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A Control Chart Approach to Monitor and Improve Production Processes: Maximum Exponentially Weighted Moving Average Using Auxiliary Variable (AV) and Multiple Measurement (ME)

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Abstract Control charts are fundamental tools in Statistical Process Control (SPC), employed to monitor and improve production processes. This study aims to evaluate the performance of the Maximum Exponentially Weighted Moving Average (Max-EWMA) chart by incorporating Auxiliary Variables (AV) and Multiple Measurements (ME). This study also evaluating the covariate method, a multiple measurement framework, and scenarios involving linearly increasing variance. The evaluation focuses on the analysis of both Type I and Type II error rates, using simulation-based methodologies. The results indicate that the Multiple Measurement approach consistently outperforms the Maximum EWMA ME (Covariate)-AV Control Chart, exhibiting lower Type II error rates and higher robustness in detecting process shifts. An increase in parameter A enhances the chart's sensitivity to mean shifts, whereas parameter B shows negligible influence. Additionally, Maximum EWMA ME (Linearly Increasing Variance)-AV Control Chart contributes to improved detection capability, particularly under conditions of high correlation. Overall, the Maximum EWMA ME (Multiple Measurement)-AV Control Chart demonstrates strong effectiveness and reliability across a variety of conditions, underscoring its practical utility in process control applications.

Keywords EWMA ME AV, Mean Shift Detection, Multiple Measurement Error, Multiple Measurement

AMS 2010 subject classifications 93C83, 93C62, 93C35

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1. Introduction

Statistical Process Control (SPC) is the application of statistical tools to monitor and improve process quality [1]. Nowadays, various controls are used not only in industrial production, but also in many other industries. Statistical quality control is an important tool widely used in the service delivery field to monitor the entire operation. Techniques such as Shewhart, CUSUM, and EWMA control charts have since proven effective in improving process performance [2]. The Exponentially Weighted Moving Average (EWMA), initially introduced by Robert in 1959 [3], gained further refinement in 1999 by Xie, allowing simultaneous monitoring of process mean and variability. This method surpasses the sensitivity of traditional Shewhart and CUSUM charts in detecting trends, leading to the development of Maximum EWMA (Max-EWMA), which provides sensitivity comparable to

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independent control methods [4]. Recently, Sanusi, Teh, and Khoo (2020) proposed the Maximum EWMA chart for simultaneous monitoring, expanding the utility of EWMA in modern process control [5].

EWMA-ME-AV represents an extension of the EWMA control chart, integrating auxiliary variables to address measurement errors in observations [6]. Despite advancements, measurement errors persist in quality characteristic assessments during control chart construction, potentially leading to misleading conclusions [7]. Assumptions of precise measurements across all variables need careful consideration, as measurement error denotes differences between actual and collected values, influencing control chart efficiency and decision-making accuracy. Effective control charting requires precise measurements to reflect true process behavior accurately. Maravelakis, Panaretos, and Psarakis (2004) utilized a covariate model to study the impact of measurement error, emphasizing its adverse effect on EWMA efficiency [8]. Their research highlighted the benefits of incorporating multiple measurements to enhance chart reliability [8]. Daryabari et al. (2017) further explored measurement error effects using a linear covariate error model, demonstrating its detrimental impact on joint monitoring techniques like Maximum EWMA and mean squared deviation charts [9]. Covariates, as independent variables associated with observed variables, are crucial in estimating parameters through regression methods, enhancing the robustness of statistical models [10, 11]. Several researchers, including Riaz (2008), Abbas, Riaz, and Does (2014), and others, leveraged auxiliary information to develop efficient control charts [12]. Auxiliary variables, strongly correlated with response variables, enhance measurement precision and improve control chart efficiency [13]. The strength of correlation, often quantified by the Pearson correlation coefficient, underscores the statistical dependence between variables [13]. Incorporating auxiliary information effectively explains quality characteristics, optimizing control chart performance across diverse applications [14, 6, 12, 13, 15, 16, 17, 18, 19].

The Maximum EWMA using ME-AV control chart integrates these insights, utilizing auxiliary information to mitigate measurement error impacts and enhance efficiency [6, 20]. This study investigates the Max-EWMA control chart by incorporating measurement error handling and the use of auxiliary information to enhance process monitoring. While similar studies have previously explored the development of such control charts, the novelty of this research lies in the evaluation method applied. Prior research has primarily assessed chart sensitivity using Average Run Length (ARL) simulations. In contrast, this study aims to evaluate the performance of the control chart in terms of Type I error (α) and Type II error (β), providing a more detailed statistical perspective on the chart's ability to accurately detect true process states. This approach offers new insights into the reliability and efficiency of control charts under measurement error conditions and auxiliary variable utilization.

2. Methods

2.1. Maximum EWMA using ME (Covariate)-AV

The observed value or the i-th measurement of a quality characteristic is denoted by Y_i. It is assumed that this quality characteristic has a true value (X_i) that follows a $\mathcal{N}(\mu_x, \sigma_x^2)$ distribution when the process is in control. The true value X is not known with certainty and is related to the observed value Y through the covariate relationship given by the equation Y = A + B X + ε. In this equation, A and B are constants, and ε is an error term that is a random variable following a $\mathcal{N}(0, \sigma_m^2)$ distribution. Using those linear combination model, the value Y follows a Normal distribution with mean $\mu_y = A + B\mu_x$ and variance $\sigma_y^2 = B^2\sigma^2 + \sigma_m^2$ [21]. Then, $M_{YW,j}^{(1)}$ and variance $V_j^{(1)}$ can be constructed as follows [22]:

$$M_{YW,j}^{(1)} = \bar{Y}_j + \rho \left(\frac{\sqrt{B^2\sigma^2 + \sigma_m^2}}{\sigma_W} \right) (\mu_W - \bar{W}_j) \tag{1}$$

$$V_j^{(1)} = \Phi^{-1} \left[H \left(\frac{(n-1)S_{Y,j}^2}{B^2\sigma^2 + \sigma_m^2}, n-1 \right) \right] - \rho^* \Phi^{-1} \left[H \left(\frac{(n-1)S_{W,j}^2}{\sigma_W^2}, n-1 \right) \right] \tag{2}$$

With expectation $\mathbb{E}(M^{(1)}_{YW,j}) = \mu_Y$ and variance $\text{Var}(M^{(1)}_{YW,j}) = \frac{1}{n}(B^2\sigma^2 + \sigma^2_m)(1-\rho^2_{YW})$ for the mean differentiation estimators. For the variance differentiation estimators, the expectation is $\mathbb{E}(V^{(1)}_j) = 0$ and the variance is $\text{Var}(V^{(1)}_j) = 1-(\rho^*)^2$, where:

$$\bar{Y}_j = \frac{1}{n}\sum_{i=1}^{n} y_{ij}, \qquad \bar{W}_j = \frac{1}{n}\sum_{i=1}^{n} W_{ij},$$

$$S^2_{Y,j} = \frac{1}{n-1}\sum_{i=1}^{n}(Y_{ij}-\bar{Y}_j)^2, \qquad S^2_{W,j} = \frac{1}{n-1}\sum_{i=1}^{n}(W_{ij}-\bar{W}_j)^2$$

The function $H(\xi, v) \sim \chi^2_v$, and $\Phi^{-1}(\cdot)$ is the inverse of the standard normal distribution function [22]. The transformation estimators for the mean (M_{je}) and variance (V_{je}) each follow the equations:

$$M^{(1)}_{je} = \frac{M_{YW,j} - (A + B\mu_x)}{\sqrt{\frac{1}{n}(B^2\sigma^2 + \sigma^2_m)(1-\rho^2_{YW})}} \tag{3}$$

$$V^{(1)}_{je} = \frac{V_j}{\sqrt{1-(\rho^*)^2}} \tag{4}$$

The EWMA for $P^{(1)}_i$ and variance $Q^{(1)}_i$ follow equations:

$$P^{(1)}_i = \theta M^{(1)}_{je} + (1-\theta)P_{i-1}, \quad i = 1, 2, \ldots \tag{5}$$

$$Q^{(1)}_i = \theta V^{(1)}_{je} + (1-\theta)Q_{i-1}, \quad i = 1, 2, \ldots \tag{6}$$

The Maximum EWMA-AV statistics follow the equation:

$$M_i = \max\left\{\left|P^{(1)}_i\right|, \left|Q^{(1)}_i\right|\right\} \tag{7}$$

The Maximum EWMA-AV statistic plotted follows Maximum EWMA statistic (M_i) is defined to have non-negative values, so that the lower control limit (LCL = 0) and the upper control limit (UCL) is defined in the following equation [4]:

$$\text{UCL} = 1.12839 + 0.602810 \cdot L \cdot \sqrt{\frac{\lambda}{2-\lambda}} \tag{8}$$

The derivation of the UCL value is obtained from the derivation of the expectation and variance of the Max EWMA statistic, which does not have a closed-form solution and therefore is resolved using a numerical computational approach (Monte Carlo simulations) [1, 4]. With λ is smoothing parameter $(0 \le \lambda \le 1)$ and determined weighted parameter L so that with combinations (λ, L) has $\text{ARL}_0 \approx 370$ [4].

