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# Short Note of Linguistic SuperHypersoft Set

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**Abstract** Soft sets provide a mathematical framework for decision-making by associating parameters with subsets of a universal set, effectively managing uncertainty and imprecision [6, 7]. Over time, various extensions of soft sets, including Hypersoft Sets, SuperHypersoft Sets, and Treesoft Sets, have been introduced to address increasingly complex decision-making processes. This paper investigates the Linguistic SuperHyperSoft Set, which is an extension of the Linguistic HyperSoft Set.

**Keywords** Superhypersoft set, Soft Set, Hypersoft set, Linguistic Soft Set

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## 1. Introduction

In recent years, the integration of soft computing techniques has played a vital role in addressing complex real-world problems across various disciplines. Applications range from neuro-fuzzy systems for reservoir operations [1], to hybrid fuzzy logic and genetic algorithms for power system stability enhancement [2], and intelligent swarm-based malware detection frameworks in cybersecurity [3]. Moreover, fuzzy optimization and learning algorithms have been instrumental in advancing autonomous systems [4], while novel mathematical structures like nigh-open sets in topology continue to enrich the theoretical foundations of soft set theory [5]. These developments emphasize the growing need for generalized models that can handle uncertainty and imprecision effectively; challenges that soft set theory is well-equipped to manage.

Uncertainty and imprecision are regarded as very essential key factors in the environments of complex decision-making. In return for these challenges, Molodtsov presented the soft sets' concept in 1999 as a generalized mathematical structure for modeling problems with uncertain parameters [7]. The definition of a soft set is based on a universal set and links subsets of this universe with each parameter, providing a malleable tool for qualitative data analysis [6].

Continuing this, hypersoft sets were established to address multi-attribute decision-making schemes by linking attribute values' combinations (in place of single parameters) with subsets of the universe. Such an improvement enables a more grainy representation of decision settings [8, 9]. Afterwards, superhypersoft sets arose to improve this flexibility by enabling mappings from power set combinations of attribute values to subsets of the universe. This

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allows accommodating greater variability in criteria evaluation and catching interactions of higher order among parameters.

To further narrow the gap between mathematical formalism and qualitative human reasoning, linguistic hypersoft sets were defined [10], incorporating into the hypersoft framework some linguistic variables, like "high", "low", "medium". Such sets are particularly successful in several scenarios in the real world where crisp numerical limits are hard to determine, for instance, service quality evaluations and public health assessments.

In consideration of such developments, this work presents and formalizes the Linguistic SuperHypersoft Set (L-SHSS) concept, an innovative hybrid structure that combines the linguistic descriptors' expressiveness with the decision capacity of superhypersoft sets of higher-dimensional. The L-SHSS expedites the representation of complex qualitative criteria with the use of combinations of linguistic term sets, allowing subtle evaluations in multi-criteria environments. The presented framework universalizes both the classical superhypersoft set and the linguistic hypersoft set, offering a robust tool for progressed decision-making problems.

This work is arranged in the subsequent manner: Section 1 exhibits the preliminary concepts and foundational definitions linked to soft sets, hypersoft sets, and superhypersoft sets, besides their linguistic variants. Section 2 presents the Linguistic SuperHypersoft Set formally, demonstrating its reduction properties and theoretical framework. Numerous demonstrative examples are given to illustrate the pertinence and expressive power of the established model in several real-world scenarios, like project management and rural health evaluation. Section 3 argues likely the broader impact of L-SHSS in decision support systems and future directions, followed by the last section that concludes the results of this work.

## 2. Preliminaries and Definitions

In this section, we introduce the fundamental concepts of Soft Sets, Hypersoft Sets, and SuperHypersoft Sets. Throughout this paper, all sets are assumed to be finite. For the basic operations associated with each concept, the reader is referred to the respective references.

### *2.1. Hypersoft Sets*

A Hypersoft Set enhances multi-attribute decision analysis by associating combinations of multiple attributes with subsets of a universal set, enabling a more nuanced and comprehensive evaluation [8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19]. The definition is stated as follows.

