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Multivariate Time Series Analysis of the USD/IQD Exchange Rate Using VAR, SVAR, and SVECM Models.

Diana Hikmat Hana [1], Sameera A. Othman [2,*]

[1] *Department of Statistics and Informatics, College of Administration and Economics, University of Dohuk, Kurdistan Region, Iraq*
[2] *Department of Mathematics, College of Basic Education, University of Duhok, Iraq*

Abstract This study examines the USD/IQD exchange rate using multivariate time series models. We implement vector autoregressive (VAR), structural VAR (SVAR), and structural vector error correction (SVEC) models using the 'vars' package in R. The analysis includes diagnostic testing, a constrained model estimation, prediction, causality analysis, impulse response functions, and forecast error variance decomposition. Variables are selected using the Granger causality test, leading to various model combinations. Model 3, which includes USD, gold, and copper, is identified as optimal for accurate forecasting. Although the oil variable has a high p-value (0.4674), its inclusion is justified based on economic intuition and statistical reasoning, given its influence on exchange rates and commodity prices that is crucial for making good investment decisions.

Keywords VAR models, Co-integration, Granger causality, Engle-Granger, Johansen test.

AMS 2010 subject classifications: 62G05; 60E05; 62N05; 62P30

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1. Introduction

*M*any time-related variables make up a multivariate time series (MTS), and it is important to realize that each variable's dependence is impacted by interactions with other variables in addition to its previous values. This dependency is used to forecast values for the future. Investigating the intricate relationships between variables and improving forecast accuracy are the objectives of multivariate time series analysis. Vector autoregressive models (VARs) became a standard tool in econometrics in the early 1980s after being criticized by the authors in [1]. Since statistical tests are frequently used to find links and complex correlations among variables, the addition of non-statistical prior information quickly improves this method. Unlike deterministic repressors, VAR models use their own histories to fully represent endogenous variables. The explicit modelling of contemporaneous interdependencies between the variables on the left is made easier by structured vector autoregressive models, or SVARs. As a result, these models make an effort to rectify the shortcomings of VAR models. The multiple structural equation model paradigm, which was first created by the Cowles Foundation in the 1940s and 1950s, was put to the test by Sims. The idea of co-integration, however, was brought to the area of econometrics by Granger in [2] and then by Engle and Granger in [3] as a potent tool for modelling and assessing economic interactions. The use of structural vector error correction models (SVEC) and vector error correction models (VECM) has recently led to a convergence in the research of these domains. Each of these models is thoroughly explained theoretically in the monographs written by Hamilton [7], Hendry [5], Johansen [6], Banerjee et al. [8], and Lütkepohl [4], [9], [10]. Examining Vector Autoregressive (VAR) techniques for multivariate time series data analysis is the main goal of

*Correspondence to: Diana Hikmat Hana (Email: diana.hana@uod.ac). Department of Statistics and Informatics, College of Administration and Economics, University of Duhok, Iraq.

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this research, using data from [https://www.investing.com], this study attempts to determine the best method for modelling four distinct time series variables associated with the Iraqi USD/IQD exchange rate. Using the vars package in R, the study investigates a number of modelling approaches, such as Vector Autoregressive (VAR), Structural Vector Autoregressive (SVAR), and Structural Vector Error Correction Models (SVECM). Additionally, it provides a comprehensive framework for diagnostic testing, model estimation, prediction, causality analysis, impulse response analysis, and forecast error variance decomposition.

1.1. Methodology

We present the theoretical foundation of the Vector Autoregressive (VAR) model.

1.2. Form of Vector Autoregressive (VAR) Model

The VAR model is a multivariate regression model in which all variables on the right side of the equation represent the dependent variables' lagged values. Let $Z_t = (Z_{1t} \cdots Z_{kt})'$ represent the vector of variables at time t and let Φ_i represent the coefficient matrices. These parameters account for cross-lagged relationships between variables and the autoregressive effect of a variable on itself across time. The intercepts of the model are designated by $c = (c_1 \cdots c_k)'$ and the white noise process, often referred to as perturbations or random shocks, is represented by $a_t = (a_{1t} \cdots a_{kt})'$. These innovations are the portions of the present data Z_t that cannot be explained by earlier observations $Z_{(t-1)}, Z_{(t-2)}, \ldots, Z_{(t-p)}$. The innovations are believed to follow a white-noise process, which means that each has a zero mean and a time-invariant, symmetric, positive-definite covariance matrix, which is frequently assumed to be block-diagonal. The basic p-lag Vector Autoregressive VAR(p) model is expressed as follows: [11]

$$Z_t = c + \Phi_1 Z_{(t-1)} + \Phi_2 Z_{(t-2)} + \cdots + \Phi_p Z_{(t-p)} + a_t; \quad t = 0, \pm 1, \pm 2, \ldots \tag{1}$$

Where:
$Z_t = (Z_{1t} \cdots Z_{kt})'$ is a $(k \times 1)$ vector of time series variables.
Φ_i are fixed $(k \times k)$ coefficient matrices.
$c = (c_1 \cdots c_k)'$ is a fixed $(k \times 1)$ vector of intercept terms.
$a_t = (a_{1t} \cdots a_{kt})'$ is a $(k \times 1)$ white noise process, representing independent samples taken from a multivariate Gaussian distribution with a variance-covariance matrix over time points.

$$\begin{pmatrix} Z_{1t} \\ Z_{2t} \\ \vdots \\ Z_{kt} \end{pmatrix} = \begin{pmatrix} \Phi^1_{11} & \Phi^1_{12} & \cdots & \Phi^1_{1k} \\ \Phi^1_{21} & \Phi^1_{22} & \cdots & \Phi^1_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \Phi^1_{k1} & \Phi^1_{k2} & \cdots & \Phi^1_{kk} \end{pmatrix} \begin{pmatrix} Z_{1(t-1)} \\ Z_{2(t-1)} \\ \vdots \\ Z_{k(t-1)} \end{pmatrix} + \begin{pmatrix} \Phi^2_{11} & \Phi^2_{12} & \cdots & \Phi^2_{1k} \\ \Phi^2_{21} & \Phi^2_{22} & \cdots & \Phi^2_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \Phi^2_{k1} & \Phi^2_{k2} & \cdots & \Phi^2_{kk} \end{pmatrix} \begin{pmatrix} Z_{1(t-2)} \\ Z_{2(t-2)} \\ \vdots \\ Z_{k(t-2)} \end{pmatrix} + \cdots +$$

$$\begin{pmatrix} \Phi^p_{11} & \Phi^p_{12} & \cdots & \Phi^p_{1k} \\ \Phi^p_{21} & \Phi^p_{22} & \cdots & \Phi^p_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \Phi^p_{k1} & \Phi^p_{k2} & \cdots & \Phi^p_{kk} \end{pmatrix} \begin{pmatrix} Z_{1(t-p)} \\ Z_{2(t-p)} \\ \vdots \\ Z_{k(t-p)} \end{pmatrix} + \begin{pmatrix} a_{1t} \\ a_{2t} \\ \vdots \\ a_{kt} \end{pmatrix} \tag{2}$$

1.3. Stationary Time Series

Stationarity of the time series is an important assumption in VAR modelling. Stationarity means that the series' statistical features, such as mean, variance, and autocorrelation, are constant over time. For a time series $Z(t)$ to be termed stationary, its marginal distribution should not change with time t. The joint distribution of $Z(t_1), \ldots, Z(t_n)$ must be identical to that of $Z(t_1 + \tau), \ldots, Z(t_n + \tau)$, for any time shift τ. [12]

1.4. Unit Root Test

To determine the stationarity of time series, there are different methods, including the augmented Dickey-Fuller unit root test (ADF), Phillips-Perron unit roots, and Kwiatkowski-Phillips-Schmidt-Shin (KPSS). An additional technique to ascertain if the data is steady is the autocorrelation function (ACF) and partial autocorrelation function (PACF) [13].

