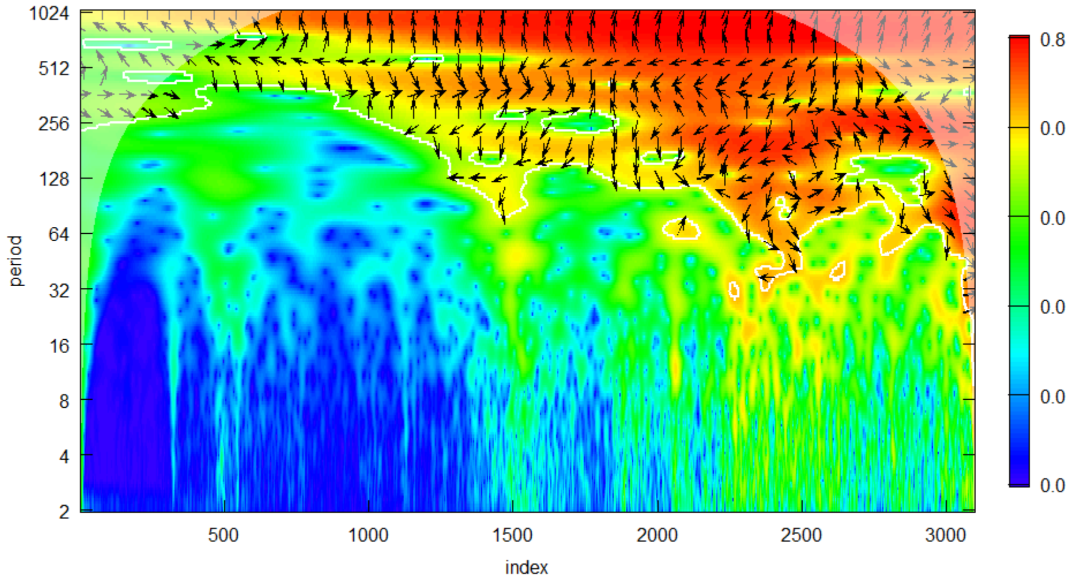

Figure 5. Wavelet Coherence Analysis of Bitcoin-Gold Relationship: Time-Frequency Dynamics

gold or vice versa, indicating that these interactions are sensitive to short-term market shocks or fluctuation in sentiment.

These findings suggest a differentiated investment strategy. Over the long term, the robust correlation between Bitcoin and gold provides a solid foundation for diversification or hedging strategies, aligning with their roles as alternatives to traditional markets. However, the short-term instability in this relationship, coupled with unpredictable shifts in influence, makes tactical decisions based on this correlation inherently risky. Investors should prioritize a structured, long-term approach and refrain from overinterpreting short-term co-movements, which are often distorted by speculative or external factors. Thus, the Bitcoin-gold connection proves to be more relevant as a strategic tool for extended time horizons rather than for reactive trading.

4.6. Stationarity Test

The unit root test, proposed by Dickey and Fuller (ADF: Augmented Dickey-Fuller), is widely used to detect non-stationarity. The test's null hypothesis assumes the presence of a unit root, indicating stochastic non-stationarity. [17]; [18].

The results of the Augmented Dickey-Fuller (ADF) test indicate that the logarithm-transformed gold prices (GLD.Ln) are not stationary, as the null hypothesis of a unit root is not rejected. However, the returns of gold prices (GLD.Return) are stationary, as the null hypothesis of a unit root is rejected (Table No 2). Similarly, for the BTC series, the test results indicate that the logarithm-transformed Bitcoin prices (BTC.Ln) are generally non-stationary. However, the returns of Bitcoin prices (BTC.Return) are stationary (Table no 3).

This shows that the logarithm-transformed series (GLD and BTC) have a unit root, whereas the returns do not, indicating that both series transformed into logs are integrated of the same order I(1).

Table 2. Augmented Dickey-Fuller test for GLD stationarity.

		GLD.Ln[0]			GLD.Return[0]		
Model	H_0^1	Stat.[2]	C_α^3	Decision	Stat.[2]	C_α^3	Decision
M3	$H_0 : \rho = 0$	-2.00	-3.41	NRH0[4]	-40.9	-3.41	RH0[4]
M3	$H_{03} : \beta = 0$	3.55	6.25	NRH0	1.81	1.96	NRH0
M2	$H_0 : \rho = 0$	-0.49	-2.86	NRH0	-40.8	-2.86	RH0
M2	$H_{02} : c = 0$	0.32	4.59	NRH0	0.63	1.96	NRH0
M1	$H_0 : \rho = 0$	1.94	-1.95	NRH0	-40.8	-1.95	RH0

[0] The Gold series taken in logarithm and then in diff, in this case it is the return.
[1] The null hypothesis of the corresponding Model.
[2] The observed test statistic.
[3] The critical value, tabulated on the Dickey-Fuller or Student table.
[4] NRH0: Non-Rejected Null Hypothesis, RH0: Rejected Null Hypothesis.

