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Compliance-and-defiance dilemma game best strategy for three and four agents

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Abstract Dilemma noncooperative games involving three or four agents are studied, where strategic interaction is based on selecting between the compliance strategy and the defiance strategy. When the agent applies the compliance strategy, it increases the agent's loss amount; contrariwise, when the agent applies the defiance strategy, it does not increase the agent's loss amount. It is assumed that if only one agent defies, it does not much affect the system, and there are no fines. When two or three agents defy, every agent is fined by the same amount. The objective is to determine and analyze the agent's best strategy in the dilemma games with three and four agents dealing with the full-defiance situation cost as a variable, where partial-defiance situation cost is deduced from this variable. The agent's best strategy is determined depending on the full-defiance situation cost. The best strategy for every agent is to defy (by applying the defiance pure strategy) with a probability that ensures the global minimum to the agent's loss on the probabilistic interval $[0;\ 1]$ by a given value of the full-defiance situation cost. In the three-agent dilemma game, the best strategy is to fully defy if this cost does not exceed $\frac{2}{3}$, whereupon the agent's loss is equal to the cost. As the cost is increased off value $\frac{2}{3}$, the best strategy probability exponentially-like decreases, while the agent's minimized loss increases in the same manner. Nevertheless, the best strategy ensures the agent's minimized loss does not exceed 1 (a conditional unit), which is the cost of full compliance. In the four-agent dilemma game, the best strategy is to fully defy if the full-defiance situation cost does not exceed $\frac{8}{9}$, whereupon the agent's loss is equal to the cost. As the cost is increased off value $\frac{8}{9}$, the best strategy probability exponentially-like decreases dropping down off value $\frac{1}{4}$, which is also the best strategy (being far more favorable for the system) by when the full-defiance situation cost is $\frac{8}{9}$. Similarly to the agent's minimized loss in the three-agent game, the agent's minimized loss in the four-agent game increases in the same manner never exceeding 1.

Keywords Dilemma game, Compliance, Defiance, Full-compliance situation, Full-defiance situation, Agent's loss, Agent's best strategy, Minimized loss

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1. Introduction

The nature is much sophisticated, but when it comes to decision making, there are almost always two alternatives like those in a philosophical dilemma "to be or not to be". One of the most prominent examples of dilemma decision making is the prisoner's dilemma game [33, 12]. This game involves two rational agents, each of whom can indirectly cooperate for mutual benefit or defect (i. e., betray the other agent) for individual reward [27, 29]. In more general, the prisoner's dilemma game models a process, where two entities could gain benefits from indirectly

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cooperating or suffer losses from failing to do so, but coordination of their decisions is either too expensive or extremely difficult [31, 12, 16]. The prisoner's dilemma game and its variations (the iterated, stochastic, continuous, impure, and asymmetric prisoner's dilemma, Axelrod's tournament) have real-life similarities in environmental studies (pollution prevention) [3, 17, 24, 25], understanding cooperative behavior of animals [11, 31, 19, 20], addiction research and psychology [1, 32], economics (advertizing, cooperating, investing) [16, 27, 21, 22], professional sport doping [10, 15], international politics (security dilemma) [13, 8, 26, 30].

Dilemma noncooperative games involving three and more entities or agents have been studied far less than the prisoner's dilemma game and its variations. Meanwhile, dilemma games with three or four agents model more common interaction processes. An example is the environmental protection model based on a three-agent dilemma noncooperative game [23, 25], where wastewater pollution and cleaning treatment are two alternatives of an industrial enterprise utilizing water in production (manufacturing) processes. Other examples are dilemma games of network peers struggling for benefiting from data sharing [28, 31, 36]. The strategy of sharing in similar dilemma game models can be substituted by supporting, investing, upgrading, updating, etc. [6, 27, 29]. In wider sense, it is a strategy of compliance that means the agent is about to comply with requirements of a system to which the agents belong. The opposite one is a strategy of defiance that means the agent is about to defy (or disregard) system requirements. Due to resource limitation (either informational or material, or both) and restricted direct cooperation among agents, consequences of complying and defying are uncertain. Basically this induces the dilemma game itself. Furthermore, a dilemma game may continue even being solved because its solutions are unstable due to asymmetry or non-favorability (non-profitability) [5, 24, 4]. Thus, imposing mandatory obligations on obligated entities for the consumption of renewable energy may lead to an evolutionary game model whose solutions never stop evolving, which either drives renewable portfolio standards out of effective implementation or requires an additional calibration of the behavioral parameters of the obligated agents [35].

When the agent applies the compliance strategy, it increases the agent's loss amount; contrariwise, when the agent applies the defiance strategy, it does not increase the agent's loss amount. Requirements of the system, in which strategic noncooperative interaction is modeled by a dilemma game, must be such that the agents would eventually come at a balance between compliance and defiance [17, 9, 3]. Even if the agents are assumed to have identical possibilities and constraints imposed, this balance does not necessarily mean equiprobable application of the compliance and defiance strategies. It rather is a quasioptimal ratio between these strategies [24, 25]. It is quite obvious that the ratio must be the same for every agent. This should ensure sustainable development of the system along with stimulating every agent to continue one's activity within the system with a perspective of individual development as well [17, 11, 23, 26]. A bond exists between sustainable development and losses assigned to compliance (additional expenses to maintain compliance, e. g., payment for equipment, new technologies, qualified personnel, etc.) and defiance (fines imposed for excessive disregard of system requirements). This bond is tried to get optimized via using the best strategy as the quasioptimal ratio between compliance and defiance.

2. Motivation and objective

While best strategies in noncooperative games have been commonly associated with the Nash equilibrium, in dilemma noncooperative game this approach fails due to stable noncooperative interaction supposes fairness in payoffs (or losses). As an equilibrium situation ensures unequal payoffs for agents, it does not attract an agent whose loss is higher. What attracts every agent, whose selfishness is taken into consideration also, is loss symmetry with subsequent loss minimization [24, 25], which every agent understands as practically the best composition of selfishness and fairness-based equilibrium. Besides, if such a symmetric situation exists and it ensures the agent's loss is lower than the compliance cost, this situation becomes a substantiated solution to the initial problem of balancing compliance and defiance in order to satisfy both the system and the group of agents. For example, the abovementioned environmental protection model as a three-agent dilemma noncooperative game is expected to stimulate industry into applying more water-cleaning technologies without declining in manufacturing processes [13, 17, 23, 24, 25]. This appears to be possible if every industrial enterprise applies cleaning technologies partially (i. e., it partially complies), leaving just a few percent of polluted wastewater. For instance, when the compliance cost is a conditional unit and the full-defiance situation costs 3 conditional units for every enterprise [23], the best strategy is to apply cleaning technologies with probability

$$\frac{3+\sqrt{7}}{6},$$

where

$$0.94095 < \frac{3+\sqrt{7}}{6} < 0.94096,$$

and thus the ecosystem is capable of restore oneself with only 5.904 % of polluted wastewater from each of the three enterprises. Some generalizations of this instance were made in [24, 25], but their main accent is on symmetric equilibria, which are still worse than the full-compliance situation [23, 30].

Another argument against Nash equilibria is multiplicity [4, 10]. Multiple asymmetric and symmetric Nash equilibria induce additional instability of selective behavior. Dilemma games may have multiple asymmetric Nash equilibria, just like the prisoner's dilemma game and its variations [11, 12, 16, 27]. Eventually, agents must independently reject equilibria and seek for a loss-minimizing symmetric situation. This situation depends on the value of defiance. In fact, it reminds an authoritarian environmental governance framework, by which local governments navigate central-local dynamics and adapt policy implementation strategies to address the challenges of resources (e. g. energy) transition within complex and evolving environmental governance systems [14]. A row of other examples is the development of practical and effective enforcer-strategies of policymakers to improve safety [7, 18] and security [2, 34]. Such strategies are intended not only to enforce agents' behavioral patterns desirable for local governments, but also to support sustainable evolution of the agents. This is a sort of closed circle whose closeness is maintained owing to an all-side balance between compliance and defiance [1, 6, 3, 27].

It is assumed in [24, 25] that if only one agent defies, it does not much affect the system, and there are no fines. Besides, when two or three agents defy, every agent (even the complying one, because it is either difficult or impossible to determine the complier) is fined by the same amount [24]. Obviously, this is a rough model, so paper [25] distinguished the fines for full defiance and 1-compliance (when there is a single complier). Similar diversification might be introduced into a four-agent dilemma noncooperative game. Therefore, the objective is to determine and analyze the agent's best strategy in the dilemma games with three and four agents dealing with the full-defiance situation cost as a variable, where partial-defiance situation cost is deduced from this variable. The paper proceeds with formalization of a set of dilemma (pure, using the game theory terminology) strategies of the agent and dilemma games themselves. Next, each game is substantialized and the pure strategy losses are defined. Then the agent's best strategy is determined depending on the full-defiance situation cost. A qualitative analysis of the best strategy will be carried out for each game. Finally, the paper is concluded with a discussion of its findings and an outlook for further research.