1.5. Stable VAR(p) Processes

Process 1 is stable if all of the matrix's roots are contained within the unit circle and the roots of matrix Φ_i have absolute values lower than 1. That is, if $\det(I_n - \Phi_1 Z - \ldots - \Phi_p Z^p) \neq 0$ for $|Z| \leq 1$, Z_t where $t = 0, \pm 1, \pm 2, \ldots$, is a stationary VAR(p) process that is stable.

a. Stable VAR(p) Process' Autocovariances

After subtracting the mean from VAR(p), the outcome is

$$Z_t - \mu = \Phi_1(Z_{t-1} - \mu) + \ldots + \Phi_p(Z_{t-p} - \mu) + a_t \tag{3}$$

Once both sides have been divided by $(Z_{t-1} - \mu)'$, the expectation is calculated, arriving at $l = 0$ using the following methods:

$$\Gamma_z(i) = \Gamma_z(-i)' \tag{4}$$

$$\Gamma_z(0) = \Phi_1(Z_{t-1} - \mu) + \ldots + \Phi_p(Z_{t-p} - \mu) + \Sigma_a = \Phi_1\Gamma_z(1)' + \ldots + \Phi_p\Gamma_z(p)' + \Sigma_a \tag{5}$$

If $\mu > 0$, then $\Gamma_z(l) = \Phi_1\Gamma_z(l-1)' + \ldots + \Phi_p\Gamma_z(l-p)' + \Sigma_a$ If $\Phi_1, \ldots, \Phi_p$ and $\Gamma_z(p-1)$ are provided, the autocovariance functions $\Gamma_z(l)$ for $l \geq p$ can be derived from these equations.

b. Stable VAR(p) Process's Autocorrelation

A stable VAR(p) process's autocorrelation may be obtained by taking information out of the matrix:

$$R_z(l) = D^{-1}\Gamma_z(l)D^{-1} \tag{6}$$

The standard deviation of the Z_t component is therefore on the major diagonal of D, a diagonal matrix. As such,

$$D^{-1} = \begin{bmatrix} \frac{1}{\sqrt{\gamma_{11}(0)}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{\sqrt{\gamma_{kk}(0)}} \end{bmatrix} \tag{7}$$

And $Z_{i,t}$ and $Z_{j,t-1}$ have the following correlation:

$$\rho_{ij}(l) = \frac{\gamma_{ij}(l)}{\sqrt{\gamma_{ii}(0)}\sqrt{\gamma_{jj}(0)}}' \tag{8}$$

This represents only the ij-th element of $R_z(l)$. The inverses of the solutions are once again the distinctive roots of the model. Consequently, all characteristic roots must have a modulus of less than one in order for stationarity to occur. For a stationary AR(p) sequence with $p \geq 0$, the ACF satisfies the difference equation $(1 - \Phi_1 B - \Phi_2 B^2 - \ldots - \Phi_p B^p)^p$. [14]

1.6. Estimation Parameters for VAR Model [12]

The VAR(1) model, which is comparable to the VAR(p) model, can be used to illustrate how to estimate model parameters and the error covariance matrix:

$$Z_t = \Phi_1 Z_{t-1} + a_t \tag{9}$$

where Z_t represents a $(k \times 1)$ vector, Φ_i denotes a fixed $(k \times k)$ coefficient matrix, and a_t denotes a white noise process such that $a_t \sim N(0, \Sigma)$.

We must estimate the model parameters Φ_1 and the covariance matrix Σ to estimate the VAR(1) model. Given certain assumptions, Equation (9) may be expressed as follows:

$$Z = XB + A \tag{10}$$

where Z is a $(T - p) \times k$ matrix with the ith row being Z'_{p+i}, X is a $(T - p) \times (kp + 1)$ matrix with the ith row being X'_{p+i}, and A is a $(T - p) \times k$ matrix with the ith row being a'_{p+i}.

1.7. Order Selection by VAR

Determining the VAR lag order is a crucial first step in developing models and conducting impulse response analyses. This investigation employs several widely utilized lag-order selection criteria, including the Akaike Information Criterion AIC $= \ln|\Sigma_u(p)| + \frac{2(pm^2)}{T}$ and Hannan-Quinn Criterion HQ $= \ln|\Sigma_u(p)| + \frac{2\ln\ln T(pm^2)}{T}$, the Final Prediction Error FPE $= \left[\frac{T+sp+1}{T-sp-1}\right]^k |\Sigma_p|$, the Bayesian Information Criterion BIC $= \ln|\Sigma_u(p)| + \frac{\ln T(pm^2)}{T}$ Here, Σ_p represents the covariance matrices, s is the number of variables, T is the number of observations, and p is the lag order. [15]

1.8. Forecasting

Forecasts can be employed if it is found that the fitted model in Section 1.2 is adequate. Forecasts are made using the following equation:

$$Z_t = c + \Phi_1 Z_{t-1} + \Phi_2 Z_{t-2} + \cdots + \Phi_p Z_{t-p} + a_t, \quad t = 0, \pm 1, \pm 2, \ldots \tag{11}$$

The predictions generated in this way have the minimum mean square error given the forecast origin t. [4]

2. Application

The methods outlined are expanded upon in this section. To illustrate the multivariate time series analysis method, we examine the USD/IQD exchange rate, symbolized as USD, in conjunction with other correlated variables, such as gold price (XAU/USD), copper price (HG), and Brent oil (WTI (F) LCOc1), symbolized as Oil. The dataset monthly time series from 2010 to to the end of April 2023 acquired from `https://www.investing.com`. These four variables' time series data are separated into two sets: 90% of the data are in the training set, while the remaining 10% are in the testing set. A range of graphical depictions of these variables are shown in Figure 1, which also offers preliminary insights into their trends and patterns.

The thorough analysis and modeling of these multivariate time series, with an emphasis on USD, gold, copper, and oil, is the main scientific challenge this study attempts to address. The primary objectives are to uncover complex correlations, identify important patterns in the data, and develop a robust multivariate model capable of producing accurate forecasts and detailed analyses.