2.2. *Maximum EWMA using ME (Multiple Measurement)-AV*

Based on the previous theory, handling measurement error can be done by taking repeated measurements k times per unit and utilizing the covariate model $Y = A + BX + \varepsilon$, with $X \sim \mathcal{N}(\mu_x, \sigma^2)$ and $\varepsilon \sim \mathcal{N}(0, \sigma^2_m)$. Therefore, the quality characteristic $\bar{Y} \sim \mathcal{N}\left(A + B\mu_x, \frac{B^2\sigma^2}{n} + \frac{\sigma^2_m}{nk}\right)$ [14]. With this information, the statistics $M^{(2)}_{YW,j}$ and $V^{(2)}_j$ can be constructed as follows [22]:

$$M^{(2)}_{YW,j} = \bar{Y}_j + \rho\left(\frac{\sqrt{(nk-1)\left(\frac{B^2\sigma^2}{n} + \frac{\sigma^2_m}{nk}\right)}}{\sigma_W}\right)\left(\mu_W - \bar{W}_j\right) \tag{9}$$

$$V_j^{(2)} = \Phi^{-1}\left[H\left(\frac{(nk-1)S_{Y,j}^2}{B^2\sigma^2+\sigma_m^2}, nk-1\right)\right] - \rho^*\Phi^{-1}\left[H\left(\frac{(n-1)S_{W,j}^2}{\sigma_W^2}, nk-1\right)\right] \tag{10}$$

The expected value of $M_{YW,j}^{(2)}$ is μ_Y, and the variance is $\left(\frac{B^2\sigma^2}{n}+\frac{\sigma_m^2}{nk}\right)(1-\rho_{YW}^2)$ for the mean differentiation estimators. For the variance differentiation estimators, the expectation is $E(V_j^{(2)})=0$ and the variance is $\text{Var}(V_j^{(2)}) = 1-(\rho^*)^2$.

Where:

$$\bar{Y}_j = \frac{1}{nk}\sum_{i=1}^{nk} y_{ij}, \quad \bar{W}_j = \frac{1}{n}\sum_{i=1}^{n} W_{ij}$$

$$S_{Y,j}^2 = \frac{1}{nk-1}\sum_{i=1}^{nk}(Y_{ij}-\bar{Y}_j)^2, \quad S_{W,j}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(W_{ij}-\bar{W}_j)^2$$

The function $H(\xi, v)$ follows a chi-squared distribution with v degrees of freedom, and $\Phi^{-1}(\cdot)$ is the inverse of the standard normal distribution function. The transformation estimators for $M_{je}^{(2)}$ and $V_{je}^{(2)}$ are given as:

$$M_{je}^{(2)} = \frac{M_{YW,j}^{(2)} - (A+B\mu_x)}{\sqrt{\left(\frac{B^2\sigma^2}{n}+\frac{\sigma_m^2}{nk}\right)(1-\rho_{YW}^2)}} \tag{11}$$

$$V_{je}^{(2)} = \frac{V_j^{(2)}}{\sqrt{1-(\rho^*)^2}} \tag{12}$$

The statistics for the Maximum EWMA-AV and the upper control limit (UCL) follow Equation (8) and Equation (9), respectively.

2.3. *Maximum EWMA ME (Lineary Increasing Variance)-AV*

In the two previous approaches, namely the Maximum EWMA ME (Covariate)-AV Control Chart and the Maximum EWMA ME (Multiple Measurement)-AV Control Chart, it is assumed that the error variance is constant, i.e., $\sigma_m^2(i) = \sigma_m^2$. In practice, however, it is rare to encounter error variances that are truly constant; instead, the error variance tends to exhibit a linear trend, referred to as the *Maximum EWMA ME (Linearly Increasing Variance)-AV Control Chart*. The observed value is denoted by Y, which has a true value X that is unknown and follows a Normal distribution with mean μ_x and variance σ_x^2. In the covariate relationship expressed by the equation $Y = A + BX + \varepsilon$, where A and B are constants, ε is a random variable that follows a Normal distribution with mean 0 and a variance that increases linearly, such that $\sigma_m^2 = C + D\mu_x$. Here, C represents the constant component of the linearly increasing variance model, and D represents the linear rate of change in error variance with respect to μ_x. Due to these relationships, the observed value $Y \sim \mathcal{N}$ with expected value $\mathbb{E}(Y) = \mu_y = A + B\mu_x$ and variance $\text{Var}(Y) = \sigma_y^2 = B^2\sigma^2 + C + D\mu_x$ [7]. Then, the mean and variance estimators $\left(M_{YW,j}^{(3)}\right)$ and $\left(V_j^{(3)}\right)$ can be constructed as follows [22]:

$$M_{YW,j}^{(3)} = \bar{Y}_j + \rho\left(\frac{\sqrt{B^2\sigma^2+C+D\mu}}{\sigma_W}\right)(\mu_W - \bar{W}_j) \tag{1}$$

$$V_j^{(3)} = \Phi^{-1}\left[H\left(\frac{(n-1)S_{Y,j}^2}{B^2\sigma^2+C+D\mu}, n-1\right)\right] - \rho^*\Phi^{-1}\left[H\left(\frac{(n-1)S_{W,j}^2}{\sigma_W^2}, n-1\right)\right] \tag{2}$$

With expectation and variance:

$$\mathbb{E}(M^{(3)}_{YW,j}) = \mu_Y, \quad \mathrm{Var}(M^{(3)}_{YW,j}) = \frac{1}{n}(B^2\sigma^2 + C + D\mu)(1-\rho^2_{YW})$$

$$\mathbb{E}(V^{(3)}_j) = 0, \quad \mathrm{Var}(V^{(3)}_j) = 1-(\rho^*)^2$$

where:

$$\bar{Y}_j = \frac{1}{n}\sum_{i=1}^{n} y_{ij}, \quad \bar{W}_j = \frac{1}{n}\sum_{i=1}^{n} W_{ij}$$

$$S^2_{Y,j} = \frac{1}{n-1}\sum_{i=1}^{n}(Y_{ij}-\bar{Y}_j)^2, \quad S^2_{W,j} = \frac{1}{n-1}\sum_{i=1}^{n}(W_{ij}-\bar{W}_j)^2$$

The function $H(\xi, v) \sim \chi^2_v$, and $\Phi^{-1}(\cdot)$ is the inverse of the standard normal distribution function [22]. The transformation estimators for $M^{(3)}_{je}$ and $V^{(3)}_{je}$ follow the equations:

$$M^{(3)}_{je} = \frac{M_{YW,j} - (A + B\mu_x)}{\sqrt{\frac{1}{n}(B^2\sigma^2 + C + D\mu)(1-\rho^2_{YW})}} \tag{3}$$

$$V^{(3)}_{je} = \frac{V_j}{\sqrt{1-(\rho^*)^2}} \tag{4}$$

Finally, the Maximum EWMA-AV statistic and the upper control limit (UCL) follow equations (8) and (9).

2.4. Data Assumption

Simulation data refers to data generated under the assumption that the distribution and parameters are known. The assumed distribution and generated parameters are presented in Table 1.

Table 1. Assumed distribution and generated parameters

Chart	**Assumption**
Max EWMA Cov	$X \sim \mathcal{N}(\mu_x, \sigma^2_x)$ $\varepsilon \sim \mathcal{N}(0, \sigma^2_m)$ $Y \sim \mathcal{N}(\mu_y = A + B\mu_x,\ \sigma^2_y = B^2\sigma^2_x + \sigma^2_m)$ $(Y, W) \sim \mathcal{N}_2(\mu_Y, \mu_W, \sigma^2_Y, \sigma^2_W, \rho_{YW})$
Max EWMA MM	$X \sim \mathcal{N}(\mu_x, \sigma^2_x)$ $\varepsilon \sim \mathcal{N}(0, \sigma^2_m)$ $\bar{Y} \sim \mathcal{N}\left(\mu_{\bar{Y}} = A + B\mu_x,\ \sigma^2_{\bar{Y}} = \frac{B^2\sigma^2_x}{n} + \frac{\sigma^2_m}{nk}\right)$ $(\bar{Y}, W) \sim \mathcal{N}_2(\mu_{\bar{Y}}, \mu_W, \sigma^2_{\bar{Y}}, \sigma^2_W, \rho_{\bar{Y}W})$
Max EWMA LIV	$X \sim \mathcal{N}(\mu_x, \sigma^2_x)$ $\varepsilon \sim \mathcal{N}(0, \sigma^2_m = C + D\mu_x)$ $Y \sim \mathcal{N}(\mu_y = A + B\mu_x,\ \sigma^2_y = B^2\sigma^2_x + C + D\mu_x)$ $(Y, W) \sim \mathcal{N}_2(\mu_Y, \mu_W, \sigma^2_Y, \sigma^2_W, \rho_{YW})$

Abbreviations: Max EWMA Cov = Maximum EWMA ME (Covariate)-AV; Max EWMA MM = Maximum EWMA ME (Multiple Measurement)-AV; Max EWMA LIV = Maximum EWMA ME (Linearly Increasing Variance)-AV. Based on Table 1, it is established that the required assumption is the assumption of a Normal distribution. A violation of the normality assumption will result in the constant value for the UCL of the Max-EWMA statistic being invalid. This occurs because the constant value is derived from the expectation and variance of the Max statistic, which in turn are obtained from the derivation of the statistics $M_{je} \sim \mathcal{N}(0,1)$ and $V_{je} \sim \mathcal{N}(0,1)$. The distribution assumptions and parameters in Table 1 will be generated using several combinations:

1. $X \sim \mathcal{N}(\mu_x = 3, \sigma_x^2 = 1.5)$
2. $\varepsilon \sim \mathcal{N}(\mu_m = 0, \sigma_m^2 = 0.5)$
3. $Y \sim \mathcal{N}(\mu_W = 4, \sigma_W^2 = 1)$
4. $\delta = 0, 0.1, 0.2, \ldots, 3$ (mean shifts)
5. $\theta = 0, 0.1, 0.2, \ldots, 3$ (variance shifts)
6. $A = 0, 1, 2, 3$
7. $B = 1, 2, 3$
8. $C = 0, 1$
9. $D = 1, 3, 5$

The selection of the range of process shifts in the mean (δ) and variance (θ) is based on the mean and variance values of the covariate variable (X) and the auxiliary variable (W), so as to cover shifts from small to large magnitudes [6].