3. Compliance-and-defiance dilemma strategies

In a three-agent dilemma game the pure strategy of agent i is denoted by x_i, where $x_i \in \{0, 1\}$ for $i = \overline{1, 3}$. Strategy $x_i = 0$ corresponds to compliance, and strategy $x_i = 1$ corresponds to defiance. Triple

$$\{x_1, x_2, x_3\} \tag{1}$$

is a pure strategy situation in a three-agent dilemma game

$$\left\langle \{\{0, 1\}\}_{i=1}^{3}, \{L_i(x_1, x_2, x_3)\}_{i=1}^{3} \right\rangle, \tag{2}$$

in which

$$L_i(x_1, x_2, x_3) \tag{3}$$

is a payoff function of agent i, $i = \overline{1, 3}$. Payoff (3) is a conditional loss of agent i in pure strategy situation (1). Three-agent dilemma game (2) has eight pure strategy situations.

In a four-agent dilemma game the pure strategy of agent i is denoted by y_i, where $y_i \in \{0, 1\}$ for $i = \overline{1, 4}$. Strategy $y_i = 0$ corresponds to compliance, and strategy $y_i = 1$ corresponds to defiance. Quadruple

$$\{y_1, y_2, y_3, y_4\} \tag{4}$$

is a pure strategy situation in a four-agent dilemma game

$$\left\langle \{\{0, 1\}\}_{i=1}^{4}, \{H_i(y_1, y_2, y_3, y_4)\}_{i=1}^{4} \right\rangle, \tag{5}$$

in which

$$H_i(y_1, y_2, y_3, y_4) \tag{6}$$

is a payoff function of agent i, $i = \overline{1, 4}$. Payoff (6) is a conditional loss of agent i in pure strategy situation (4). Four-agent dilemma game (5) has 16 pure strategy situations.

In games (2) and (6), every pure strategy situation should be estimated by the respective conditional loss for each of the agents. In fact, the payoff function of agent i in game (2) can be represented as a $2 \times 2 \times 2$ matrix, and thus game (2) can be represented as a numbered set of three $2 \times 2 \times 2$ matrices. The payoff function of agent i in game (5) can be represented as a $2 \times 2 \times 2 \times 2$ matrix, and thus game (5) can be represented as a numbered set of four $2 \times 2 \times 2 \times 2$ matrices.

4. Pure strategy losses

In both games, pure strategy losses are naturally defined such that in any symmetric pure strategy situation, the conditional loss is the same [23]. In game (2), in the full-compliance situation

$$\{0, 0, 0\} \tag{7}$$

the cost of the compliance is a conditional unit:

$$L_i(0, 0, 0) = 1 \ \forall i = \overline{1, 3}. \tag{8}$$

In the full-defiance situation

$$\{1, 1, 1\} \tag{9}$$

the cost of the defiance is a conditional units [24]:

$$L_i(1, 1, 1) = a \ \forall i = \overline{1, 3}. \tag{10}$$

It is still assumed that, if only one agent defies, it does not affect the system. There are three such pure strategy situations:

$$\{1, 0, 0\}, \ \{0, 1, 0\}, \ \{0, 0, 1\}. \tag{11}$$

Each of situations (11) is called a 1-defiance situation. The payoff of agent i in a 1-defiance situation is 0 if the defiance strategy $x_i = 1$ is applied, and it is 1 if the compliance strategy $x_i = 0$ is applied:

$$L_i(x_1, x_2, x_3) = 1 - x_i \ \forall i = \overline{1, 3}. \tag{12}$$

It is also assumed that, of only one agent complies, it badly affects the system so that every agent additionally loses a half of the full-defiance situation cost regardless of whether the compliance strategy is applied or not. Being inverted to (8), there are three such 1-compliance situations:

$$\{0, 1, 1\}, \ \{1, 0, 1\}, \ \{1, 1, 0\}. \tag{13}$$

Each of situations (13) can be called a 1-compliance situation. The payoff of agent i in a 1-compliance situation is $\frac{a}{2}$ conditional units if the defiance strategy $x_i = 1$ is applied, and it is $1 + \frac{a}{2}$ if the compliance strategy $x_i = 0$ is applied:

$$L_i(x_1,\ x_2,\ x_3) = 1 + \frac{a}{2} - x_i \ \ \forall\, i = \overline{1,\ 3}. \tag{14}$$

If game (2) with its eight pure strategy situations (7), (9), (11), (13) and respective payoffs (8), (10), (12), (14) is studied without its mixed extension [4, 10, 23], there are one to seven equilibria depending on $a > 0$. For $a \in (0;\ 2)$, full-defiance situation (9) is the single equilibrium, in which every agent loses a conditional units. There are seven equilibria for $a = 2$: any situation except for full-compliance situation (7) is equilibrium. For $a > 2$, all three 1-defiance situations (11) are equilibria. Every 1-defiance situation is an efficient equilibrium, in which the sum of the agents' losses is 2 (the single agent that defies loses nothing, whereas the other two agents lose 1 unit each), but this equilibrium is asymmetric. Its asymmetry destroys the equilibrium presumptive stability, as it has been mentioned above.

In game (5), in the full-compliance situation

$$\{0,\ 0,\ 0,\ 0\} \tag{15}$$

the cost of the compliance is the very conditional unit:

$$H_i(0,\ 0,\ 0,\ 0) = 1 \ \ \forall\, i = \overline{1,\ 4}. \tag{16}$$

In the full-defiance situation

$$\{1,\ 1,\ 1,\ 1\} \tag{17}$$

the cost of the defiance is a conditional units [24]:

$$H_i(1,\ 1,\ 1,\ 1) = a \ \ \forall\, i = \overline{1,\ 4}. \tag{18}$$

It is still assumed that, if only one agent defies, it does not affect the system. There are four such 1-defiance situations in game (5):

$$\{1,\ 0,\ 0,\ 0\},\ \ \{0,\ 1,\ 0,\ 0\},\ \ \{0,\ 0,\ 1,\ 0\},\ \ \{0,\ 0,\ 0,\ 1\}. \tag{19}$$

Identically to game (2), the game (5) payoff of agent i in a 1-defiance situation is 0 if the defiance strategy $y_i = 1$ is applied, and it is 1 if the compliance strategy $y_i = 0$ is applied:

$$H_i(y_1,\ y_2,\ y_3,\ y_4) = 1 - y_i \ \ \forall\, i = \overline{1,\ 4}. \tag{20}$$

Next, it is also assumed that, of only one agent complies, it badly affects the system so that every agent additionally loses an amount of a conditional units regardless of whether the compliance strategy is applied or not. Being inverted to (19), there are four such 1-compliance situations:

$$\{0,\ 1,\ 1,\ 1\},\ \ \{1,\ 0,\ 1,\ 1\},\ \ \{1,\ 1,\ 0,\ 1\},\ \ \{1,\ 1,\ 1,\ 0\}. \tag{21}$$

In each of 1-compliance situations (21) there are three defying agents, unlike just two defying agents in a 1-compliance situation of set (13) in game (2). Therefore, the game (5) payoff of agent i in a 1-compliance situation is now a conditional units if the defiance strategy $y_i = 1$ is applied, and it is $1 + a$ if the compliance strategy $y_i = 0$ is applied:

$$H_i(y_1,\ y_2,\ y_3,\ y_4) = 1 + a - y_i \ \ \forall\, i = \overline{1,\ 4}. \tag{22}$$

The six remaining pure strategy situations are

$$\{1,\ 1,\ 0,\ 0\},\ \ \{1,\ 0,\ 1,\ 0\},\ \ \{1,\ 0,\ 0,\ 1\},\ \ \{0,\ 1,\ 1,\ 0\},\ \ \{0,\ 1,\ 0,\ 1\},\ \ \{0,\ 0,\ 1,\ 1\}. \tag{23}$$

Having two defying agents, each of situations (23) can be called a 2-defiance or 2-compliance situation. Therefore, the game (5) payoff of agent i in a 2-defiance situation is $\frac{a}{2}$ conditional units if the defiance strategy $y_i = 1$ is applied, and it is $1+\dfrac{a}{2}$ if the compliance strategy $y_i=1$ is applied:

$$H_i\left(y_1,\ y_2,\ y_3,\ y_4\right)=1+\frac{a}{2}-y_i\ \ \forall\, i=\overline{1,\ 4}. \tag{24}$$

If game (5) with its 16 pure strategy situations (15), (17), (19), (21), (23) and respective payoffs (16), (18), (20), (22), (24) is studied without its mixed extension [4, 10, 23], there are one to 11 equilibria depending on $a>0$. For $a\in(0;\ 2)$, just like in game (2), full-defiance situation (17) is the single equilibrium, in which every agent loses a conditional units. There are 11 equilibria for $a=2$: full-defiance situation (17), all four 1-defiance situations (19), and all six 2-defiance situations (23). For $a>2$, full-defiance situation (17) and all four 1-defiance situations (19) are equilibria. It is noteworthy that the full-defiance situation in game (2) for $a>2$ is not equilibrium. Every 1-defiance situation is an efficient equilibrium, in which the sum of the agents' losses is 3 (the single agent that defies loses nothing, whereas the other three agents lose 1 unit each), but this equilibrium is asymmetric. Its asymmetry destroys the equilibrium presumptive stability, as it has been mentioned above for game (2). All six 2-defiance situations (23) are asymmetric as well, where the sum of the agents' losses depends on a.