A VCEM model will be applied if the two series are cointegrated; otherwise, we need to apply the VAR model to the differenced series, which corresponds to the returns.

Table 3. Augmented Dickey-Fuller test for BTC stationarity.

		BTC.Ln[0]			BTC.Return[0]		
Model	H_0^1	Stat.[2]	C_α^3	Decision	Stat.[2]	C_α^3	Decision
M3	$H_0 : \rho = 0$	-2.15	-3.41	NRH0[4]	-25.9	-3.41	RH0[4]
M3	$H_{03} : \beta = 0$	3.13	6.25	NRH0	-1.36	1.96	NRH0
M2	$H_0 : \rho = 0$	-1.95	-2.86	NRH0	-25.8	-2.86	RH0
M2	$H_{02} : c = 0$	5.55	4.59	NRH0	2.75	1.96	RH0
M1	$H_0 : \rho = 0$	1.94	-1.95	NRH0	-25.7	-1.95	RH0

[0] The Gold series taken in logarithm and then in diff, in this case it is the return.
[1] The null hypothesis of the corresponding Model.
[2] The observed test statistic.
[3] The critical value, tabulated on the Dickey-Fuller or Student table.
[4] NRH0: Non-Rejected Null Hypothesis, RH0: Rejected Null Hypothesis.

4.7. Cointegration Test

We recall that two or more time series are said to be cointegrated if a linear combination of these series is stationary, even if the individual series themselves are non-stationary. This implies the existence of a stable long-term relationship between them [19]; [20].

The Johansen test is used to test for cointegration. The results suggest that there is insufficient evidence to reject the null hypothesis, which states either that there is no cointegration vector ($r = 0$) or that there is only one cointegration vector ($r = 1$). This indicates that the logarithmic series are not cointegrated. As a result, there is no long-term relationship between Bitcoin and gold prices. Statistically, this means we should use a VAR model to analyze the returns of the series (see Table No. 4).

The table's results show that (see Table No: 5), for the *BTC.Return* variable, the coefficient associated with the *GLD* lag (0.126) is positive but not statistically significant (p-value = 0.22), indicating that we cannot conclude a direct effect of gold's past returns on Bitcoin's returns. Similarly, the coefficient for *BTC*'s own lag (-0.005) is not

Table 4. Johansen cointegration test for the series in logarithm.

Null hypothesis	Test Statistic	$C_\alpha(10\%)$	$C_\alpha(5\%)$	$C_\alpha(1\%)$
$H_0 : r = 1$	0.04	6.50	8.18	11.65
$H_0 : r = 0$	8.21	15.66	17.95	23.52

significant (p-value = 0.17), suggesting the absence of notable short-term autocorrelation in Bitcoin returns. For the *GLD.Return* variable, the coefficient for *GLD*'s own lag (-0.039) is significant (p-value $\approx$ 0.025) and negative, indicating a mean reversion pattern in gold returns: an increase in gold's returns in one period is associated with a slight decrease in the next. Finally, the coefficient associated with *BTC*'s lag (0.041) is not significant (p-value = 0.171), implying that Bitcoin's past returns do not statistically influence gold's current returns.

Table 5. Bivariate VAR model estimation.

	BTC.Return[1]			GLD.Return[1]		
Lag.[2]	Coefficient	S.E	Sig.	Coefficient	S.E	Sig.
GLD. L1	0.126	0.10	0.22	-0.039	0.02	0.025*
BTC. L1	-0.005	0.02	0.76	0.004	0.01	0.171

[1] The coefficients, standard error, and significance levels for BTC.return and GLD.Return as dependent variables.
[2] The lags of the variables BTC.return and GLD.return as independent variables in the VAR model.

4.8. Impact of Halving Events

The Bitcoin halving event, which involves halving the mining rewards for new blocks in the Bitcoin blockchain, is a crucial moment for the cryptocurrency. It has significant implications, especially for its relationship with gold, one of the most precious traditional assets.

The four halving dates are:

- November 8, 2012
- July 9, 2016
- May 11, 2020
- April 20, 2024

In this section, we will examine in detail the impact of the halving event on the relationship between Bitcoin and gold, highlighting the similarities and differences between these assets in terms of scarcity, perceived value, and behavior in financial markets.

4.8.1. Cointegration test for each period

For all halving periods (Table No 6), the test statistics are below the critical values for both null hypotheses ($H_0 : r = 1$ and $H_0 : r = 0$). This means the tests do not reject the respective null hypotheses, suggesting that there is no significant cointegration between BTC.ln and GLD.ln for any of the halving periods analyzed.

In summary, the results indicate that there is no statistically significant evidence of cointegration between the logarithms of Bitcoin and gold during the halving periods.