*Definition 2.1* (Set)
[20] A *set* is a well-defined collection of distinct elements or objects. If $a$ is an element of a set $A$, we write $a \in A$; otherwise, we write $a \notin A$.

*Definition 2.2* (Subset)
[20] Let $A$ and $B$ be sets. $A$ is called a *subset* of $B$, denoted $A \subseteq B$, if every element of $A$ is also an element of $B$. If $A \subseteq B$ but $A \neq B$, then $A$ is called a *proper subset* of $B$, denoted $A \subset B$.

*Definition 2.3* (Empty Set)
[20] The *empty set*, denoted by $\varnothing$, is the unique set containing no elements. Formally, for any set $A$, $\varnothing \subseteq A$.

*Definition 2.4* (Power Set)
[20] Let $A$ be a set. The *power set* of $A$, denoted by $\mathcal{P}(A)$, is the set of all subsets of $A$. That is,

$$\mathcal{P}(A) = \{X \mid X \subseteq A\}.$$

*Definition 2.5* (Universal Set)
A *universal set*, denoted by $U$, is the set that contains all elements under consideration in a particular context. Every set discussed is assumed to be a subset of $U$.

*Definition 2.6* (Soft Set)
[7] Let $U$ be a finite universal set and $A$ be a set of attributes. Let $S \subseteq A$ denote a chosen subset of parameters. A

*soft set* over $U$ is defined as a pair $(\mathcal{F}, S)$ where

$$\mathcal{F} : S \to \mathcal{P}(U)$$

is a function that assigns to each parameter $\alpha \in S$ a subset $\mathcal{F}(\alpha) \subseteq U$. Formally,

$$(\mathcal{F}, S) = \{ (\alpha, \mathcal{F}(\alpha)) \mid \alpha \in S, \mathcal{F}(\alpha) \subseteq U \}.$$

*Definition 2.7* (Hypersoft Set)
[8] Let $U$ be a finite universal set and let $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_m$ be $m$ distinct attribute domains. Define the Cartesian product

$$\mathcal{C} = \mathcal{A}_1 \times \mathcal{A}_2 \times \cdots \times \mathcal{A}_m,$$

so that each element $\gamma \in \mathcal{C}$ is an $m$-tuple

$$\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_m) \quad \text{with } \gamma_i \in \mathcal{A}_i \text{ for } i = 1, 2, \ldots, m.$$

A *hypersoft set* over $U$ is a pair $(G, \mathcal{C})$ where

$$G : \mathcal{C} \to \mathcal{P}(U)$$

assigns to each $\gamma \in \mathcal{C}$ a subset $G(\gamma) \subseteq U$. Formally,

$$(G, \mathcal{C}) = \{ (\gamma, G(\gamma)) \mid \gamma \in \mathcal{C}, G(\gamma) \subseteq U \}.$$

### 2.2. Linguistic Hypersoft Set

The definition of the Linguistic Hypersoft Set is presented as follows [10, 21]. Related concepts include linguistic soft sets, which have been studied in various contexts [22, 23, 24, 25, 26].

*Definition 2.8* (Linguistic Hypersoft Set)
[10] Let $\Omega$ be a finite universe of objects (for example, rural health service centers). Let $t \geq 1$ be a positive integer, and let $\{\alpha_1, \alpha_2, \ldots, \alpha_t\}$ be $t$ distinct parameters (or criteria) relevant to a decision-making problem. For each parameter $\alpha_i$, define a finite, strictly ordered set of linguistic values $\Upsilon_i$, such that

$$\Upsilon_i = \{\kappa_{i,1}, \kappa_{i,2}, \ldots, \kappa_{i,m_i}\},$$

where the elements are arranged in increasing order (for example, from "very low" to "very high") and $\Upsilon_i \cap \Upsilon_j = \emptyset$ for $i \neq j$.

Define the *linguistic parameter space* as the Cartesian product

$$\Lambda = \Upsilon_1 \times \Upsilon_2 \times \cdots \times \Upsilon_t.$$

An element $\lambda \in \Lambda$ is a $t$-tuple

$$\lambda = (\kappa_{1,j_1}, \kappa_{2,j_2}, \ldots, \kappa_{t,j_t}),$$

where $\kappa_{i,j_i} \in \Upsilon_i$.