5. Mixed extension

In games (2) and (5), denote the mixed strategy of agent i by p_i and q_i, respectively, where $p_i\in[0;\ 1]$ and $q_i\in[0;\ 1]$ are probabilities of that agent i applies the defiance strategy. Consequently, $1-p_i$ is the probability of that agent i applies the compliance strategy in game (2), and $1-q_i$ is the probability of that agent i applies the compliance strategy in game (5). So, $p_i=x_i=1$ when agent i defies and

$$1-p_i=1-x_i=1$$

when agent i complies in game (2); $q_i=y_i=1$ when agent i defies and

$$1-q_i=1-y_i=1$$

when agent i complies in game (5).

In game (2), in a symmetric mixed strategy situation

$$\{p,\ p,\ p\} \tag{25}$$

with a probability $p\in[0;\ 1]$ of defiance, the expected payoff of agent i is calculated as follows [24]:

$$\begin{aligned} l_i\left(p,\ p,\ p\right)=&\left(1-p\right)^3L_i\left(0,\ 0,\ 0\right)+\\ &+\left(1-p\right)^2pL_i\left(0,\ 0,\ 1\right)+\\ &+\left(1-p\right)^2pL_i\left(0,\ 1,\ 0\right)+\\ &+\left(1-p\right)p^2L_i\left(0,\ 1,\ 1\right)+\\ &+\left(1-p\right)^2pL_i\left(1,\ 0,\ 0\right)+\\ &+\left(1-p\right)p^2L_i\left(1,\ 0,\ 1\right)+\\ &+\left(1-p\right)p^2L_i\left(1,\ 1,\ 0\right)+\\ &+p^3L_i\left(1,\ 1,\ 1\right)\ \text{ for }\ i=\overline{1,\ 3}. \end{aligned} \tag{26}$$

Plugging (8), (10), (12), (14) into (26) expectedly reveals that in game (2) situation (25) every agent loses the same amount of conditional units, denoted by $l\left(p,\ a\right)$:

$$l\left(p,\ a\right)=l_i\left(p,\ p,\ p\right)=-\frac{a}{2}p^3+\frac{3a}{2}p^2-p+1\ \text{ for }\ i=\overline{1,\ 3}. \tag{27}$$

In game (5), in a symmetric mixed strategy situation

$$\{q,\ q,\ q,\ q\} \tag{28}$$

with a probability $q \in [0;\ 1]$ of defiance, the expected payoff of agent i is calculated as follows:

$$\begin{aligned} h_i(q,\ q,\ q,\ q) = (1-q)^4 H_i(0,\ 0,\ 0,\ 0) + \\ +(1-q)^3 qH_i(0,\ 0,\ 0,\ 1) + \\ +(1-q)^3 qH_i(0,\ 0,\ 1,\ 0) + \\ +(1-q)^2 q^2 H_i(0,\ 0,\ 1,\ 1) + \\ +(1-q)^3 qH_i(0,\ 1,\ 0,\ 0) + \\ +(1-q)^2 q^2 H_i(0,\ 1,\ 0,\ 1) + \\ +(1-q)^2 q^2 H_i(0,\ 1,\ 1,\ 0) + \\ +(1-q) q^3 H_i(0,\ 1,\ 1,\ 1) + \\ +(1-q)^3 qH_i(1,\ 0,\ 0,\ 0) + \\ +(1-q)^2 q^2 H_i(1,\ 0,\ 0,\ 1) + \\ +(1-q)^2 q^2 H_i(1,\ 0,\ 1,\ 0) + \\ +(1-q) q^3 H_i(1,\ 0,\ 1,\ 1) + \\ +(1-q)^2 q^2 H_i(1,\ 1,\ 0,\ 0) + \\ +(1-q) q^3 H_i(1,\ 1,\ 0,\ 1) + \\ +(1-q) q^3 H_i(1,\ 1,\ 1,\ 0) + \\ +q^4 H_i(1,\ 1,\ 1,\ 1) \text{ for } i = \overline{1,\ 4}. \end{aligned} \tag{29}$$

Just like in game (2), plugging (16), (18), (20), (22), (24) into (29) expectedly reveals that in game (5) situation (28) every agent loses the same amount of conditional units, denoted by $h(q,\ a)$:

$$h(q,\ a) = h_i(q,\ q,\ q,\ q) = -2aq^3 + 3aq^2 - q + 1 \text{ for } i = \overline{1,\ 4}. \tag{30}$$

The task is to minimize each of functions (27) and (30) by the probability variable depending on the full-defiance situation cost a.

6. Best strategy in three-agent games

As an optimal situation in three-agent dilemma game (2) with payoffs (8), (10), (12), (14) is based on its symmetry and minimized equal loss, a symmetric situation

$$\{p^*,\ p^*,\ p^*\} \tag{31}$$

is searched such that

$$l^*(a) = l(p^*,\ a) = \min_{p\in[0;\ 1]} l(p,\ a) \text{ by } a > 0. \tag{32}$$

If

$$l^*(a) < 1\ \ \forall a > 0 \tag{33}$$

then situation (31) is acceptable having clear advantage over full-compliance situation (7).

The first partial derivative of surface (27) with respect to p is

$$\frac{\partial l}{\partial p} = -\frac{3a}{2}p^2 + 3ap - 1. \tag{34}$$

The discriminant of quadratic equation

$$-\frac{3a}{2}p^2 + 3ap - 1 = 0 \tag{35}$$

is

$$D = 9a^2 - 6a = 3a\left(3a - 2\right).$$

Hence, the roots of equation (35) are

$$p = p^{(1)} = \frac{3a + \sqrt{3a\left(3a-2\right)}}{3a} = 1 + \sqrt{\frac{3a-2}{3a}} \tag{36}$$

and

$$p = p^{(2)} = \frac{3a - \sqrt{3a\left(3a-2\right)}}{3a} = 1 - \sqrt{\frac{3a-2}{3a}}. \tag{37}$$

Roots (36) and (37) exist by $a \geqslant \frac{2}{3}$, where $p^{(1)} \geqslant p^{(2)}$. Besides, it is easy to see that

$$1 - \sqrt{\frac{3a-2}{3a}} \leqslant 1 \;\; \text{by} \;\; a \geqslant \frac{2}{3}$$

and

$$1 - \sqrt{\frac{3a-2}{3a}} > 0 \;\; \text{by} \;\; a \geqslant \frac{2}{3}$$

owing to

$$\frac{2}{3a} > 0,$$

$$1 > \frac{3a-2}{3a} > 0,$$

$$1 > \sqrt{\frac{3a-2}{3a}},$$

so $p^{(2)} \in (0;\ 1]$, i. e. root (37) is a mixed strategy in game (2). The second partial derivative of surface (27) with respect to variable p is

$$\frac{\partial^2 l}{\partial p^2} = -3ap + 3a = 3a\left(1 - p\right), \tag{38}$$

where

$$\left.\frac{\partial^2 l}{\partial p^2}\right|_{p=p^{(1)}} = 3a\left(1 - 1 - \sqrt{\frac{3a-2}{3a}}\right) = -\sqrt{3a\left(3a-2\right)} < 0$$

and

$$\left.\frac{\partial^2 l}{\partial p^2}\right|_{p=p^{(2)}} = 3a\left(1 - 1 + \sqrt{\frac{3a-2}{3a}}\right) = \sqrt{3a\left(3a-2\right)} > 0$$