2.5. Simulation Studies

Type I error (α) and Type II error (β) are critical indicators of a control chart's ability to accurately reflect the true condition of a process in the context of statistical hypothesis testing. A Type I error occurs when the chart incorrectly signals that the process is out of control when it is actually stable ($\delta = 0$; $\theta = 1$), while a Type II error happens when the chart fails to signal a true shift in the process ($\delta = \delta_i$; $\theta = \theta_i$). These errors provide a direct measure of control chart efficiency, as a reliable chart should minimize both false alarms (α) and missed detections (β). A smaller β (Type II error) indicates that the control chart is more effective in detecting actual shifts or changes in the process. This means the likelihood of failing to signal when the process is truly out of control is reduced. In other words, the control chart has higher detection power, making it more reliable in identifying deviations from the normal condition and ensuring better process quality monitoring. The Auxiliary Variables (AV) and Multiple Measurements (ME) are integrated into the Max-EWMA control chart in the flowchart shown in Figure 1.

Monte Carlo simulation is a computational technique that uses repeated random sampling to model and analyze complex systems or processes that are influenced by uncertainty. The simulation using the Type II error (β) through Monte Carlo simulations is carried out in the following steps:

1. Generate random data for $m = 1000$ subgroups with known parameters and a specified process shift.
2. Calculate the control chart statistics.
3. Determine the control limits.
4. Compute Type II error (β) as the proportion of observations falling within the control limits.
5. Repeat steps 1–4 for $r = 10{,}000$ replications.
6. Calculate the average Type II error (β) across all repetitions.
7. Repeat the above procedure for all parameter combinations.

3. Result and Discussion

In addressing measurement error, two approaches can be employed: the covariate model and multiple measurement. A covariate variable X is defined as the true value of the observed characteristic Y, which is not known with certainty. The additive error covariate model can be expressed as:

$$Y = A + BX + \varepsilon,$$

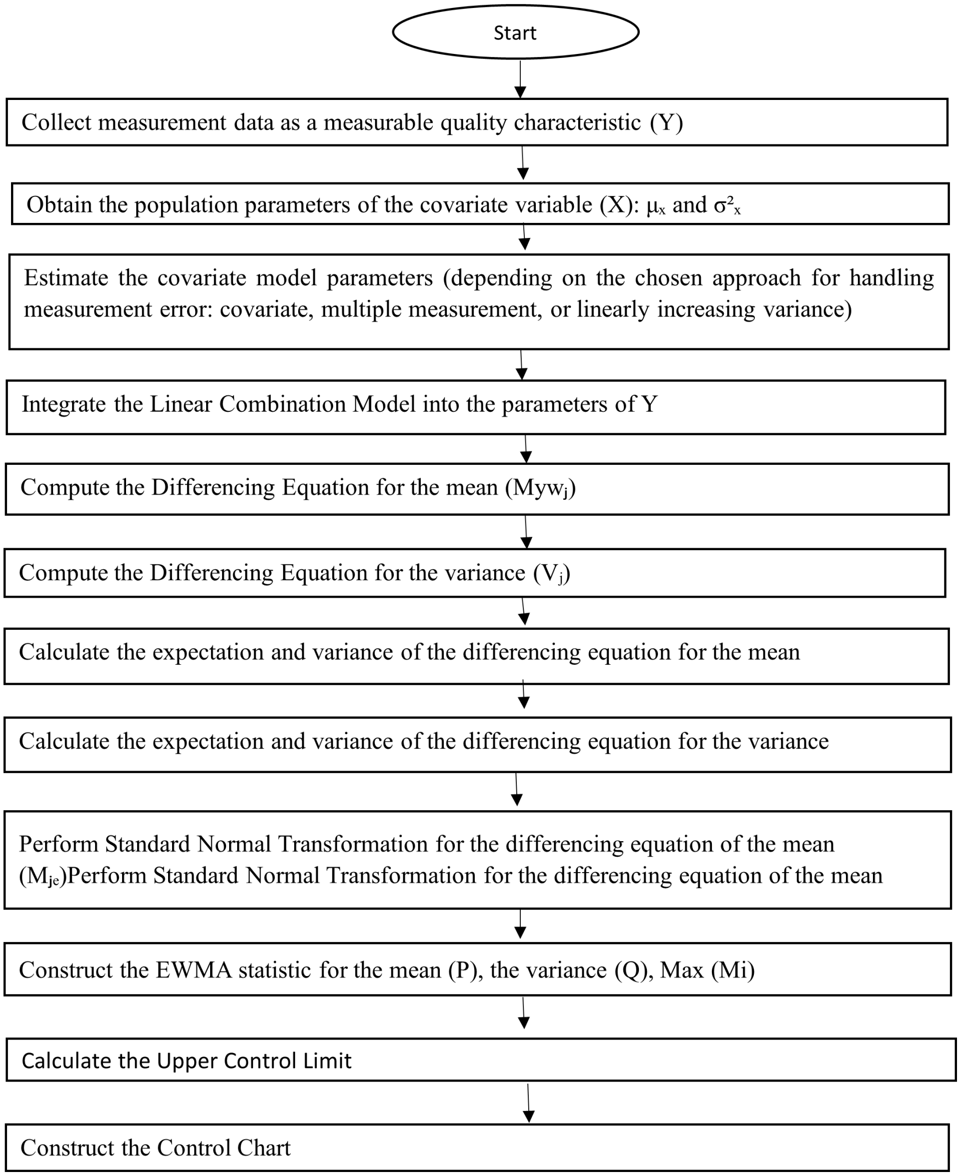


Figure 1. Flowchart integration of Auxiliary Variables (AV) and Multiple Measurements (ME) into Max-EWMA.

where A and B are constants and ε is a random variable following a normal distribution, denoted as $\varepsilon \sim \mathcal{N}(0, \sigma_m^2)$. Under in-control process conditions, the population characteristic X is assumed to be known, so $X \sim \mathcal{N}(\mu_x, \sigma_x^2)$. In relation to the covariate variable X, the variable Y follows a normal distribution with mean $\mu_y = A + B\mu_x$ and variance $\sigma_y^2 = B^2\sigma_x^2 + \sigma_m^2$. To mitigate the impact of measurement error, a repeated measurement approach can be taken, with k measurements. Therefore, the average of each measurement follows a normal distribution with mean: $\mu_{\bar{y}} = A + B\mu_x$ and variance: $\sigma_{\bar{y}}^2 = \frac{B^2\sigma_x^2}{n} + \frac{\sigma_m^2}{nk}$. An auxiliary variable is one that is correlated with the observed variable, and in process control, it is used as a consideration. Under in-control conditions, the characteristic Y and the auxiliary variable W are assumed to follow a bivariate normal distribution:

$$(Y, W) \sim \mathcal{N}_2(\mu_y, \mu_w, \sigma_y^2, \sigma_w^2, \rho).$$

If a process shift occurs in the mean by δ, the mean of the covariate variable X changes from μ_x to $\mu_x + \delta\sigma_x^2$ [21]. If a process shift occurs in the variance by θ, the variance of the covariate variable X changes from σ_x^2 to $\theta^2\sigma_x^2$ [15]. Consequently, if there is a shift in either the mean or the variance, Y follows the assumptions based on the mean and variance of the covariate variable X with the shift. Type I error and Type II error are crucial metrics that depict the control chart's effectiveness in accurately detecting the true state of the process within the framework of statistical hypothesis testing. For a known population, the values of α and β can be calculated by performing simulations, generating data for m subgroups, each consisting of n observations and k repetitions. Under in-control conditions, the population is characterized as:

$$X \sim \mathcal{N}(\mu_x = 3, \sigma_x^2 = 1.5), \quad \varepsilon \sim \mathcal{N}(0, \sigma_m^2 = 0.5), \quad (Y, W) \sim \mathcal{N}_2(\mu_y, \mu_w = 4, \sigma_y^2, \sigma_w^2 = 1, \rho).$$

Simulations are conducted for various parameter combinations:

