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The local stability of a compartmental corruption epidemic model under the impact of personal willingness

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Abstract The given work establishes a compartmental corruption epidemic model that consists of the susceptible compartment, exposed compartment, corrupt compartment, jailed compartment, reformed compartment, and honest compartment under the impact of personal willingness. In this model, it is assumed that corruption spreads like an infectious disease if there is an interaction between a susceptible individual and a corrupt individual, hence the epidemiology theory can be used to analyze the behavior of the model. The local stability analysis of the model is established. The study shows that the corruption-free fixed point and the corruption-endemic fixed point depend on the basic reproduction number. The numerical simulations that demonstrate the local stability of the corruption-free fixed point and the corruption-endemic fixed point under influence of the personal willingness are conducted.

Keywords Stability, epidemic model, basic reproduction number, fixed point

AMS 2010 subject classifications 34A34, 34D05

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1. Introduction

Mathematical modeling is a systematic approach used in various disciplines to analyze real-world phenomena or systems using mathematical equations, differential equation systems, or dynamic systems. One of the most important studies in dynamic systems is stability of the model. Stability of the model is the instrument for knowing the long-term behavior of a dynamic system.

Corruption is a social behavior that exists in almost every country. In the literature, it is mentioned that corruption is an act against the law carried out to gain personal or group gain by abuse of authority or power by public (government) or private officials [1, 2]. Corruption control policies and strategies have been developed in various countries, yet corruption cases continue. Even in parts of the world, cases of corruption have become an epidemic in society. Various scientific studies related to this corrupt behavior have been carried out by various researchers, one of which is the use of dynamic models.

The study of corrupt behavior using dynamic models is a very active research topic among mathematical modeling researchers. Some recent studies on the use of dynamic models associated with corruption are found in the literatures [3, 4, 5, 6, 7, 8, 9, 10], where authors mentioned that corrupt behavior can spread like the spread of infectious diseases if individuals who are not corrupt always interact with perpetrators of corruption. In the mentioned literature above, the population of the observed region is grouped into several compartments, such as the compartment of vulnerable individuals who have not committed corruption, the compartment of individuals who are exposed (suspected) to corruption but have not been sentenced, the individual compartment that is corrupt,

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the corrupt compartment that is serving the sentence, and the corrupted compartment that is already free from punishment. From these existing models, they studied the stability of the fixed point of the model with various variations of the parameters involved without regard to the influence of the individual parameters that may corrupt by their own will, although they do not interact with the corrupt individual.

In this paper, we study the local stability of fixed points for the corruption epidemic model which consist of six compartments, that are the compartment of the susceptible individuals (symbolyzed as S), the compartment of the exposed individuals (symbolyzed as E), the compartment of the corrupt individuals (symbolyzed as C), the compartment of the jailed individuals (symbolyzed as J), the compartment of individuals who are already free from punishment (symbolyzed as R), and the compartment of the honest individuals (symbolyzed as H). The model is constructed taking into account the influence of the individual parameters that may corrupt by their own will, although they do not interact with the corrupt individual. We establish the local stability criteria for the considered model. As far as the authors know, there is no mathematical model in this form and its local stability analysis. As a result, the findings of this study represent both a novel and a fresh advancement in the field of epidemic dynamics.

The paper is organized as follows: Section 2 presents the model formulation and its properties. The local stability analysis of the fixed points is presented in the section 3. Section 4 concludes the paper.

2. Model Formulation and Its Properties

In this section, we go over the model's assumptions, schematic, and system of equations. Furthermore, we prove that the model equations' solutions are both positive and bounded in order to demonstrate the model's mathematical soundness and biological significance. The work done in [11] has been expanded to include (i) the reformed compartment, (ii) parameter of personal willingness to practice corruption, and (iii) permitting people to remain susceptible after being released from reform in order to create a mathematical model that describes the dynamics of corruption.

The total population at time t, denoted as $N(t)$, is split into six compartments, as follows:

i. Susceptible compartment, denoted as $S(t)$, is the class of individuals who have not committed corruption but vulnerable to do. As an example, an official with low salaries and limited opportunities for income growth while corruption offers significant benefits may interact with corrupt individuals and be encouraged to adopt similar behavior. Thus, if α denote the effective corruption contact rate between the susceptible individual and the corrupt individual, then a number of $\dfrac{\alpha SI}{N}$ susceptible people per time will move to become exposed. Moreover, some of them can also leave this compartment and move to honest compartment and corrupt compartment due to personal desires. This compartment derived from the natural birth of populations and individuals that reformed of the corruption.

ii. Exposed compartment, denoted as $E(t)$, is the class of individuals who reported corruption but could not influence the vulnerable to corruption. This compartment comes from vulnerable individuals and corrupt individuals who have interacted. However, some of them can leave this compartment and move to corrupted and honest compartments.

iii. Corrupt compartment, denoted as $C(t)$, is the class of individuals who involved in corrupt practices. It is assumed that corrupt behavior can be transmitted to susceptible individuals if there is frequent interaction between corrupt individuals and susceptible individuals. In addition, corruption can also occur due to personal desires without influence from other individuals. Thus, this compartment comes from susceptible individuals and exposed individuals. However, some of them can leave this compartment and move to jailed and recovered compartments.

iv. Jailed compartment, denoted as $J(t)$, is the class of individuals jailed because of corrupt practices. This compartment contains individuals who are generated only from the corrupt compartment.

v. Reformed compartment, denoted as $R(t)$, is the class of individuals who have changed (recovered) from corruption, either because of their awareness or completed their imprisonment. Subpopulation R can become susceptible again and can also become honest individuals.

vi. Honest compartment, denoted as $H(t)$, is the class of individuals who honest. Class honesty comes from susceptible individuals, exposed individuals, and individuals who have already recovered.