2.1. Test of Stationarity

Testing the original time series data's stationarity is the first stage in the analytical process, using the Augmented Dickey-Fuller test in R (ADF.test). According to the findings, the variables (USD, Gold, Copper, and Oil) are not

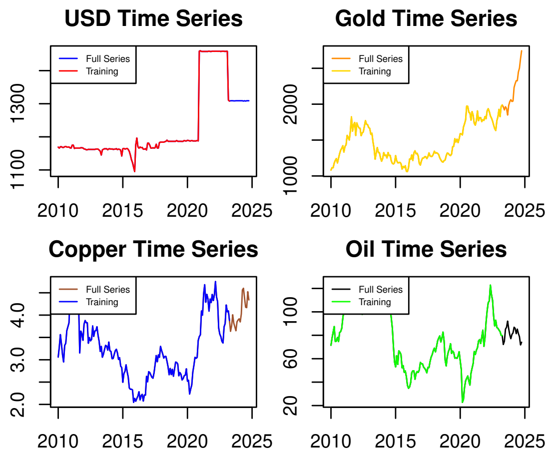

Figure 1. Actual Data for Four Time Series (USD, Gold, Copper, and Oil).

stationary in their initial state. To ascertain whether univariate time series datasets are stationary, the Unit Root test is employed. This test is based on the idea that a trend-lined series will show a significant p-value and a unit root. Null Hypothesis (H_0) : The data is non-stationary and has a unit root. Table 1 displays the comprehensive findings of the stationarity tests. The autocorrelation function (ACF) and partial autocorrelation function (PACF) plots for the time series are also shown in figure 2

Table 1. Data Stationarity Testing

Datasets	**Augmented Dickey-Fuller**		**Phillips-Perron Unit Root Test**		**KPSS Level**	
	Test Statistic	p-value	Test Statistic	p-value	Test Statistic	p-value
USD	-2.1159	0.528	-9.3088	0.5809	1.8192	0.01
Gold	-1.3876	0.8318	-5.5958	0.7938	1.1021	0.01
Copper	-1.7329	0.6877	-7.4402	0.6881	0.56493	0.02704
Oil	-1.9186	0.6103	-8.3144	0.6379	1.0331	0.01

Based on the p-value in table 1 , we do not reject the null hypothesis (p-value > 0.05) for the Augmented Dickey–Fuller (ADF) test and Phillips-Perron Unit Root Test. Similarly, we do not reject the null hypothesis for the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test, because the p-value < 0.05 indicates that the series does have a unit root and is thus not stationary.
We take first-order differencing to the non-stationary series. As a result, all four differenced series exhibit stationarity. As a result, each stationary series that follows an I(1) process has been created from the time series. Table 2 displays the comprehensive findings of the stationarity tests.

Table 2. Data Stationarity Testing After First-Order Differencing

Datasets	**Augmented Dickey-Fuller**		**Phillips-Perron Unit Root Test**		**KPSS Level**	
	Test Statistic	p-value	Test Statistic	p-value	Test Statistic	p-value
USD	-4.6114	0.01	-152.92	0.01	0.077163	0.1
Gold	-4.4426	0.01	-176.87	0.01	0.13449	0.1
Copper	-5.0993	0.01	-178.09	0.01	0.08711	0.1
Oil	-5.0691	0.01	-120.48	0.01	0.086416	0.1

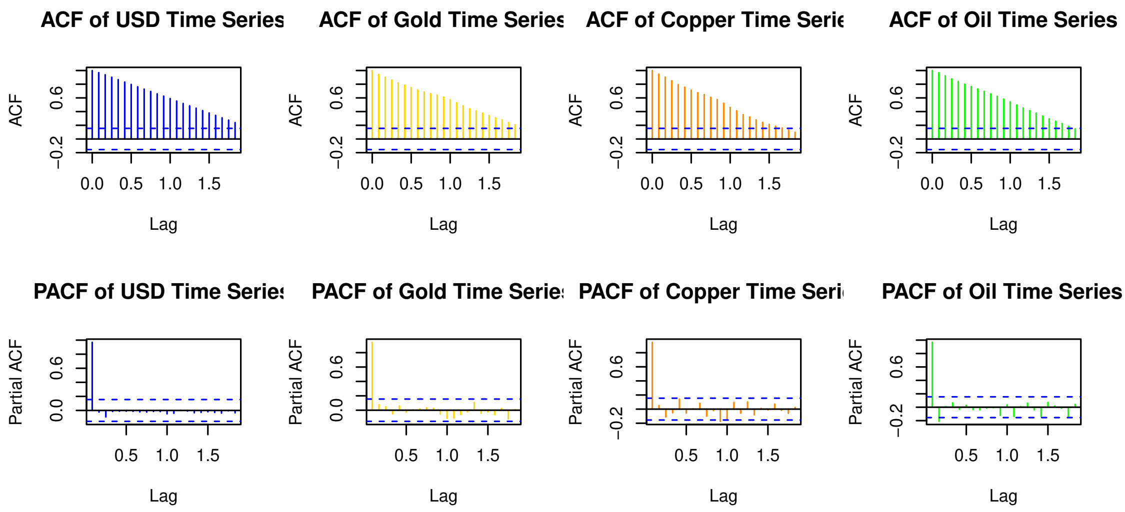

Figure 2. Autocorrelation function (ACF) and partial autocorrelation function (PACF) plots for the ((USD, Gold, Copper, and Oil).

Based on the p-value in table 2, we reject the null hypothesis (p-value < 0.05) for the Augmented Dickey–Fuller (ADF) test and Phillips-Perron Unit Root Test. Similarly, we reject the null hypothesis for the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test because p-value > 0.05, indicating that the series does not have a unit root and is thus stationary.

2.2. *Granger Causality Test*

In the event that variable x influences variable Z, the former ought to contribute to enhancing the latter's projections. Let us assume that Ω_t is the information set that encompasses all pertinent data in the universe up to and including period t. Define $Z_t(h \mid \Omega_t)$ to be the best (least MSE) h-step predictor of process Z_t at origin t, given the data in Ω_t. Conduct Granger causality tests to identify causal relationships between variables [16].

$$Z_t = \beta_0 + \sum_{i=1}^{p_1} \beta_i^{(z)} Z_{t-i} + \sum_{i=p_0}^{p_2} \beta_i^{(x)} x_{t-i} + \epsilon_i \tag{12}$$

Where: Z_t and x_t are the time series variables, p_i is the number of lags, ϵ_i are error terms, and $\beta_0, \beta_i^{(Z)}, \beta_i^{(x)}$ are parameters of the model. The hypotheses are: $H_0 : \beta_{p_0}^{(x)} = \ldots = \beta_{p_2}^{(x)} = 0 \quad \text{vs} \quad H_1 : \beta_{p_0}^{(x)} \neq \ldots \neq \beta_{p_2}^{(x)} \neq 0 \quad \text{where } p_0 \leq p \leq p_2$

Accepting the null hypothesis indicates that x is not a Granger cause of Z. The interdependent structure of the underlying systems of multivariate time series was examined using Granger causality. The Granger causality test should be conducted to determine which variables should be included in the VAR model. This test is a key tool in multivariate analysis, used to assess the relationships between the model variables. The results are presented in Table 3 using data after taking the first difference. In R(causality) We obtain the following outcomes:

According to Table 3, since the p-value for oil is greater than 0.05, it is not a Granger cause of the other variables (USD, gold, and copper). However, as their p-values are less than 0.05, the other variables (USD, gold, and copper) are Granger causes of each other. Therefore, for further analysis, we select the three variables (USD, gold, and copper). To test stability, the modulus of each characteristic root must be less than one. The log-likelihood is -1590.619, and the roots are 0.1609, 0.1609, and 0.1379.