A mapping

$$\Gamma : \Lambda \to \mathcal{P}(\Omega)$$

assigns to each $\lambda \in \Lambda$ a subset $\Gamma(\lambda) \subseteq \Omega$ representing the collection of objects that satisfy the linguistic description given by $\lambda$. The pair $(\Gamma, \Lambda)$ is called a *Linguistic Hypersoft Set* (LHSS) over $\Omega$.

In practical applications, $\Gamma$ is constructed based on expert judgments or data analysis, thereby linking linguistic evaluations to concrete subsets of the decision universe.

*Theorem 2.9* (Generalization of Hypersoft Set by LHSS)
Let $(\Gamma, \Lambda)$ be a Linguistic Hypersoft Set over a finite universe $\Omega$, where

$$\Lambda = \Upsilon_1 \times \Upsilon_2 \times \cdots \times \Upsilon_t,$$

and each $\Upsilon_i$ is a finite, strictly ordered set of linguistic terms. If, for each $i$, the set $\Upsilon_i$ is treated as a crisp set (i.e., the linguistic ordering and other qualitative nuances are disregarded so that each term is simply a distinct element), then the pair $(\Gamma, \Lambda)$ reduces to a standard Hypersoft Set. Thus, the Linguistic Hypersoft Set is a proper generalization of the Hypersoft Set.

*Proof*
Assume that for every $i \in \{1, 2, \dots, t\}$, the linguistic set $\Upsilon_i$ is crisp, meaning that we consider $\Upsilon_i$ as a set of distinct elements without any additional linguistic structure (such as fuzzy membership degrees or qualitative ordering beyond mere distinctness). In this case, we can identify $\Upsilon_i$ with a crisp set $A_i$. Consequently, the Cartesian product

$$\Lambda = \Upsilon_1 \times \Upsilon_2 \times \cdots \times \Upsilon_t$$

becomes

$$\Lambda = A_1 \times A_2 \times \cdots \times A_t,$$

which is exactly the parameter space employed in the standard definition of a Hypersoft Set.

The mapping

$$\Gamma : \Lambda \to \mathcal{P}(\Omega)$$

then assigns to each tuple $(a_1, a_2, \dots, a_t)$ (with $a_i \in A_i$) a subset $\Gamma((a_1, a_2, \dots, a_t)) \subseteq \Omega$. This assignment conforms precisely to the definition of a Hypersoft Set.

Therefore, when the linguistic sets $\Upsilon_i$ are interpreted as crisp sets, the Linguistic Hypersoft Set $(\Gamma, \Lambda)$ is equivalent to the standard Hypersoft Set. This proves that the Linguistic Hypersoft Set generalizes the Hypersoft Set. □

*Example 2.10* (LHSS for Evaluating Rural Health Service Centers)
Consider a decision-making scenario where a government agency must evaluate rural health service centers to determine which centers to fund for improvements. Let the universe be:

$$\Omega = \{o_1, o_2, o_3, o_4, o_5\},$$

where each $o_i$ represents a different health service center.

Suppose we consider three evaluation criteria:

1. **Quality of Service** ($\alpha_1$) with linguistic values:

   $$\Upsilon_1 = \{\text{Poor}, \text{Fair}, \text{Good}\}.$$

   Here, ”Poor” < ”Fair” < ”Good”.
2. **Accessibility** ($\alpha_2$) with linguistic values:

   $$\Upsilon_2 = \{\text{Low}, \text{Medium}, \text{High}\}.$$

   Here, ”Low” < ”Medium” < ”High”.
3. **Resource Availability** ($\alpha_3$) with linguistic values:

   $$\Upsilon_3 = \{\text{Insufficient}, \text{Sufficient}, \text{Excellent}\}.$$

   Here, ”Insufficient” < ”Sufficient” < ”Excellent”.

