STATISTICS, OPTIMIZATION AND INFORMATION COMPUTING
Stat., Optim. Inf. Comput., Vol. **14**, August 2025, pp 628–635.
Published online in International Academic Press (www.IAPress.org)

Decoupled PI Design for the Second-Order Model of a Magnetic Levitation System

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Abstract This paper presents a decoupled control strategy for stabilizing a second-order magnetic levitation system (MLS) based on a nonlinear feedback linearization approach combined with a proportional-integral (PI) controller. The proposed methodology transforms the nonlinear dynamics of the MLS into an equivalent linear system using an exact feedback linearization scheme, enabling the application of classical control techniques. Stability of the closed-loop system is formally demonstrated through a Lyapunov-based analysis, ensuring asymptotic convergence to the equilibrium point. The controller structure permits flexible tuning of gains to shape the system's dynamic response, achieving both critically damped and underdamped behaviors. The performance of the control scheme is validated through extensive numerical simulations under varying gain configurations, demonstrating fast convergence and high precision in position tracking. While the study assumes ideal conditions, the findings provide a foundation for future developments that address robustness against model uncertainties, disturbances, and practical implementation constraints. Overall, this work contributes a theoretically grounded and computationally efficient control design for MLS applications.

Keywords Decoupled control design, nonlinear control approach, magnetic levitation system, stability test, numerical simulations

AMS 2010 subject classifications 62J07

DOI:10.19139/soic-2310-5070-2390

1. Introduction

Developing advanced control strategies for nonlinear systems remains a fundamental challenge in control engineering [1]. Nonlinear systems are prevalent in numerous real-world applications, including robotics, aerospace, power systems, and magnetic levitation systems (MLS) [2]. These systems often exhibit complex dynamics, high sensitivity to parameter variations, and strong coupling between variables, making the traditional linear control approaches insufficient [3]. MLS stand out as a benchmark problem for testing control methodologies due to their inherently unstable nature and nonlinear behavior [4]. Consequently, designing robust and efficient control methods for such systems helps to advance theoretical research and enhances their practical deployment in various technological domains [5].

The study of dynamical systems is a cornerstone of modern control theory, as it provides the foundation for understanding and managing the behavior of complex systems over time [6]. In this context, second-order systems play a significant role due to their wide applicability in modeling physical processes and engineering systems [7]. In particular, a MLS serves as a representative example of a second-order nonlinear system with unique characteristics, *e.g.*, inherent instability, nonlinearity, and coupling effects [8]. These features make them an ideal

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ISSN 2310-5070 (online) ISSN 2311-004X (print)

platform for exploring innovative control strategies, allowing researchers to address theoretical challenges while evaluating practical implementation scenarios [2]. The second-order MLS model offers a simplified yet insightful representation of system dynamics, providing a critical stepping stone for developing and validating robust control techniques [4].

The specialized literature includes multiple approaches to dealing with the control of MLS. Some of the most relevant contributions are explained below.

The authors of [2] introduced a classical proportional-integral-derivative (PID) control strategy, demonstrating its effectiveness in achieving basic stabilization, though limited in handling nonlinearities. Similarly, the work by [9] employed a linear-quadratic regulator for MLS, showcasing its ability to optimize system performance by minimizing a cost function. However, this approach required precise system modeling, making it less robust to uncertainties.

Adaptive control methods have also been extensively explored. In [10], an adaptive sliding mode controller was developed to improve robustness against parameter variations and external disturbances. Meanwhile, [4] presented an adaptive backstepping control strategy to handle strong nonlinearities, achieving superior tracking performance, albeit at the cost of increased computational complexity.

Fuzzy logic controllers (FLCs) have emerged as a popular alternative due to their ability to handle nonlinearities without requiring an explicit mathematical model. The study by [11] demonstrated the effectiveness of FLCs in maintaining system stability under varying load conditions. Similarly, the authors of [12] combined fuzzy logic with PID control (Fuzzy-PID) to enhance robustness and simplify implementation.

