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Analysing Volatility Persistence in the Nairobi Securities Exchange: The Role of Exchange and Interest Rates

Edwin Moyo[1,2,*], Stanley Jere[1], Christian Kasumo[1], Peter Chidiebere Nwokolo[3], Anthony Mulinge[4], Clement Mwaanga[5], Wamulume Mushala[5]

[1] *Department of Mathematics and Statistics, Mulungushi University, Kabwe, Zambia*
[2] *Department of Statistics, University of Botswana, Gaborone, Botswana*
[3] *Department of Mathematics, Nnamdi Azikiwe University, Awka, Nigeria*
[4] *Department of Economics and Statistics, KCA University, Nairobi, Kenya*
[5] *Department of Business Studies, Mulungushi University, Kabwe, Zambia*

Abstract In this paper, the main objective was to analyse the influence of exchange and interest rates on volatility persistence using asymmetric GARCH models (EGARCH and TGARCH) on NSE data. The analysis of the relationship between stock return volatility, exchange, and interest rates on volatility persistence was performed using the models ARMA(1, 2)-EGARCH(1, 1) and ARMA(1, 2)-TGARCH(1, 1) under the student t distribution and the generalised error distribution assumption using the NSE daily 20-share price index, interest rates, and exchange rates from 02/01/2015 to 31/12/2024 accounting for 3106 observations. The degree of persistence in the conditional variance equations slightly increased for the ARMA(1, 2)-TGARCH(1, 1) model and there was a slight reduction for the ARMA(1, 2)-EGARCH(1, 1) with the inclusion of interest rate and exchange rate which was consistent regardless of the error term distribution assumption. Generally, information shocks increase volatility persistence, and negative shocks have a greater impact than positive shocks. The coefficient of the exchange rate (δ_2) is positive and statistically significant for ARMA(1, 2)-TGARCH(1, 1). Hence, we deduce that the volatility in the NSE can be explained by the exchange rate, and there exists a positive relationship. Therefore, it is evident that stock returns are positively related to changes in exchange rates. The government should implement policy measures to control the exchange rate, such as real-time disclosure of financial information, trading volumes, and corporate actions, as these affect stock returns.

Keywords Stock return, Volatility, Macroeconomic variables, Asymmetric GARCH models, Leverage effect

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1. Introduction

The study of volatility in financial markets is of great importance to investors in the management of risk, as it provides a degree of uncertainty on their investment. For financial development within today's globalised economy, a strong capital market is crucial. In the study by [9], it is argued that national economic activity is stimulated by a functioning stock market that efficiently channels savings into investments. Adesina [1] articulates that financial analysts and investors in financial markets are concerned with the

*Correspondence to: Edwin Moyo (Email: emoyo@mu.edu.zm). Department of Mathematics and Statistics, Mulungushi University, Kabwe, Zambia.

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unpredictability of asset return investments attributed to the instability of business performance and the varying prices of the market. A stock return is what an investor gains or losses on investing in a particular stock or portfolio that is dependent on the inherent risk in the market that the stock is listed. Sharpe [26] articulates that variations in investment returns are mainly dependent on the investor's willingness to take risks. That is, the higher the risk, the higher the returns and vice versa.

Generally, in financial markets, the main concern is often the spread of asset returns. Kumar et al. [16] articulates that for any stock market, volatility and returns are two important factors around which the entire stock market revolves. Volatility is defined as a measure of variability or dispersion for a given security or market index [2]. It can be measured using the standard deviation or variance and volatility is associated with the uncertainty of the price. However, it is not really the same as risk. The undesirable outcome is linked to risk, whereas a strict measure of uncertainty which can be attributed to either positive or negative outcome is volatility. For example, higher volatility implies higher risk in the market.

In the study by [22], the focus was on the effect of the fluctuation of the foreign exchange rate, the interest rate, and the inflation rate on the volatility of the returns of stocks on the Nairobi Stock Exchange (NSE). The empirical analysis used was Exponential Generalised Autoregressive Conditional Heteroscedasticity (EGARCH) and Threshold Generalised Conditional Heteroscedasticity (TGARCH). The main findings of the research study are as follows: the stock returns are symmetric but leptokurtic and not normally distributed. The results showed evidence that the foreign exchange rate, the interest rate, and the inflation rate affect the volatility of the return of the stocks. However, the current study seeks to study the effects of the inclusion of exchange and interest rates on the degree of persistence in the conditional variance equation.

Moyo et al. [21] modelled the effects of trading volume on stock return volatility using the AR(2)-EGARCH(1,1) and AR(2)-TGARCH(1,1) models under the student t distribution and GED. The AR(2)-EGARCH(1,1) model without trading volume was suggested to be a more suitable model to capture the main features of the NSE return, such as the volatility clustering, the stock returns volatility and the leverage effect. The current study seeks to extend this study by exploring the effects of exchange rate and interest rate on the persistence of volatility in the conditional variance equation.

Ayele et al. [4] modelled and forecasted the volatility of the gold price using the exponentially weighted moving average (EWMA) and the generalised autoregressive conditional heteroscedasticity (GARCH) models. The gold series showed the classical characteristics of financial time series, such as leptokurtic distributions, data dependence and strong serial correlation in squared returns. Hence, the series can be modelled using both EWMA and GARCH-type models. Among GARCH-type models, GARCH-M(2,2) with Student's t distribution for residuals was found to be the best fit model. In addition, interest rates, exchange rates, and crude oil prices were found to have a significant impact on gold volatility.