We evaluate the accuracy of four models, as presented in Table 4, and select the best model to focus on in our study.

Table 3. Granger causality test results.

$Z\backslash X$	USD	Gold	Copper	Oil
USD	NA	0.4475	0.0214	0.1869
Gold	0.02819	NA	0.7031	0.8309
Copper	0.0111	0.0274	NA	0.3777
Oil	0.4701	0.7016	0.8194	NA

Table 4. Comparing the performance of models for (USD, Gold, Copper, and Oil).

Models	Series	MAE	RMSE	MAPE
Model 1	USD	10.1455	11.20563	0.7751695
	Copper	0.4286295	0.528262	10.07754
Model 2	USD	7.400091	8.743237	0.5654111
	Gold	190.0743	271.8869	7.926284
Model 3	USD	3.774073	4.338757	0.2883554
	Gold	190.8006	273.1234	7.951968
	Copper	0.2470051	0.3250794	5.82204
Model 4	USD	3.769328	4.340726	0.287995
	Gold	190.7362	273.0184	7.9497
	Copper	0.2464144	0.3224625	5.816971
	Oil	4.874683	5.759674	5.963201

Model 4 is the best option for our VAR analysis based on the variables listed in Table 4. However, when looking at Table 3, Model 3, which has three variables (USD, copper, and gold), turns out to be the best option. This VAR(1) model balances significant causal relationships with robust accuracy metrics, making it a comprehensive and effective option for our thesis on multivariate time series analysis.

2.3. Co-integration Test

It is the $I(0)$ process that is particularly the stationary series. A series is considered co-integrated if it is composed of two or more non-stationary, $I(d)$ processes, and a stationary linear combination of these series. Multivariate variables $\mathbf{x}_t = (x_{1t}, \ldots, x_{kt})$ are said to be co-integrated if they satisfy the following conditions: x_{it} is an $I(d)$ process, $\forall i = 1, \ldots, k$; and $\exists\, \boldsymbol{\beta} = (\beta_1, \ldots, \beta_k) \neq 0$, such that $\beta_1 x_{1t} + \ldots + \beta_k x_{kt}$ is stationary [17].

There are two main tests for co-integration:

a. Engle-Granger two-step method: Time series are considered integrated if they have the same order of integration and can be combined linearly to form a stationary time series (integrated of order one). Co-integration relationship estimation is done using the two-step Engle and Granger test, which is suitable for a single co-integrating relation when the second series x_t regresses the first series Z_t. Stability is checked on the resultant error series Z_t following the first step [18].

$$Z_t = \mu + b x_t + x_t \tag{13}$$

b. Johansen test: When looking for a relationship between the dependent and explanatory variables over an extended period of time, Johansen co-integration is utilized. As a crucial tool for estimating models involving time series data, the Johansen technique provides estimates of all co-integrating relationships that could be present in a vector of stationary or non-stationary variables, and it can be applied to analyze multiple co-integrating relations [19].

$$\mathbf{X}_t = \Pi_1 \mathbf{X}_{t-1} + \ldots + \Pi_k \mathbf{X}_{t-k} + \epsilon_t, \quad t = 1, \ldots, T \tag{14}$$

$$\Delta \mathbf{X}_t = \sum_{i=1}^{k-1} \Gamma_i \Delta \mathbf{X}_{t-i} + \Pi \mathbf{X}_{t-1} + \epsilon_t, \quad t = 1, \ldots, T \tag{15}$$

Where: $\Gamma_i = -I + \Pi_1 + \ldots + \Pi_i$ with $i = 1, \ldots, k-1$, $\Pi = -(I - \Pi_1 - \ldots - \Pi_k)$, $\lambda_{\text{trace}} = -T \sum_{i=r+1}^{p} \ln(1 - \hat{\lambda}_i)$, $\lambda_{\max} = -T \ln(1 - \hat{\lambda}_{r+1})$

The long-term link between the variables in the nexus is ascertained by applying the co-integration test (Johansen's), and the hypotheses are H_0 : no co-integration among variables vs H_1 : co-integration among variables. The results in Table 5 represent Johansen's co-integration result in two methods: the trace and maximum eigenvalue for three series' time series after taking the first difference (USD, Gold, Copper). In R (ca.jo).

Table 5. Results of Johansen's Co-integration test for three series' time series after taking first difference (USD, Gold, Copper)

Unrestricted Co-integration Rank Test (Trace)

Co-integration Rank (r)	Eigenvalue	Trace Stat.	Critical Value (5%)
r = 0*	0.4476093	200.54	42.44
r $\leq$ 1$*$	0.3541572	107.36	25.32
r $\leq$ 2$*$	0.2185695	38.72	12.25

Unrestricted Co-integration Rank Test (Maximum Eigenvalue)

Co-integration Rank (r)	Eigenvalue	Max Eigenvalue Stat.	Critical Value (5%)
r = 0*	0.4476093	93.18	25.54
r $\leq$ 1$*$	0.3541572	68.64	18.96
r $\leq$ 2$*$	0.2185695	38.72	12.25

We reject the null hypotheses in both Rank Test (Trace) and Rank Test (Maximum Eigenvalue), indicating there are co-integration relations.

2.4. Cross-Correlation Matrices

A statistical technique called cross-correlation may be used to assess how similar time series variables are to one another and establish whether one series is ahead of or behind another. The degree of the association between the time series reflects how strongly they are related.[20] For every sample CCM, a basic matrix $S_\phi[S_{\phi,ij}]$ is created in this way:

$$S_{\phi,ij} = \begin{cases} (+) & \text{if } \phi_{ij} \geq \frac{2}{\sqrt{T}} \\ (-) & \text{if } \phi_{ij} \leq \frac{2}{\sqrt{T}} \\ (.) & \text{if } \phi_{ij} < \frac{2}{\sqrt{T}} \end{cases} \tag{16}$$

where $\hat{\rho}_\phi$ is a consistent estimate of $\hat{\rho}_\phi$, and T is the total number. [21]

Table 6 displays the simplified CCM for the month of (USD, Gold, Copper). It is evident that significant cross-correlations at the estimated 5% stage are mostly visible at lag 1.

2.5. Model Selection

AIC, BIC, and HQC at various lags are shown in Table 7, which represents the data. A three-selection process reaches the minimal values (the bolded values). VAR(1) is, therefore, the model of choice in Table 7: Empirical Lag Selection.