4.8.2. The model VAR for each period

When examining the coefficients, standard errors (S.E.), and significance values (Sig.) indicated in table no. 6 for the returns of Bitcoin (BTC.Return) and gold (GLD.Return), the following main results and conclusions are observed (Table No: 7):

Table 6. Test of cointegration between BTC.ln and GLD.ln for each halving period.

	Halving 1[1]		Halving 2[1]		Halving 3[1]	
Null hypothesis	Stat.	$C_\alpha(5\%)^2$	Stat.	$C_\alpha(5\%)$	Stat.	$C_\alpha(5\%)$
$H_0 : r = 1$	1.96	8.18	0.13	8.18	1.62	8.18
$H_0 : r = 0$	8.36	17.95	5.41	17.95	6.07	17.95

[1] The three halving periods between the dates 2012, 2016, 2020, and 2024.
[2] The critical value, tabulated by simulation of Johansen and Juselius.

Table 7. Bivariate VAR model estimation for each period of Halving.

		BTC.Return[1]			GLD.Return[1]		
Halving	Lag.[2]	Coeff.	S.E	Sig.	Coeff.	S.E	Sig.
Halving 1	GLD. L1	0.322	0.20	0.11	−0.086	0.03	0.01*
	BTC. L1	0.013	0.03	0.56	0.002	0.01	0.62
Halving 2	GLD. L1	−0.096	0.17	0.58	−0.006	0.03	0.85
	BTC. L1	−0.031	0.03	0.33	0.012	0.01	0.03*
Halving 3	GLD. L1	0.035	0.13	0.77	−0.011	0.03	0.71
	BTC. L1	−0.041	0.03	0.19	−0.001	0.01	0.91

[1] The coefficients, standard error, and significance levels for BTC.return and GLD.Return as dependent variables.
[2] The lags of the variables BTC.return and GLD.return as independent variables in the VAR model.

- Halving 1: Gold returns at lag 1 significantly affect their own returns but not Bitcoin's returns. Bitcoin returns at lag 1 do not significantly affect their own returns or gold's returns.
- Halving 2: Bitcoin returns at lag 1 have a positive and significant effect on gold returns, but other relationships are not significant.
- Halving 3: None of the coefficients are significant, indicating no significant effects of gold or Bitcoin returns at lag 1 on their own returns or each other's returns.

These results suggest that the dynamic interactions between gold and Bitcoin returns vary across different halving periods. While some periods show significant interactions, others do not, highlighting the changing nature of the relationship between these two assets over time.

4.9. The results with control variables

4.9.1. Johansen cointegration test for 4 time series .
First, the Augmented Dickey-Fuller (ADF) test for the two control variables (Table No. 8) confirms that both series are non-stationary and integrated of order 1 (I(1)). As a result, all four series, Bitcoin, Gold, S&P 500, and Oil, share the same integration order I(1). This condition satisfies the requirement for conducting a cointegration test, validating the next step in the analysis.

Table No. 9 presents the results of the Johansen cointegration test to assess the number of cointegration relationships $\mathbf{r}$ among the variables studied. The hypotheses tested range from $\mathbf{r} \leq \mathbf{3}$ to $\mathbf{r} = \mathbf{0}$, with test statistics compared to critical values at three significance levels (10%, 5%, 1%). The cointegration test results reveal a stable long-term relationship among the variables. Specifically:

Table 8. ADF Test Statistics for GSPC.Ln and OIL.Ln Series

Series	ADF Statistic	Lag Order	p-value	Decision
GSPC.Ln	-1.934	14	0.2121	Non-stationary ($p > 0.05$)
OIL.Ln	-1.918	14	0.6132	Non-stationary ($p > 0.05$)
$\Delta(GSPC.Ln)$	-15.071	14	0.01	Stationary ($p < 0.05$)
$\Delta(OIL.Ln)$	-13.896	14	0.01	Stationary ($p < 0.05$)

Rejecting the null hypothesis $H_0 : r = 0$ (The test statistic (12.64) remains below the critical values (17.85 at 10%, 19.96 at 5%, 24.60 at 1%): indicates the presence of at least one cointegration relationship.
Failing to reject $H_0 : r = 1$ (the test statistic (26.79) is lower than the critical value 10% (30.00) but also lower than those in 5% and 1%.): confirms that there is exactly one cointegration equation in the system.

This implies that, despite short-term fluctuations, the variables converge to a common equilibrium over the long run. These findings justify the use of a VECM (Vector Error Correction Model), which is suitable for analyzing:

- Short-term dynamics (instantaneous variations in the variables),
- Adjustment mechanisms toward the long-term equilibrium.

The model incorporates a single cointegration equation $(r = 1)$, where the Error Correction Term (ECT) measures the speed at which deviations from equilibrium are corrected after a shock. This process ensures that temporary imbalances are gradually resolved, highlighting the structural interdependence of the variables.

