STATISTICS, OPTIMIZATION AND INFORMATION COMPUTING
Stat., Optim. Inf. Comput., Vol. **14**, November 2025, pp 2228–2254.
Published online in International Academic Press (www.IAPress.org)

A Generalized Approach to Time-Fuzzy Soft Expert Sets for Decision-Making

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Abstract As an extension of the fuzzy soft set, Ayman A. Hazaymeh presented the idea of the time-fuzzy soft set in his doctoral thesis in 2013. By introducing the Generalized Time-Fuzzy Soft Expert Set (GT-FSES) as an additional extension of the fuzzy soft set, we expand on this concept in this paper. We examine the most important functions of this new architecture, such as intersection, complement, union, and the logical operations "AND" and "OR." Additionally, we show how GT-FSES may be used practically to solve decision-making issues, providing a fresh method for managing complexity and ambiguity in decision-making processes.

Keywords Soft set, Fuzzy soft set, Generalized fuzzy , Time-fuzzy soft set

AMS 2010 subject classifications 03E72, 03E75, 90B50

DOI: 10.19139/soic-2310-5070-2666

1. General Introduction

The majority of problems in engineering, medical research, economics, and the environment are fraught with uncertainty. Molodtsov [2] introduced the notion of soft set theory as a tool in math for coping with such uncertainty. Following Molodtsov's work, [3], Maji et al. [4] and Maji et al.[5] researched several soft set operations and applications. Also Maji et al. [6] they presented the notion of fuzzy soft set as a more broad concept, as well as a combination of fuzzy set and soft set, and investigated its features. Roy and Maji [7] also applied this idea to handle decision-making challenges. [8] introduced generalized fuzzy soft sets and studied some of their properties. Application of generalized fuzzy soft sets in decision making problem and medical diagnosis problem is introduced by them also. Furthermore, in 2010 Çağman et al. [9], [10] proposed the notion of soft expert sets and fuzzy soft expert sets, which allow users to get the views of all experts in one model without any procedures. A strong mathematical tool, soft set theory has many uses in optimization, decision-making, and other domains. Over time, the idea has undergone significant changes that have improved its application across a range of fields. [13] expanded on this by introducing *Time Fuzzy Soft Sets* and investigating their efficacy in decision-making, providing a fresh viewpoint for developing systems that take time-related uncertainty into account. In order to further expand the versatility of fuzzy sets in dynamic environments, [12] introduced the concept of the *Time-Shadow Soft Set*, outlining its essential ideas and illustrating its practical applications. [15] presented the concept of *Time Effective Fuzzy Soft Sets*, discussing their applications with and without neutrosophic elements. To further improve the capacity to model and evaluate complex systems with time-varying parameters, [16] suggested *Time Fuzzy Parameterized Fuzzy Soft Expert Sets*. By include time-related factors—which are essential for simulating real-world systems that change over time—these works jointly advance the expanding subject of fuzzy soft set

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ISSN 2310-5070 (online) ISSN 2311-004X (print)

theory. Additionally, [14] investigated how the *Time Factor* affected *Fuzzy Soft Expert Sets*, demonstrating how temporal factors affect expert system performance and dependability. In a similar field.

Expert systems and fuzzy soft sets are now effective methods for handling difficult decision-making issues, especially in dynamic and unpredictable contexts. These ideas have been developed by several scholars who have broadened and improved the theory to better fit practical uses. For example, the notion of *Fuzzy Soft Set and Fuzzy Soft Expert Set: Some Generalizations and Hypothetical Applications* was introduced by [18], who provided insightful information about how these sets might be expanded for more intricate problem-solving situations. Expanding upon this basis, [17] presented the idea of *N-valued refined neutrosophic soft sets* and investigated their uses in medical diagnosis and decision-making issues, showcasing the effectiveness of neutrosophic soft sets in managing uncertainty in dynamic and imprecise contexts. Additionally, by offering a more comprehensive framework for their use, [19] introduced the *Generalized Fuzzy Soft Expert Set*, which further improves the adaptability and flexibility of fuzzy soft sets in expert systems. It expands on the fuzzy soft expert set concept by using parameterized structures to model more complex scenarios and systems, especially when parameter variation is crucial to decision-making. Together, these advances enhance the theory of fuzzy soft sets and enable more reliable and useful applications in a variety of fields, especially those that call for the modeling of imprecision and uncertainty.