Researchers have recently explored model predictive control (MPC) for MLS. For example, the authors of [13] highlighted the potential of this approach for providing real-time optimal control by predicting future system behavior. This method, however, requires significant computational resources, limiting its applicability in low-cost hardware.

Neural network-based controllers have also gained attention. The authors of [14] utilized artificial neural networks (ANN) to design an intelligent control system capable of learning system dynamics and adapting to changes. Similarly, [15] proposed a hybrid approach that combines neural networks with sliding mode control, resulting in improved robustness and adaptability.

Finally, recent advancements in robust control techniques have been applied to MLS. The authors of [16] proposed an H_∞ controller to ensure system stability under worst-case disturbance scenarios. This method provided excellent robustness but required conservative design parameters, which could affect overall performance.

These studies highlight the diversity of the control methods available for MLS, as well as the need for innovative approaches that balance robustness, computational efficiency, and practical implementation. In this vein, this article proposes a novel methodology for controlling an MLS based on its second-order model by means of a decoupled control strategy. The second-order model was selected because the current flow can be utilized as the control input, as demonstrated by the authors of [4].

The proposed decoupled control strategy is based on a nonlinear design that transforms the original nonlinear dynamics of the MLS into an equivalent linear model through exact feedback linearization. This transformation is achieved by introducing an auxiliary control variable that decouples the system's dynamics. A proportional-integral (PI) controller is then applied to the linearized model, ensuring asymptotic stability of the closed-loop system in the sense of Lyapunov. Numerical simulations validate the effectiveness of the proposed control scheme under various gain configurations, revealing both critically damped and underdamped responses depending on the tuning. However, the current analysis assumes ideal conditions without accounting for model uncertainties, external disturbances, actuator limitations, or noise, which may affect the controller's performance in practical implementations. These limitations highlight the need for future work focused on robustness analysis and experimental validation.

The remainder of this contribution is organized as follows. Section 2 introduces the mathematical modeling of the MLS, focusing on its second-order dynamic model. Additionally, the equilibrium point is analyzed to determine the desired state variable values and the corresponding control input. Section 3 details the design of the PI controller for the linear equivalent dynamic model, leveraging Lyapunov-based stability theory to ensure the stabilization of the MLS around its equilibrium point. Section 4 presents numerical validations of the proposed controller, evaluating its performance under various scenarios via computer simulation. Finally, Section 5 summarizes the key conclusions of this work and outlines potential directions for future research.

2. Mathematical modeling

This section presents the mathematical formulation of the studied MLS using a third-order model, as described in [2]. Subsequently, the state-space representation is employed to define the error model dynamics around the equilibrium point. Figure 1 provides a schematic representation of the MLS, where F_{mag} denotes the magnetic force generated by the current i flowing through the inductor L; F_{g} represents the gravitational force acting on the metallic mass m; R accounts for the equivalent resistance of the inductor; v is the input voltage; and e is the equivalent voltage measured via the Hall effect, as described in [2].

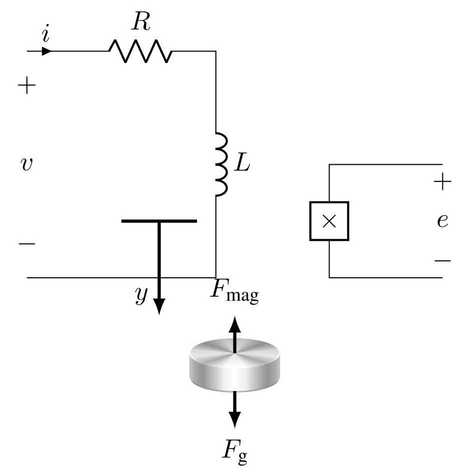

Figure 1. Schematic representation of a MLS

To determine the interaction of the gravitational and magnetic forces applied to the metallic mass illustrated in Figure 1, Newton's laws of motion are applied, which yields the following:

$$m\ddot{y} = F_{\text{g}} - F_{\text{mag}}. \tag{1}$$

The representation of the equivalent magnetic force varies across different studies. This work adopts the model proposed in [2], which characterizes the magnetic force acting on the metallic ball as inversely cubic with regard to the distance and directly proportional to the current:

$$F_{\text{mag}} = k_{\text{mag}} \left(\frac{1}{y^3} \right) i, \tag{2}$$

where k_{mag} is a constant that depends on the electromagnet's turn ratio and cross-sectional area.