by $a > \frac{2}{3}$. Therefore, point (37) is a local minimum point of function (27) of variable p with parameter $a > \frac{2}{3}$. Inasmuch as point (36) is a local maximum point and $p^{(1)} > 1$ by $a > \frac{2}{3}$, then on unit segment $[0;\ 1]$ by $a > \frac{2}{3}$ point (37) is the global minimum point of function (27) of variable p:

$$p^* = 1 - \sqrt{\frac{3a-2}{3a}}. \tag{39}$$

The global minimum value of function (27) by $p \in [0;\ 1]$ is

$$
\begin{aligned}
l^*(a) &= \min_{p\in[0;\ 1]} l(p,\ a) = l\left(1-\sqrt{\frac{3a-2}{3a}},\ a\right) = \\
&= -\frac{a}{2}\left(1-\sqrt{\frac{3a-2}{3a}}\right)^3 + \frac{3a}{2}\left(1-\sqrt{\frac{3a-2}{3a}}\right)^2 - 1 + \sqrt{\frac{3a-2}{3a}} + 1 = \\
&= \left(1-\sqrt{\frac{3a-2}{3a}}\right)^2 \cdot \left(a + \frac{a}{2}\cdot\sqrt{\frac{3a-2}{3a}}\right) + \sqrt{\frac{3a-2}{3a}} = \\
&= \left(\frac{6a-2}{3a} - 2\sqrt{\frac{3a-2}{3a}}\right)\cdot\left(a + \frac{a}{2}\cdot\sqrt{\frac{3a-2}{3a}}\right) + \sqrt{\frac{3a-2}{3a}} = \\
&= \frac{6a-2}{3} - 2a\sqrt{\frac{3a-2}{3a}} + \frac{6a-2}{6}\cdot\sqrt{\frac{3a-2}{3a}} - \frac{3a-2}{3} + \sqrt{\frac{3a-2}{3a}} = \\
&= a + \sqrt{\frac{3a-2}{3a}}\cdot\left(-2a + \frac{6a-2}{6} + 1\right) = \\
&= a + \sqrt{\frac{3a-2}{3a}}\cdot\left(\frac{2}{3} - a\right) \ \text{ by } \ a > \frac{2}{3}.
\end{aligned}
\tag{40}
$$

At $a = \frac{2}{3}$ local minimum point (37) and local maximum point (36) coincide: $p^{(1)} = p^{(2)} = 1$. As parabola (34) is negative by $a < \frac{2}{3}$, then surface (27) is a decreasing function of p by $a \in \left(0;\ \frac{2}{3}\right)$ and so $p^* = 1$, whereas

$$
l^*(a) = \min_{p\in[0;\ 1]} l(p,\ a) = l(1,\ a) = a \ \text{ by } \ a \in \left(0;\ \frac{2}{3}\right]. \tag{41}
$$

The full-defiance situation cost $a \in \left(0;\ \frac{2}{3}\right]$ is too low and thus the agent's best strategy is to defy owing to loss (41) satisfies condition (33). Now, it is to ascertain whether loss (40) for higher full-defiance situation costs satisfies condition (33).

The first derivative of the global minimum value (40) of function (27) as a function of a is:

$$
\begin{aligned}
\frac{dl^*}{da} &= 1 + \frac{1}{2\sqrt{\frac{3a-2}{3a}}}\cdot\left(\frac{2}{3a^2}\right)\cdot\left(\frac{2}{3}-a\right) - \sqrt{\frac{3a-2}{3a}} = \\
&= 1 + \frac{2-3a}{3a\sqrt{3a}\sqrt{3a-2}} - \sqrt{\frac{3a-2}{3a}} = \\
&= 1 - \sqrt{\frac{3a-2}{3a}}\cdot\left(\frac{3a+1}{3a}\right).
\end{aligned}
\tag{42}
$$

Inasmuch as

$$
1 - \sqrt{\frac{3a-2}{3a}}\cdot\left(\frac{3a+1}{3a}\right) > 0,
$$

$$
1 > \sqrt{\frac{3a-2}{3a}}\cdot\left(\frac{3a+1}{3a}\right),
$$

$$
1 > \left(\frac{3a-2}{3a}\right)\cdot\left(\frac{9a^2+6a+1}{9a^2}\right),
$$

$$1 > \frac{27a^3 - 18a^2 + 18a^2 - 12a + 3a - 2}{27a^3},$$

$$27a^3 > 27a^3 - 9a - 2,$$

$$0 > -9a - 2 \text{ for any } a > 0,$$

derivative (42) is positive, and so (40) is an increasing function. Therefore, the maximum of minimized loss (40) is

$$\sup_{a > \frac{2}{3}} l^*(a) = \lim_{a \to \infty} l^*(a) = \lim_{a \to \infty} \left[a + \sqrt{\frac{3a-2}{3a}} \cdot \left(\frac{2}{3} - a \right) \right].$$

The minimized loss by $a \geqslant \frac{2}{3}$ is function (40) also. So, the minimum of the minimized loss by $a \geqslant \frac{2}{3}$ is

$$\min_{a \geqslant \frac{2}{3}} l^*(a) = l^*\left(\frac{2}{3}\right) = a.$$

Meanwhile,

$$a \leqslant l^*(a) \leqslant l^*(1) = 1 - \frac{1}{3\sqrt{3}} < 1 \text{ for } a \in \left[\frac{2}{3};\ 1\right].$$

Next, it is easy to prove that

$$a + \sqrt{\frac{3a-2}{3a}} \cdot \left(\frac{2}{3} - a \right) < 1 \text{ for } a > 1. \tag{43}$$

Indeed, from inequality (43) it follows that

$$a - 1 < \sqrt{\frac{3a-2}{3a}} \cdot \left(a - \frac{2}{3} \right),$$

$$a^2 - 2a + 1 < \frac{3a-2}{3a} \cdot \left(\frac{3a-2}{3} \right)^2,$$

$$a^2 - 2a + 1 < \frac{27a^3 - 54a^2 + 36a - 8}{27a},$$

$$27a^3 - 54a^2 + 27a < 27a^3 - 54a^2 + 36a - 8,$$

$$0 < 9a - 8,$$

which is true for $a > 1$. Therefore,

$$\max_{a > \frac{2}{3}} l^*(a) < 1$$

and the minimized loss

$$l^*(a) \in [a;\ 1) \text{ by } a > 0 \tag{44}$$

along with (41).

While the minimized loss is constant by $a \in \left(0;\ \frac{2}{3}\right)$ and increases as the full-defiance situation cost is increased off value $\frac{2}{3}$, the best strategy (39) being valid for $a \geqslant \frac{2}{3}$ is a decreasing function of a. This follows from that

$$\frac{dp^*}{da} = -\frac{1}{2\sqrt{\frac{3a-2}{3a}}} \cdot \frac{2}{3a^2} = -\frac{1}{a\sqrt{3a}\sqrt{3a-2}} < 0.$$

It is quite natural as high full-defiance situation costs force agents to defy less.

7. Best strategy in four-agent games

In four-agent dilemma game (5) with payoffs (16), (18), (20), (22), (24), an optimal situation is a symmetric situation

$$\{q^*,\ q^*,\ q^*,\ q^*\} \tag{45}$$

is searched such that

$$h^*(a) = h(q^*,\ a) = \min_{q\in[0;\ 1]} h(q,\ a) \ \text{ by } \ a > 0. \tag{46}$$

If

$$h^*(a) < 1 \ \ \forall\, a > 0 \tag{47}$$

then situation (45) is acceptable having clear advantage over full-compliance situation (16).