Financial time series returns often display clustering of volatility. Vasudevan and Vetrivel [30] outlines the most essential financial time series features as; they tend to have leptokurtic distribution, leverage effect, skewness, and volatility clustering. Hence, the standard ARCH/GARCH model can model the clustering of leptokurtosis, skewness, and volatility. Coffie [8] shows that the standard model is unable to capture the dynamics of an important feature of financial time series known as leverage effect, that is, cannot model this asymmetric behaviour of stock returns.

However, there is a fair amount of empirical evidence in the relationship between selected macroeconomic variables and volatility of the return of stocks for emerging stock markets in developing countries. In this study, traditional GARCH models are improved by directly integrating interest and exchange rates into the conditional variance equation. This modification enables an analysis of how macroeconomic variables affect volatility persistence in the Nairobi Securities Exchange (NSE). Thus, we analyse the relationship between the selected macroeconomic variables and stock return volatility. This study assesses whether, with the inclusion of the selected macroeconomic variables in conditional variance equation, volatility persistence disappears using the generalised autoregressive conditional heteroscedasticity models; EGARCH and TGARCH. The use of TGARCH and EGARCH models offers significant advantages for analysing volatility persistence. Both models account for asymmetry in shock responses, enabling them to differentiate between the effects of positive and negative shocks on volatility [29, 23]. TGARCH employs an intuitive threshold-based approach, while EGARCH models the log-variance, ensuring positivity and accommodating varying shock magnitudes [11]. Therefore, these models effectively quantify persistence and volatility clustering, providing valuable insights into the dynamics of financial markets, especially in emerging markets like Kenya.The remaining parts of this paper are organised as follows. We discuss the methodology considered in Section 2. The results and discussion are contained in Section 3 and, lastly, we conclude the paper in Section 4.

2. Methodology

To aid in analysis, the R software environment [24] and RStudio [25] was used to analyse the daily NSE 20-share price index, interest rates, and the exchange rate from 01/01/2015 to 31/12/2024, accounting for 3106 observations obtained from Central Bank of Kenya website and Nairobi Securities Exchange (NSE) website. The Asymmetry and leverage effects were modeled using EGARCH and TGARCH specifications implemented in the `rugarch` package [12] and the ARIMA specification was fitted using auto.arima function in the package [13]. The series of the stock return is defined as:

$$R_t = \ln P_t - \ln P_{t-1} = (1 - B) \ln P_t, \tag{1}$$

where P_t and P_{t-1} are the values of the stock index at the close of the current day and the previous day, respectively. R_t is the logarithmic returns of the stocks, and the letter B is the backward shift operator. Similarly, we define the exchange rate (ER_t) and the interest rate (IR_t) as the logarithmic transformation. We let ϵ_t be the shock at time t and F_t be the available information through time t. The modeling includes the estimation of the mean and conditional variance equations. We define the model as,

$$\begin{aligned} R_t &= \mathbb{E}(R_t|F_{t-1}) + \epsilon_t, \quad \epsilon_t \sim N(0,1) \\ R_t &= \mu_t + \epsilon_t, \end{aligned} \tag{2}$$

where μ_t is the conditional mean of R_t given information through time $t-1$ and R_t is the return at time index t. ϵ_t is a non constant term with respect to time and is defined as,

$$\epsilon_t = \sigma_t a_t. \tag{3}$$

where,

$$\begin{aligned} \sigma_t &= \sqrt{Var(R_t|F_{t-1})} \\ &= \sqrt{\mathbb{E}\left[(R_t - \mu_t)^2 | F_{t-1}\right]} \end{aligned}$$

and F_{t-1} denotes the information set available at time $t-1$, σ_t is the volatility that evolves over time and $a_t \sim N(0,1)$ which are independently and identically distributed (i.i.d).

2.1. Conditional mean equation

Jere and Moyo [14] articulated that the Box-Jenkins modeling procedure can be used to fit a time series model to the data so as to determine an ARIMA (p, d, q) model which is simple and provides a sufficiently accurate description of the behavior of the data. In modeling the conditional mean equation of R_t, we will employ the general Box Jenkins ARMA(p, q) model defined as:

$$R_t = \mu + \sum_{i=1}^{p} \phi_i R_{t-i} + \epsilon_t + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} \tag{4}$$

where μ is a constant, ϕ_i and θ_j are parameters of the ARMA(p, q) model for $i = 1, 2, \cdots, p$ and $j = 1, 2, \cdots, q$. Then, ϵ_t is the disturbance term. This study adopted the use of the Auto ARIMA in the [24] package [13] as it helps in choosing the optimal set of parameters and integrating them into the ARIMA model [7]. The study by [3] articulates that the auto ARIMA model simplifies the ARIMA modeling process by automatically identifying the best combination of p, q, and d values, thereby enhancing the model's performance.