By incorporating the magnetic force defined in (2) and the gravitational force expressed as $F_{\text{g}} = mg$ in Equation (1), the resulting dynamical model can be formulated as follows:

$$\ddot{y} = g - \frac{k_{\text{mag}}}{m} \left(\frac{1}{y^3} \right) i, \tag{3}$$

To derive the dynamical model of the MLS presented in (3), the following state variables are introduced: $\eta = y - y_\star$, $\zeta = \dot{y}$, and the control input $\omega = i$. Using these definitions, the equivalent dynamical system can be expressed as follows:

$$\dot{\eta} = \zeta, \tag{4}$$

$$\dot{\zeta} = g - \left(\frac{k_{\text{mag}}}{m}\right)\left(\frac{1}{(\eta + y_\star)^3}\right)\omega, \tag{5}$$

where $y_\star$ represents the desired equilibrium position of the metallic mass, ensuring that $\eta_\star = 0$ at this point.

Note that, in order to calculate the equilibrium point, it is assumed that the desired position of the metallic mass (*i.e.*, $y_1^\star$) is known. This implies that, under steady-state conditions, $\dot{\eta} = 0$ and $\dot{\zeta} = 0$, which yields

$$\zeta_\star = 0, \tag{6}$$

$$\omega_\star = \left(\frac{mg}{k_{\text{mag}}}\right)(y_\star)^3. \tag{7}$$

3. PI design

To design decoupled PI controllers for the second-order model of MLS, the incremental model is typically employed. This approach considers an equivalent equilibrium point at the origin of the coordinate system. Accordingly, the control input ω is defined as follows:

$$\omega = (\eta + y_\star)^3\left(\frac{m}{k_{\text{mag}}}\right)(g - u), \tag{8}$$

Here, u represents an auxiliary control input used to regulate the state variables. Substituting Equation (8) into (5) yields the following incremental model:

$$\dot{\eta} = \zeta, \tag{9}$$

$$\dot{\zeta} = u. \tag{10}$$

When applied to the decoupled system in (9)–(10), the primary objective of the PI approach is to achieve the asymptotic stabilization of the system at the origin of coordinates. To design the PI controller, the PI Structure (11) is defined.

$$u = -k_p\zeta - k_o\eta - k_i\int_0^t \zeta d\tau, \tag{11}$$

To prove that the PI control law in (11) asymptotically stabilizes the Dynamical System (9)–(10), a candidate quadratic Lyapunov function is employed, as defined in (12), which fulfills the first two conditions of Lyapunov's theorem, *i.e.*, $\mathcal{V}(\eta,\zeta,\gamma) \succ 0\,\forall\,(\eta,\zeta,\gamma) \neq (0,0,0)$ and $\mathcal{V}(0,0,0) = 0$ [17].

$$\mathcal{V}(\eta,\zeta,\gamma) = \frac{1}{2}\left(k_o\eta^2 + \zeta^2 + k_i\gamma^2\right), \tag{12}$$

where $\dot{\gamma} = -\zeta$, k_o and k_i are two positive-definite constants.