All parameters related to the formation of the desired model that shows the change in the number of individuals from one compartment to another are given in Table 1. In light of the aforementioned factors, Figure 1 depicts a compartmental flow diagram as the basis for forming a corruption epidemic model.

Table 1. Parameters involved in the formation of the corruption epidemic model

Parameter	**Description**	**Unit**
β	Natural birth rate	time^{-1}
α	Effective corruption contact rate	time^{-1}
μ	Natural mortality rate	time^{-1}
γ	The rate at which susceptible individuals become corrupt due to personal willingness	time^{-1}
κ	The proportion of exposed individuals entering into the corruption subpopulation	%
ϕ	The rate of transition from exposed individuals to corrupt individuals	time^{-1}
η	The rate of susceptible individual migration to becoming honest	time^{-1}
δ	The rate of imprisonment for individuals who commit corruption	time^{-1}
τ	The rate at which corrupt individuals become reformed	time^{-1}
ρ	The rate at which jailed individuals become reformed	time^{-1}
θ	The rate at which reformed individuals become honest	time^{-1}
ω	The rate at which reformed individuals become susceptible	time^{-1}

According to the flow chart, the model will be governed by the following system of differential equations:

$$\begin{aligned}
\dot{S} &= \beta N - \frac{\alpha S C}{N} - (\gamma + \eta + \mu)S + \omega R \\
\dot{E} &= \frac{\alpha S C}{N} - (\phi + \mu)E \\
\dot{C} &= \kappa\phi E + \gamma S - (\delta + \tau + \mu)C \\
\dot{J} &= \delta C - (\rho + \mu)J \\
\dot{R} &= \rho J + \tau C - (\omega + \theta + \mu)R \\
\dot{H} &= \theta R + (1-\kappa)\phi E + \eta S - \mu H,
\end{aligned} \tag{1}$$

where

$$N(t) = S(t) + E(t) + C(t) + J(t) + R(t) + H(t), \tag{2}$$

with all of the initial conditions $S(0) = S_0, E(0) = E_0, C(0) = C_0, J(0) = J_0, R(0) = R_0, H(0) = H_0$ are nonnegative.

In order to verify the validity of the model, we need to show that the solution of the model (1) is nonnegative and bounded, thus the presented model is epidemiologically and mathematically meaningful. First let us show that the solution of the model (1) is nonnegative, We state the following result.

Theorem 2.1
The solution of the model (1) is nonnegative for the all nonnegative initial conditions $S_0, E_0, C_0, J_0, R_0, H_0$.

Proof
Take a look at system (1)'s corrupt compartment equation, provided by $\dot{C} = \kappa\phi E + \gamma S - (\delta + \tau + \mu)C$. We can

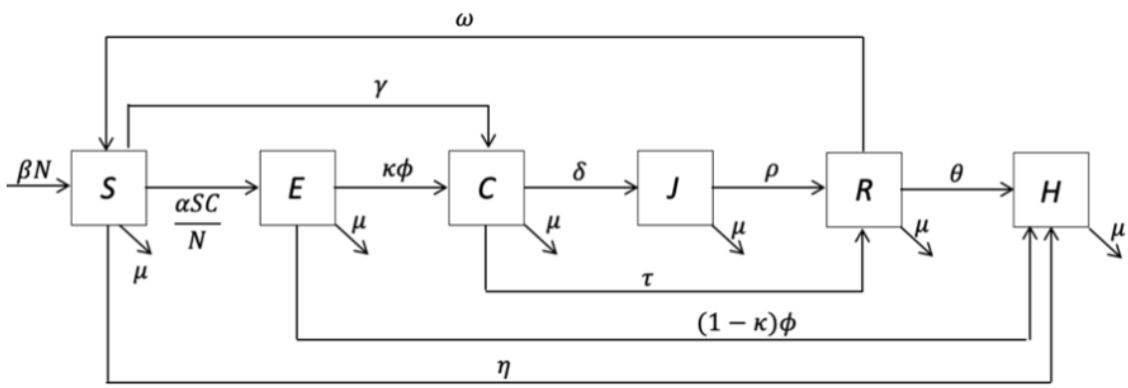

Figure 1. Compartmental Diagram of Corruption Epidemic Model

rearrange it as $\dfrac{\dot{C}}{C} = \dfrac{\kappa\phi E}{C} + \dfrac{\gamma S}{C} - (\delta + \tau + \mu)$ and upon integration it gives the general solution as

$$C(t) = K_1 \exp\left(\int \left(\frac{\kappa\phi E}{C} + \frac{\gamma S}{C} - (\delta + \tau + \mu)\right) dt\right)$$

where K_1 is a constant integration. Upon utilize the initial condition $C(0) = C_0 \geq 0$, we obtain

$$K_1 = C_0 \exp\left(-\int \left(\frac{\kappa\phi E}{C} + \frac{\gamma S}{C} - (\delta + \tau + \mu)\right) dt\Big|_{t=0}\right).$$