Table 9. Johansen Cointegration Test (Trace Statistic)

Nullhypothesis	Test Statistic	$C_\alpha(10\%)$	$C_\alpha(5\%)$	$C_\alpha(1\%)$
$r \leq 3$	3.82	7.52	9.24	12.97
$r \leq 2$	12.64	17.85	19.96	24.60
$r \leq 1$	26.79	32.00	34.91	41.07
$r = 0$	56.87	49.65	53.12	60.16

4.9.2. VECM Results with Control Variables .

long-run cointegration .
The long-term cointegration relationship, identified by the Johansen test and represented by the cointegration equation ($Coint_t$), shows that Gold (XAU) and Bitcoin (BTC) contribute positively to equilibrium, while the S&P500 (GSPC) and Oil (OIL) exert downward pressure. Specifically, a 1% increase in the S&P500 reduces equilibrium by 1.48%, a significantly stronger effect compared to oil's contribution (-0.16%). This suggests that stock market movements (S&P500) dominate systemic imbalances, likely due to its role as a benchmark for capital flows. The positive constant term (3.89) indicates a high baseline equilibrium level, reflecting a structural upward trend in the studied assets. These findings highlight the disproportionate influence of equity markets in disrupting long-term stability, while gold and Bitcoin act as stabilizing forces within the system.

$$\text{Coint}_t = 1.00 \cdot \text{XAU.Ln}_{t-1} + 0.16 \cdot \text{BTC.Ln}_{t-1} - 1.48 \cdot \text{GSPC.Ln}_{t-1} - 0.16 \cdot \text{OIL.Ln}_{t-1} + 3.887$$

Regarding short-term adjustment dynamics, the ECT (Error Correction Term) coefficient (see Table No 10) measures how quickly a system returns to equilibrium after a shock. For Bitcoin, the positive ECT coefficient

($+0.0117$) is unusual: instead of correcting imbalances, it amplifies them. This behavior could stem from Bitcoin's inherent volatility or speculative trading patterns that prolong deviations from equilibrium. In contrast, Gold shows a slightly negative ECT (-0.0006), signaling a slow but stabilizing correction mechanism. Meanwhile, S&P500 and Oil have modestly positive ECT coefficients (0.0065 and 0.0046, respectively), suggesting they temporarily drift further from equilibrium before gradually adjusting in later periods.

Table 10. VECM Estimation Results

Variable	XAU (Gold)	BTC (Bitcoin)	GSPC (S&P 500)	OIL (Crude Oil)
ECT_1	-0.0006267	0.0117369	0.0064734	0.0045218
$XAU.Ln.d1$	-0.0429082	0.1391973	0.0232211	-0.0307188
$BTC.Ln.d1$	0.0021856	0.0366985	0.0008376	0.0090267
$GSPC.Ln.d1$	-0.0018372	-0.1151222	-0.1331822	-0.0109655
$OIL.Ln.d1$	0.0070482	-0.0019091	-0.0054679	0.0032640

Direct Impacts of the S&P 500 and Oil on Bitcoin and Gold .

Short-term coefficients, as seen in rows `XAU.Ln.d1`, `BTC.Ln.d1`, `GSPC.Ln.d1`, and `OIL.Ln.d1` (refer to Table 10), demonstrate how past fluctuations influence the current variations of each asset. For instance, the S&P 500 has a negative impact on Bitcoin: a 1% increase in the S&P 500 is associated with a 0.115% decrease in Bitcoin's value. This trend may reflect a substitution effect, where investors tend to favor traditional assets in rising stock markets. In contrast, the effect of the S&P 500 on gold is minimal, with a change of –0.0018%, highlighting gold's role as a safe haven that is relatively insensitive to fluctuations in the stock market.

Regarding oil, it shows a marginal effect on Bitcoin, resulting in a decrease of –0.0019%, while it has a slight positive influence on gold, increasing its value by $+0.0070\%$. This relationship may be explained by inflationary expectations: rising oil prices, which are often associated with upward pressure on overall price levels, enhance gold's appeal as a hedge. Meanwhile, Bitcoin, perceived as a riskier asset, may suffer indirectly from these inflationary repercussions.

The results highlight two key dynamics:

1. **Dominance of the S&P 500:** Its short-term negative influence on Bitcoin, coupled with its central role in long-term imbalances, makes it a major determinant. Investors appear to reallocate their portfolios toward equities during market upswings, often at the expense of Bitcoin.
2. **Role of Oil:** Although less pronounced, oil's slight positive impact on gold and negative effect on Bitcoin reflect indirect macroeconomic channels, such as inflation or geopolitical risk.