Now, by taking the time derivative of the Candidate Lyapunov Function (12), the following is obtained:

$$\dot{\mathcal{V}}(\eta, \zeta, \gamma) = k_o \eta \dot{\eta} + \zeta \dot{\zeta} + k_i \gamma \dot{\gamma}, \tag{13}$$

wherein the Dynamical System (9)–(10) can be replaced:

$$\begin{aligned} \dot{\mathcal{V}}(\eta, \zeta, \gamma) &= k_o \eta \zeta + \zeta \left(-k_p \zeta - k_o \eta - k_i \int_0^t \zeta d\tau \right) - k_i \gamma \zeta, \\ &= -k_p \zeta^2 \leq 0. \end{aligned} \tag{14}$$

This fulfills the second condition of Lyapunov's theorem and ensures that the System (9)–(10) asymptotically converges to the origin of coordinates [6].

4. Numerical validation

This section analyzes the performance of the proposed decoupled PI controller applied to the second-order MLS model. To conduct the evaluation, simulation parameters were taken from [2], including, among others, the operating and current limits, the equivalent metallic mass, and the magnetic force constant. A detailed summary of these parameters can be found in Table 1.

Table 1. Parameters of the MLS used in the simulations

Parameter	Value	Unit	Parameter	Value	Unit	Parameter	Value	Unit
k_{mag}	2.40×10^{-6}	kgm^{45}/s^{2}A	$y_\star$	20×10^{-3}	m	$i_{\min}$	0	A
m	0.02985	kg	$i_\star$	0.9758	A	$i_{\max}$	4	A

The proposed decoupled PI controller requires the tuning of three parameters: k_p, k_o, and k_i. To assess the performance of our proposal, this work considers three distinct tuning scenarios, which are detailed in Table 2.

Table 2. PI gain parameters for the simulations

Case	k_p	k_o	k_i	Case	k_p	k_o	k_i	Case	k_p	k_o	k_i
I	20	700	0.01	II	30	150	0.1	III	40	100	0.1

Three different position references for the metallic mass were considered. From 0 to 2 s, the reference position was $y_\star = 20$ mm; from 2 to 4 s, it changed to $y_\star = 24$ mm; and, from 4 to 6 s, it shifted to $y_\star = 16$ mm. Figure 2 illustrates the dynamic response of the metallic mass and the current input.

From Figure 2, it can be concluded that:

i. All of the three parameter combinations successfully converge to the desired reference for the magnetic ball (see Figure 2a). In Case I, the system exhibits an under-damped response, with noticeable oscillations before reaching equilibrium. In Cases II and III, the responses of the MLS are critically damped. However, Case II ensures a rapid convergence to the reference with minimal overshoot, while Case III demonstrates the slowest stabilization.

ii. The inductor current reaches distinct steady-state values corresponding to the desired equilibrium points of the metallic mass ($y_\star$). These equilibrium points, shown in Figure 2b, are 0.9758 A, 1.6862 A, and 0.4996 A for the first (from 0 to 2 s), second (from 2 to 4 s), and third (from 4 to 6 s) periods under analysis, respectively.

When comparing the results shown in Figure 2a against the parameter values presented in Table 2, it becomes evident that the gain k_p influences whether the system response is under-damped or critically damped. As k_p