- $A = 0, 1, 2, 3,$
- $B = 1, 2, 3,$
- $\rho = 0, 0.25, 0.5, 0.95,$
- $\delta = 0, 0.1, 0.25, \ldots, 3,$
- $\theta = 0.25, 0.5, 0.75, \ldots, 3.$

For the in-control process, the control chart parameters are set to $L = 2.709$ and $\lambda = 0.05$ to achieve an $ARL_0 = 370$. The simulation involves 10,000 repetitions, and the values of α and β are calculated for the known parameters, with the results presented in Figures 2–5. The values of Type I error (α) and Type II error (β) for various parameter combinations are used to assess the sensitivity of the control chart under each parameter set, specifically A, B, and ρ, in detecting process shifts in both mean and variance. In the covariate model $Y = A + BX + \varepsilon$, if $B \approx 1$, the covariate model simplifies to: $Y = A + X + \varepsilon$, allowing for the observation of the impact of the parameter A on the Maximum EWMA-AV and Maximum EWMA ME (Multiple Measurement)-AV control charts. To observe its effect on mean shift detection, variance changes are assumed insignificant. Given $\theta = 1$, the variance of the covariate variable X remains constant at σ_x^2. Figure 2 illustrates the Type II error (β) values for the Maximum EWMA-AV and Maximum EWMA ME-AV control charts, with parameter A taking values $0, 1, 2, 3$, for mean shifts (δ) ranging from 0 to 3, and different values of correlation (ρ) between the auxiliary variable W and observed variable Y, from low to high correlation in each graph.

In Figure 2, if there is no correlation between the auxiliary variable and Y, Type II error (β) as a function of the shift in means (δ) can be depicted as follows: As δ increases, the Type II error decreases for both methods, indicating improved detection of out-of-control conditions. The Maximum EWMA ME (Multiple Measurement)-AV Control Chart consistently shows lower Type II errors compared to the Maximum EWMA ME (Covariate)-AV Control Chart, signifying its superior effectiveness. Additionally, higher values of A significantly reduce the Type II error for the Maximum EWMA ME (Covariate)-AV Control Chart, indicating a strong influence of A. However, for the Maximum EWMA ME (Multiple Measurement)-AV Control Chart, the values of A do not have a significant impact, as many of the β values overlap, showing less sensitivity to changes in A.

In Figure 2, the Type II error (β) is illustrated as a function of the shift in means (δ) for various values of A between two methods when there is a small correlation between the auxiliary variable (W) and the observed variable (Y), that is $\rho = 0.25$. As δ increases, both methods exhibit a decrease in Type II error, indicating enhanced ability to detect out-of-control conditions. The Maximum EWMA ME (Multiple Measurement)-AV Control Chart demonstrates consistently lower Type II errors than the Maximum EWMA ME (Covariate)-AV Control Chart, underscoring its greater effectiveness. For the Maximum EWMA ME (Covariate)-AV Control Chart, increasing A values substantially lower the Type II error, highlighting a significant influence of A. In contrast, the Maximum EWMA ME (Multiple Measurement)-AV Control Chart shows minimal sensitivity to changes in A, as the β values for different A values largely coincide. This indicates that while the Maximum EWMA ME (Covariate)-AV Control Chart's performance improves with higher A values, the Maximum EWMA ME (Multiple Measurement)-AV Control Chart remains equally effective across different A values when $\rho = 0.25$. Consequently, it is observed that a small correlation value does not yield different results compared to a correlation value of 0.

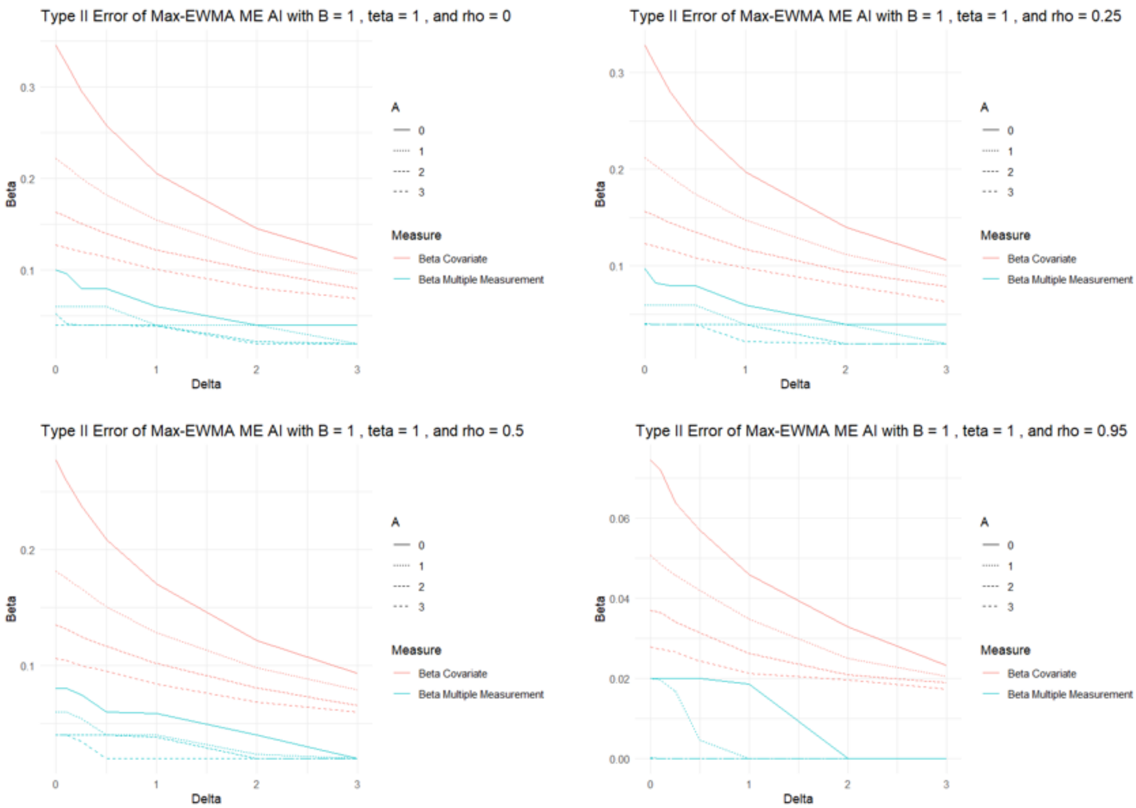


Figure 2. Type II Error of Maximum EWMA-AV and Maximum EWMA ME-AV with $B = 1$, $\theta = 1$, and different values of A.

In Figure 2.C, with $\rho = 0.5$, the overall Type II error decreases further, meaning the correlation between the auxiliary variable and Y enhances the detection capability of both methods, reducing the likelihood of false negatives. The Maximum EWMA ME (Multiple Measurement)-AV Control Chart consistently results in lower Type II errors compared to the Maximum EWMA ME (Covariate)-AV Control Chart, indicating its superior effectiveness. The Maximum EWMA ME (Covariate)-AV Control Chart shows a significant reduction in Type II error with increasing values of A, while the Maximum EWMA ME (Multiple Measurement)-AV Control Chart remains less sensitive to changes in A, as many β values overlap. In Figure 2.D, Type II error (β) is calculated with the correlation between the auxiliary variable (W) and the observed variable (Y) being high, with a value of 0.95. For the Maximum EWMA ME (Multiple Measurement)-AV Control Chart, it has successfully detected a shift in mean (δ) of 1 or greater perfectly, as indicated by the β values dropping to 0, but it creates false alarms when $\delta = 0$ for $A = 2$ and $A = 3$, where β is also 0. To avoid false alarms, it is recommended to keep A no higher than 1 when ρ is high. Overall, the Maximum EWMA ME (Covariate)-AV Control Chart is not as effective as the Maximum EWMA ME (Multiple Measurement)-AV Control Chart. To observe its effect on variances shift detection, mean changes are assumed to be insignificant. Given $\delta = 0$, the mean of the covariate variable X remains constant at μ_x.