The first partial derivative of surface (30) with respect to q is

$$\frac{\partial h}{\partial q} = -6aq^2 + 6aq - 1. \tag{48}$$

The discriminant of quadratic equation

$$-6aq^2 + 6aq - 1 = 0 \tag{49}$$

is

$$D = 36a^2 - 24a = 12a(3a-2).$$

Hence, the roots of equation (49) are

$$q = q^{(1)} = \frac{3a + \sqrt{3a(3a-2)}}{6a} = \frac{\sqrt{3a} + \sqrt{3a-2}}{2\sqrt{3a}} = \frac{p^{(1)}}{2} \tag{50}$$

and

$$q = q^{(2)} = \frac{3a - \sqrt{3a(3a-2)}}{6a} = \frac{\sqrt{3a} - \sqrt{3a-2}}{2\sqrt{3a}} = \frac{p^{(2)}}{2}. \tag{51}$$

It is trivially obvious that, just like roots (36) and (37), roots (50) and (51) exist by $a \geqslant \frac{2}{3}$, where $q^{(1)} \geqslant q^{(2)}$. Besides, as root (37) $p^{(2)} \in (0;\ 1]$, it is easy to see that $q^{(2)} \in \left(0;\ \frac{1}{2}\right]$, i. e. root (51) is a mixed strategy in game (5). However, unlike root (36) in game (2), it turns out that root (50) is a mixed strategy in game (5) as well. Indeed, $q^{(1)} = q^{(2)} = \frac{1}{2}$ at $a = \frac{2}{3}$, and

$$q^{(1)} = \frac{\sqrt{3a} + \sqrt{3a-2}}{2\sqrt{3a}} = \frac{1}{2} + \frac{\sqrt{3a-2}}{2\sqrt{3a}},$$

where

$$\frac{\sqrt{3a-2}}{2\sqrt{3a}} \in \left[0;\ \frac{1}{2}\right),$$

so $q^{(1)} \in \left[\frac{1}{2};\ 1\right)$.

The second partial derivative of surface (30) with respect to variable q is

$$\frac{\partial^2 h}{\partial q^2} = -12aq + 6a = 6a(1-2q), \tag{52}$$

where

$$\left.\frac{\partial^2 h}{\partial q^2}\right|_{q=q^{(1)}} = 6a\left(1 - 2\cdot\frac{\sqrt{3a} + \sqrt{3a-2}}{2\sqrt{3a}}\right) = -2\sqrt{3a(3a-2)} < 0$$

and

$$\left.\frac{\partial^2 h}{\partial q^2}\right|_{q=q^{(2)}} = 6a\left(1 - 2\cdot\frac{\sqrt{3a}-\sqrt{3a-2}}{2\sqrt{3a}}\right) = 2\sqrt{3a\,(3a-2)} > 0$$

by $a > \frac{2}{3}$. Therefore, points (50) and (51) are local maximum and minimum points, respectively, of function (30) of variable q with parameter $a > \frac{2}{3}$. At $a = \frac{2}{3}$ these local extrema coincide, whereupon

$$h^*\left(\frac{2}{3}\right) = h\left(q^*,\ \frac{2}{3}\right) = \min_{q\in[0;\ 1]} h\left(q,\ \frac{2}{3}\right) = \min\left\{h\left(\frac{1}{2},\ \frac{2}{3}\right),\ h\left(1,\ \frac{2}{3}\right)\right\} =$$
$$= \min\left\{-2\cdot\frac{2}{3}\cdot\left(\frac{1}{2}\right)^3 + 3\cdot\frac{2}{3}\cdot\left(\frac{1}{2}\right)^2 - \frac{1}{2} + 1,\ -2\cdot\frac{2}{3} + 3\cdot\frac{2}{3} - 1 + 1\right\} =$$
$$= \min\left\{\frac{5}{6},\ \frac{2}{3}\right\} = \frac{2}{3} = a \tag{53}$$

and thus the best strategy at $a = \frac{2}{3}$ is $q^* = 1$.

By $a > \frac{2}{3}$ the local minimum value of function (30) of variable q is

$$h\left(q^{(2)},\ a\right) = h\left(\frac{\sqrt{3a}-\sqrt{3a-2}}{2\sqrt{3a}},\ a\right) =$$
$$= a\cdot\left(\frac{\sqrt{3a}-\sqrt{3a-2}}{2\sqrt{3a}}\right)^2\cdot\left(3 - 2\cdot\frac{\sqrt{3a}-\sqrt{3a-2}}{2\sqrt{3a}}\right) - \frac{\sqrt{3a}-\sqrt{3a-2}}{2\sqrt{3a}} + 1 =$$
$$= \frac{3a - 1 - \sqrt{3a}\cdot\sqrt{3a-2}}{6}\cdot\frac{2\sqrt{3a}+\sqrt{3a-2}}{\sqrt{3a}} + \frac{\sqrt{3a}+\sqrt{3a-2}}{2\sqrt{3a}} =$$
$$= \frac{6a\sqrt{3a} + 3a\sqrt{3a-2} - 2\sqrt{3a} - \sqrt{3a-2} - 6a\sqrt{3a-2} - \sqrt{3a}\cdot(3a-2) + 3\sqrt{3a} + 3\sqrt{3a-2}}{6\sqrt{3a}} =$$
$$= \frac{3a\sqrt{3a} - 3a\sqrt{3a-2} + 3\sqrt{3a} + 2\sqrt{3a-2}}{6\sqrt{3a}} =$$
$$= \frac{3\sqrt{3a}\,(a+1) - \sqrt{(3a-2)^3}}{6\sqrt{3a}} = \frac{a+1}{2} - \frac{\sqrt{(3a-2)^3}}{6\sqrt{3a}}. \tag{54}$$

The first derivative of the local minimum value (54) as a function of a is:

$$\frac{dh\left(q^{(2)},\ a\right)}{da} = \frac{1}{2} - \frac{1}{6}\cdot\left(\frac{9}{2}\cdot\sqrt{\frac{3a-2}{3a}} - \frac{3}{2}\cdot\sqrt{\frac{(3a-2)^3}{(3a)^3}}\right) =$$
$$= \frac{1}{2} - \frac{3}{4}\cdot\sqrt{\frac{3a-2}{3a}} + \frac{1}{4}\cdot\sqrt{\frac{(3a-2)^3}{(3a)^3}} = \frac{1}{2} - \frac{3}{4}\cdot\sqrt{\frac{3a-2}{3a}} + \frac{(3a-2)}{4\cdot 3a}\cdot\sqrt{\frac{3a-2}{3a}} =$$
$$= \frac{1}{2} + \sqrt{\frac{3a-2}{3a}}\left(\frac{3a-2}{12a} - \frac{3}{4}\right) = \frac{1}{2} - \sqrt{\frac{3a-2}{3a}}\left(\frac{1+3a}{6a}\right). \tag{55}$$

If (55) is positive then function (54) is increasing. Consider inequality

$$\frac{1}{2} - \sqrt{\frac{3a-2}{3a}}\left(\frac{1+3a}{6a}\right) > 0 \ \text{ by } \ a > \frac{2}{3}, \tag{56}$$

whence

$$1 > \sqrt{\frac{3a-2}{3a}}\left(\frac{1+3a}{3a}\right),$$

$$\frac{3a}{1+3a} > \sqrt{\frac{3a-2}{3a}},$$

$$\frac{9a^2}{1+6a+9a^2} > \frac{3a-2}{3a},$$

$$27a^3 > (3a-2)\left(1+6a+9a^2\right),$$

$$27a^3 > 3a-2+18a^2-12a+27a^3-18a^2,$$

$$0 > -2-9a,$$

which is true. So, inequality (56) is true and the local minimum value of function (30) of variable q by $a > \frac{2}{3}$ is an increasing function. Therefore, the maximum of the local minimum value is never reached as

$$\sup_{a\in\left(\frac{2}{3};\,\infty\right)} h\left(q^{(2)},\ a\right) = \lim_{a\to\infty} h\left(q^{(2)},\ a\right).$$

Nevertheless, it is easy to prove that

$$h\left(q^{(2)},\ a\right) = \frac{a+1}{2} - \frac{\sqrt{(3a-2)^3}}{6\sqrt{3a}} < 1 \text{ by } a \geqslant \frac{2}{3}. \tag{57}$$

Indeed, from inequality (57) it follows that

$$\frac{a+1}{2} - \frac{\sqrt{(3a-2)^3}}{6\sqrt{3a}} < 1,$$

$$a-1 < \frac{\sqrt{(3a-2)^3}}{3\sqrt{3a}},$$

$$3\sqrt{3a}\,(a-1) < \sqrt{(3a-2)^3}. \tag{58}$$

Inequality (58) holds for $a \in \left[\frac{2}{3};\ 1\right]$. If $a > 1$ then squaring both sides of inequality (58) gives:

$$27a^3-54a^2+27a < 27a^3-18a^2-36a^2+24a+12a-8, \tag{59}$$

$$0 < 9a-8,$$

i. e. inequality (58) holds for $a > 1$, whence inequality (57) holds.