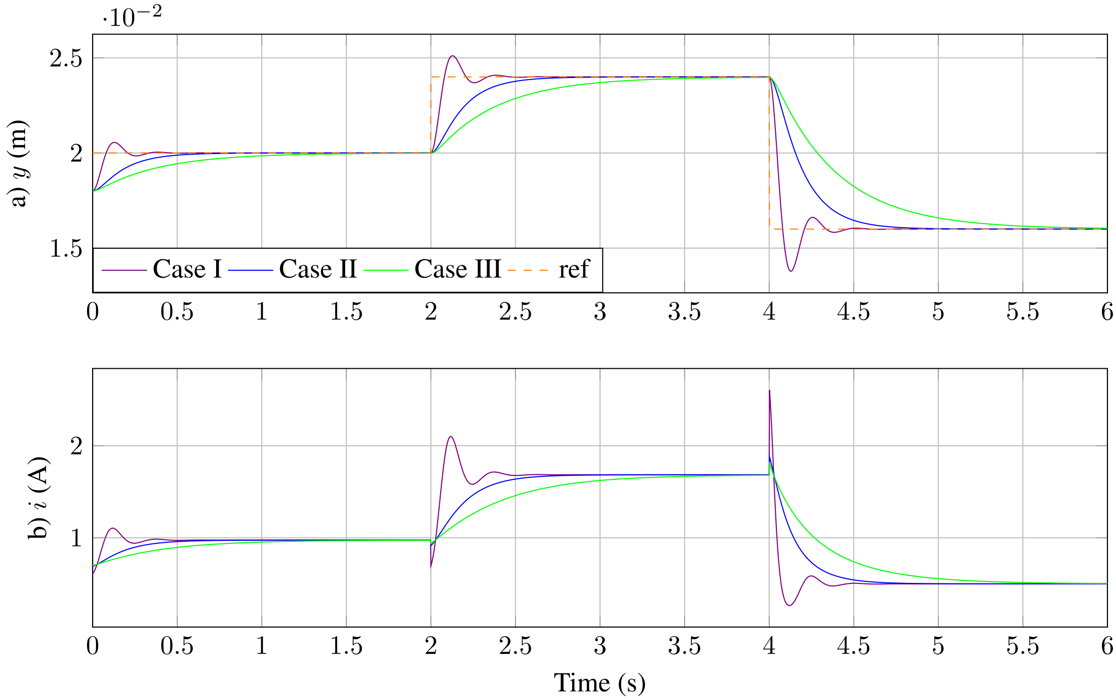

Figure 2. Position of the levitating metallic mass

increases, the system transitions from under-damped to critically damped. On the other hand, k_o affects the settling time of the system; as k_o increases, the system's response becomes faster.

To evaluate the performance of each simulation case, three traditional metrics were employed: the integral time square error (ITSE), the integral absolute error (IAE), and the integral time absolute error (ITAE). These indices, along with the average stabilization times, are summarized in Table 3.

Table 3. Summary of the analyzed indices

Case	ITSE	IAE	ITAE	T_s [s]
I	1.144×10^{-5}	1.058×10^{-3}	3.119×10^{-3}	0.5
II	3.445×10^{-5}	2.810×10^{-3}	8.495×10^{-3}	0.9
III	6.426×10^{-5}	5.579×10^{-3}	17.960×10^{-3}	1.5

Case I demonstrates superior performance across all evaluated indices. Specifically:

- This case reports the smallest ITSE (1.144×10^{-5}), indicating minimal deviation in the squared error over time. This suggests that the system reaches its desired position with higher precision compared to Cases II (3.445×10^{-5}) and III (6.426×10^{-5}).
- The IAE is also the smallest in Case I (1.058×10^{-3}), underscoring its accuracy in maintaining minimal deviations over time.
- Case I achieves the lowest ITAE (3.119×10^{-3}), highlighting its efficiency in quickly reducing the error relative to time when compared to Cases II (8.495×10^{-3}) and III (17.960×10^{-3}).
- Case I reports the shortest stabilization time ($T_s = 0.5$ s), reflecting its ability to reach a steady state faster than Cases II ($T_s = 0.9$ s) and III ($T_s = 1.5$ s).

Overall, Case I constitutes the most desirable balance between precision, efficiency, and speed. On the other hand, the results reveal a trade-off between stabilization time (T_s) and the error indices (ITSE, IAE, ITAE):

- As the stabilization time increases (from $T_s = 0.5$ s in Case I to $T_s = 1.5$ s in Case III), the error indices become significantly worse. For example, the ITAE in Case III (17.960×10^{-3}) is over five times higher than that of Case I (3.119×10^{-3}).
- The results suggest that the critically damped cases (II and III) sacrifice accuracy and error minimization for slower and more gradual stabilization. This is evident in their higher ITSE and IAE values, indicating larger cumulative errors during their transition to the steady state.
- The under-damped behavior in Case I constitutes the best overall trade-off, with shorter stabilization times and lower cumulative errors across all metrics. However, the slightly faster dynamics may introduce small overshoots, which should be considered in applications where overshoot is critical.