Gold retains a stabilizing role, albeit a limited one, while Bitcoin emerges as a reactive and speculative asset, amplifying imbalances rather than correcting them. These insights are crucial for investors as they underscore the importance of monitoring both stock market trends and commodity prices to anticipate Bitcoin movements, and confirm gold as a more reliable hedge during periods of volatility.

5. Discussion

This study analyzes the dynamic relationship between Bitcoin (BTC) and gold, focusing on the halving periods from January 1, 2012, until the most recent halving date on April 30, 2024. The results reveal a dual interaction over time, suggesting that, in the long term, Bitcoin and gold exhibit a significant statistical concordance. This implies that Bitcoin, despite its speculative nature, has the potential to replace gold as a hedging instrument. These findings align with the results of previous studies conducted by Dyhrberg (2016) [2] and Corbet et al. (2019) [28], which argue that Bitcoin behaves similarly to gold as a safe-haven asset in the long term.

In contrast, short-term dynamics diverge significantly. Analysis using vector autoregression (VAR) frameworks and cointegration reveals no lasting connection between the two assets. Additionally, DCC-GARCH models and wavelet coherence metrics highlight Bitcoin's increased sensitivity to idiosyncratic shocks, such as regulatory changes, technological disruptions, or sudden shifts in market sentiment, whereas gold tends to show price inertia. This observation aligns with the findings of Selmi et al. (2020) [6] and Klein et al. (2018) [30], who attribute Bitcoin's volatility to its speculative appeal and its vulnerability to external catalysts.

Incorporating macroeconomic controls, such as the S&P 500 and oil prices, into our VECM framework adds further context to these dynamics. Bitcoin returns exhibit an inverse relationship with movements in the equity market, supporting the notion presented by Bouri et al. (2017) [25] that a substitution dynamic exists between risk-on assets and safe havens during downturns. Conversely, gold's muted response to oil price fluctuations reinforces its identity as a resilient hedge, which is consistent with the findings of Baur and Lucey (2010) [3].

In conclusion, while Bitcoin may serve as a complement to gold in long-term diversification strategies, especially as a hedge during crises, its high volatility and short-term sensitivity necessitate careful portfolio management. Investors should not only allocate these assets responsibly, but also dynamically adjust their exposures in response to changing macroeconomic and regulatory conditions, balancing specific risks against the potential for rewards.

6. Conclusion

This study has provided an in-depth econometric investigation into the dynamic interplay between Bitcoin and gold, particularly around Bitcoin halving events. Our analysis demonstrates that while strong long-term coherence exists, suggesting that Bitcoin can exhibit safe haven characteristics akin to gold, the short-term relationship remains volatile and sensitive to transient shocks. The application of diverse methodologies, including VAR, VECM, DCC-GARCH, and wavelet coherence analysis, underscores the complex and evolving nature of these assets. In periods of economic uncertainty, the complementary roles of Bitcoin and gold become more apparent, offering potential hedging benefits in diversified portfolios.

However, the findings also highlight important caveats. Bitcoin's high volatility and susceptibility to exogenous events, such as regulatory shifts and market sentiment changes, limit its reliability as a short-term hedge compared to the more stable gold market. Furthermore, the influence of broader macroeconomic factors, evidenced by the incorporation of control variables, suggests that external market conditions play a critical role in shaping the relationship between these assets. Future research should aim to refine these models and incorporate additional variables to further elucidate the nuances of the Bitcoin-gold dynamic, ensuring that investors can better navigate the inherent risks and opportunities.

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7. Appendix: The R code used to reproduce the results

7.1. Preprocessing data

Load necessary packages

```
library(quantmod)
library(urca)
library(CADFtest)
library(bootUR)
library(tseries)
library(readr)
library(GGally)
library(highcharter)
library(tidyverse)
library(vars)
library(stargazer)
library(lpirfs)
library(qrmtools)
library(readxl)
library(xts)
library(lubridate)
```

Gold data download

```
XAU.0=getSymbols(Symbols = "GC=F",
                 from = "2012-01-01",to = "2024-05-01",
                 warnings = TRUE, src = "yahoo",auto.assign = FALSE)
head(XAU.0)
XAU.1=XAU.0[,4] # select the Close Price
names(XAU.1)=c("XAU.Price")
XAU.1$XAU.Price=as.numeric(XAU.1$XAU.Price)
str(XAU.1)
```

Bitcoin data download.
Link for downloading Bitcoin data :
https://coincodex.com/crypto/bitcoin/historical-data/
you must choose the period concerned on the site from 01-01-2012 to 30-04-2024, Once the data is downloaded, it must be inserted into the same working folder in R with the name ”Bitcoin Historical Data.csv”

```
Btc.Hist<- read.csv("Bitcoin␣Historical␣Data.csv")
#  Reverse the lines
btc_H <- Btc.Hist[nrow(Btc.Hist):1, ]
# Reset line numbers
row.names(btc_H) <- NULL
# Selection of columns required
btc_H=btc_H[,1:2]
# transform data into xts format
BTC.0=xts(btc_H$Price,order.by = as.Date(btc_H$Date, format = "%m/%d/%Y"))
names(BTC.0)=c("BTC.Price")
BTC.1=BTC.0["2012-01-02/2024-04-30",]
head(BTC.1)
#  only weekday----------------
btc.wday <-BTC.1[.indexwday(BTC.1)%in% 1:5]
```