2.2. Conditional variance equation

To model the daily stock returns volatility, we used asymmetric models since shocks of equal magnitude, which may either be positive or negative, are considered to have different effects on the volatility in future. in the asymmetric models the shocks of the same magnitude, positive or negative, have different effect on future volatility. Maqsood et al. [18] articulates that asymmetric models are extensively motivated by the need to distinguish between negative and positive shocks and their impact on volatility in financial markets. In this paper, we used the standard EGARCH(r, s) and TGARCH(r, s) models that we discuss below:

2.2.1. Exponential GARCH (EGARCH) Models. In this model, the asymmetric responses of the time-varying variance to shocks is captured. The model ensures a positive variance and uses standardized value of $\frac{\epsilon_{t-i}}{\sigma_{t-i}}$. The EGARCH (r, s) specification is given by,

$$\log \sigma_t^2 = \alpha_0 + \sum_{i=1}^{r} \alpha_i \frac{|\epsilon_{t-i}| + \gamma_i \epsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{s} \beta_j \log(\sigma_{t-j}^2) \tag{5}$$

where the asymmetric or leverage parameter is γ_i. In most empirical cases, the leverage parameter is expected to be positive so that a negative shock increases future volatility or uncertainty while a positive shock eases the impact on future uncertainty. When ϵ_{t-i} is positive (i.e. good news), its contribution to the log volatility is $\alpha_i(1+\gamma_i)|\epsilon_{t-i}|$ while if ϵ_{t-i} is negative (i.e. bad news) then, the total impact is $\alpha_i(1-\gamma_i)|\epsilon_{t-i}|$. If γ_i is :

- i $\gamma_i = 0$, there is symmetry i.e. no asymmetric volatility
- ii $\gamma_i < 0$, then negative shocks (bad news) will increase the volatility more than positive shocks (good news).
- iii $\gamma_i > 0$, then positive shocks (good news) will increase the volatility more than negative shocks (bad news)

The persistence $\hat{P}$ of the model is given by,

$$\hat{P} = \sum_{j=1}^{s} \beta_j$$

In this paper, two macro economic variables are included in the initial model in equation (5). The following modification of the conditional variance equation (5) is used:

$$\log \sigma_t^2 = \alpha_0 + \sum_{i=1}^{r} \alpha_i \frac{|\epsilon_{t-i}| + \gamma_i \epsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{s} \beta_j \log(\sigma_{t-j}^2) + \delta_1 IR_t + \delta_2 ER_t \tag{6}$$

where IR_t is the interest rate and ER_t is the exchange rate of the Kenyan Shilling (KSh) to the United States Dollar (USD) and δ_1 and δ_2 are their respective coefficients. The rest of the parameters are as defined in equation (5).

2.2.2. Threshold GARCH (TGARCH) Models. The TGARCH (r, s) conditional variance specification is given by,

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{r} (\alpha_i + \gamma_i N_{t-i}) \epsilon_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 \tag{7}$$

where N_{t-i} is given by,

$$N_{t-i} = \begin{cases} 1, & \text{if} \quad \epsilon_{t-i} < 0 \\ 0, & \text{if} \quad \epsilon_{t-i} \geq 0 \end{cases} \tag{8}$$

where γ_i is the asymmetric response parameter or leverage parameter and α_i and β_i are non-negative parameters satisfying conditions similar to those of GARCH models. The model collapses to the classical GARCH (p, q) process If $\gamma_i = 0$. In this model, positive shocks (good news) and negative shocks (bad news) have different effects on the conditional variance σ_t^2 that is when,

- $\epsilon_{t-i}^2 > 0$, the effect on volatility is α_i.
- $\epsilon_{t-i}^2 < 0$, then the effect on volatility is $\alpha_i + \gamma_i$.

Thus, we can generally say that for $\alpha_i > 0$, the effect of negative shocks have larger impact on conditional variance than does positive shocks. The persistence $\hat{P}$ of the model is given by,

$$\hat{P} = \sum_{i=1}^{r} \alpha_i + \sum_{j=1}^{s} \beta_j + \sum_{i=1}^{r} \gamma_i k$$

where k is the expected value of the standardized residuals a_t below zero (effectively the probability of being below zero),

$$k = \mathbb{E}[N_{t-i} a_{t-i}^2] = \int_{-\infty}^{0} f(a, 0, 1, \cdots) da \tag{9}$$

where f is the standardized conditional density with any additional skew and shape parameters $(\cdots)$. For example, the value of k is 0.5 in the case of symmetric distributions.
In this study, two macroeconomic variables are included in the initial model in equation (7). The following modification of the conditional equation (7) is used,

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{r} (\alpha_i + \gamma_i N_{t-i}) \epsilon_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 + \delta_1 IR_t + \delta_2 ER_t \tag{10}$$

where IR_t is the interest rate and ER_t is the exchange rate of the Kenyan Shilling (KSh) to the United States dollar (USD) while meaning of the rest of the parameters are as defined in equation (7).