By $a > \frac{2}{3}$ the global minimum of function (30) of variable q can be reached at $q = 1$, so the best strategy is

$$q^* \in \left\{q^{(2)},\ 1\right\} = \left\{\frac{\sqrt{3a}-\sqrt{3a-2}}{2\sqrt{3a}},\ 1\right\} \tag{60}$$

and the lowest loss is

$$h^*(a) = \min\left\{h\left(q^{(2)},\ a\right),\ h(1,\ a)\right\} =$$

$$= \min\left\{\frac{a+1}{2} - \frac{\sqrt{(3a-2)^3}}{6\sqrt{3a}},\ a\right\}. \tag{61}$$

Owing to (57), the lowest loss

$$h^*(a) < 1 \ \ \forall\, a \geqslant \frac{2}{3}. \tag{62}$$

The local minimum value (54) becomes the global minimum value if

$$\frac{a+1}{2} - \frac{\sqrt{(3a-2)^3}}{6\sqrt{3a}} < a. \tag{63}$$

Inequality (63) is equivalent to inequality

$$1 - a < \frac{\sqrt{(3a-2)^3}}{3\sqrt{3a}}, \tag{64}$$

which is true $\forall\, a \geqslant 1$. For $a \in \left[\frac{2}{3};\ 1\right)$ inequality (64), similarly to (58) for $a > 1$, is equivalent to (59), i. e. inequality (64) holds for $a > \frac{8}{9}$. Consequently, inequality (63) holds by $a > \frac{8}{9}$ and probability (51) is the global minimum point of function (30) of variable q by $a > \frac{8}{9}$, where the minimized loss is (54).

At $a = \frac{8}{9}$ the minimized loss is a and it is reached both at probability $q^* = 1$ and at probability (51) that turns out to be

$$q^* = \frac{\sqrt{3\cdot\frac{8}{9}} - \sqrt{3\cdot\frac{8}{9} - 2}}{2\sqrt{3\cdot\frac{8}{9}}} = \frac{1}{2} - \frac{\sqrt{\frac{8}{3} - 2}}{2\sqrt{\frac{8}{3}}} = \frac{1}{2} - \frac{\sqrt{\frac{2}{3}}}{4\sqrt{\frac{2}{3}}} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}.$$

The minimized loss a is reached at probability $q^* = 1$ by $a \in \left[\frac{2}{3};\ \frac{8}{9}\right)$.

By $a \in \left(0;\ \frac{2}{3}\right)$ first derivative (48) does not turn into 0 and it is negative $\forall\, q \in [0;\ 1]$ due to quadratic equation (49) does not have roots and

$$-6aq^2 + 6aq - 1 < 0$$

owing to $-6a < 0$. Therefore, surface (30) with respect to q is decreasing by $a \in \left(0;\ \frac{2}{3}\right)$ and its minimum on interval $[0;\ 1]$ is reached at point $q = q^* = 1$, i. e.

$$h^*(a) = h(1,\ a) = a < \frac{2}{3}.$$

This means that by $a \in \left(0;\ \frac{8}{9}\right]$ the minimized loss a is reached at probability $q^* = 1$ being the best strategy. Probability $q^* = \frac{1}{4}$ is also the best strategy at $a = \frac{8}{9}$. By $a > \frac{8}{9}$ the minimized loss is (54) reached at probability (51) being the best strategy. Overall, the minimized loss

$$h^*(a) \in [a;\ 1) \ \text{ by } \ a > 0 \tag{65}$$

along with

$$h^*(a) = \min_{q \in [0;\ 1]} h(q,\ a) = h(1,\ a) = a \text{ by } a \in \left(0;\ \frac{8}{9}\right]. \tag{66}$$

While the minimized loss is constant by $a \in \left(0;\ \frac{8}{9}\right)$ and increases as the full-defiance situation cost is increased off value $\frac{8}{9}$, the best strategy (51) being valid for $a \geqslant \frac{8}{9}$ is a decreasing function of a. This follows from that the best strategy (51) is half the best strategy (39) being a decreasing function in game (2) for $a \geqslant \frac{2}{3}$.

8. Discussion and conclusion

In the considered compliance-and-defiance dilemma games with three or four agents, the best strategy for every agent is to defy (by applying the defiance pure strategy) with a probability that ensures the global minimum to the agent's loss on the probabilistic interval $[0;\ 1]$ by a given value a of the full-defiance situation cost. In three-agent dilemma game (2) with payoffs (8), (10), (12), (14), the best strategy is to fully defy if this cost does not exceed $\frac{2}{3}$. This is the straightforward consequence of a too low cost of full-defiance situation (9), in which the best strategy ensures the agent's loss equal to the cost. As the cost is increased off value $\frac{2}{3}$, the best strategy probability exponentially-like decreases (Figure 1), while the agent's minimized loss increases (Figure 2) in the same manner. Nevertheless, the best strategy ensures the agent's minimized loss does not exceed 1 (a conditional unit), which is the cost of full compliance.

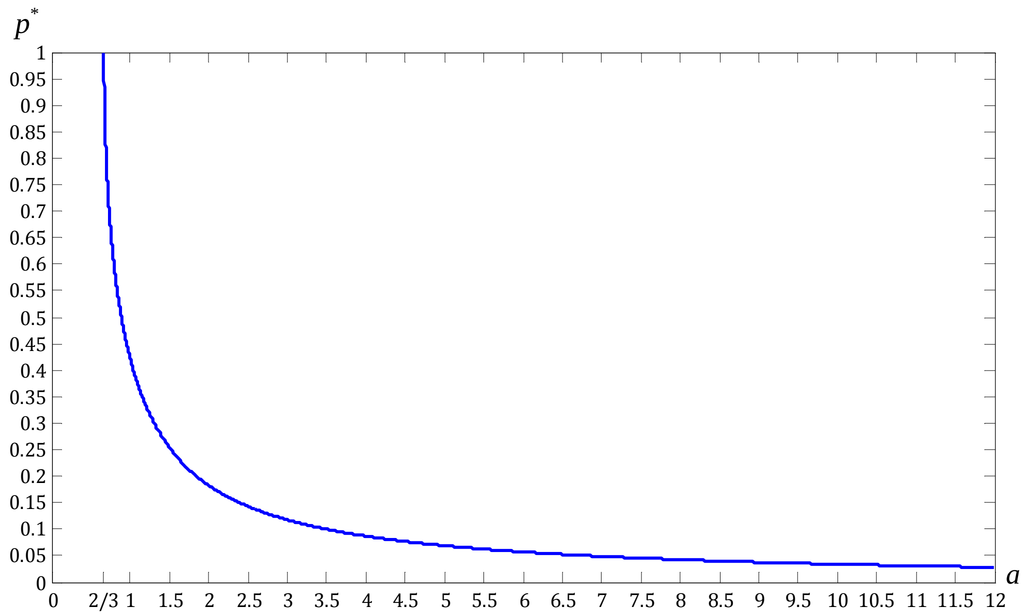

Figure 1. The best strategy probability in three-agent dilemma game (2) versus the full-defiance situation cost

In four-agent dilemma game (5) with payoffs (16), (18), (20), (22), (24), the consequence of too low costs of full-defiance situation (17) is a little bit severer. The best strategy in this game is to fully defy if the full-defiance situation cost does not exceed $\frac{8}{9}$, whereupon the agent's loss is equal to the cost. As the cost is increased off value $\frac{8}{9}$, the best strategy probability exponentially-like decreases (Figure 3) dropping down off value $q^* = \frac{1}{4}$, which is

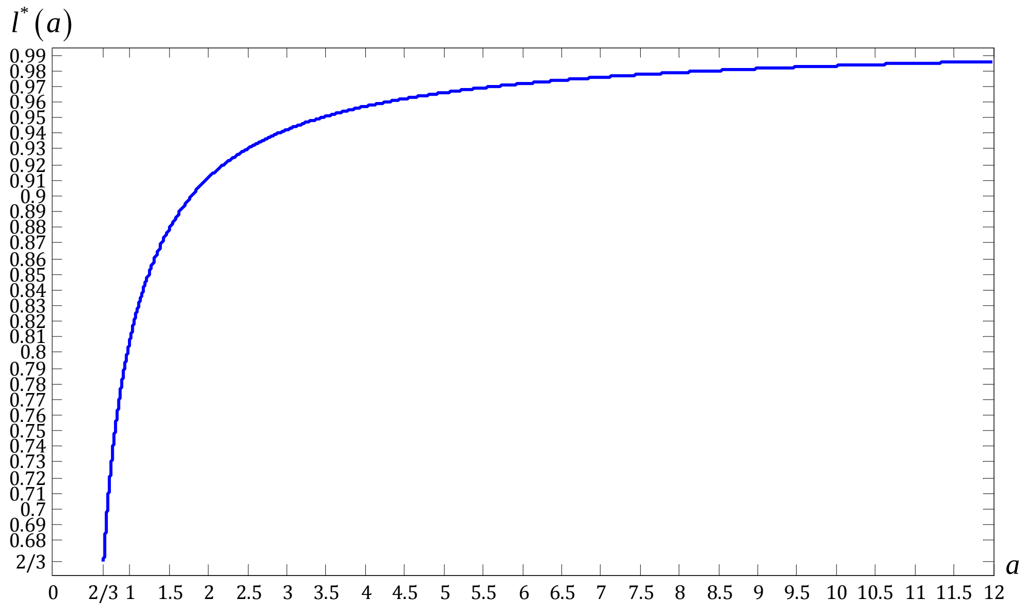

Figure 2. The agent's minimized loss in three-agent dilemma game (2) versus the full-defiance situation cost

also the best strategy (being far more favorable for the system) at $a = \frac{8}{9}$. Unlike the three-agent game, which does not have a discontinuity jump in the best strategy probability, in the four-agent game the best strategy probability does have such a jump at $a = \frac{8}{9}$. Similarly to the agent's minimized loss in the three-agent game, the agent's minimized loss in the four-agent game increases (Figure 4) in the same manner never exceeding 1.