Eliminating the constant integration K_1 the particular solution is obtained as follows:

$$C(t) = C_0 \exp\left(\int \left(\frac{\kappa\phi E}{C} + \frac{\gamma S}{C} - (\delta + \tau + \mu)\right) dt - \int \left(\frac{\kappa\phi E}{C} + \frac{\gamma S}{C} - (\delta + \tau + \mu)\right) dt\Big|_{t=0}\right).$$

Since the exponential function is positive, we conclude that $K(t)$ is nonnegative. By the same procedure, the other state variables $S(t), E(t), J(t), R(t), H(t)$ are nonnegative for all time $t \geq 0$. □

Next, let us show that the solution of the model (1) is bounded. By differentiating both sides the equation (2) with respect to time t and on using the equation (1) and after algebraic simplifications, we obtain the rate of change of the total population is given by

$$\dot{N} = (\beta - \mu)N. \tag{3}$$

Solution of the equation (3) is

$$N(t) \;\; = \;\; N_0 e^{(\beta-\mu)t}. \tag{4}$$

Form (4), one can see that $N(t) \leq N_0$ for all time $t > 0$ if $\beta \leq \mu$, and this means the solution of the model (1) is bounded.

Next, let us make model (1) a dimensionless model for simplicity. By defining $s = \dfrac{S}{N}, e = \dfrac{E}{N}, c = \dfrac{C}{N}, j = \dfrac{J}{N}, r = \dfrac{R}{N}, h = \dfrac{H}{N}$ as the fractions of the class susceptible, exposed, corrupt, jailed, reformed, and honest in the population, respectively, from equation (1) we get,

$$\begin{aligned}
\dot{s} &= \beta - \alpha sc - (\gamma + \eta + \mu)s + \omega r \\
\dot{e} &= \alpha sc - (\phi + \mu)e \\
\dot{c} &= \kappa\phi e + \gamma s - (\delta + \tau + \mu)c \\
\dot{j} &= \delta c - (\rho + \mu)j \\
\dot{r} &= \rho j + \tau c - (\omega + \theta + \mu)r \\
\dot{h} &= \theta r + (1-\kappa)\phi e + \eta s - \mu h
\end{aligned} \tag{5}$$

with all of the initial conditions $s(0) = s_0, e(0) = e_0, c(0) = c_0, j(0) = j_0, r(0) = r_0, h(0) = h_0$ are nonnegative.

3. Local Stability Analysis

In accordance with the theory of the dynamical system [12], the local stability of the model (5) is the behavior of the model solution at infinity around the fixed points. The fixed points are determined by making

$$\dot{s} = \dot{e} = \dot{c} = \dot{j} = \dot{r} = \dot{h} = 0. \tag{6}$$

Using (6) for (5), we have

$$s = \frac{\beta + \omega r}{\alpha c + \gamma + \eta + \mu}, \tag{7}$$

$$e = \frac{\alpha sc}{\phi + \mu}, \tag{8}$$

$$c = \frac{\kappa\phi e + \gamma s}{\delta + \tau + \mu}, \tag{9}$$

$$j = \frac{\delta c}{\rho + \mu}, \tag{10}$$

$$r = \frac{\rho j + \tau c}{\omega + \theta + \mu}, \tag{11}$$

$$h = \frac{\theta r + (1-\kappa)\phi e + \eta s}{\mu}. \tag{12}$$

There are two fixed points: the corruption-free fixed point and the corruption-endemic fixed point.

3.1. Stability around of the corruption-free fixed points

The corruption-free fixed point, denoted by

$$\mathcal{E}^0 = (s^0, e^0, c^0, j^0, r^0, h^0),$$

describes a situation where the population is free from corrupt practices. At this fixed point, the relation $c^0 = 0$ is hold. Using this we have the free corruption fixed point

$$\mathcal{E}^0 = \left(\frac{\beta}{\gamma + \eta + \mu}, 0, 0, 0, 0, \frac{\eta\beta}{\mu(\gamma + \eta + \mu)}\right).$$