2.3. Distribution Assumptions of the Error (a_t) *in the GARCH-type Model*

In the study by [27], it is argued that often non-normality patterns such as excess kurtosis and skewness are exhibited by financial time series. The residuals of conditional heteroscedasticity models may generally show excess kurtosis, heavy tails, and skewness. To account for skewness, excess kurtosis, and heavy tails of return distributions, this study will employ the Student-t distribution and the Generalized Error distribution (GED)

2.3.1. Student's t-Distribution. Bollerslev [5] proposed that in fitting the GARCH model for the standardized error of the return series, the Student's t distribution can be used to better capture the observed fat tails. The probability density function for a random variable that has a Student t distribution with ν degrees of freedom is given by,

$$f_X(t;\nu) = \frac{\Gamma(\frac{(\nu+1)}{2})}{\sqrt{\nu\pi}\Gamma(\frac{\nu}{2})\left(1+\frac{t^2}{\nu}\right)^{\frac{(v+1)}{2}}} \tag{11}$$

The density of the standardized Student's t-Distribution with $\nu > 2$ degrees of freedom is given by;

$$f(a_t) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\pi(\nu-2)}}\left(1+\frac{a_t^2}{\nu-2}\right)^{-\frac{\nu+1}{2}} \tag{12}$$

where $a_t = \frac{\epsilon_t}{\sigma_t}$ is the standardized error, $\Gamma(x) = \int_0^\infty e^{-x}x^{v-1}$ is the gamma function , ν is the parameter that measures the tail thickness. The log likelihood function is given by,

$$L_N = N\ln\left[\frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\pi(\nu-2)}}\right] - \frac{1}{2}\sum_{t=1}^{N}\left[\ln\sigma_t^2 + (\nu+1)\ln\left(1+\frac{a_t^2}{\sigma_t^2(\nu-2)}\right)\right] \tag{13}$$

2.3.2. Generalized Error Distribution The probability density function of the Generalized Error Distribution (GED) is given by,

$$f_X(x;\nu) = \frac{\nu\exp\left(\frac{-1}{2}|\frac{x}{\nu}|^\nu\right)}{\lambda 2^{(\frac{\nu+1}{\nu})}\Gamma(\frac{1}{\nu})}, -\infty < x < \infty, 0 < \nu \leq \infty, \tag{14}$$

where $\Gamma(.)$ is the gamma function and

$$\lambda = \left[\frac{2^{(\frac{-2}{\nu})}\Gamma(\frac{1}{\nu})}{\Gamma(\frac{3}{\nu})}\right]$$

The log likelihood function is given by,

$$L_N = \sum_{t=1}^{N}\left[\ln\left(\frac{\nu}{\lambda}\right) - \frac{1}{2}\left|\frac{x}{\sigma_t^2\nu}\right|^\nu - (1+\nu^{-1})\ln 2 - \ln\Gamma\left(\frac{1}{\nu}\right) - \frac{1}{2}\ln\sigma_t^2\right] \tag{15}$$

To maximize the log-likelihood function, the quasi-maximum likelihood function estimator (QMLE) will be used concerning the unknown parameters. This is a preferred methodology because it is said to provide asymptotic standard errors that are valid under nonnormality, are generally consistent, and have a normal limiting distribution [6].

2.4. Model comparison

The criterion is to choose a model with minimum values of AIC and BIC and with largest value of the log likelihood function as the model of best fit [21, 3, 31]. The AIC value is computed as follows:

$$AIC = -2\log L + 2k,$$

the BIC is defined as,

$$BIC = -2\log L + k\log n,$$

and the Log-Likelihood (LL) denoted as D is given by,

$$D = 2\{\log(\text{ likelihood for alternative model }) - \log(\text{ likelood for the null model })\}$$

where $k = p + q$ are the number of unknown parameters, L is the likelihood function of the model and n is the number of observations.

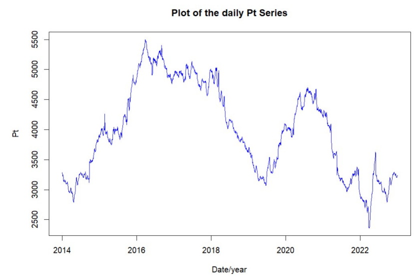
3. Results and discussion

The descriptive statistics of the variables considered in this study are presented in Table 1. We observe that the interest rate series has a negative daily mean, suggesting that it decreases slightly over time. In contrast, the average mean daily exchange rate and stock return series are positive, which implies that they increase slightly with time. The stock return and interest rate are left skewed while the exchange rate is right skewed implying that they have asymmetric distributions, as can be seen from the coefficient of the skewness. The values of skewness and kurtosis are different from 'zero' and 'three', respectively. This indicates that the stock return, interest rate and exchange rate for the selected period of study are leptokurtic, that is the series for all the variables are not normally distributed. This is confirmed by the Jarque-Bera (JB) test statistic at 5% level of significance. [10] argues that while common due to its historical use and practical convenience, a 5%significance level provides enough statistical power to detect real differences while controlling for random chance.

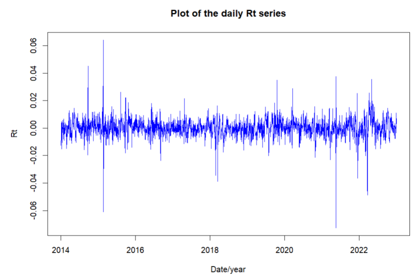
Table 1. Descriptive statistics of the daily stock returns and trading volume.