Figure 2 demonstrates the Type II error (β) as a function of the shift in variances (θ) for various values of A (0, 1, 2, 3) using two methods: the Maximum EWMA ME (Covariate)-AV Control Chart (red lines) and the Maximum EWMA ME (Multiple Measurement)-AV Control Chart (blue lines), with different values of correlation (ρ) between the auxiliary variable (W) and the observed variable (Y) at $\rho = 0$, 0.25, 0.5, and 0.95. At low correlation ($\rho = 0$ and 0.25), shown in Figure 2.A and Figure 2.B, the results are similar for both correlations, but the Maximum EWMA ME (Multiple Measurement)-AV Control Chart consistently shows lower Type II errors (β), indicating better effectiveness in detecting variance shifts. Increasing A decreases Type II error in the Maximum EWMA ME (Covariate)-AV Control Chart, enhancing control chart sensitivity. However, at high correlation ($\rho = 0.95$), A values of 2 and 3 result in β values of 0 for $\theta = 0$, indicating false alarms when no variance shift occurs. This suggests that for high ρ values, large A values can lead to over-sensitive detection. In contrast, the Maximum EWMA ME (Multiple Measurement)-AV Control Chart shows less significant impact from A, with overlapping β values, indicating robustness and less sensitivity to changes in A compared to the Maximum EWMA ME (Covariate)-AV Control Chart. To observe its effect on mean shift detection, variance changes are

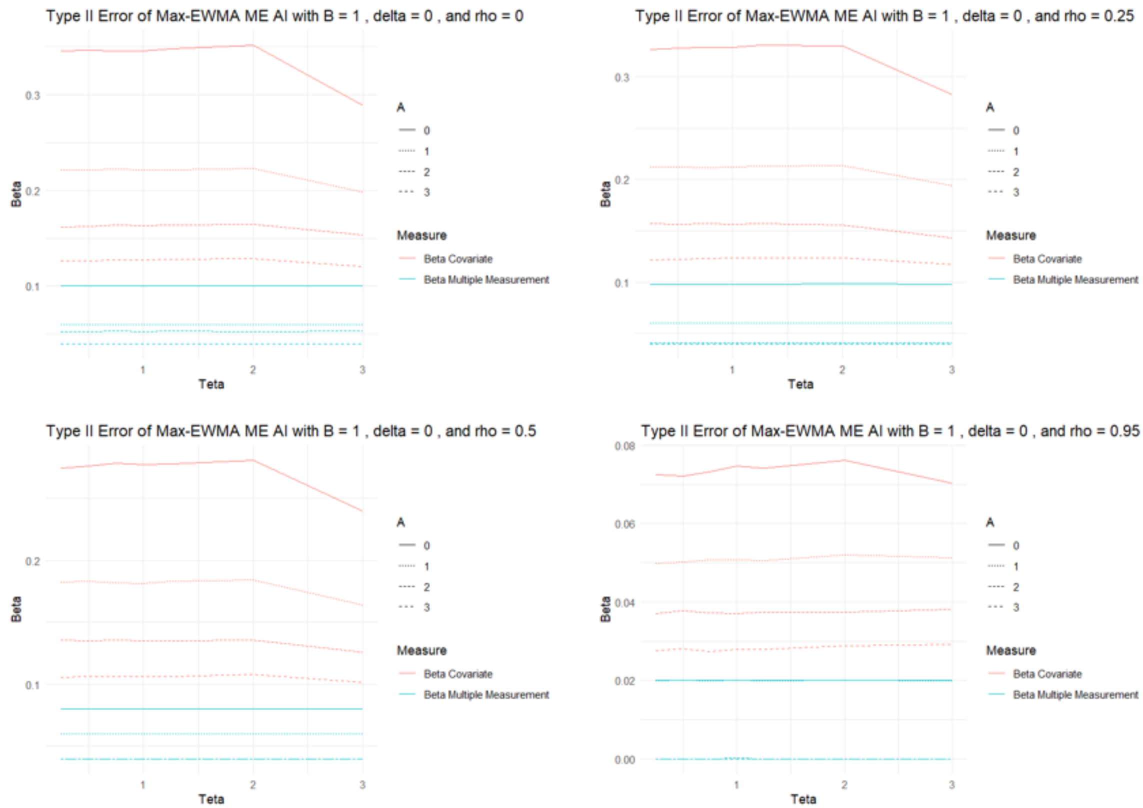


Figure 3. Type II Error of Maximum EWMA-AV and Maximum EWMA ME-AV with $B = 1$, $\delta = 0$, and different values of A.

assumed insignificant. Given $\theta = 1$, the variance of the covariate variable X remains constant at σ_x^2. To observe its effect on variance shift detection, mean changes are assumed to be insignificant. Given $\delta = 0$, the mean of the covariate variable X remains constant at μ_x.

Figure 3 demonstrates the Type II error (β) as a function of the shift in means (δ) for various values of B (i.e., $B = 1, 2, 3$) in terms of two methods comparison: Covariate (depicted by red lines) and Multiple Measurement (depicted by blue lines), with different values of correlation (ρ) between the auxiliary variable (W) and the observed variable (Y), ranging from low to high correlation in each graph. Furthermore, Figure 3 illustrates the Type II error (β) values for the Maximum EWMA AV and Maximum EWMA ME AV control charts, with parameter B taking values 1, 2, and 3, for variance shifts (θ) ranging from 0 to 3.

In Figure 4, the Maximum EWMA ME (Multiple Measurement)-AV Control Chart (blue lines) consistently shows lower Type II errors (β) compared to the Maximum EWMA ME (Covariate)-AV Control Chart (red lines) for the same values of δ and B, indicating that the Maximum EWMA ME (Multiple Measurement)-AV Control Chart is more effective in identifying out-of-control conditions. Additionally, for both methods and across all values of ρ, the differences in parameter B are not significant, as indicated by the overlapping lines for different B values. As the variance shift increases, the Type II error decreases for all schemes, which aligns with the theory that larger

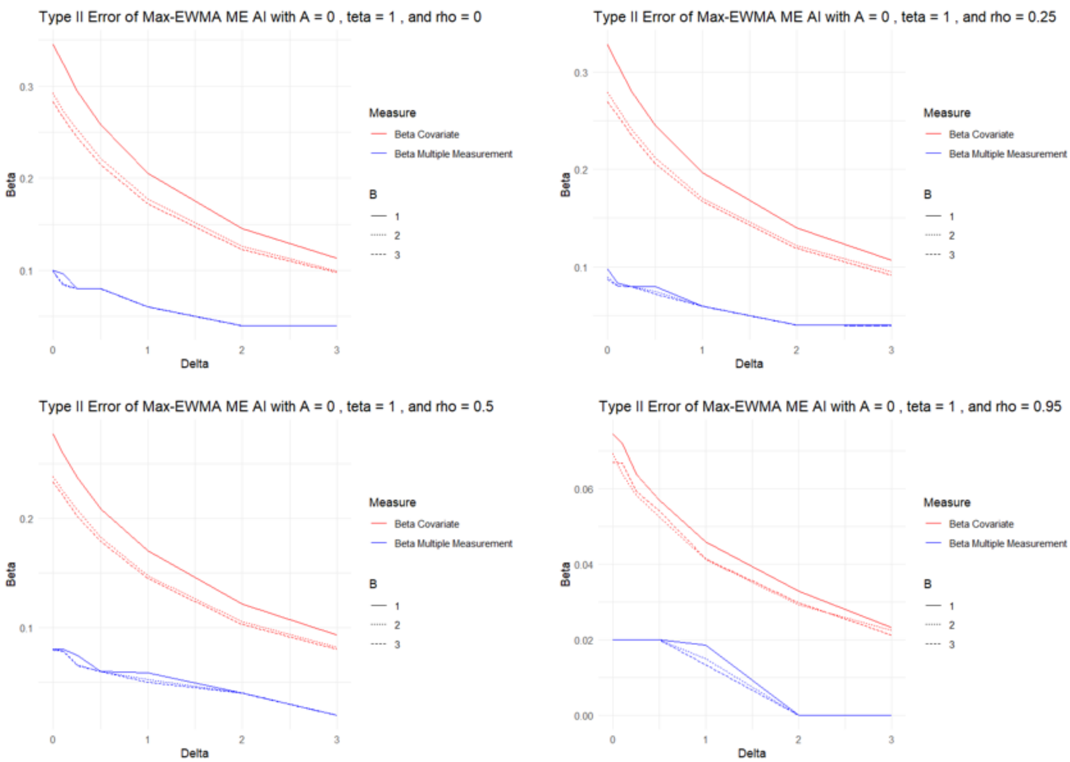


Figure 4. Type II Error of Maximum EWMA-AV and Maximum EWMA ME-AV with $A = 0$, $\theta = 1$, and different values of B.

shifts are easier to detect than smaller ones. However, in the Maximum EWMA ME (Multiple Measurement)-AV Control Chart, mean shifts (δ) greater than two are detected perfectly by the control chart, resulting in β values of 0, demonstrating the method's superior sensitivity and accuracy in detecting larger shifts.
Figure 5.A illustrates the Type II error (β) of the Maximum EWMA control chart for different methods (Beta Covariate and Beta Multiple Measurement) under the conditions $A = 0$, $\delta = 0$, and $\rho = 0$. The x-axis represents the shift in variance (θ), while the y-axis indicates the Type II error, which measures the probability of failing to detect an out-of-control condition in Phase II. The blue lines depict the performance of the Maximum EWMA ME (Multiple Measurement)-AV Control Chart, showing consistently low Type II errors across all values of θ and different values of B, indicating this method's robustness and reliability in detecting shifts in variance. In contrast, the red lines represent the Maximum EWMA ME (Covariate)-AV Control Chart, which shows higher Type II errors for smaller shifts ($\theta = 1$) and a decreasing trend as θ increases. This suggests that the Maximum EWMA ME (Covariate)-AV Control Chart becomes more effective with larger shifts but is initially less sensitive compared to the Maximum EWMA ME (Multiple Measurement)-AV Control Chart. Furthermore, the Type II error for the Maximum EWMA ME (Covariate)-AV Control Chart is affected by the value of B, with higher values of B leading to higher errors for smaller shifts but greater sensitivity to larger shifts. Overall, the Maximum EWMA ME (Multiple Measurement)-AV Control Chart is more reliable across all conditions, making it a preferable choice for maintaining process control.