Building a database by merging the two datasets

```
df.0=merge(XAU.1,BTC.1,join='left', retclass = "xts")
head(df.0,5)
tail(df.0,5)

df.0$XAU.Ln=log(df.0$XAU.Price)
df.0$BTC.Ln=log(df.0$BTC.Price)
```

Daily return calculation

```
BTC_R=returns(df.0$BTC.1)*100
XAU_R=returns(df.0$XAU.1)*100
names(BTC_R)=c("BTC.Return")
names(XAU_R)=c("XAU.Return")
df.R=merge.xts(XAU_R,BTC_R,join='left')
head(df.R)
```

7.2. Step-by-Step Implementation of DCC-GARCH in R

Step 0: Load the necessary packages and and data preparation

```
library(rugarch)
library(rmgarch)
```

Step 1: Specify univariate GARCH models for each series:

```
ugarch_spec <- ugarchspec(
  variance.model = list(model = "sGARCH",
                        garchOrder = c(1,1)),
  mean.model = list(armaOrder = c(0,0)))
```

Step 2: Define the DCC-GARCH specification:

```
dcc_spec <- dccspec(
  uspec = multispec(replicate(2, ugarch_spec)),
  dccOrder = c(1,1), distribution = "mvnorm")
```

Step 3: Estimate the DCC-GARCH model:

```
dcc_fit <- dccfit(dcc_spec, data = df.R)
```

Step 4: Extract and visualize dynamic correlations:

```
dcc_cor <- rcor(dcc_fit)
plot(dcc_fit)
```

Step-by-Step Implementation ofwavelet coherence analysis in R

```
library(WaveletComp)
Do=data.frame( df.0$XAU.Price, df.0$BTC.Price)
C.ond=na.omit(Do)
```

```
# Calculate wavelet coherence ===== :
result <- analyze.coherency(C.ond,
                            my.pair = c("XAU.Price", "BTC.Price"))
# View the results:
wc.image(result)
```

7.3. *Step-by-Step Implementation ADF-Test in R*

```
# determination of the optimal lags order p
head(df.0)
N=nrow(df.0)
P.max=floor(4*((N/100)^0.25)) ## the maximum theoretical lags number
```

```
## ----------- ADF test Strategies  GOLD  ---------------

# Model 3 model complet with Trend and Drift
adf.3=ur.df(df.0$XAU.Price, type = c("trend"), lags =P.max,
            selectlags = "BIC")
summary(adf.3)

# Model 2 model with Drift
adf.2=ur.df(df.0$XAU.Price, type = c("drift"), lags = P.max,
            selectlags = "BIC")
summary(adf.2)

# Model 1 model contains neither trend nor drift
adf.1=ur.df(df.0$XAU.Price, type = c("none"), lags = P.max,
            selectlags = "BIC")
summary(adf.1)
```

The same steps for the stationarity test of the Bitcoin series, it was enough to replace 'XAU.Price' with 'BTC.Price' data.

7.4. *Cointegration Analysis (Johansen Test)*

We use the Johansen cointegration test to investigate the long-term equilibrium relationship between Bitcoin and gold prices. The test is performed in R using the following commands:

```
# Load required package
library(urca)
# Prepare data as time series
data_ts <- df.0[,c(3,4)]
```

```
# Determine optimal lag length
library(vars)
lagselect <- VARselect(data, lag.max=12, type="const")
```

```
# Johansen Cointegration Test
library(urca)
jotest=ca.jo(data_ts, type="trace", K=8, ecdet="none", spec="longrun")
summary(jotest)
```

```
# Johansen Cointegration Test _ Another way
library(urca)
jotest <- ca.jo(data_ts, type="trace", K=8, ecdet="const", spec="transitory")
summary(jotest)
```

7.5. *Step-by-Step Estimation of a VAR model in R*

Step 1: Checking for Stationarity
Firstly, verify the stationarity of your time series data using the Augmented Dickey-Fuller (ADF) test:

```
library(tseries)
adf.test(Gold)
adf.test(Bitcoin) # repeat for each variable
```

Step 2: Optimal Lag Selection

```
library(vars)
lagselect <- VARselect(data, lag.max = 10, type = "const")
lagselect$selection
```

Step 3: Estimate VAR Model

```
var.model <- VAR(data, p = 2, type = "const")
summary(varmodel)
```