Measure	R_t	IR_t	ER_t
Number of observations	3106	3106	3106
Mean	0.000046	-0.000029	0.000123
Median	0.000023	0.001697	0.000094
Maximum	0.064127	0.537097	0.095927
Minimum	-0.07269	-0.482895	-0.094581
Std. deviation	0.007291	0.066801	0.004524
Skewness	-0.496612	-0.073850	0.053820
Kurtosis	13.631036	9.936019	190.729625
Jarque Bera (JB)	6428.9218	8686.44	3198.248
P-value (JB)	0.0000	0.0000	0.0000

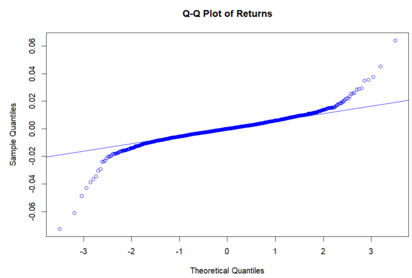
Figure 1, 2 and 3 shows the time series plot of the daily stock return,interest rate and exchange rate of NSE. From the plots, it is observed that periods of high volatility, occasional extreme movements and volatility clustering, as upward movements tend to be followed by other upward movements and downward movements also followed by other downward movements.



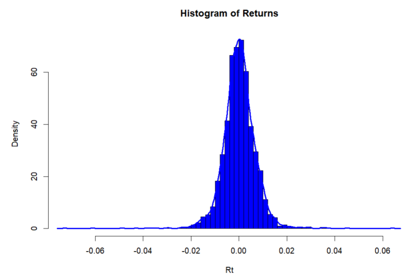
(a) plot of the NSE 20 share index price series.



(b) plot of the NSE 20 share index return series.

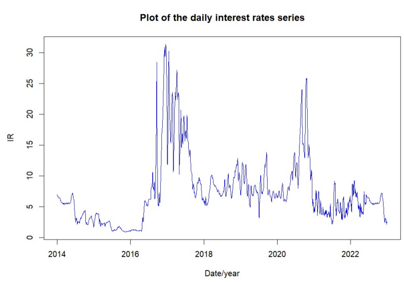


(c) QQ plot of the NSE 20 share index return series.

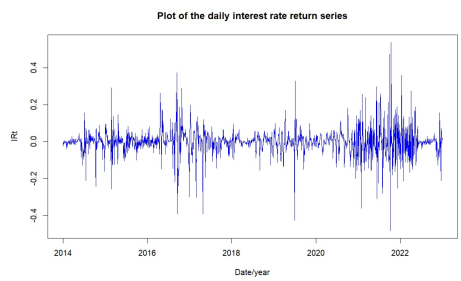


(d) Histogram of the NSE 20 share index return series.

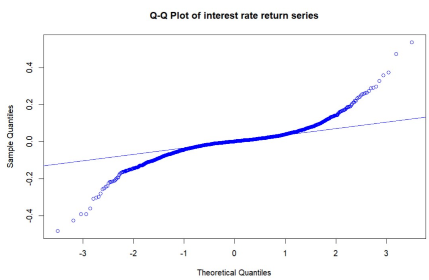
Figure 1. Plots of the NSE 20 share index series.



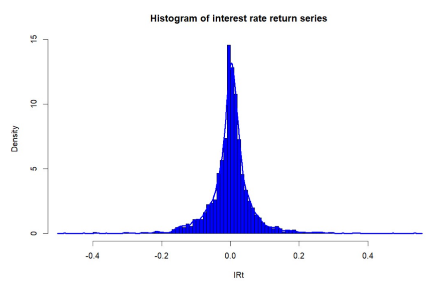
(a) plot of the interest rate series.



(b) plot of the interest rate return series.