Thus, the linguistic parameter space is given by:

$$\Lambda = \Upsilon_1 \times \Upsilon_2 \times \Upsilon_3.$$

An element of $\Lambda$ might be

$$\lambda = (\text{Good}, \text{High}, \text{Excellent}),$$

which represents a center with very favorable evaluations in all three criteria.

Now, suppose that experts have determined the following assignment:

$$\Gamma(\text{Good}, \text{High}, \text{Excellent}) = \{o_2, o_4\},$$

meaning that health service centers $o_2$ and $o_4$ are classified as having excellent service quality, accessibility, and resource availability. Similarly, one could define:

$$\Gamma(\text{Fair}, \text{Medium}, \text{Sufficient}) = \{o_1, o_3\},$$

and

$$\Gamma(\text{Poor}, \text{Low}, \text{Insufficient}) = \{o_5\}.$$

Thus, the LHSS $(\Gamma, \Lambda)$ provides a clear mapping from linguistic descriptions to specific sets of health service centers. Decision-makers can then use these mappings to aggregate information and make informed funding decisions.

This example demonstrates how LHSS can concretely model complex linguistic evaluations in real-world multi-criteria decision-making problems.

### *2.3. SuperHypersoft Sets*

SuperHypersoft Sets extend the concept of Hypersoft Sets by mapping power set combinations of multiple attributes to subsets of a universal set. This extension supports high-dimensional decision-making and captures intricate interdependencies among attributes, offering significant flexibility for addressing advanced decision-making challenges [27, 28, 29, 30, 31, 32, 33, 34, 35]. The definition is stated as follows.

*Definition 2.11* (SuperHypersoft Set)
[28, 36] Let $U$ be a finite universal set and let $a_1, a_2, \ldots, a_n$ be $n$ distinct attributes with corresponding finite sets of values $A_1, A_2, \ldots, A_n$ that are pairwise disjoint:

$$A_i \cap A_j = \emptyset \quad \text{for all } i \neq j.$$

For each $i$, let $\mathcal{P}(A_i)$ denote the power set of $A_i$. Define the Cartesian product

$$\mathcal{C} = \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \cdots \times \mathcal{P}(A_n).$$

A *SuperHypersoft Set* over $U$ is a pair $(F, \mathcal{C})$ where

$$F : \mathcal{C} \to \mathcal{P}(U)$$

assigns to each element $\gamma = (\alpha_1, \alpha_2, \ldots, \alpha_n) \in \mathcal{C}$ (with $\alpha_i \in \mathcal{P}(A_i)$) a subset $F(\gamma) \subseteq U$. Formally,

$$(F, \mathcal{C}) = \{\, (\gamma, F(\gamma)) \mid \gamma \in \mathcal{C},\, F(\gamma) \subseteq U \,\}.$$

*Example 2.12* (Product Selection in E-Commerce)
Consider an e-commerce platform that offers 10 different products:

$$U = \{p_1, p_2, \ldots, p_{10}\}.$$

The decision-makers evaluate products based on three disjoint attributes:

- **Color** ($a_1$) with $A_1 = \{\text{red}, \text{blue}, \text{green}\}$,
- **Size** ($a_2$) with $A_2 = \{\text{small}, \text{medium}, \text{large}\}$,
- **Brand** ($a_3$) with $A_3 = \{\text{BrandA}, \text{BrandB}\}$.

Since the attribute values are pairwise disjoint, we can form the power sets:

$$\mathcal{P}(A_1), \quad \mathcal{P}(A_2), \quad \mathcal{P}(A_3).$$

The parameter space is

$$\mathcal{C} = \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \mathcal{P}(A_3).$$

For instance, one may consider the tuple

$$\gamma = (\{\text{red}, \text{blue}\}, \{\text{medium}\}, \{\text{BrandA}\}).$$

If the mapping $F$ is defined by

$$F(\{\text{red}, \text{blue}\}, \{\text{medium}\}, \{\text{BrandA}\}) = \{p_2, p_5, p_7\},$$

this means that products $p_2$, $p_5$, and $p_7$ satisfy the composite criteria of being either red or blue, of medium size, and from BrandA. This detailed structure allows the platform to effectively filter and recommend products based on complex customer preferences.