Table 7 displays the AIC, SC, and HQC at various delays for the three series' time series after taking the first difference (USD, Gold, Copper).

Table 6. Sample Cross-Correlation Matrices of three series time series after taking first difference (USD, Gold, Copper)

Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6
$\begin{bmatrix} . & . & + \\ . & . & . \\ . & - & . \end{bmatrix}$	$\begin{bmatrix} . & . & . \\ . & . & . \\ + & . & . \end{bmatrix}$	$\begin{bmatrix} . & . & . \\ . & . & . \\ . & . & . \end{bmatrix}$	$\begin{bmatrix} . & . & . \\ . & . & . \\ . & . & . \end{bmatrix}$	$\begin{bmatrix} . & + & . \\ . & . & . \\ . & . & . \end{bmatrix}$	$\begin{bmatrix} . & . & . \\ . & . & . \\ . & . & . \end{bmatrix}$
Lag 7	**Lag 8**	**Lag 9**	**Lag 10**	**Lag 11**	**Lag 12**
$\begin{bmatrix} . & . & . \\ . & . & . \\ . & . & . \end{bmatrix}$	$\begin{bmatrix} . & . & . \\ . & . & . \\ . & . & . \end{bmatrix}$	$\begin{bmatrix} . & . & . \\ . & . & . \\ . & . & . \end{bmatrix}$	$\begin{bmatrix} . & . & . \\ . & . & . \\ . & + & + \end{bmatrix}$	$\begin{bmatrix} . & . & . \\ . & . & . \\ . & . & . \end{bmatrix}$	$\begin{bmatrix} . & . & . \\ . & . & . \\ . & . & . \end{bmatrix}$

Table 7. Empirical Lag Selection (a) for the data after first difference.

Lag (n)	AIC(n)	HQ(n)	SC(n)	FPE(n)
1	**11.82958**	**11.92787**	**12.07151**	**137258.26521**
2	11.83924	12.01125	12.26262	138614.52585
3	11.89480	12.14053	12.49962	146591.42179
4	11.94435	12.26379	12.73061	154149.71944
5	11.90397	12.29714	12.87169	148221.92715
6	11.97747	12.44436	13.12663	159797.33736
7	12.04680	12.58740	13.37741	171669.05708
8	12.09451	12.70883	13.60656	180613.55157
9	12.18556	12.87361	13.87906	198615.34375
10	12.19611	12.95787	14.07105	201713.18596

2.6. Model Presentation

The VAR(1) model with significant parameters is presented in matrix form using equation 2 in the technique, as indicated in table 7 that reflects the difference data the best lag number, based on the criteria, is $p = 1$. This can be expressed explicitly as follows for the three variables (USD,Gold,Copper) after take first difference as (Z_{1t},Z_{2t}, Z_{3t} respectively.

$$\begin{pmatrix} Z_{1t} \\ Z_{2t} \\ Z_{3t} \end{pmatrix} = \begin{pmatrix} 1.0000302 \\ 6.29920671 \\ 0.0066103439 \end{pmatrix} + \begin{pmatrix} 0.01158580 & -0.05326705 & 28.6601909 \\ -0.04091837 & -0.14349430 & 10.50426879 \\ 0.0007577117 & -0.0005912646 & -0.0297217793 \end{pmatrix} \begin{pmatrix} Z_{(1t-1)} \\ Z_{(2t-1)} \\ Z_{(3t-1)} \end{pmatrix} + \begin{pmatrix} a_{1t} \\ a_{2t} \\ a_{3t} \end{pmatrix}$$

From this, we can derive the following equations:

$$\begin{aligned} Z_{1t} &= 1.0000302 + 0.01158580 Z_{1t-1} - 0.05326705 Z_{2t-1} + 28.6601909 Z_{3t-1} + a_{1t} \\ Z_{2t} &= 6.29920671 - 0.04091837 Z_{1t-1} - 0.14349430 Z_{2t-1} + 10.50426879 Z_{3t-1} + a_{2t} \\ Z_{3t} &= 0.0066103439 + 0.0007577117 Z_{1t-1} - 0.0005912646 Z_{2t-1} - 0.0297217793 Z_{3t-1} + a_{3t} \end{aligned}$$

Lagged copper prices play a substantial effect in determining the USD exchange rate, as evidenced by the big coefficient (28.6601909 Z_{3-1}). This relationship emphasizes the need of including commodity prices in economic models, as well as the potential benefits of using copper price swings to predict currency fluctuations. By completely comprehending the ramifications and validity of this result, stakeholders can make more informed decisions and negotiate the intricacies of the global economic scene.

2.7. Diagnostic Testing

We must confirm that the VAR(1) model fit is accurate. To this purpose, the diagnostic methods listed below are employed.

a. Residual Autocorrelation Function: The hypothesis is: $H_0 : \rho_{uv,i} = 0 \quad \text{vs} \quad H_1 : \rho_{uv,i} \neq 0$ We reject H_0 if $|r_{uv,i}| > \frac{2}{\sqrt{N}}$

b. Test for Autocorrelation for Serial Correlation (PT): ACF and PACF of the discrepancies are displayed on the graphs, one for each equation, together with a realistic distribution chart and a discrepancy plot. The plot method offers more explanations for altering its layout.

c. The Covariance and Correlation Matrix of Residuals:

$$\text{Cov(residual)} = \begin{pmatrix} Z_{1t} \\ Z_{2t} \\ Z_{3t} \end{pmatrix} \begin{pmatrix} 656.13739 & 101.328 & 0.02021 \\ 101.32788 & 4772.346 & 4.69206 \\ 0.02021 & 4.692 & 0.04317 \end{pmatrix}$$

$$\text{Corr(residual)} = \begin{pmatrix} Z_{1t} \\ Z_{2t} \\ Z_{3t} \end{pmatrix} \begin{pmatrix} 1 & 0.057262 & 0.003798 \\ 0.057262 & 1 & 0.326906 \\ 0.003798 & 0.2690 & 1 \end{pmatrix}$$

The residual autocorrelation function for the three series' [22] time series after taking the first difference (Z_{1t} ,Z_{2t} , Z_{3t}) is shown below in Figure 3, the Top figure show considerable differences between fitted and actual values, there is no discernible partial autocorrelation at lower delays. And the middle figure suggests independent residuals and model adequacy due to its zero residuals and low autocorrelation. And the lowest figure shows substantial swings in residuals, with minimal autocorrelation.

2.8. *Autoregressive Conditional Heteroskedasticity Test (ARCH)*

When it comes to studying and forecasting the volatility of macroeconomic and financial variables, scholars and practitioners have focused especially on the ARCH models. This test is used to determine the heteroscedasticity of the VAR(1) model [23].