Using the Hartman's Theorem [12], the corruption-free fixed points $\mathcal{E}^0$ is locally asymptotically stable if the real part of all eigenvalues of the Jacobian matrix $\mathcal{J}$ at $\mathcal{E}^0$ (symbolized by $\mathcal{J}_{\mathcal{E}^0}$) is negative provided $\mathcal{E}^0$ is a hyperbolic fixed point. A simple calculation yields

$$\mathcal{J}_{\mathcal{E}^0} = \begin{bmatrix} -k_1 & 0 & -\frac{\alpha\beta}{\gamma+\eta+\mu} & 0 & \omega & 0 \\ 0 & -k_2 & \frac{\alpha\beta}{\gamma+\eta+\mu} & 0 & 0 & 0 \\ 0 & \kappa\phi & -k_3 & 0 & 0 & 0 \\ 0 & 0 & \delta & -k_4 & 0 & 0 \\ 0 & 0 & \tau & \rho & -k_5 & 0 \\ \eta & (1-\kappa)\phi & 0 & 0 & \theta & -\mu \end{bmatrix}, \tag{13}$$

with

$$k_1 = \gamma+\eta+\mu, \tag{14}$$
$$k_2 = \phi+\mu, \tag{15}$$
$$k_3 = \delta+\tau+\mu, \tag{16}$$
$$k_4 = \rho+\mu, \tag{17}$$
$$k_5 = \omega+\theta+\mu. \tag{18}$$

The characteristic polynomial of $\mathcal{J}_{\mathcal{E}^0}$ is given by

$$p(\lambda) = (-\mu-\lambda)(-k_1-\lambda)(-k_5-\lambda)(-k_4-\lambda)\Big[\lambda^2 + (k_2+k_3)\lambda + k_2k_3 - \frac{\kappa\phi\alpha\beta}{\gamma+\eta+\mu}\Big]. \tag{19}$$

The eigenvalues of the Jacobian matrix $\mathcal{J}_{\mathcal{E}^0}$ are the roots of $p(\lambda)$, namely, $\lambda_1 = -\mu, \lambda_2 = -k_1, \lambda_3 = -k_5, \lambda_4 = -k_4$, and the remaining two roots, let's say λ_5, λ_6, are the roots of the following polynomial

$$p_1(\lambda) = \lambda^2 + (k_2+k_3)\lambda + k_2k_3 - \frac{\kappa\phi\alpha\beta}{\gamma+\eta+\mu}. \tag{20}$$

It is obvious that $\lambda_i < 0$, for $i = 1,2,3,4$. Using Routh-Hurwitz criterion [13], the real part of λ_5, λ_6 is negative if $k_2 + k_3 > 0$ and $k_2k_3 - \frac{\kappa\phi\alpha\beta}{\gamma+\eta+\mu} > 0$. It is obvious that $k_2 + k_3 > 0$ due to (15) and (16). Moreover, $k_2k_3 - \frac{\kappa\phi\alpha\beta}{\gamma+\eta+\mu} > 0$ if and only if $\mathcal{R}_0 < 1$, where

$$\mathcal{R}_0 = \frac{\kappa\phi\alpha\beta}{(\gamma+\eta+\mu)(\phi+\mu)(\delta+\tau+\mu)} \tag{21}$$

is the basic reproduction number that can be determined using the next-generation matrix method [14]. This explanation leads to the conclusion that the corruption-free fixed point is locally asymptotically stable if $\mathcal{R}_0 < 1$, which is convenient to the epidemiology theory.

3.2. Stability around of the corruption-endemic fixed point

The corruption-endemic fixed point, denoted by

$$\mathcal{E}^* = (s^*, e^*, c^*, j^*, r^*, h^*),$$

describes a situation in which corrupt practices persist in the population. At this fixed point, the relation $c^* > 0$ is hold. By simplifying the eqution (7) - (12), we have the following corruption-endemic fixed point:

$$
\begin{aligned}
s^* &= \frac{\beta(\omega+\theta+\mu)(\rho+\mu)+\omega\left[\rho\delta+\tau(\rho+\mu)\right]\left(\frac{-Y\pm\sqrt{Y^2-4XZ}}{2X}\right)}{\left(\alpha\left(\frac{-Y\pm\sqrt{Y^2-4XZ}}{2X}\right)+\gamma+\eta+\mu\right)(\omega+\theta+\mu)(\rho+\mu)},\\
e^* &= \frac{U}{(\phi+\mu)(\alpha c+\gamma+\eta+\mu)(\omega+\theta+\mu)(\rho+\mu)},\\
c^* &= \frac{-Y\pm\sqrt{Y^2-4XZ}}{2X},\\
j^* &= \frac{\delta}{(\rho+\mu)}\left(\frac{-Y\pm\sqrt{Y^2-4XZ}}{2X}\right),\\
r^* &= \frac{\rho\delta+\tau(\rho+\mu)}{(\omega+\theta+\mu)(\rho+\mu)}\left(\frac{-Y\pm\sqrt{Y^2-4XZ}}{2X}\right),\\
h^* &= \frac{P+Q+T}{\mu},
\end{aligned}
\tag{22}
$$