*Example 2.13* (Weather Forecasting for Severe Conditions)
Consider a meteorological department monitoring weather conditions across 8 observation stations:

$$U = \{s_1, s_2, \ldots, s_8\}.$$

Suppose that weather conditions are characterized by three distinct attributes:

- **Temperature Level** ($a_1$) with $A_1 = \{\text{low}, \text{medium}, \text{high}\}$,
- **Humidity Level** ($a_2$) with $A_2 = \{\text{dry}, \text{normal}, \text{humid}\}$,
- **Wind Speed** ($a_3$) with $A_3 = \{\text{calm}, \text{breezy}, \text{stormy}\}$.

The power sets $\mathcal{P}(A_1)$, $\mathcal{P}(A_2)$, and $\mathcal{P}(A_3)$ are then considered, and the parameter space becomes

$$\mathcal{C} = \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \mathcal{P}(A_3).$$

For example, the tuple

$$\gamma = (\{\text{high}\}, \{\text{humid}\}, \{\text{stormy}\})$$

represents the condition of high temperature, humid air, and stormy wind speed. If the mapping $F$ is specified by

$$F(\{\text{high}\}, \{\text{humid}\}, \{\text{stormy}\}) = \{s_3, s_7\},$$

this indicates that stations $s_3$ and $s_7$ are experiencing conditions that may trigger a severe weather warning. Such a concrete example aids meteorologists in identifying areas at risk and issuing timely alerts.

*Example 2.14* (Project Management in Construction)
Consider a construction company managing 7 projects:

$$U = \{P_1, P_2, \ldots, P_7\}.$$

Projects are evaluated based on three disjoint attributes:

- **Budget Category** ($a_1$) with $A_1 = \{\text{low}, \text{medium}, \text{high}\}$,
- **Timeline** ($a_2$) with $A_2 = \{\text{short}, \text{medium}, \text{long}\}$,
- **Risk Level** ($a_3$) with $A_3 = \{\text{low}, \text{high}\}$.

We form the power sets $\mathcal{P}(A_1)$, $\mathcal{P}(A_2)$, and $\mathcal{P}(A_3)$, and the overall parameter space is

$$\mathcal{C} = \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \mathcal{P}(A_3).$$

For instance, consider the tuple

$$\gamma = (\{\text{medium}, \text{high}\}, \{\text{long}\}, \{\text{high}\}).$$

If we define

$$F(\{\text{medium}, \text{high}\}, \{\text{long}\}, \{\text{high}\}) = \{P_3, P_5, P_7\},$$

this indicates that projects $P_3$, $P_5$, and $P_7$ are characterized by a medium to high budget, long timeline, and high risk level. This detailed classification allows project managers to prioritize projects that require more intensive monitoring and risk mitigation strategies.

## 3. Result of this Paper

This section presents the results of this paper.

### 3.1. Linguistic SuperHypersoft Set

The definition of the Linguistic SuperHypersoft Set is described as follows.

*Definition 3.1* (Linguistic $n$-Superhypersoft Set)
Let $\Omega$ be a finite universe of discourse, consisting of objects under evaluation (e.g., hospitals, cities, products). Let $n \geq 1$ be an integer representing the number of evaluation criteria. For each $i = 1, 2, \ldots, n$, let $\Upsilon_i$ be a finite, totally ordered set of linguistic terms associated with the $i$th criterion, such as:

$$\Upsilon_i = \{\kappa_{i,1}, \kappa_{i,2}, \ldots, \kappa_{i,m_i}\}, \quad \text{with } \kappa_{i,1} < \kappa_{i,2} < \cdots < \kappa_{i,m_i}.$$

Define the linguistic parameter space as the Cartesian product of the linguistic term sets:

$$\Lambda = \Upsilon_1 \times \Upsilon_2 \times \cdots \times \Upsilon_n.$$

Define the *linguistic super attribute space* as:

$$\Lambda^* = \mathcal{P}(\Upsilon_1) \times \mathcal{P}(\Upsilon_2) \times \cdots \times \mathcal{P}(\Upsilon_n),$$

where $\mathcal{P}(\Upsilon_i)$ denotes the power set of $\Upsilon_i$.