Figures 5.B (with $\rho = 0.25$), 5.C (with $\rho = 0.5$), and Figure 5.D (with $\rho = 0.95$) show the Type II error (β) of the Maximum EWMA control chart for the β Covariate and β Maximum EWMA ME (Multiple Measurement)-AV Control Charts, with $A = 0$ and $\delta = 0$. Across those figures, there is no significant difference observed in the overall patterns. However, it is evident that the Type II error gradually decreases for all scenarios as θ increases, indicating an improvement in the detection capability for larger shifts in variance. The values of B do not show

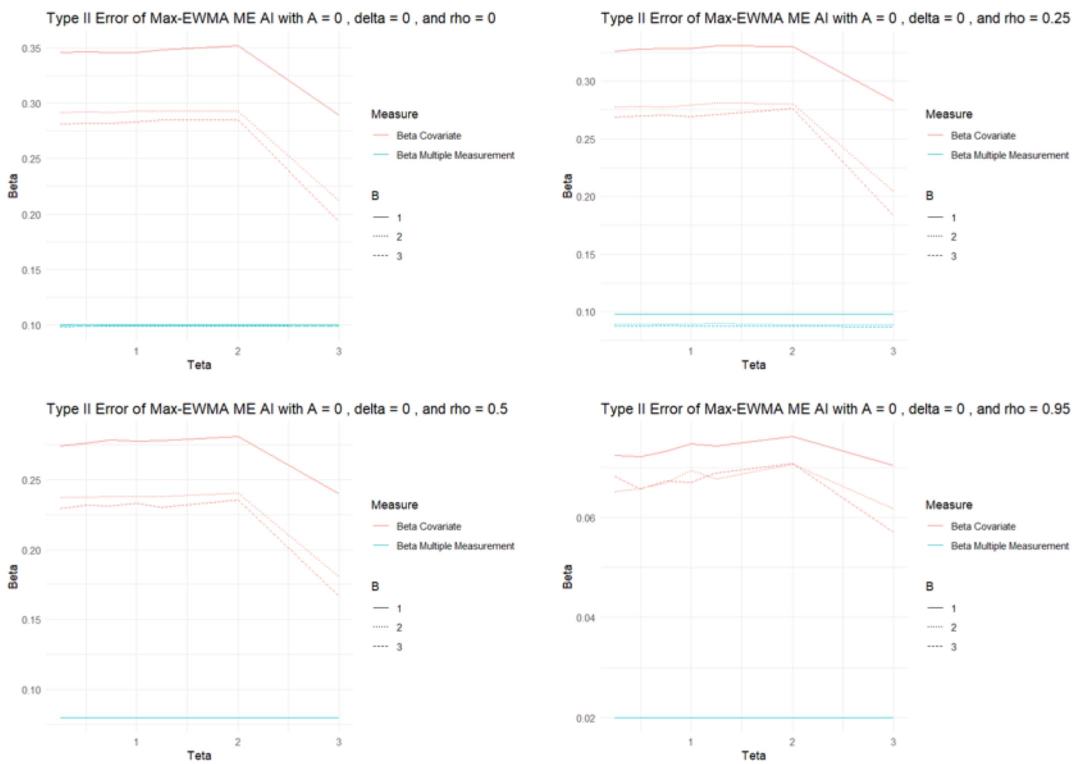


Figure 5. Type II Error of Maximum EWMA-AV and Maximum EWMA ME-AV with $A = 0$, $\theta = 1$, and different values of B.

a significant impact on the Type II error, as the lines for different B values overlap considerably. This suggests that the effect of B on the Type II error is minimal in these scenarios. Additionally, the β Maximum EWMA ME (Multiple Measurement)-AV Control Chart consistently maintains a lower Type II error compared to the β Maximum EWMA ME (Covariate)-AV Control Chart across all values of θ and ρ, highlighting its robustness and effectiveness in detecting shifts in variance. Overall, increasing ρ from 0.25 to 0.5 does not lead to substantial changes in the performance of either method, reinforcing the stability of the detection methods against varying correlations between the auxiliary variable and Y.

In real-world populations, it is often observed that the variance of errors is not constant. The error variance can vary linearly with the covariate variable X. Considering the covariate model $Y = A + BX + \varepsilon$, under in-control process conditions, X is normally distributed with mean μ_x and variance σ_x^2, denoted as $X \sim \mathcal{N}(\mu_x, \sigma_x^2)$. The error follows a normal distribution with mean 0 and variance $\sigma_m^2 = C + D\mu_x$, denoted as $\varepsilon \sim \mathcal{N}(0, C + D\mu_x)$. In relation to the covariate variable X, the variable Y follows a normal distribution with mean $\mu_y = A + B\mu_x$ and variance $\sigma_y^2 = B^2\sigma_x^2 + \sigma_m^2$. To mitigate the impact of measurement error, a repeated measurement approach can be employed, with k measurements. Consequently, the average of each measurement follows a normal distribution with mean $\bar{\mu}_y = A + B\mu_x$ and variance $\bar{\sigma}_y^2 = \frac{B^2\sigma_x^2}{n} + \frac{\sigma_m^2}{nk}$. Each σ_m^2 adheres to a linearly increasing variance rule. Additionally, the auxiliary variable W and the observed variable Y follow a bivariate normal distribution: $(Y, W) \sim \mathcal{N}_2(\mu_y, \mu_w, \sigma_y^2, \sigma_w^2, \rho)$. Type I error ($\alpha$) and Type II error ($\beta$) are crucial metrics that depict the control chart's effectiveness in accurately detecting the true state of the process within the framework of statistical hypothesis testing. For a known population, the values of α and β can be calculated by performing simulations. Under in-control conditions, the population is characterized as $X \sim \mathcal{N}(\mu_x = 3, \sigma_x^2 = 1.5)$, $\varepsilon \sim \mathcal{N}(0, \sigma_m^2)$, and $(Y, W) \sim \mathcal{N}_2(\mu_y, \mu_w = 4, \sigma_y^2, \sigma_w^2 = 1, \rho)$. The values of A, B, C, and D are simulated for several criteria.

Since this chapter focuses on examining the effect of linear changes in error variance, values of $A = 0$ and $B = 1$ are applied for $C = 0, 1$ and $D = 1, 3, 5$. If a process shift occurs in the mean by δ, the mean of the covariate variable X changes from μ_x to $\mu_x + \delta\sigma_x^2$. If a process shift occurs in the variance by θ, the variance of the covariate variable X changes from σ_x^2 to $\theta^2\sigma_x^2$. Consequently, if there is a shift in either the mean or the variance, Y follows the assumptions based on the shifted mean and variance of the covariate variable X. In simulations, various parameter combinations are tested: ρ values of 0, 0.25, 0.5, and 0.95; mean shifts δ ranging from 0, 0.1, 0.25 up to 3; and variance shifts θ ranging from 0.25, 0.5, 0.75 up to 3. For the in-control process, the control chart parameters are set to $L = 2.709$ and $\lambda = 0.05$ to achieve an ARL_0 of 370. The simulation involves 10,000 repetitions, and the α and β values are calculated for the known parameters, with the results presented in Figures 5–6. The values of Type I error (α) and Type II error (β) for various parameter combinations are used to assess the sensitivity of the control chart under each parameter set when the variance error of the population is linearly increasing, specifically parameters C, D, and ρ, in detecting process shifts in both mean and variance.

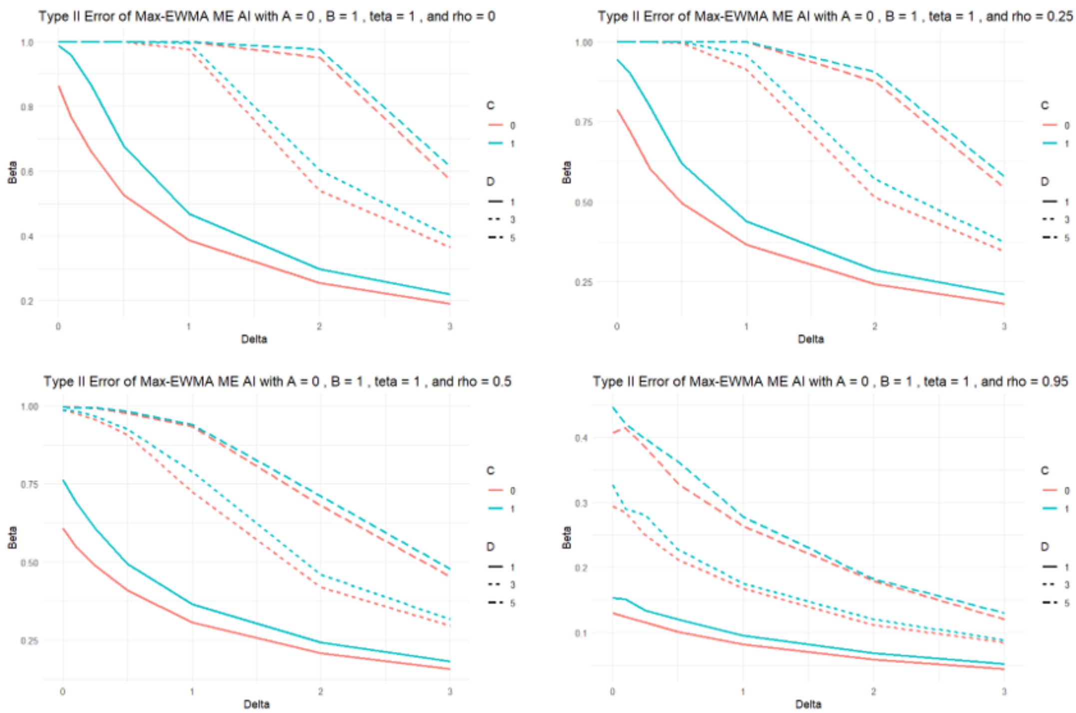


Figure 6. Type II Error of Maximum EWMA ME (Linearly Increasing Variance) with $A = 0$, $B = 1$, $\theta = 1$, and different values of C and D.