Step 4: Model Diagnostic Checks

```
serial.test(VARmodel)
arch.test(VARmodel)
normality.test(VARmodel)
```

Step 5: Forecasting and Visualization

```
forecast <- predict(VARmodel, n.ahead=5)
plot(forecast)
```

7.6. Implementation of the Econometric Model in R (VECM with Macroeconomic Controls)

Load Necessary Packages and Prepare the Data

```
library(vars)
library(urca)
library(tseries)

# --- Download S&P 500 data from Yahoo Finance -----
getSymbols("^GSPC", src = "yahoo", from = "2012-01-01", to = "2024-05-01")
GSPC.0=na.omit(GSPC)
GSPC.1=GSPC.0[,4] # Close Price
names(GSPC.1)=c("GSPC.Price")
GSPC.1$GSPC.Price=as.numeric(GSPC.1$GSPC.Price)
# --- Download Brent Oil Data (BZ=F) --------
getSymbols("BZ=F", src = "yahoo", from = "2012-01-01", to = "2024-05-01")
BZ_F.0=na.omit(`BZ=F`)
BZ_F.1=BZ_F.0[,4] # Close Price
names(BZ_F.1)=c("OIL.Price")
BZ_F.1$OIL.Price=as.numeric(BZ_F.1$OIL.Price)

# --- Building a database by merging the two datasets -----.
D.Ctrl=merge(BZ_F.1,GSPC.1,join='left', retclass = "xts")
D.Ctrl=na.omit(D.Ctrl)
D.Ctrl$OIL.Ln=log(D.Ctrl$OIL.Price)
D.Ctrl$GSPC.Ln=log(D.Ctrl$GSPC.Price)

# --- Building final database for in Log ---
data.ts=merge.xts(df.0,D.Ctrl,join='left')
data.ts=na.omit(data.ts[, c(3,4,7,8)])
```

Stationarity analysis

```
#-- Check stationarity with Augmented Dickey-Fuller Test (ADF Test)
names(data.ts)
adf.test(data.ts[, "XAU.Ln"])
adf.test(data.ts[, "BTC.Ln"])
adf.test(data.ts[, "GSPC.Ln"])
adf.test(data.ts[, "OIL.Ln"])
```

Cointegration analysis

```
## --- Perform Johansen Cointegration Test
johansen_test <- ca.jo(data.ts,
                       type = "trace",
                       ecdet = "const",
                       K = 2)  # Choose optimal lag length
summary(johansen_test)
```

```
## Estimate the VECM Model
# Convert Johansen test result to a VECM model
vecm_model <- cajorls(johansen_test, r = 1)  # r = number of cointegrating
    relations
vecm_model
```

REFERENCES

1. BAUR ET AL. (2018) *Bitcoin: Medium of exchange or speculative assets?. Journal of International Financial Markets, (54) 187-189.*
2. DYHRBERG, A. H. (2016) *Bitcoin, gold and the dollar–A GARCH volatility analysis. Finance Research Letters, (16) 85-92.*
3. BAUR, D. G., & LUCEY, B. M. (2010) *Is gold a hedge or a safe haven? An analysis of stocks, bonds and gold. Financial Review, 45(2), 217-229.*
4. CORBET, S., LUCEY, B., URQUHART, A.,& YAROVAYA, L. (2020) *Cryptocurrencies as a financial asset: A systematic analysis. International Review of Financial Analysis, (62), 182-199.*
5. SELMI, R., MENSI, W., HAMMOUDEH, S., & BOUOIYOUR, J. (2018) *Is Bitcoin a hedge, a safe haven or a diversifier for oil price movements? A comparison with gold. Finance Research Letters, (74), 787-801.*
6. SELMI, R., MENSI, W., HAMMOUDEH, S., & BOUOIYOUR, J. (2020) *Bitcoin's Halving Events and Market Dynamics. Finance Research Letters, (32), 85-92.*
7. KYRIAZIS, N. A. (2020) *The Role of Bitcoin in Investment Portfolios. Journal of Risk and Financial Management, 13(4), 84.*
8. KYRIAZIS, N. A. (2020) *Is Bitcoin similar to gold? An integrated overview of empirical findings. Journal of Risk and Financial Management, 13(5), 88-110.*
9. SHEHZAD, K., BILGILI, F., ZAMAN, U., KOCAK, E., & KUSKAYA, (2021) *Is gold favourable than bitcoin during the COVID-19 outbreak? Comparative analysis through wavelet approach. Resources Policy, (73), 88-110*, doi:10.1016/j.resourpol.2021.102163
10. KAHNEMAN, DANIEL AND TVERSKY, AMOS, (2013) *Prospect theory: An analysis of decision under risk. Handbook of the fundamentals of financial decision making: Part I, 99-127*, https://doi.org/10.1142/9789814417358_0006
11. BOURI, ELIE AND GUPTA, RANGAN AND ROUBAUD, DAVID (2019) *Herding behaviour in cryptocurrencies. Finance Research Letters, (29), 216–221*, https://doi.org/10.1016/j.frl.2018.12.012
12. DICKEY, DAVID A AND FULLER, WAYNE A (1979) *Distribution of the Estimators for Autoregressive Time Series with a Unit Root. Journal of the American Statistical Association, 74(366), 427–431*, https://doi.org/10.1080/01621459.1979.10482531
13. ROBERT ENGLE (2002) *Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models Journal of business & economic statistics, 20(3), 339–350*, publisher=Taylor & Francis, https://doi.org/10.1198/073500102288618487
14. ENDERS. WALTER. Applied econometric time series (4th ed.). *JOHN WILEY & SONS. INC. lowa State University,*
15. JOHANSEN. S. Likelihood-based inference in cointegrated vector autoregressive models (1st ed.). *Oxford University Press.*, Feb. 1996.
16. LÜTKEPOHL, H. Vector autoregressive models. *Springer,*.
17. Dickey, David A., and Wayne A. Fuller: Distribution of the Estimators for Autoregressive Time Series with a Unit Root. Journal of the American Statistical Association, **74**(366), 427-431 (1979)
18. Jalil, A., & Rao, N. H.: Time Series Analysis (Stationarity, Cointegration, and Causality). Environmental Kuznets Curve (EKC), 85-99 (2019). doi:10.1016/b978-0-12-816797-7.00008-4
19. Johansen, S.: Statistical Analysis of Cointegration Vectors. Journal of Economic Dynamics and Control. (1988)
20. Juselius, K. : "The Cointegrated VAR Model: Methodology and Applications". Oxford University Press.. (2006)