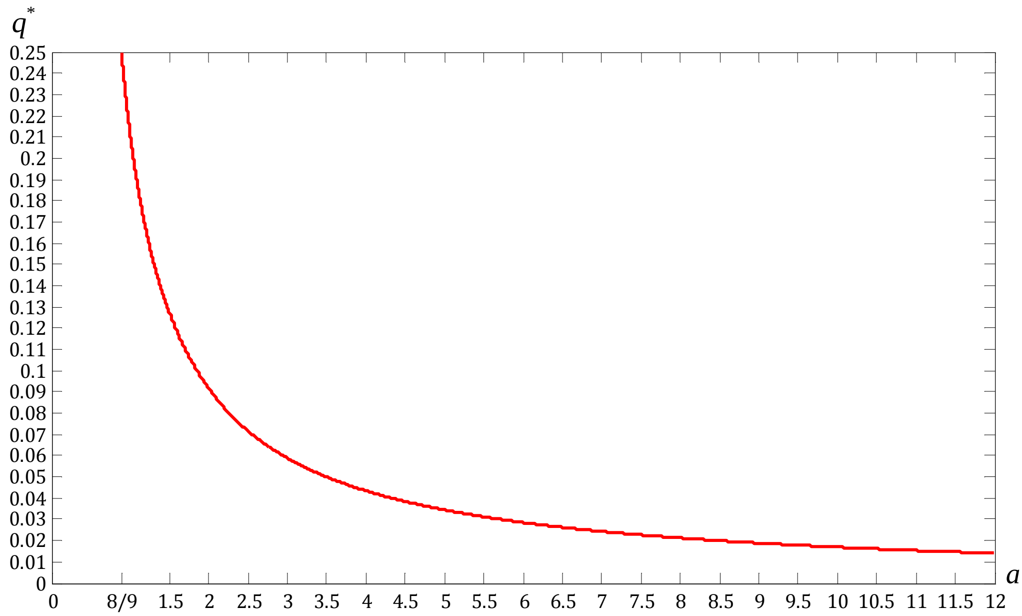

Figure 3. The best strategy probability in four-agent dilemma game (5) versus the full-defiance situation cost $a \geqslant \frac{8}{9}$

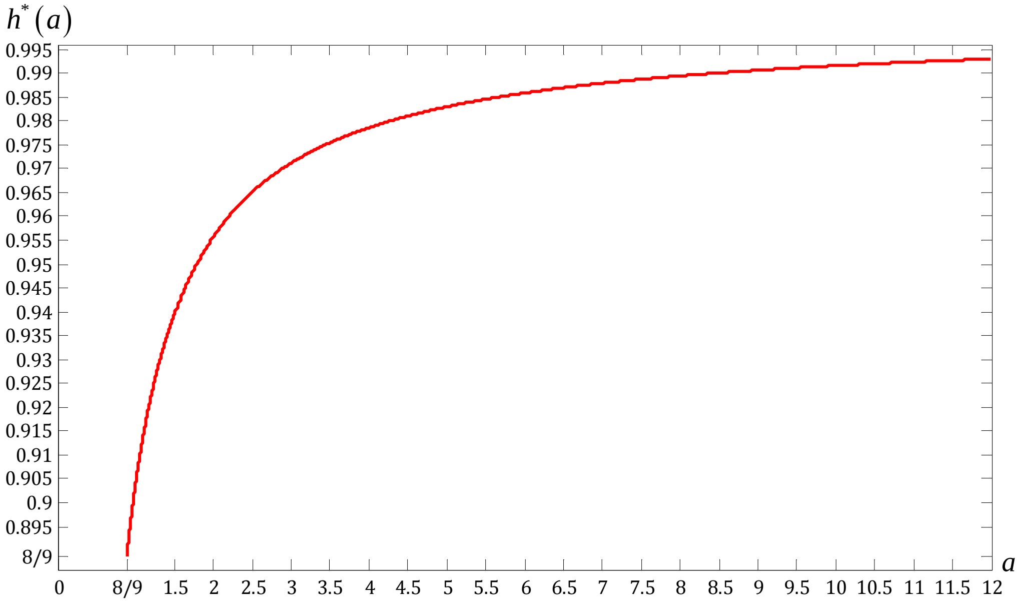

Figure 4. The agent's minimized loss in four-agent dilemma game (5) versus the full-defiance situation cost $a \geqslant \frac{8}{9}$

It is easy to get convinced that

$$l^*(a) < h^*(a) \quad \text{for} \quad a \geqslant \frac{8}{9} \tag{67}$$

and easier to see that

$$p^* > q^* \quad \text{for} \quad a \geqslant \frac{8}{9}$$

by recalling that the respective relationship $q^* = \frac{p^*}{2}$ is true from (51). Indeed,

$$l^*(a) - h^*(a) = a + \sqrt{\frac{3a-2}{3a}} \cdot \left(\frac{2}{3} - a\right) - \frac{a+1}{2} + \frac{\sqrt{(3a-2)^3}}{6\sqrt{3a}} =$$
$$= \frac{a-1}{2} + \sqrt{\frac{3a-2}{3a}} \cdot \left(\frac{2}{3} - a\right) + \frac{\sqrt{(3a-2)^3}}{6\sqrt{3a}} =$$
$$= \frac{a-1}{2} + \frac{4\sqrt{3a-2} - 6a\sqrt{3a-2} + \sqrt{(3a-2)^3}}{6\sqrt{3a}} =$$
$$= \frac{a-1}{2} + \frac{(4 - 6a + 3a - 2)\sqrt{3a-2}}{6\sqrt{3a}} =$$
$$= \frac{a-1}{2} - \frac{\sqrt{(3a-2)^3}}{6\sqrt{3a}}.$$

Inequality

$$\frac{a-1}{2} < \frac{\sqrt{(3a-2)^3}}{6\sqrt{3a}} \tag{68}$$

is true for $a \in \left[\frac{8}{9};\ 1\right]$. For $a > 1$ inequality (68) is equivalent to inequality (58), which holds for $a > 1$, and, therefore, inequality (67) is true.

The results, although being much theorized, are believed to serve as behavioral or governmental policy for balancing compliance and defiance in order to harmonize both the system global purposes and local entrepreneurship endeavors of its components (agents). In particular, the compliance-and-defiance dilemma game best strategy eventually focuses on government, legislation, and law enforcement for environmental protection. This should mostly serve as a comprehensively legal and institutional framework to guide environmental-management decisions. This is why the model perceives agents as homogeneous ones in terms of their strategies and payoffs, rather than assuming them to be such.

Some heterogeneousness of agents might be considered, but the model perceives agents as homogeneous ones (in terms of their strategies and payoffs) because, in reality, the environmental protection framework must not strongly distinguish among pollution or damage subjects. First, it is almost always impossible to determine and prove the source of pollution or damage. Even if it was possible, it would take years to prove it in court and implement the respective court adjudication. Second, the collective responsibility must dominate here over individual responsibility inasmuch as tracing individual responsibility would dissipate additional resources which might be directed towards environmental protection development instead (e. g., subsidizing low-income subjects of pollution that cannot afford compliance in full measure).

This system set-up is considered static as it is a proper assumption for a relatively short period of time. Adaptivity of strategies is shown, in fact, with the best strategy, towards which the agent strives for. All the more, the present-day environmental protection policies provide for management stability rather than management dynamics. Because stability means predictability, and the latter implies better management and control. Even when the compliance-and-defiance dilemma game models network security or, speaking generally, resource allocation, prioritizing subjects takes significant resources whose dissipation may weaken system protection, security, defense, etc. The system sustainable development has higher priority, after all. The suggested model is a simplified scenario to support the sustainable development under administrable conditions of regulation. This is the main practical contribution of the research.