In the covariate model $Y = A + BX + \varepsilon$, if $A = 0$ and $B = 1$, the model simplifies to $Y = X + \varepsilon$, allowing for the observation of the impact of the random variable ε that has variance linearly increasing ($\sigma_m^2 = C + D\mu_x$) on the sensitivity of the Maximum EWMA AV and Maximum EWMA ME AV control charts. To observe its effect on mean shift detection, variance changes are assumed insignificant. Given $\theta = 1$, the variance of the covariate variable X remains constant at σ_x^2. Figure 5 illustrates the Type II error (β) values for the Maximum EWMA AV and Maximum EWMA ME AV control charts, with combinations of parameter C taking values 0 and 1, and parameter D taking values 1, 3, and 5 for mean shifts (δ) ranging from 0 to 3, with different values of correlation (ρ) between the auxiliary variable (W) and the observed variable (Y) from low correlation to high correlation in each graph. In Figure 5.A and Figure 5.B, which depict the Type II error (β) as a function of mean shift (δ) with a low correlation ($\rho = 0$ or 0.25) between the auxiliary variable and the observed variable, the red lines represent $C = 0$ and the blue lines represent $C = 1$. The different line patterns indicate various values of D (1, 3, 5). As δ

increases, β decreases for both values of C, indicating that larger mean shifts are easier to detect. This aligns with theoretical expectations. For $C = 1$ (blue lines), the Type II error is generally lower across all values of δ and D, suggesting that setting C to 1 improves the sensitivity of the control chart in detecting mean shifts.

In Figure 5.C and Figure 5.D, with relatively high correlation ($\rho = 0.5$ or 0.95), a similar trend is observed: β decreases with increasing δ for both $C = 0$ (red lines) and $C = 1$ (blue lines). However, the difference between $C = 0$ and $C = 1$ is more pronounced in this high-correlation scenario. For $C = 1$, the Type II error is consistently lower, especially for larger values of δ, indicating that the control chart is more effective in detecting mean shifts when C is set to 1. Additionally, for higher values of D, the lines tend to converge, suggesting that the impact of D diminishes with high correlation. Comparing both figures, the improvement in detection with $C = 1$ is evident in both low and high-correlation scenarios, but the advantage is more significant when the correlation is high ($\rho = 0.95$). This indicates that setting $C = 1$ enhances the control chart's performance, particularly in environments with strong correlations between auxiliary and observed variables.

To observe its effect on variances shift detection, mean changes are assumed to be insignificant. Given $\delta = 0$, the mean of the covariate variable X remains constant at μ_x. Figure 6 illustrates the Type II error (β) values for the Maximum EWMA ME AV and Maximum EWMA AV control charts, with combinations of parameter C taking values 0 and 1, and parameter D taking values 1, 2, and 3 for variance shifts (θ) ranging from 0 to 3. The graphs also incorporate different values of correlation (ρ) between the auxiliary variable (W) and the observed variable (Y), specifically $\rho = 0, 0.25, 0.5$, and 0.95 in each subplot. In the first two graphs in Figure 7, where $\rho = 0$ and $\rho = 0.25$

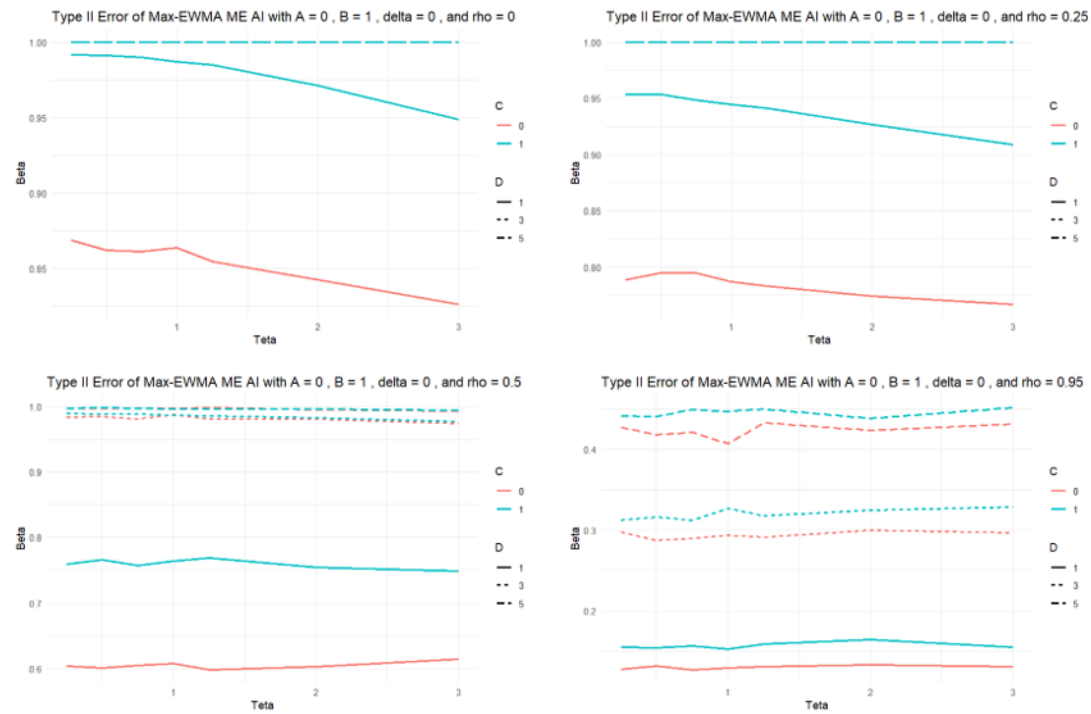


Figure 7. Type II Error of Maximum EWMA ME (Linearly Increasing Variance) with $A = 0$, $B = 1$, $\delta = 0$, and different values of C and D.

(low correlation), both the blue ($C = 1$) and red ($C = 0$) lines show a general decrease in β as θ increases. The blue line starts at a higher β compared to the red line, indicating that when $C = 1$, the method is less sensitive to variance shifts initially but becomes more sensitive as θ increases. The different patterns of the lines, representing different D values (1, 3, 5), show minor variations but follow the same trend.

In Figure 7.D, where $\rho = 0.95$ (high correlation), the Type II error values are overall lower than in the first figure, suggesting higher sensitivity to variance shifts. Here, the blue lines ($C = 1$) and red lines ($C = 0$) do not show as much separation as in the first figure, indicating that the high correlation makes the distinction between C values less pronounced. Overall, the figures demonstrate that higher correlation between the auxiliary and observed variables decreases the Type II error, enhancing sensitivity to variance shifts, with the parameter C having a more significant effect at lower correlation levels.

3.1. Impact Each Parameters

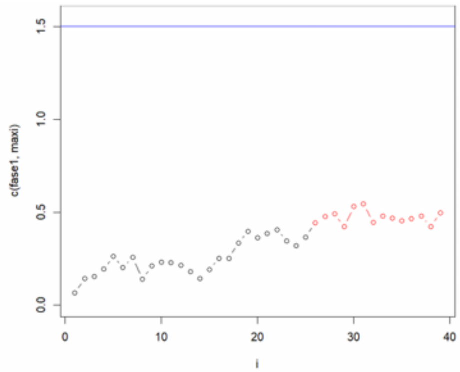
The impact of each parameter based on the simulation results on the Type II Error (β) value is presented in the Table 2.