where

$$
\begin{aligned}
X &= -\kappa\phi\alpha\omega\left[\rho\delta+\tau(\rho+\mu)\right]+\alpha(\delta+\tau+\mu)(\phi+\mu)(\omega+\theta+\mu)(\rho+\mu),\\
Y &= -\kappa\phi\alpha\beta(\omega+\theta+\mu)(\rho+\mu)-\gamma\omega(\phi+\mu)\left[\rho\delta+\tau(\rho+\mu)\right]\\
&\quad +(\delta+\tau+\mu)(\phi+\mu)(\gamma+\eta+\mu)(\omega+\theta+\mu)(\rho+\mu),\\
Z &= -\gamma\beta(\omega+\theta+\mu)(\rho+\mu)(\phi+\mu),
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
P =& \theta\frac{\rho\delta+\tau(\rho+\mu)}{(\omega+\theta+\mu)(\rho+\mu)}\left(\frac{-Y\pm\sqrt{Y^2-4XZ}}{2X}\right),\\
Q =& \frac{(1-\kappa)\phi U}{(\phi+\mu)(\alpha c+\gamma+\eta+\mu)(\omega+\theta+\mu)(\rho+\mu)}\\
T =& \eta\frac{\beta(\omega+\theta+\mu)(\rho+\mu)+\omega\left[\rho\delta+\tau(\rho+\mu)\right]\left(\frac{-Y\pm\sqrt{Y^2-4XZ}}{2X}\right)}{\left(\alpha\left(\frac{-Y\pm\sqrt{Y^2-4XZ}}{2X}\right)+\gamma+\eta+\mu\right)(\omega+\theta+\mu)(\rho+\mu)},\\
U =& \alpha\beta\left(\frac{-Y\pm\sqrt{Y^2-4XZ}}{2X}\right)(\omega+\theta+\mu)(\rho+\mu)\\
&+\alpha\omega\left(\frac{-Y\pm\sqrt{Y^2-4XZ}}{2X}\right)^2\left[\rho\delta+\tau(\rho+\mu)\right].
\end{aligned}
$$

Using the Hartman's Theorem again, the corruption-endemic fixed point is locally asymptotically stable if the real part of all eigenvalues of the Jacobian matrix $\mathcal{J}$ at $\mathcal{E}^*$ (symbolized by $\mathcal{J}_{\mathcal{E}^*}$) is negative. One can see that Z in the equation (23) is negative due to all of the parameters are positive. Moreover

$$
\begin{aligned}
X =& \alpha(\delta+\tau+\mu)(\phi+\mu)(\omega+\theta+\mu)(\rho+\mu)-\kappa\phi\alpha\omega\tau(\rho+\mu)-\kappa\phi\alpha\omega\rho\delta\\
=& \delta+\tau+\mu-\kappa\delta\frac{\phi}{(\phi+\mu)}\frac{\omega}{(\omega+\mu)}\frac{\rho}{(\rho+\mu)}-\kappa\tau\frac{\phi}{(\phi+\mu)}\frac{\omega}{(\rho+\mu)}\\
=& \delta\left[1-\kappa\frac{\phi}{(\phi+\mu)}\frac{\omega}{(\omega+\mu)}\frac{\rho}{(\rho+\mu)}\right]+\tau\left[1-\kappa\frac{\omega}{(\omega+\mu)}\right]+\mu\\
>& 0.
\end{aligned}
$$

Note that if $\mathcal{R}_0 > 1$ then $Y < 0$, due to

$$\begin{aligned} Y =& -\kappa\phi\alpha\beta(\omega+\theta+\mu)(\rho+\mu) - \gamma\omega(\phi+\mu)\left[\rho\delta+\tau(\rho+\mu)\right] \\ &+ (\delta+\tau+\mu)(\phi+\mu)(\gamma+\eta+\mu)(\omega+\theta+\mu)(\rho+\mu) < 0 \\ =& \left(\frac{\gamma+\eta+\mu}{\gamma+\eta+\mu} - \frac{\kappa\phi\alpha\beta}{(\phi+\mu)(\delta+\tau+\mu)(\gamma+\eta+\mu)}\right)(\omega+\theta+\mu)(\rho+\mu) \\ &- \frac{\gamma\omega(\rho\delta+\tau(\rho+\mu))}{(\delta+\tau+\mu)(\gamma+\eta+\mu)} < 0 \\ =& \left(1-\Re_0\right)(\omega+\theta+\mu)(\rho+\mu) - \frac{\gamma\omega(\rho\delta+\tau(\rho+\mu))}{(\delta+\tau+\mu)(\gamma+\eta+\mu)} \\ <& \ 0, \end{aligned} \tag{24}$$

thus one gets $c^* > 0$ if $\mathcal{R}_0 > 1$ and $\sqrt{Y^2 - 4XZ} < -Y$. Furthermore, the Jacobian matrix $\mathcal{J}_{\mathcal{E}^*}$ is given by

$$\mathcal{J}_{\mathcal{E}^*} = \begin{bmatrix} -k_6 & 0 & -\alpha s^* & 0 & \omega & 0 \\ \alpha c^* & -k_2 & \alpha s^* & 0 & 0 & 0 \\ \gamma & \kappa\phi & -k_3 & 0 & 0 & 0 \\ 0 & 0 & \delta & -k_4 & 0 & 0 \\ 0 & 0 & \tau & \rho & -k_5 & 0 \\ \eta & (1-\kappa)\phi & 0 & 0 & \theta & -\mu \end{bmatrix}, \tag{25}$$