21. Nakamoto, S. (2008). Bitcoin: A Peer-to-Peer Electronic Cash System. Retrieved from. https://bitcoin.org/bitcoin.pdf]
22. Regis Bourbonnais, (2018), Econmetrie ; Ed.10, Paris, Dunod 2018, Chap. 9-10-11, Element d'analyse des series temporelles, page : 257-339.
23. S. Sathyanarayana, (2019) ; Modeling Cryptocurrency (Bitcoin) using Vector Autoregressive (Var) Model ; SDMIMD Journal of Management, September 2019, page : 47-64.
24. Baur, D. G., Hong, K., & Lee, A. D. (2018). Bitcoin: Medium of exchange or speculative assets? Journal of International Financial Markets, Institutions and Money, 54, 177-189. https://doi.org/10.1016/j.intfin.2017.12.004
25. Bouri, E., Molnár, P., Azzi, G., Roubaud, D., & Hagfors, L. I. (2017). On the hedge and safe haven properties of Bitcoin: Is it really more than a diversifier? Finance Research Letters, 20, 192-198. https://doi.org/10.1016/j.frl.2016.09.025
26. Bouoiyour, J., & Selmi, R. (2016). Bitcoin: Between legal and financial regulation. International Journal of Economics and Financial Issues, 6(4), 1353-1362.
27. Junpeng Wang, Yubo Xue, Minghao Liu, 2016, An Analysis of Bitcoin Price Based on VEC Model. (International Conference on Economics and Management Innovations) AnAnalysisofBitcoinPriceBasedonVECModel|AtlantisPress(atlantis-press.com)
28. Corbet, Shaen and Lucey, Brian and Peat, Maurice and Vigne, Samuel (2018) *Bitcoin Futures—What use are they?. Economics Letters, (172), 23-27*, publisher=Elsevier, https://doi.org/10.1016/j.econlet.2018.07.031
29. Corbet, Shaen and Lucey, Brian and Yarovaya, Larisa (2018) *Datestamping the Bitcoin and Ethereum bubbles. Finance Research Letters, (26), 81–88*, publisher=Elsevier, https://doi.org/10.1016/j.frl.2017.12.006
30. Klein, T., Pham Thu, H., & Walther, T. (2018) *Bitcoin is not the New Gold – A comparison of volatility, correlation, and portfolio performance. International Review of Financial Analysis, (59), 105-116*, publisher=Elsevier, https://doi.org/10.1016/j.irfa.2018.07.010
31. Gajardo, Gabriel and Kristjanpoller, Werner D and Minutolo, Marcel (2018) *Does Bitcoin exhibit the same asymmetric multifractal cross-correlations with crude oil, gold and DJIA as the Euro, Great British Pound and Yen? Chaos, Solitons & Fractals, (109), 195–205*, publisher=Elsevier, https://doi.org/10.1016/j.chaos.2018.02.029
32. Auer, Raphael and Claessens, Stijn (September 2018) *Regulating cryptocurrencies: assessing market reactions. BIS Quarterly Review September*,
https://ssrn.com/abstract=3288097