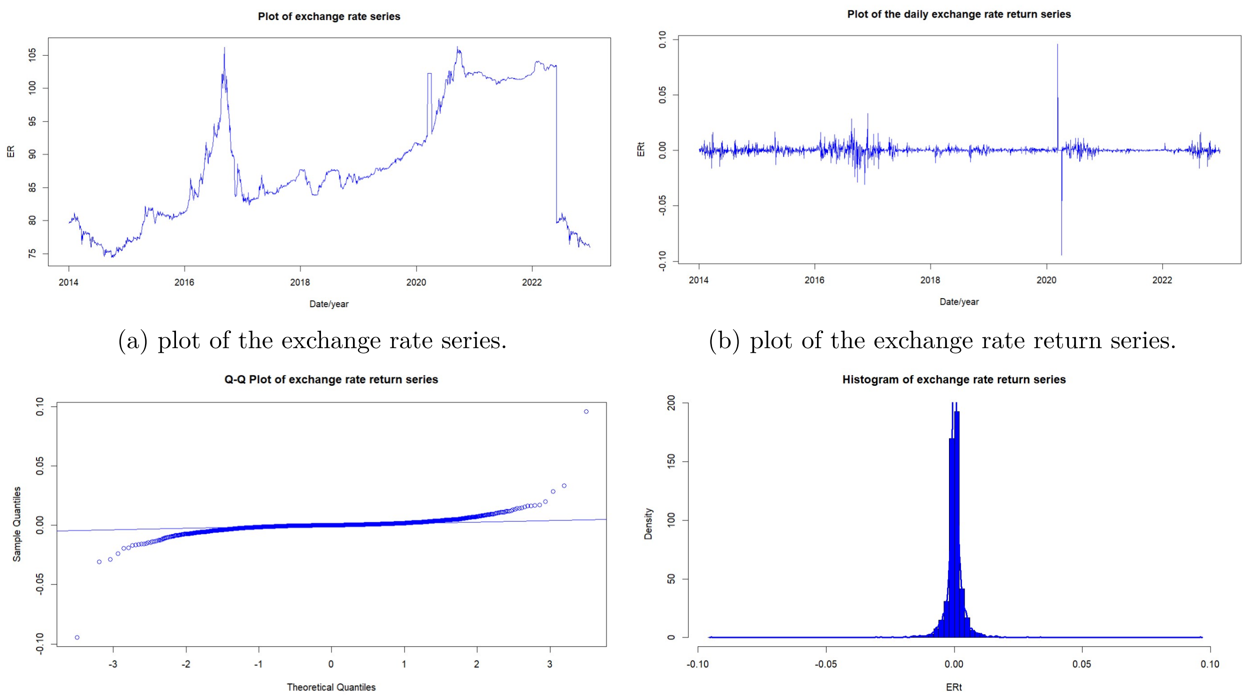


(c) QQ plot of the interest rate return series.



(d) Histogram of the interest rate return series.

Figure 2. Plots of the interest rate series.



(a) plot of the exchange rate series. (b) plot of the exchange rate return series.

(c) QQ plot of the exchange rate return series. (d) Histogram of the exchange rate return series.

Figure 3. Plots of the exchange rate series.

Table 2. Augmented Dickey-Fuller (ADF) test for the daily NSE 20- share index return series and explanatory variables

	Test statistic	Critical Value		
		1%	5%	10%
R_t	-24.2949	-2.58	-1.95	-1.62
IR_t	-23.8884			
ER_t	-31.5208			

To test for stationarity of the time series, the Augmented Dickey Fuller (ADF) test was considered. From the computed test statistics in Table 2, we observe that the computed test statistics for the daily stock returns, exchange rate and interest rate is stationary and thus,the null hypothesis is rejected at 5% level of significance.

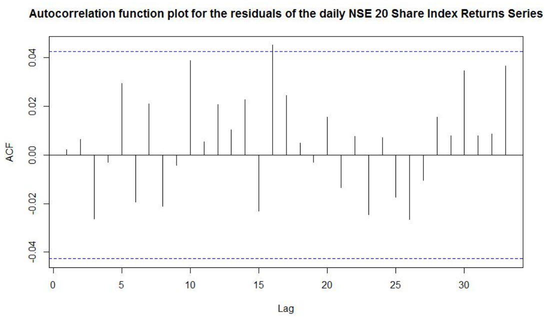
3.1. Estimated Mean Equation

In this study, an ARMA(p, q) model was used to fit the mean returns because it is said to provide approximations to the conditional mean dynamics that are flexible and parsimonious. According to Tsay and Tiao [28], to deduce the order of an ARMA(p, q) model the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) are used. In this study, it was suggested that the stock returns can be modelled by an ARMA(1, 2) process using the auto ARIMA similar with the idea used in the study by [3, 20, 19] since the auto ARIMA does not depend on manual visual observation of the ACF and

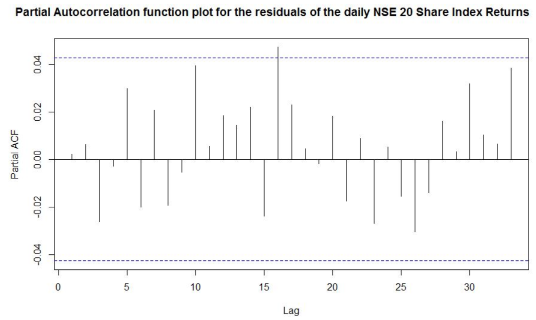
PACF. The test for ARCH effects on the residuals of the ARMA(1, 2) model resulted in the rejection of the null hypothesis at 5% level of significance and the results of the tests considered are given in Table 3. The lack of fit can also be observed from the plot of ACF and PACF in Figures 4a and 4b. Therefore, the implementation of the GARCH-type models is valid in the modeling of the stock return volatility.

Table 3. ARCH effect test on residuals of the ARMA(1,2) model.

	m	2	4	6	8
Ljung Box	Q_m^2	158.92	161.45	161.81	161.91
	$P-value$	$2.2e-16$	$2.2e-16$	$2.2e-16$	$2.2e-16$
Lagrange Multiplier	Q_m	134.84	138.55	139.21	139.12
	$P-value$	$2.2e-16$	$2.2e-16$	$2.2e-16$	$2.2e-16$



(a) ACF plot for the residuals of the daily NSE 20 share index return series.



(b) PACF plot for the residuals of the daily NSE 20 share index return series.

3.2. Estimated Volatility Models

The results of the parameter estimates for ARMA(1, 2)-EGARCH(1, 1) and ARMA(1, 2)-TGARCH(1, 1) without and with interest and exchange rates respectively under the Student t-distribution assumption and generalised error distribution of the error term distribution are presented in Table 4 and Table 5. The p-values are given in parentheses.