2.9. *Jarque-Bera Test*

This test, which was introduced by Jarque and Bera in 1987, determines whether or not the residuals are normally distributed. The test statistics are as follows: [24]

$$JB = \frac{\hat{S}^2(r)}{6/T} + \frac{(\hat{K}(r) - 3)^2}{24/T} \tag{17}$$

Here, T represents the sample size, skewness $\hat{S}(r)$ is defined as $\hat{S}(r) = \frac{1}{(T-1)\hat{\sigma}_r^3} \sum_{t=1}^{T} (r_t - \bar{r})^3$ and kurtosis $\hat{K}(r)$ is given by $\hat{K}(r) = \frac{1}{(T-1)\hat{\sigma}_r^4} \sum_{t=1}^{T} (r_t - \bar{r})^4$ These are calculated from sample data as $\hat{S}^2(r)$, $\hat{K}(r)$, and $\{r_1, \ldots, r_T\}$ is a variable containing T observations, in R(Serial.test, ARCH.test, Normality.test)

Table 8. Diagnostic tests of VAR (1) for three series' time series (Z_{1t} , Z_{2t} , Z_{3t}) Null Hypothesis test Statistic p-value

Null Hypothesis Test	Statistic	p-value
No autocorrelation PT	120.4	0.8111
No heteroscedasticity ARCH	282.74	1.53e-06
Not normality JB	32848	2.2e-16
	Kurtosis 32111	2.2e-16
	Skewness 736.73	2.2e-16

The null hypothesis of no autocorrelation is accepted because $0.8111 > 0.05$, while the null hypothesis of no heteroscedasticity is rejected because $1.53e - 06 < 0.05$. The null hypothesis of not normality (for JB, Kurtosis, Skewness) is rejected because $2.2e - 16 < 0.05$. The rejection of the normalcy assumption, as seen by substantial Jarque-Bera, kurtosis, and skewness test results is a prevalent problem with financial data. Non-normal residuals can affect the effectiveness of parameter estimates and inference processes in Vector Autoregression (VAR) models.

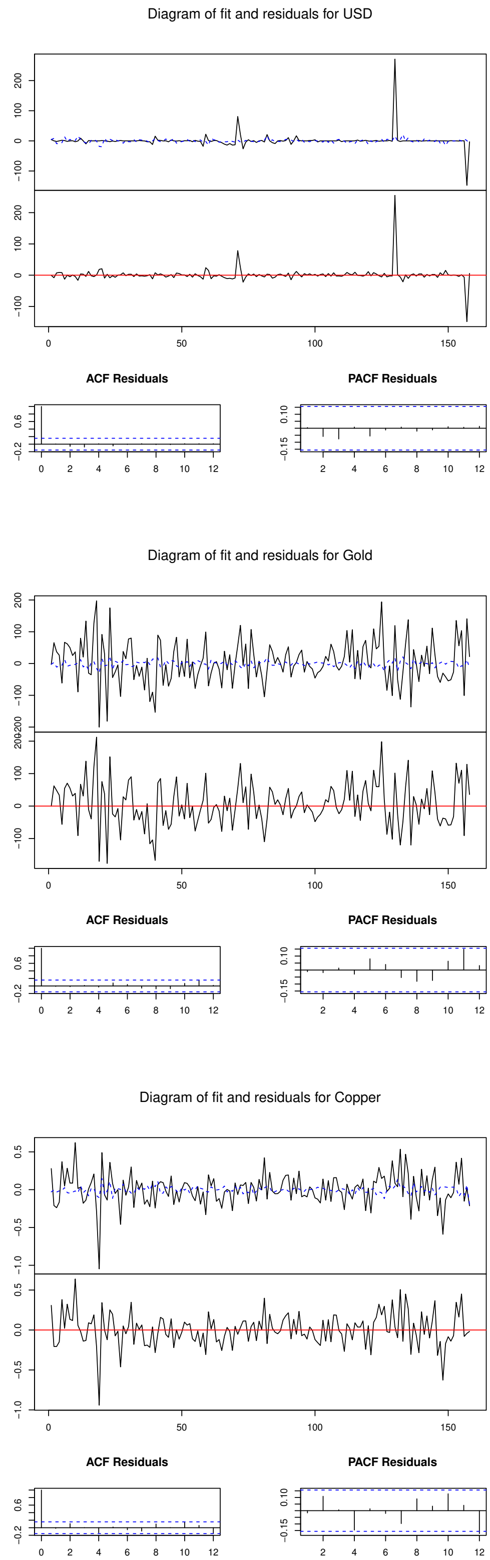

Figure 3. Residual autocorrelation function for three series' time series after taking the first difference (Z_{1t} ,Z_{2t} , Z_{3t}).

2.10. Structural Stability (SVC)

The stability test evaluates the presence of structural breakdowns, which, if neglected, can result in erroneous estimates. To overcome this issue, we use the CUSUM (Cumulative total of Recursive Residuals) test, which visually examines the cumulative total of residuals over time. When data points exceed predetermined criteria, structural alterations are detected. Figure 4 shows the outcomes for the three time series (USD, Gold, and Copper) following the initial difference. The results confirm the system's stability, since the cumulative sum remains inside the red border lines, indicating no substantial structural fractures [25].
In addition to the VAR model, the SVAR and SVECM techniques provide useful insights. SVAR improves analysis by including structural constraints and allows for simultaneous interactions, which can better capture immediate economic shocks and increase model interpretability if structural cracks are discovered. In contrast, SVECM is advantageous when variables have long-term equilibrium relationships, as it allows for a better differentiation between short-term fluctuations and long-run adjustments if cointegration exists.
The stability test findings indicate that the existing VAR model produces accurate estimates with no structural fractures. However, additional study with SVAR could improve the model by integrating economic constraints, whereas SVECM could test the robustness of long-run correlations. Exploring these models could lead to a better understanding of the dynamic interactions between financial factors, hence improving the analysis' predictive potential. In R (stability).

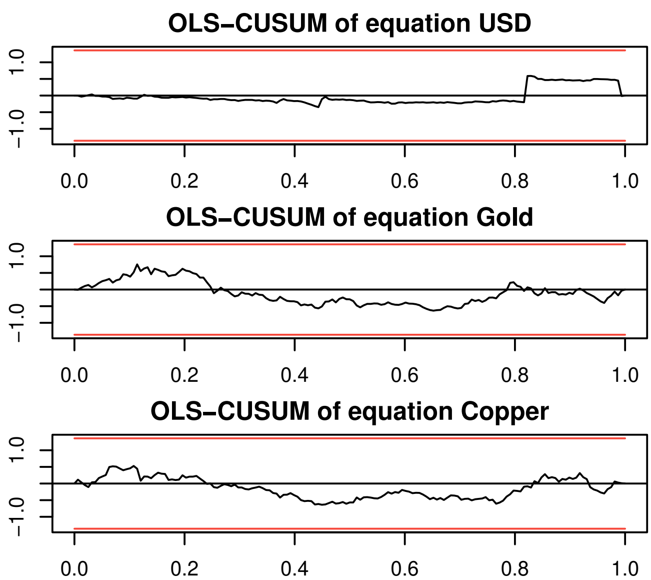

Figure 4. CUSUM Test for (Z_{1t} , Z_{2t} , Z_{3t})

This figure of cumulative sum control (CUSUM) shows that the system is stable because there are no points that go beyond the two red lines.