where

$$k_6 = \alpha c^* + \gamma + \eta + \mu. \tag{26}$$

The characteristic polynomial of $\mathcal{J}_{\mathcal{E}^*}$ is given by

$$p_{\mathcal{E}^*}(x) = (-\mu - x)(x^5 + a_1x^4 + a_2x^3 + a_3x^2 + a_4x + a_5), \tag{27}$$

where

$$\begin{aligned} a_1 =& k_2 + k_3 + k_4 + k_5 + k_6, \\ a_2 =& k_5k_4 + (k_5+k_4)(k_6+k_2+k_3) + k_6k_2 + k_6k_3 + k_2k_3 + \alpha s^*\gamma, \\ a_3 =& -\omega\alpha c^*\tau\gamma + k_5k_4(k_6+k_2+k_3) + k_5k_4(k_6k_2 + k_6k_3 + k_2k_3 + \alpha s^*\gamma) \\ &+ k_6k_2k_3 + \kappa\phi\alpha s^* + \alpha^2 s^* c^* \kappa\phi + \alpha s^*\gamma k_2, \\ a_4 =& -\omega\alpha c^*\Big(\delta\rho\gamma + \tau k_4\gamma + \tau(\kappa\phi + \gamma k_2)\Big) + k_5k_4(k_6k_2 + k_6k_3 + k_2k_3 + \alpha s^*\gamma) \\ &+ (k_5+k_4)(k_6k_2k_3 + \kappa\phi\alpha s^* + \alpha^2 s^* c^*\kappa\phi + \alpha s^*\gamma k_2), \\ a_5 =& -\omega\alpha c^*(\delta\rho + \tau k_4)(\kappa\phi + \gamma k_2) + k_5k_4(k_6k_2k_3 + \kappa\phi\alpha s^* + \alpha^2 s^* c^*\kappa\phi + \alpha s^*\gamma k_2). \end{aligned} \tag{28}$$

The eigenvalues of the Jacobian matrix $\mathcal{J}_{\mathcal{E}^*}$ are the roots of $p_{\mathcal{E}^*}(x)$, namely, $x_1 = -\mu$, and the remaining five roots, let's say $x_i, i = 2, 3, 4, 5$, are the roots of the following polynomial

$$p_2(x) = x^5 + a_1x^4 + a_2x^3 + a_3x^2 + a_4x + a_5. \tag{29}$$

It is obvious that $x_1 < 0$. Using Routh-Hurwitz criterion [13], the real part of $x_i < 0$, for $i = 2, 3, 4, 5$ is negative if

(i). $a_1 > 0$,
(ii). $a_1a_2 - a_3 > 0$,
(iii). $a_3(a_1a_2 - a_3) - a_1^2a_4 + a_5a_1 > 0$,
(iv). $a_4\Big(a_3(a_1a_2 - a_3) - a_1^2a_4 + a_5a_1\Big) - a_5\Big(a_1a_2^2 + a_5 - a_1a_4 - a_2a_3\Big) > 0$,
(v). $a_5 > 0$.

This explanation leads to the conclusion that the corruption-endemic fixed point is locally asymptotically stable if $\mathcal{R}_0 > 1$ and the conditions (i), (ii), (iii), (iv), and (v), are held.

3.3. Numerical Simulation

In this section we are going to present results obtained from simulations based on equation (5). The numerical simulation is carried out with the help of MATLAB software which is simulated using the Runge Kutta method order-4 to see each sub population changes for cases $\gamma = 0$ and $\gamma > 0$. All of the initial conditions and parameter values are estimated. The used initial conditions are $s(0) = 0.5$, $e(0) = 0.2$, $c(0) = 0.2$, $j(0) = 0.1$, $r(0) = 0$, $h(0) = 0$. For the model (5), the estimated of parameter values are presented in Table 2.

Table 2. Reasonable values of the parameters

Parameter	β	α	ω	δ	τ	ρ	μ	ϕ	κ	η	θ
Value	0.02	0.85	0.0021	0.05	0.01	0.6	0.08	0.9	0.6	0.01	0.001

A simple calculation using the values of the parameter in Table 2 results the corruption-free fixed points:

(i). $\mathcal{E}^0 = (0.2222, 0, 0, 0, 0, 0.0278)$ where $\mathcal{R}_0 = 0.7434$ for $\gamma = 0$,
(ii). $\mathcal{E}^0 = (0.2198, 0, 0, 0, 0, 0.0275)$ where $\mathcal{R}_0 = 0.7353$ for $\gamma = 0.01$, and
(iii). $\mathcal{E}^0 = (0.2128, 0, 0, 0, 0, 0.0266)$ where $\mathcal{R}_0 = 0.7118$ for $\gamma = 0.04$.

Graphs of the susceptible subpopulation, the exposed subpopulation, the corrupt subpopulation, the jailed subpopulation, the reformed subpopulation, and the honest subpopulation for several values of the personal willingness γ are given in Figure 2. Based on Figure 2, one can see that when $\mathcal{R}_0 < 1$, the corrupt curves converge to zero, showing the population is free from corrupt practices.