This analysis indicates that, while longer stabilization times may offer smoother transitions, they come at the cost of increased errors, highlighting the need for application-specific prioritization of speed and precision.

5. Conclusions and future work

This article presented the design of a nonlinear controller for stabilizing a magnetic levitation system modeled using a second-order equivalent model. The proposed controller employed a decoupled control design, integrating feedback linearization with a proportional-integral approach to stabilize the metallic mass around its equilibrium point while ensuring asymptotic stability in the sense of Lyapunov. Numerical simulations demonstrated that the behavior of the metallic mass — be it critically damped or under-damped —depends on the selection of the control gains.

The analysis presented in this study highlights several key findings regarding the control of MLS using decoupled PI controllers. First, the under-damped behavior observed in Case I demonstrates superior performance across all evaluated metrics, *i.e.*, ITSE, IAE, ITAE, and stabilization time. The faster convergence and reduced cumulative error achieved in Case I make it the most efficient approach for applications where speed and precision are critical.

Moreover, the results reveal a clear trade-off between stabilization time and accuracy. Cases II and III, which exhibit a critically damped behavior, provide smoother stabilization, albeit at the cost of a significantly higher cumulative error and longer stabilization times. This trade-off underscores the need to prioritize performance indices based on specific application requirements, such as minimizing overshoot or achieving rapid stabilization.

Finally, the analysis highlights the impact of control gain tuning on the system's dynamic behavior. The proportional gain (k_p) plays a pivotal role in determining whether the system exhibits an under-damped or a critically damped behavior, while the derivative gain (k_o) significantly influences the system's settling time. Proper tuning of these controller parameters is essential to optimizing the balance between speed, accuracy, and stability, ensuring a robust and efficient system operation.

Future research could explore several key directions to address the limitations identified and enhance the practical applicability of decoupled control strategies for MLS:

- Developing optimization-based methods (e.g., genetic algorithms, particle swarm optimization) for systematic tuning of PI controller gains, aiming to minimize standard performance indices (ITSE, ITAE) while ensuring fast and reliable stabilization across varying operating conditions.
- Designing advanced robust control strategies for the second-order nonlinear model, including adaptive and disturbance-rejection schemes capable of maintaining system stability and performance under parameter uncertainties, external disturbances, and unmodeled dynamics.
- Conducting experimental validations on physical MLS testbeds to evaluate real-time performance, assess implementation feasibility, and account for practical constraints such as sensor quantization, actuator saturation, and computational complexity.
- Extending the current second-order model to higher-order formulations by including coil dynamics, actuator nonlinearities, and electromagnetic hysteresis. This would enable a more realistic simulation environment for controller benchmarking and robustness testing.

- Comparing alternative magnetic force models (e.g., inverse-square and quadratic current dependencies) through both numerical and experimental analysis to identify the most suitable formulations for control design and implementation in practical settings.
- Performing a comprehensive comparative study of the proposed decoupled PI method against conventional and intelligent control techniques (PID, LQR, sliding mode, fuzzy logic, neural networks), using unified performance metrics and benchmarking scenarios.

Acknowledgments

The authors acknowledge the support provided by Thematic Network 723RT0150, titled *Red para la integración a gran escala de energías renovables en sistemas eléctricos* (RIBIERSE-CYTED), funded by the 2022 call for thematic networks of the CYTED (Ibero-American Program of Science and Technology for Development). This research was also supported by Project No. 6-24-9, titled *Desarrollo de una metodología de control secundario para microrredes de corriente continua aisladas empleando control predictivo basado en el modelo*, under Universidad Tecnológica de Pereira's Vicerrectoría de Investigación, Innovación y Extensión.

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