Table 2. Impact of Each Parameter Based on Simulation Study Results

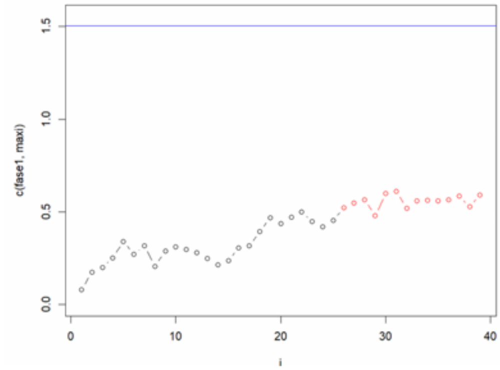
Control Chart	Parameter	Detection	Impact
Max EWMA ME (Covariate)-AV	A	Mean shift (δ)	Increasing A decreases Type II error (β) in the Covariate method for detecting mean shifts, enhancing control chart sensitivity (Figure 2).
	A	Variance shift (θ)	Increasing A decreases Type II error (β) in detecting variance shifts (Figure 3).
	B	Mean shift (δ)	Increasing B slightly decreases Type II error (β) in detecting mean shifts (Figure 4).
	B	Variance shift (θ)	Increasing B slightly decreases Type II error (β) in detecting variance shifts (Figure 5).
	ρ	Mean shift (δ)	Higher ρ reduces Type II error (β), improving detection of mean shifts (Figure 2).
	ρ	Variance shift (θ)	Higher ρ improves detection of variance shifts by reducing Type II error (β) (Figure 3).
Max EWMA ME (Multiple Measurement)-AV	A	Mean shift (δ)	Increasing A reduces Type II error (β) for detecting mean shifts (Figure 2).
	A	Variance shift (θ)	Increasing A reduces Type II error (β) for detecting variance shifts (Figure 3).
	B	Mean shift (δ)	Increasing B slightly reduces Type II error (β), but with limited sensitivity improvement (Figure 4).
	B	Variance shift (θ)	Similar slight decrease in Type II error (β) with increasing B (Figure 5).
	ρ	Mean shift (δ)	Higher ρ improves mean shift detection by reducing Type II error (β) (Figure 2).
	ρ	Variance shift (θ)	Higher ρ enhances detection of variance shifts (Figure 3).
Max EWMA ME (Linearly Increasing Variance)-AV	C	Mean shift (δ)	Decreasing C lowers Type II error (β), with moderate sensitivity improvement (Figure 6).
	C	Variance shift (θ)	Lower C improves detection of variance shifts slightly (Figure 7).
	D	Mean shift (δ)	Decreasing D reduces Type II error (β) and enhances detection sensitivity (Figure 6).
	D	Variance shift (θ)	Similar improvement observed for variance shifts with lower D (Figure 7).

3.2. Data Real Implementation

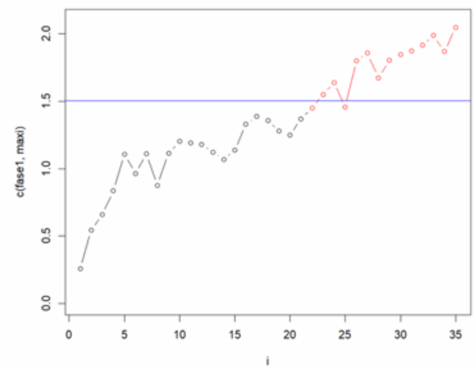
The results of real data implementation of the three approaches to the presence of measurement error in control charts for monitoring real process data shown in following the figure 8.



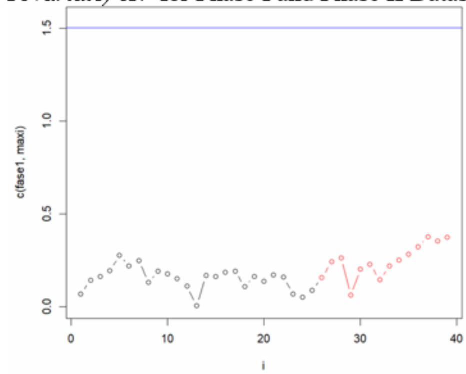
Implementation Control Chart Max-EWMA for Phase I and Phase II Dataset



Implementation Control Chart Maximum *EWMA ME (Covariate)-AV* for Phase I and Phase II Dataset



Implementation Control Chart Maximum *EWMA ME (Multiple Measurement)-AV* for Phase I and Phase II Dataset



Implementation Control Chart Maximum *EWMA ME (Linearly Increasing Variance)-AV* for Phase I and Phase II Dataset

Figure 8. Implementation of Control Chart.

The results of the sensitivity comparison of the three approaches to the presence of measurement error in control charts for monitoring real process data can be interpreted as follows.

1. Control charts without handling measurement error and without involving auxiliary variables exhibit lower sensitivity compared to the covariate and Maximum EWMA ME (Multiple Measurement)-AV Control Chart approaches, as indicated by the slower ability of the control chart to detect out-of-control (OOC) observations.
2. The control chart with the highest sensitivity is the Max-EWMA ME (Multiple Measurement) AV chart, which is capable of detecting shifts in both the process mean and variability simultaneously.
3. The Max-EWMA ME (Covariate) AV control chart is more sensitive than the standard Max-EWMA control chart, as demonstrated by the shift in the EWMA statistic that can indicate a monotonically increasing pattern of process changes.
4. When the non-constant error variance is considered as a controlled variable, the Max-EWMA ME (Linearly Increasing Variance) control chart can show that both the mean and variance of the production process are jointly stable and statistically in control.

4. Conclusion

Evaluate Performance of Control Chart Using Type I and Type II Errors in Control Charts: Type I error (α) and Type II error (β) are essential metrics for evaluating control chart effectiveness in accurately detecting the true state of a process within statistical hypothesis testing. By performing simulations with known populations, generating data for multiple subgroups and repetitions, these errors can be quantified to assess the performance of control charts. Under in-control conditions, the study used various parameter combinations (A, B, ρ, δ, and θ) to simulate different scenarios, revealing the sensitivity of control charts to shifts in mean and variance.

Impact of Parameter A on Mean Shift Detection: The study examined the effect of parameter A on the Maximum EWMA AV and Maximum EWMA ME AV control charts for different mean shifts (δ) and correlation values (ρ). Results indicated that higher values of A significantly reduced Type II error for the Covariate method, enhancing detection sensitivity. However, the Maximum EWMA ME (Multiple Measurement)-AV Control Chart method showed lower Type II errors across all values of A and ρ, demonstrating its superior effectiveness and robustness in detecting mean shifts.

Impact of Parameter A on Variance Shift Detection: When analyzing variance shifts (θ), the Covariate method's sensitivity improved with increasing values of A, particularly at low correlations (ρ). However, at high correlations ($\rho = 0.95$), large A values led to false alarms, indicating over-sensitivity. The Maximum EWMA ME (Multiple Measurement)-AV Control Chart exhibited consistently lower Type II errors and showed less sensitivity to changes in A, highlighting its robustness in variance shift detection across different correlations.

Impact of Parameter B on Mean Shift Detection: For mean shift detection, the study found that the Maximum EWMA ME (Multiple Measurement)-AV Control Chart consistently outperformed the Covariate method, with lower Type II errors across different values of B and ρ. The impact of B on detection sensitivity was minimal, as indicated by overlapping lines for different B values. This suggests that while both methods detect larger mean shifts more effectively, the Maximum EWMA ME (Multiple Measurement)-AV Control Chart remains more reliable and robust.

Impact of Parameter B on Variance Shift Detection: Similar to mean shift detection, the Maximum EWMA ME (Multiple Measurement)-AV Control Chart showed lower Type II errors compared to the Covariate method across various values of B and ρ for variance shift detection. The impact of B was negligible, with overlapping lines indicating minimal effect. This consistency across different scenarios reinforces the Maximum EWMA ME (Multiple Measurement)-AV Control Chart's reliability and robustness in detecting variance shifts.

Effect of Maximum EWMA ME (Linearly Increasing Variance)-AV Control Chart on Mean Shift Detection: The study investigated the impact of linearly increasing variance ($\sigma_m^2 = C + D\mu_x$) on mean shift detection using different combinations of parameters C and D. Results showed that setting C to 1 generally improved control chart sensitivity, particularly at high correlations (ρ). As δ increased, Type II error decreased, indicating enhanced detection capability for larger mean shifts. This trend was more pronounced at high correlations, highlighting the importance of considering linear variance changes in control chart design.

Effect of Linearly Increasing Variance on Variance Shift Detection: When examining variance shifts under linearly increasing variance conditions, the study found that higher correlations (ρ) improved detection sensitivity, reducing Type II errors. The impact of parameter C was more significant at lower correlations, with $C = 1$ showing higher initial Type II errors but improved sensitivity for larger variance shifts (θ). High correlation reduced the distinction between different C values, emphasizing the importance of correlation in control chart performance. Overall Effectiveness of Maximum EWMA ME (Multiple Measurement)-AV Control Chart: Across all scenarios and parameter combinations, the Maximum EWMA ME (Multiple Measurement)-AV Control Chart consistently demonstrated lower Type II errors compared to the Covariate method. This method's robustness and reliability in detecting both mean and variance shifts make it a preferable choice for maintaining process control. The study's findings highlight the superiority of the Maximum EWMA ME (Multiple Measurement)-AV Control Chart in various conditions, providing valuable insights for improving control chart design.

The study evaluated the performance of Maximum EWMA using auxiliary variable and covariate control charts using Type I and Type II errors, analyzing various parameter combinations for mean and variance shifts. Findings showed that the Maximum EWMA ME (Multiple Measurement)-AV Control Chart consistently outperformed the Covariate method, demonstrating lower Type II errors and greater robustness in detecting shifts. Higher values of parameter A improved mean shift detection sensitivity, while parameter B had minimal impact. Maximum EWMA ME (Linearly Increasing Variance)-AV Control Chart enhanced detection capability, particularly at higher correlations. Overall, the Maximum EWMA ME (Multiple Measurement)-AV Control Chart proved effective across diverse conditions, emphasizing its reliability in process control applications.

Limitations and Suggestions

The control chart in this study was developed under the limitation of assuming a Normal distribution and monitoring a univariate quality characteristic. It is recommended that future research focus on developing robust control charts that can accommodate distributions other than Normal and are capable of monitoring more than one observed quality characteristic (multivariate monitoring).

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