Furthermore, replacing the value of β and μ in Table 2 with $\beta = 0.04$ and $\mu = 0.05$ results the corruption-endemic fixed point:

(i). $\mathcal{E}^* = (0.2277, 0.0279, 0.1377, 0.0107, 0.1456, 0.2496)$ and $\mathcal{R}_0 = 2.9282$ for $\gamma = 0$,
(ii). $\mathcal{E}^* = (0.2243, 0.0279, 0.1396, 0.0109, 0.1477, 0.2472)$ and $\mathcal{R}_0 = 2.8802$ for $\gamma = 0.01$, and
(iii). $\mathcal{E}^* = (0.2154, 0.0279, 0.1450, 0.0112, 0.1533, 0.2475)$ and $\mathcal{R}_0 = 2.7452$ for $\gamma = 0.04$.

Graphs of the susceptible subpopulation, the exposed subpopulation, the corruption subpopulation, the jailed subpopulation, the reformed subpopulation, and the honest subpopulation for several values of the personal willingness γ are given in Figure 3. Based on Figure 3, one can see that when $\mathcal{R}_0 > 1$, the corrupt curves converge to the corruption- endemic fixed point $\mathcal{E}^*$, showing the corrupt practices persist in the population. In this case, we also see that increasing the value γ can increase the number of corrupt practices in the population n the population.

4. Conclusion

A corruption epidemic compartment model that consist of the susceptible compartment, exposed compartment, corrupt compartment, jailed compartment, reformed compartment and honest compartment, has been established. Under the assumption that the corruption can spreads like an infectious disease, the local stability analysis of of the corruption-free fixed point and the corruption-endemic fixed point under influence of the personal willingnes has been established. The study shows that the corruption-free fixed point is locally asymptotically stable if $\mathcal{R}_0 < 1$, and unstable if $\mathcal{R}_0 > 1$. Otherwise, the corruption-endemic fixed point is locally asymptotically stable if $\mathcal{R}_0 > 1$, and unstable if $\mathcal{R}_0 < 1$. The numerical simulations demonstrate that increasing the value γ can increase the number of corrupt practices in the population. This means that the existence of the personal willingness parameter in the model can increase the number of corrupt practice.