2.11. Forecast Error Variance Decomposition (FEVD)

An Estimated Error Variance Decomposition uses the VAR model to analyze how variables affect each other, and FEVD is a method used in econometrics and other multivariate time series analytic applications to aid in the understanding of a fitted VAR model [26]. The moving average formulation of the VAR model serves as the mathematical basis for the FEVD. The moving average is given by:

$$Z_t = \mu + \sum_{i=0}^{\infty} \theta_i \omega_{t-i} \tag{18}$$

With $\Sigma_\omega = I_k$, the error of the optimal h-step forecast is:

$$Z_{t+h} - Z_t(h) = \sum_{i=0}^{h-1} \phi_i u_{t+h-i} = \sum_{i=0}^{h-1} \phi_i P P^{-1} u_{t+h-i} = \sum_{i=0}^{h-1} \phi_i \omega_{t+h-i} \tag{19}$$

Using $\theta_{mn,i}$ to denote the mn-th element of θ_i, as previously, the h-step forecast error of the j-th component of Z_t is:

$$Z_{j,t+h} - Z_{j,t}(h) = \sum_{i=0}^{h-1} \left(\theta_{j1,i}\omega_{1,t+h-i} + \cdots + \theta_{jk,i}\omega_{k,t+h-i}\right) \tag{20}$$

$$Z_{j,t+h} - Z_{j,t}(h) = \sum_{i=0}^{h-1} \left(\theta_{jk,0}\omega_{k,t+h-i} + \cdots + \theta_{jk,i}\omega_{k,t+h-i}\right) \tag{21}$$

Consequently, all of the innovations $\omega_{1,t}, \ldots, \omega_{k,t}$ may be included in the forecast error of the j-th component. It is possible for some of the $\theta_{mn,i}$ to be zero, of course. Since the unit variances and lack of correlation between the $\omega_{k,t}$, the MSE of $Z_{j,t}(h)$ is:

$$E\left((Z_{j,t+h} - Z_{j,t}(h))^2\right) = \sum_{k=1}^{K} \left(\theta_{jk,0}^2 + \cdots + \theta_{jk,h-1}^2\right) \tag{22}$$

$$\theta_{jk,0}^2 + \cdots + \theta_{jk,h-1}^2 = \sum_{i=0}^{h-1} \left(e_j' \theta_i e_k\right)^2 \tag{23}$$

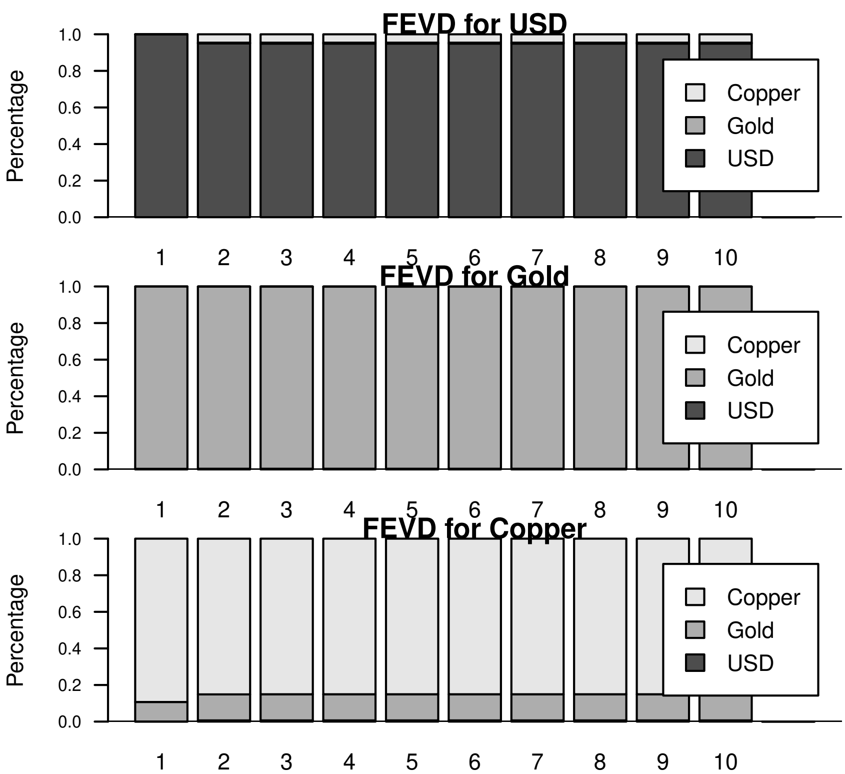

Figure 5. Forecast Error Variance Decomposition (FEVD) from VAR(1).

This is occasionally understood to be the part that innovations in variable k contribute to the MSE or prediction error variance of the variable j's h-step forecast. The k-th column of I_k is represented here by the letter e_k. Partitioning via:

$$\text{MSE}[Z_{j,t}(h)] = \sum_{i=0}^{h-1} \sum_{k=1}^{K} \theta_{jk,i}^2 \tag{24}$$

The FEVD is:

$$\omega_{jk,h} = \frac{\sum_{i=0}^{h-1} \left(e_j' \theta_i e_k\right)^2}{\text{MSE}[Z_{j,t}(h)]} \tag{25}$$

This represents the percentage of variable j's h-step forecast error variation that may be explained by $\omega_{k,t}$ innovations [4]. FEVD is a method used in econometrics to aid in the understanding of a fitted vector autoregression (VAR) model, as shown in Figure 5, which depicts three series' time series after taking the first difference (USD, Gold, Copper).

The primary plot highlights the FEVD of the USD set in figure 5, illustrating how shocks in copper and gold can affect USD projection values. The middle plot reveals how gold price changes are affected by USD and copper shocks, while the bottom plot represents the FEVD for the Copper_diff training set.

2.12. Impulse Response Function (IRF)

With an emphasis on adjusting endogenous variables and identifying dynamic correlations among contemporaneous values, the impulse response test examines how an exogenous shock affects a process over time [27]. The VAR can be represented as an infinite-order moving average:

$$Z_t = \mu + \varepsilon_t + \Psi_1 \varepsilon_{t-1} + \Psi_2 \varepsilon_{t-2} + \cdots \tag{26}$$

where Ψ_s are the matrices of $(n \times n)$ moving averages. The (i, j)-th element of the matrix Ψ_s, denoted as $\Psi_{s,ij}$, can be interpreted as the dynamic multiplier or impulse response:

$$\frac{\partial Z_{i,t+s}}{\partial \varepsilon_{j,t}} = \frac{\partial Z_{i,t}}{\partial \varepsilon_{j,t-s}} = \psi_{ij}^s, \quad i, j = 1, \ldots, n \tag{27}$$

This interpretation is only valid when var$(\varepsilon_t) = \Sigma$ is a diagonal matrix, indicating that the elements of ε_t are uncorrelated.