3.2.1. QMLE parameter estimates and p-values(in parentheses) for the ARMA(1, 2)-EGARCH(1, 1) and ARMA(1, 2)-TGARCH(1, 1) Models without interest rate and exchange rate. In Table 4, the mean parameter (μ) for all distribution assumptions are not statistically significant at 5% level of significance for both with and without the inclusion of interest rate and exchange rate. The autoregressive term is consistently significant across specifications, indicating that past returns influence current return dynamics. In contrast, the mean return parameter is statistically insignificant, which aligns with the weak-form efficiency of financial markets [15]. It can be seen that γ_1 the leverage parameter is positive and highly significant for the ARMA(1, 2)-EGARCH(1, 1) under both distributional assumptions. This means that bad news (negative shocks) increases volatility more than good news (positive shocks) of the same magnitude, whereas it was not significant for the GED and Student's t distributional assumptions for ARMA(1, 2)-TGARCH(1, 1). This finding underscores the EGARCH model's superior capability to model volatility asymmetry in the context of the NSE. Similar to the study by [17], the EGARCH model was the most effective for capturing volatility in the sampled indices. Thus, the EGARCH specification can deliver more reliable forecasts of future volatility and is better suited for practical applications, such as Value-at-Risk estimation and portfolio risk management. The model accuracy evaluation was performed using the Ljung-Box test for Q(14) and $Q^2(9)$ for residuals and squared residuals, respectively, and ARCH (7) for all models under the all error term distribution assumptions. The null hypothesis of no significant correlations and no arch effects is accepted for all the cases, implying that the fitted models were adequate.

Table 4. Parameter estimation of the ARMA(1, 2)-EGARCH(1, 1) and ARMA(1, 2)-TGARCH(1, 1) Models without exchange rate and interest rates.

	ARMA(1, 2)-EGARCH(1, 1)		ARMA(1, 2)-TGARCH(1, 1)	
Conditional distribuion	Student t	GED	Student t	GED
μ	0.0000 (0.8807)	0.0001 (0.1047)	0.0000 (0.8575)	0.0001 (0.3548)
AR(1)	0.4530 (0.0453)	0.4139 (0.0000)	0.4500 (0.0000)	0.4145 (0.0000)
MA(1)	-0.1759 (0.4535)	-0.1394 (0.0043)	0.1726 (0.0203)	-0.1396 (0.0000)
MA(2)	0.0579 (0.1917)	0.0744 (0.0006)	0.0570 (0.4485)	0.0716 (0.0002)
α_0	-2.1590 (0.0000)	-2.2315 (0.0000)	0.0014 (0.0006)	0.0014 (0.0001)
α_1	0.0046 (0.8811)	0.0114 (0.7133)	0.2178 (0.0000)	0.2388 (0.0000)
β_1	0.7893 (0.0000)	0.7817 (0.0000)	0.6120 (0.0000)	0.5974 (0.0000)
γ_1	0.4118 (0.0000)	0.4381 (0.0000)	-0.0208 (0.8203)	-0.0450 (0.6047)
Shape	6.7455 (0.0000)	1.3304 (0.0000)	6.4600 (0.0000)	1.3109 (0.0000)
$\hat{P}$	0.7894	0.7817	0.7764	0.7765
$Q(14)$	10.8617 (0.0658)	11.1580 (0.0528)	9.6247 (0.1524)	10.1873 (0.1057)
$Q^2(9)$	2.5856 (0.8249)	1.9271 (0.9135)	21.289 (0.0001)	17.353 (0.0009)
$ARCH(7)$	0.9794 (0.9169)	1.0673 (0.9024)	0.8127 (0.9420)	0.8978 (0.9296)
AIC	-7.4396	-7.4164	-7.4338	-7.4090
BIC	-7.41550	-7.3922	-7.4096	-7.3848
LL	7842.9250	7818.471	7836.7630	7810.651

3.2.2. QMLE parameter estimates and p-values(in parentheses) for the ARMA(1,2)-EGARCH(1, 1) and ARMA(1, 2)-TGARCH(1, 1) Models with interest rate and exchange rate. Table 5 gives the results of the parameter estimates for ARMA(1, 2)-EGARCH(1, 1) and ARMA(1, 2)-TGARCH(1, 1) with the interest rate and exchange rate under the GED and Student t distribution. The estimates of ARMA(1,

2) are significant, therefore supporting the implementation in modelling of the NSE stock returns with an ARMA(1, 2) model.

Table 5. Parameter estimation of the ARMA(1, 2)-EGARCH(1, 1) and ARMA(1, 2)-TGARCH(1,1) Models with interest rate and exchange rates.