Obviously, the best strategy for further research is to study compliance-and-defiance dilemma games with five and more agents, i. e. to obtain the fairest-minimum-loss solution to a dilemma game with any number of agents. However, it is worth remembering that the obtained results are not directly extendable to larger systems. A qualitative analysis of the solution, once it is obtained for such systems, should be carried out depending on the number of agents. Trends and other relationships among the solutions must be ascertained thereupon. Another pathway to be trodden out for further research is to loose the assumption and simplification about that the system is unaffected if only one agent defies. Besides, non-uniformity of fining should be considered as well due to exploiting more complex penalty structures would have a better impact of making the model more realistic.

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REFERENCES

1. S. Adlakha, R. Johari, and G. Y. Weintraub, *Equilibria of dynamic games with many players: Existence, approximation, and market structure*, Journal of Economic Theory, vol. 156, pp. 269–316, 2015.
2. E. M. Alrawhani, A. Romli, and M. A. Al-Sharafi, *Evaluating the role of protection motivation theory in information security policy compliance: Insights from the banking sector using PLS-SEM approach*, Journal of Open Innovation: Technology, Market, and Complexity, vol. 11, iss. 1, Article no. 100463, 2025.
3. C. Arguedas, *Pollution standards, technology investment and fines for non-compliance*, Journal of Regulatory Economics, vol. 44, iss. 2, 156–176, 2013.
4. E. N. Barron, *Game theory: An introduction*, 2nd ed. Hoboken, New Jersey, USA, Wiley, 2013.

5. J. P. Benoit and V. Krishna, *Finitely repeated games*, Econometrica, vol. 53, iss. 4, pp. 905–922, 1985.
6. J. P. Casasnovas, *Evolutionary Games in Complex Topologies: Interplay Between Structure and Dynamics*, Springer-Verlag Berlin Heidelberg, 2012.
7. T. P. Choudhari, B. K. Pathivada, A. Maji, M. Jayakumar, J. Damani, V. Perumal, N. R. Velaga, and K. V. K. Rao, *Helmet non-compliance among motorized two-wheelers: Insights from public opinion surveys*, Case Studies on Transport Policy, Article no. 101439, 2025.
8. R. Christmann, *Plea bargaining and investigation effort: inquisitorial criminal procedure as a three-player game*, European Journal of Law and Economics, vol. 56, pp. 497–532, 2023.
9. P. Clayton, *Introduction to electromagnetic compatibility*, Wiley, 2008.
10. T. Fujiwara-Greve, *Non-Cooperative Game Theory, in: Monographs in Mathematical Economics*, Springer Tokyo, 2015.
11. P. Kairon, K. Thapliyal, R. Srikanth, and A. Pathak, *Noisy three-player dilemma game: robustness of the quantum advantage*, Quantum Information Processing, vol. 19, Article no. 327, 2020.
12. M. Kim, *Strategy inference using the maximum likelihood estimation in the iterated prisoner's dilemma game*, Journal of the Korean Physical Society, vol. 84, pp. 102–107, 2024.
13. Q. Li and T. Fujita, *Stabilizing International Environmental Agreements*, in: S. Hori, Y. Takamura, T. Fujita, and N. Kanie (Eds.), *International Development and the Environment. Sustainable Development Goals Series*, Springer, Singapore, 2020, pp. 91–103.
14. K. Lin, *Navigating ambiguity: Local strategies in China's energy policy and governance*, Energy Research & Social Science, vol. 122, Article no. 103985, 2025.
15. A. McEachern, *Game Theory: A Classical Introduction, Mathematical Games, and the Tournament*, in: *Synthesis Lectures on Games and Computational Intelligence*, Springer Cham, Springer Nature Switzerland, 2022.
16. M. A. Ramírez, M. Smerlak, A. Traulsen, and J. Jost, *Diversity enables the jump towards cooperation for the Traveler's Dilemma*, Scientific Reports, vol. 13, Article no. 1441, 2023.
17. J. L. Regens, B. J. Seldon, and E. Elliott, *Modeling compliance to environmental regulation: Evidence from manufacturing industries*, Journal of Policy Modeling, vol. 19, iss. 6, pp. 683–696, 1997.
18. M. Rezapour, S. S. Wulff, and K. Ksaibati, *Effectiveness of enforcement resources in the highway patrol in reducing fatality rates*, IATSS Research, vol. 42, iss. 4, pp. 259–264, 2018.
19. V. V. Romanuke, *Deep clustering of the traveling salesman problem to parallelize its solution*, Computers & Operations Research, vol. 165, Article no. 106548, pp. 1–20, 2024.
20. V. V. Romanuke, *Fast approximation of the traveling salesman problem shortest route by rectangular cell clustering pattern to parallelize solving*, Statistics, Optimization and Information Computing, vol. 12, no. 5, pp. 1425–1459, 2024.
21. V. V. Romanuke, *Pure strategy saddle points in the generalized progressive discrete silent duel with identical linear accuracy functions*, Journal of Information and Organizational Sciences, vol. 48, no. 1, pp. 81–98, 2024.
22. V. V. Romanuke, *Pure strategy solutions of the progressive discrete silent duel with generalized identical quadratic accuracy functions*, Discrete Applied Mathematics, vol. 349, pp. 215–232, 2024.
23. V. V. Romanuke, *Recommendations on using the nonequilibrium symmetric situation in a dyadic game as a model of the environment preservation with the three subjects of pollution*, Ecological Safety and Nature Management, no. 5, pp. 144–159, 2010.
24. V. V. Romanuke, *Environment guard model as dyadic three-person game with the generalized fine for the reservoir pollution*, Ecological Safety and Nature Management, no. 6, 77–94, 2010.
25. V. V. Romanuke, *Ecological-economic balance in fining environmental pollution subjects by a dyadic 3-person game model*, Applied Ecology and Environmental Research, vol. 17, no. 2, pp. 1451–1474, 2019.
26. M. M. Samaan, *The Nile Development Game: Tug-of-War or Benefits for All?*, Springer Cham, Springer Nature Switzerland AG, 2019.
27. U. Schulz, W. Albers, and U. Mueller, *Social Dilemmas and Cooperation*, Springer-Verlag Berlin, Heidelberg, 1994.
28. M. B. Shareh, H. Navidi, H. H. S. Javadi, and M. HosseinZadeh, *Preventing Sybil attacks in P2P file sharing networks based on the evolutionary game model*, Information Sciences, vol. 470, 94–108, 2019.
29. J. Sostrin, *The Manager's Dilemma: Balancing the Inverse Equation of Increasing Demands and Shrinking Resources*, Palgrave Macmillan New York, 2015.
30. C. Süring and H. P. Weikard, *Coalition stability in international environmental matching agreements*, Group Decision and Negotiation, vol. 33, pp. 587–615, 2024.
31. S. Tan and J. Lü, *Strategy Selection in Networked Evolutionary Games: Structural Effect and the Evolution of Cooperation*, in: J. Lü, X. Yu, G. Chen, and W. Yu (Eds.), *Complex Systems and Networks. Understanding Complex Systems*, Springer, Berlin, Heidelberg, 2016, pp. 439–458.
32. J. Tanimoto, *Fundamentals of Evolutionary Game Theory and its Applications*, in: *Series Title: Evolutionary Economics and Social Complexity Science*, Springer Tokyo, 2015.
33. J. Wang, R. Wang, F. Yu, Z. Wang, and Q. Li, *Learning continuous and consistent strategy promotes cooperation in prisoner's dilemma game with mixed strategy*, Applied Mathematics and Computation, vol. 370, Article no. 124887, 2020.
34. L.-W. Wong, V.-H. Lee, G. W.-H. Tan, K.-B. Ooi, A. Sohal, *The role of cybersecurity and policy awareness in shifting employee compliance attitudes: building supply chain capabilities*, International Journal of Information Management, vol. 66, Article no. 102520, 2022.
35. J. Wu, Y. Liang, S. Yu, and L. Yu, *System dynamics emerging from compliance behaviors coordination of electricity consumers under renewable portfolio standards in China: An agent-based analysis*, Energy, vol. 321, Article no. 135478, 2025.
36. D. Ye and M. Zhang, *A Study on the Evolution of Cooperation in Networks*, in: X. Lin, Y. Manolopoulos, D. Srivastava, and G. Huang (Eds.), *Web Information Systems Engineering – WISE 2013. Lecture Notes in Computer Science*, vol. 8181, Springer, Berlin, Heidelberg, 2013, pp. 285–298.