The VAR equations can be expressed as:

$$Z_{1t} = c_1 + \gamma_{11}' Z_{t-1} + \cdots + \gamma_{1p}' Z_{t-p} + \eta_{1t} \tag{28}$$

$$Z_{2t} = c_2 + \beta_{21} Z_{1t} + \gamma_{21}' Z_{t-1} + \cdots + \gamma_{2p}' Z_{t-p} + \eta_{2t} \tag{29}$$

$$\vdots$$

$$Z_{nt} = c_n + \beta_{n1} Z_{1t} + \cdots + \beta_{n,n-1} Z_{n-1,t} + \gamma_{n1}' Z_{t-1} + \cdots + \gamma_{np}' Z_{t-p} + \eta_{nt} \tag{30}$$

The triangular structural VAR(p) model is represented as:

$$B Z_t = c + \Gamma_1 Z_{t-1} + \cdots + \Gamma_p Z_{t-p} + \eta_t \tag{31}$$

where:

$$B = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \beta_{21} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{n1} & \beta_{n2} & \cdots & 1 \end{bmatrix}$$

In practice, the triangular VAR(p) orthogonal IRF can be calculated from the non-triangular VAR(p) parameters. The residual covariance matrix Σ is decomposed as $\Sigma = ADA'$, where A is a lower triangular matrix that can be inverted, and D is a diagonal matrix with positive diagonal elements. The structural errors are defined as:

$$\eta_t = A^{-1}\varepsilon_t \tag{32}$$

These structural errors are orthogonal by construction since $\text{var}(\eta_t) = A^{-1}\Sigma A^{-1'} = A^{-1}DA'A^{-1'} = D$. Finally, the process can be expressed as:

$$Z_t = \mu + AA^{-1}\varepsilon_t + \Psi_1 AA^{-1}\varepsilon_{t-1} + \Psi_2 AA^{-1}\varepsilon_{t-2} + \cdots \tag{33}$$

$$= \mu + \Theta_0\eta_t + \Theta_1\eta_{t-1} + \Theta_2\eta_{t-2} + \cdots \tag{34}$$

where $\Theta_j = \Psi_j A$. The impulse responses to the orthogonal shocks $\eta_{j,t}$ are given by:

$$\frac{\partial Z_{i,t+s}}{\partial \eta_{j,t}} = \frac{\partial Z_{i,t}}{\partial \eta_{j,t-s}} = \theta_{ij}^s, \quad i, j = 1, \ldots, n \tag{35}$$

Here, θ_{ij}^s represents the (i,j)-th element of Θ_s. The orthogonal impulse response function (IRF) of Z_i with respect to η_j is shown by plotting θ_{ij}^s against s. With n variables, n^2 alternative impulse response functions can be obtained. The impulse response test explores how an exogenous shock influences a process over time.

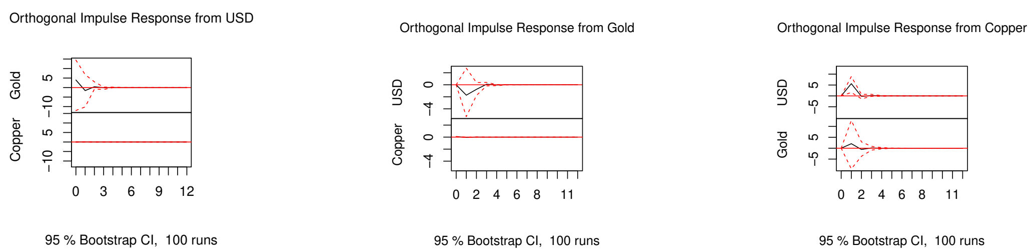

Figure 6. Impulse response function (IRF) form VAR(1).

In figure 6, which depicts the time series for three variables after taking their first differences (USD, Gold, Copper). In IRF from USD show that Gold initially experiences a sudden drop in response to USD shock, but this impact fades over several months before stabilizing around zero. Copper appears to not respond to the shock, and in IRF from Gold show that Gold shocks USD initially negatively but eventually becomes positive and zero after a few months. Copper responds negligibly or almost nonexistent to Gold shocks, with confidence intervals close to zero, and the IRF from Copper show in the first response to a shock, either positive or negative, is significant and rapidly decreases over several months.

2.13. Validation

When the predictions of Model 3 and Model 4 are compared, it is clear that the introduction of the oil variable in Model 4, despite exhibiting white noise (p-value = 0.4674), improves prediction accuracy. This shows that, despite the presence of white noise, the oil variable adds value to the model's performance by offering extra insights for prediction. Oil prices are widely recognized as a critical economic indicator influencing currency rates and commodity prices; therefore, their inclusion offers a larger and more thorough framework for anticipating USD developments.

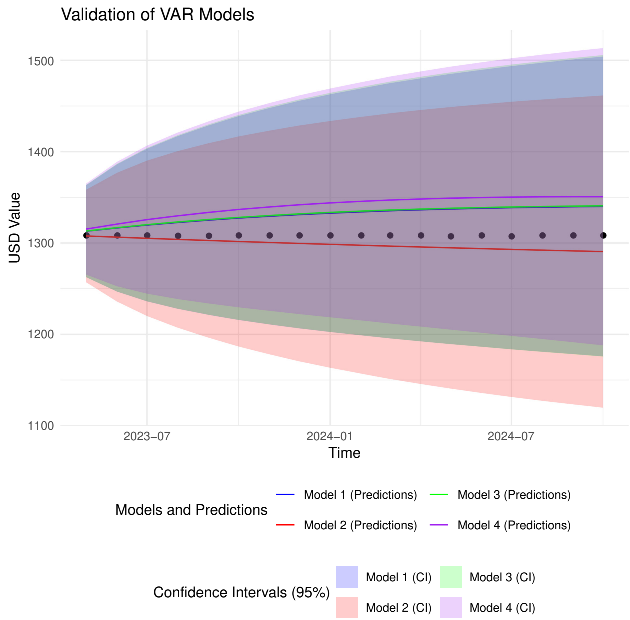

Figure 7. The comparison of model 1-4's predictions.

3. Conclusion

The Granger causality test was performed to determine the variables in the models. Despite the high p-value =0.4674, the inclusion of the oil variable is justified by both economic intuition and statistical analysis. Oil prices are an important economic indicator that influence exchange rates and commodity prices, offering a broader context for USD forecasts. Residual analysis further confirms its role in minimizing model bias and improving residual stability.
When comparing Model 3 and Model 4 predictions, the addition of white noise enhances accuracy, resulting in the lowest error metrics and a tight alignment with actual values. While Model 4 is the most accurate for USD predictions, Model 3 captures crucial causal links and has a broader use for predicting USD, Gold, and Copper. These findings highlight the relevance of including relevant economic variables to improve forecast accuracy and dependability. The findings emphasize the need of including varied factors, such as oil prices, into economic models in order to improve prediction accuracy and resilience. Policymakers, investors, and businesses can use these insights to create efficient hedging strategies, make informed investment decisions, and develop policies that encourage economic stability. Understanding the intricate effects of many factors on currency movements allows stakeholders to traverse the global economic landscape more successfully.

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