	ARMA(1, 2)-EGARCH(1, 1)		ARMA(1, 2)-TGARCH(1, 1)	
Conditional distribuion	Student t	GED	Student t	GED
μ	0.0000 (0.8411)	0.0001 (0.0054)	0.0001 (0.6981)	0.0002 (0.4516)
AR(1)	0.4518 (0.0000)	0.4137 (0.0000)	0.4463 (0.0000)	0.4123 (0.0000)
MA(1)	-0.1743 (0.0002)	-0.1392 (0.0000)	-0.1692 (0.0000)	-0.1379 (0.0000)
MA(2)	0.0583 (0.0025)	0.0741 (0.0000)	0.0578 (0.0032)	0.0711 (0.0010)
α_0	-2.2417 (0.0001)	-2.4063 (0.0000)	0.0000 (0.9944)	0.0000 (0.9985)
α_1	0.0054 (0.8636)	0.0123 (0.6918)	0.2128 (0.0000)	0.2352 (0.0000)
β_1	0.7887 (0.0000)	0.7809 (0.0000)	0.6529 (0.0000)	0.6293 (0.0000)
γ_1	0.4121 (0.0000)	0.4388 (0.0000)	-0.0383 (0.6718)	-0.0614 (0.4807)
δ_1	0.000027 (0.9879)	-0.0008 (0.7050)	0.0000 (0.9999)	0.0000 (0.9999)
δ_2	0.0008 (0.5416)	0.0019 (0.3208)	0.000013 (0.0000)	0.000014 (0.0000)
Shape	6.7318 (0.0000)	1.3351 (0.0000)	6.3442 (0.0000)	1.3161 (0.0000)
$\hat{P}$	0.7887	0.7809	0.8133	0.8058
$Q(14)$	11.0228 (0.0584)	11.6168 (0.0371)	11.0819 (0.0559)	11.6561 (0.0360)
$Q^2(9)$	2.8421 (0.7847)	2.3450 (0.8601)	23.566 (0.0000)	18.692 (0.0004)
$ARCH(7)$	0.9114 (0.9144)	0.9709 (0.8968)	0.7641 (0.9097)	0.8315 (0.8980)
AIC	-7.4379	-7.4152	-7.4288	-7.4059
BIC	-7.4084	-7.3857	-7.3992	-7.3764
LL	7843.084	7819.209	7833.477	7809.392

The leverage parameter γ_1 is positive and significant for ARMA(1, 2)-EGARCH(1, 1) under both distributional assumptions. This indicates that bad news (negative shocks) increases volatility more than good news (positive shocks) of the same magnitude, whereas it was not significant for the GED and Student's t distributional assumptions for ARMA(1, 2)-TGARCH(1, 1). The parameter δ_2 is positive and statistically significant for the ARMA(1, 2)-TGARCH(1, 1) hence we deduce that the volatility in the NSE can be explained by the exchange rate. In contrast, the parameter δ_1 is not significant for both models implying minimal effect on the degree of persistence in the conditional variance equation. The degree of persistence in the conditional variance equations slightly increased for the ARMA(1, 2)-TGARCH(1, 1) model and there was a slight reduction for the ARMA(1, 2)-EGARCH(1, 1) with the inclusion of interest rate and exchange rate which was consistent regardless of the error term distribution assumption. The persistence implies that today's volatility shocks have an impact on the future expected volatility. The ARMA(1, 2)–EGARCH(1, 1) model with Student-t innovations consistently outperforms the ARMA(1, 2)-TGARCH(1, 1) model, regardless of whether interest rates are included. The EGARCH model is more effective at capturing essential characteristics of exchange rate volatility, such as volatility clustering, heavy tails, and leverage effects, while also providing robust residual diagnostics. In contrast, the TGARCH model has difficulty modeling asymmetry and exhibits weaker persistence. Although incorporating interest rates improves both models, the superiority of the EGARCH model is evident, establishing it as the most suitable framework for modeling exchange rate return risk in this context. Since accurate volatility modeling is central to Value-at-Risk (VaR) estimation, portfolio allocation, and hedging strategies, the evidence suggests that EGARCH should be preferred in risk management applications. Its ability to incorporate asymmetry and persistence makes it more reliable for capturing downside risk, particularly during turbulent market conditions [17].

4. Conclusion

In this study, an analysis of the relationship between stock return volatility, exchange, and interest rates was conducted using the ARMA(1, 2)-EGARCH (1, 1) and ARMA(1, 2)-TGARCH(1, 1) models under the student's t distribution and GED assumptions. It was observed that both the models ARMA(1, 2)-TGARCH(1, 1) and ARMA(1, 2)-EGARCH(1, 1) capture the leverage effect of NSE returns, which indicates that negative shocks imply a higher conditional variance for the next period than positive shocks of the same magnitude. The degree of persistence in the conditional variance equations increases slightly for the ARMA(1, 2)-TGARCH(1, 1) model and a slight reduction for the ARMA(1, 2)-EGARCH(1, 1) regardless of the error distribution assumption with the inclusion of interest rate and exchange rate was observed. Thus, the arrival of information is considered a major source of volatility. As new information is released, the prices of financial assets exhibit persistence of volatility, which affects financial risk analysis and risk management strategies. The parameter δ_2 is positive and statistically significant for the ARMA(1, 2) -TGARCH(1, 1) hence, it was established that only the exchange rate had an effect on the stock returns and there is a positive relationship. Thus, we deduce that the volatility in the NSE can be explained by the exchange rate, that is, it has a positive effect on stock returns. It suffices to say that the interest rate does not explain stock return volatility in this case due to its insignificance. This study lays the foundation for analysing the persistence of volatility while incorporating macroeconomic variables. The study focused on exchange and interest rates, in a way overlooking the effect of other key macroeconomic factors such as inflation, GDP growth, and political stability that may significantly affect the volatility of stock returns. Hence, future studies need to address this limitation to improve the applicability of the results. In addition, future research could benefit from exploring a broader range of GARCH-type models, such as APARCH, GJR-GARCH, Integrated GARCH, regime-switching models as these alternatives may offer additional insights into volatility persistence and long-memory dynamics. Therefore, the government should implement targeted interventions such as adjustments to monetary policy, strategic foreign exchange operations, fiscal controls, and regulatory measures, to offer clearer guidance for policymakers.

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Conflict of interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

Declaration of generative AI

The authors disclose that no generative AI tools were used to compile this research work.

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