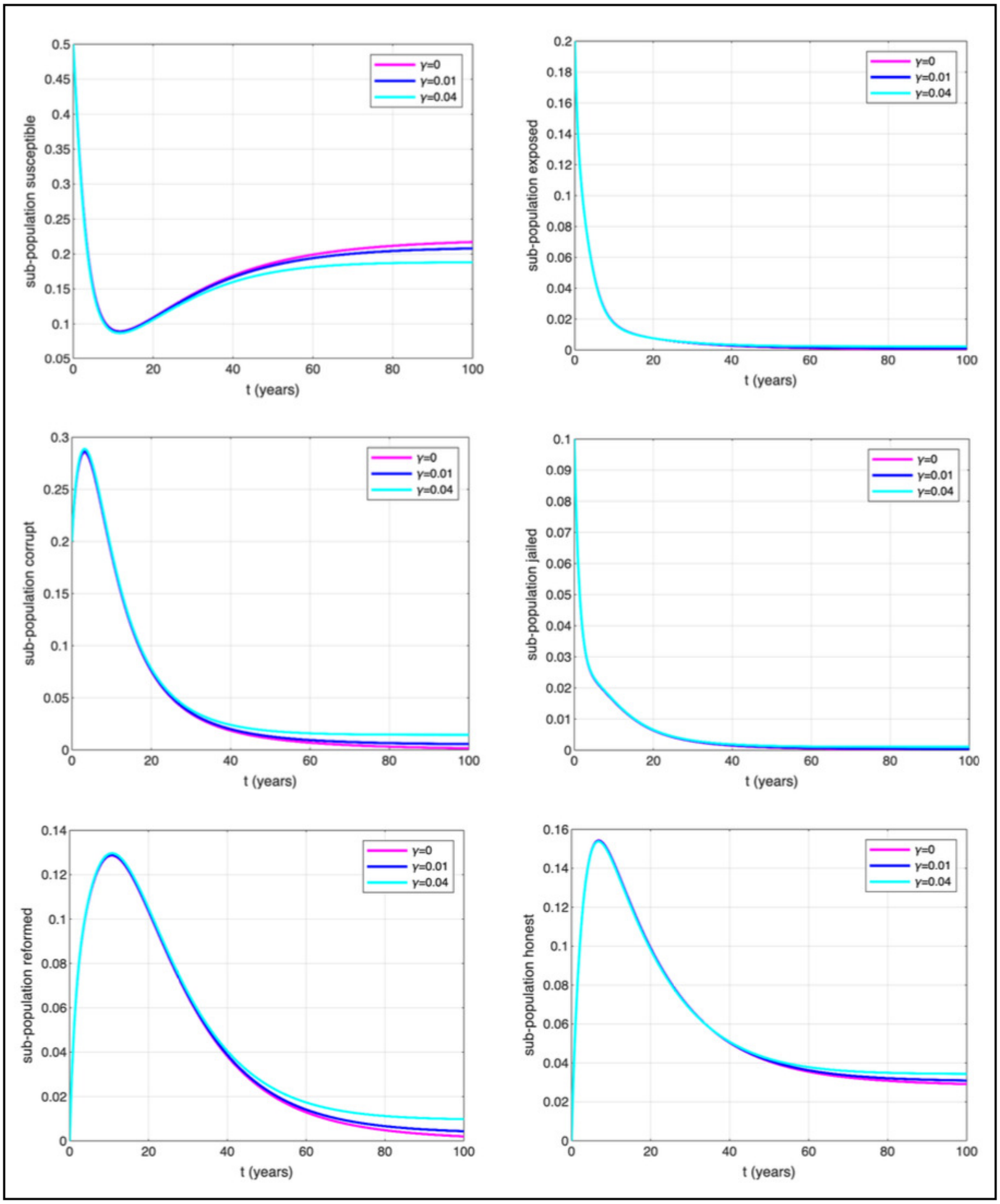

Figure 2. Curves of each subpopulation of the corruption-free cases for several values γ

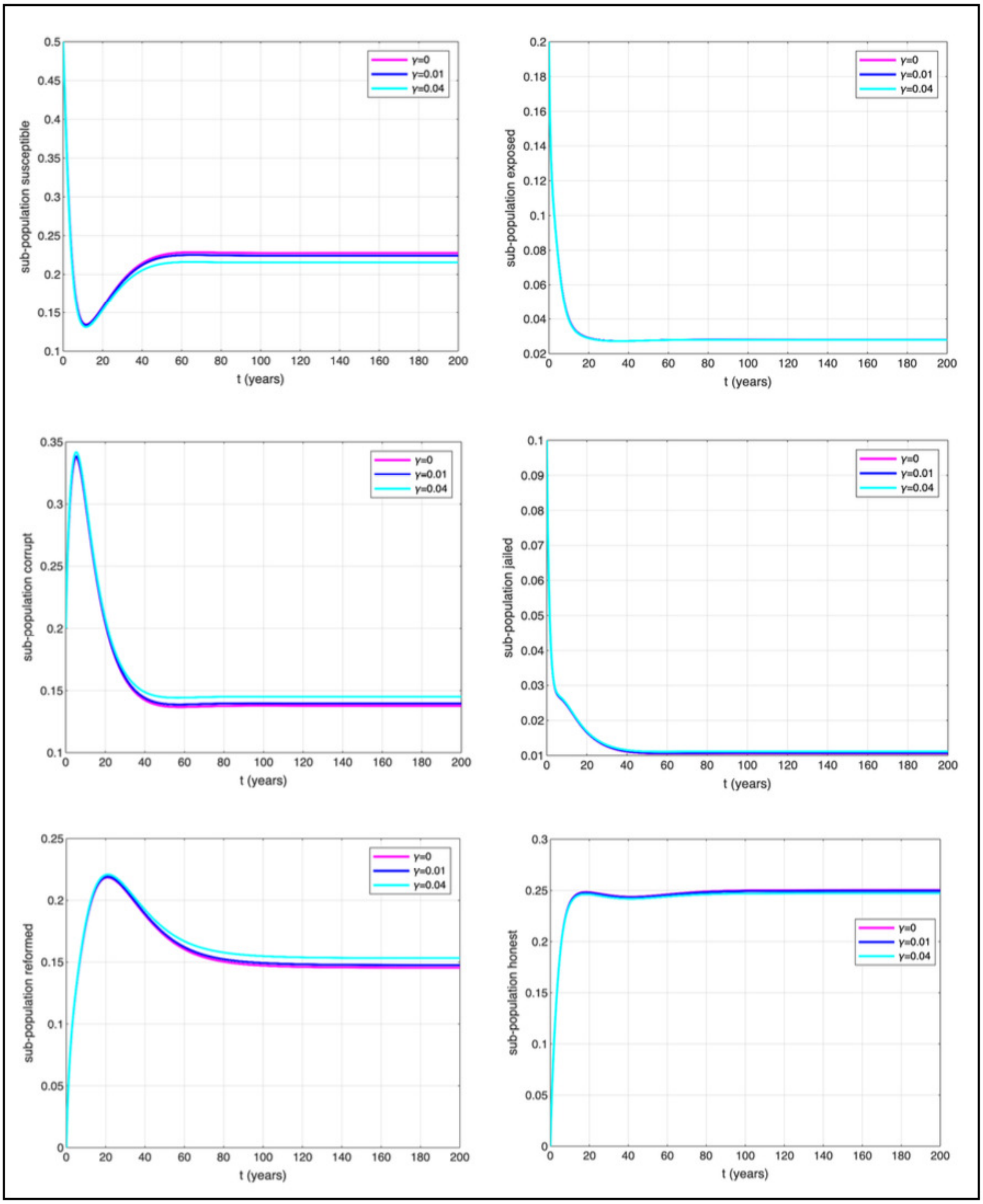

Figure 3. Curves of each subpopulation of the corruption-endemic cases for several values γ

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