A *Linguistic $n$-Superhypersoft Set (L-$n$-SHSS)* is a pair $(\Gamma, \Lambda^*)$ where $\Gamma$ is a mapping:

$$\Gamma : \Lambda^* \to \mathcal{P}(\Omega),$$

assigning to each linguistic super parameter $\lambda = (A_1, A_2, \ldots, A_n)$, with $A_i \subseteq \Upsilon_i$, a subset $\Gamma(\lambda) \subseteq \Omega$ of objects that satisfy the specified combination of linguistic terms. This set-based mapping allows multiple levels of linguistic fuzziness per attribute and enables multi-criteria qualitative analysis.

*Theorem 3.2* (Reduction to Linguistic Hypersoft Set)
If $n = 1$, then the L-$n$-SHSS reduces to the standard Linguistic Hypersoft Set.

*Proof*
When $n = 1$, we have $\Lambda^* = \mathcal{P}(\Upsilon_1)$, and $\Gamma : \mathcal{P}(\Upsilon_1) \to \mathcal{P}(\Omega)$. This exactly matches the definition of a Linguistic Hypersoft Set, where each subset of linguistic terms corresponds to a group of objects in $\Omega$. Thus, the model coincides with the LHS model. □

*Theorem 3.3* (Reduction to $n$-Superhypersoft Set)
If each $\Upsilon_i$ consists of crisp, discrete (non-linguistic) terms, then the L-$n$-SHSS reduces to the $n$-Superhypersoft Set.

*Proof*
If all $\Upsilon_i$ are crisp (e.g., numerical or label-based attributes), then the semantic layer of linguistic interpretation is removed. The power set $\mathcal{P}(\Upsilon_i)$ in this case behaves identically to the power set of a classical attribute domain. Hence, the mapping $\Gamma$ becomes equivalent to that in an $n$-Superhypersoft Set, which associates subsets of power set-attribute tuples with subsets of objects in $\Omega$. □

*Example 3.4* (Evaluation of Rural Health Service Centers)
Let

$$\Omega = \{o_1, o_2, o_3, o_4, o_5\}$$

represent five rural health service centers. Consider $n = 3$ criteria for assessment:

- **Service Quality:** $\Upsilon_1 = \{\text{poor}, \text{fair}, \text{good}\}$,
- **Accessibility:** $\Upsilon_2 = \{\text{low}, \text{medium}, \text{high}\}$,
- **Resource Availability:** $\Upsilon_3 = \{\text{insufficient}, \text{sufficient}, \text{excellent}\}$.

A linguistic decision-maker might evaluate centers based on composite linguistic evaluations. For instance, define the tuple:

$$\lambda = (\{\text{fair}, \text{good}\}, \{\text{high}\}, \{\text{excellent}\}) \in \Lambda^*.$$

Let the mapping $\Gamma$ assign:

$$\Gamma(\lambda) = \{o_2, o_5\},$$

indicating that health centers $o_2$ and $o_5$ satisfy the given composite linguistic evaluation.

Additionally, suppose another tuple

$$\lambda' = (\{\text{poor}, \text{fair}\}, \{\text{medium}, \text{high}\}, \{\text{sufficient}, \text{excellent}\})$$

maps to

$$\Gamma(\lambda') = \{o_1, o_3\},$$

meaning those centers meet relaxed criteria. The resulting linguistic 3-Superhypersoft set captures nuanced groupings of health centers under multi-linguistic conditions.

## 4. Conclusion

This paper introduced the Linguistic SuperHypersoft Set (L-SHSS), a novel generalization that combines linguistic evaluation with the structural richness of superhypersoft sets. We showed that the L-SHSS reduces to known models under specific conditions, enhancing its theoretical consistency. Examples demonstrated its utility in modeling complex decision-making problems involving qualitative and high-dimensional criteria. Future research may explore computational algorithms and real-world applications of the proposed model.

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