New avenues for nonlinear analytic study have been made possible by the interaction between uncertainty-based mathematical frameworks and fixed point theory. The combination of fixed point theory, fuzzy set theory, and neutrosophic logic is particularly important since it offers strong tools for dealing with ambiguous and imprecise data in metric spaces. These advancements have recently spread to other generalized metric spaces, such as fuzzy and neutrosophic metric spaces [24, 25]. These hybrid frameworks offer a strong foundation for examining contraction mappings and proving the presence and uniqueness of fixed points in uncertain situations. The introduction of simulation functions [23], auxiliary functions [27], and various contraction types, including Geraghty-type [26] and quasi-contractions [28], in these settings has led to a number of improvements. Neutrosophic ψ-quasi contractions [29] and NF-L contractions [21] are two examples of new methods for dealing with incomplete information that resulted from the combination of fuzzy set theory with fixed point findings. Furthermore, the study of T-distance spaces [22] and ωt-distance mappings [26] has improved our comprehension of fixed point theorems in non-classical metric contexts.

Fuzzy logic principles may be used to improve classical fixed point theory, as recent advancements have shown, especially with integral contraction methods [25] and extended metric space approaches [27]. With a focus on the interaction between fuzzy sets, neutrosophic logic, and classical fixed point theory, this study adds to this expanding body of work by examining novel fixed point findings under different contraction conditions in generalized metric spaces. The results provide new tools for applications in computational mathematics and nonlinear analysis, where uncertainty is crucial, while also extending and unifying a number of current theorems. To further enhance decision-making models under uncertainty, Hazaymeh and Bataihah [38] introduced a fuzzy soft expert set framework capable of incorporating multiple expert opinions, offering a more flexible structure for handling real-world problems.

Recent developments in neutrosophic metric spaces [35, 36], and demonstrate how fuzzy set theory and neutrosophic logic can work in concert with fixed point theory, numerical analysis [30], operator theory [31], topological algebra [33, 34], fractional calculus [37], and computational mathematics [43] to create novel frameworks for solving complex problems in nonlinear analysis under uncertainty. On the algebraic side, complex hesitant fuzzy graphs [20] describe interdependent expert judgments, and structures such as (γ, ϑ)-fuzzy HX-subgroups [32] allow hierarchical splitting of expert sets. The application of decision-making algorithms has evolved significantly with the introduction of advanced mathematical frameworks. Fixed point theory has long served as a foundational tool in various branches of mathematical analysis and its applications. In particular, the development of generalized contraction mappings using Ω-distance and simulation functions has provided deeper insights into the existence and uniqueness of fixed points [46, 39, 40, 41]. These contributions are instrumental when studying fuzzy and soft set-based frameworks, where uncertainty and approximate reasoning play central roles. When combined with fuzzy soft set theory, such fixed point results offer a powerful mechanism for establishing solution stability in complex decision-making systems. The recent work by Al-Qudah and Al-Sharqi

[42], which introduces a decision-making algorithm based on similarity measures within possibility interval-valued neutrosophic soft settings, exemplifies this synergy between fixed point approaches and soft computing paradigms.

The fixed-point results in [45], along with the generalized ω-distance mappings in [47], provide rigorous tools for analyzing imprecise systems. These frameworks extend naturally to fuzzy metric spaces, where they can model uncertain decision-making processes. Such theorems enable the formal verification of convergence in fuzzy-logic-based control systems or multi-criteria optimization under vagueness.

In this paper, we will present the notion of the effect of the generalized time fuzzy soft expert sets, which is more effective and valuable, as we will see and the decisions made will be more precise, this means we will take the component time value of the information in our consideration when we are making decision. We will also define and investigate the attributes of its basic operations, which are complement, union and intersection. Finally, we'll apply this approach to decision-making difficulties.

2. Preliminary

In this part, we cover several fundamental concepts in soft set theory. Molodtsov [2] defined soft sets over U as follows: Let U be a universe set and E set of parameters, $P(U)$ denotes the power set of U and $A \subseteq E$.

Definition 2.1. [2] consider this mapping

$$F : A \to P(U).$$

Any A pair (F, A) is considered a *soft set* over U. In other terms, a soft set over U is a parameterized collection of subsets of the universe set U. For $\varepsilon \in A, F(\varepsilon)$ can be viewed as the set of ε-approximate members of the soft set (F, A).

Definition 2.2. [6] Let U be the initial universal set, and E be the set of parameters. Let I^U be the power set of all fuzzy subsets of U. Let $A \subseteq E$, and F be the mapping

$$F : A \to I^U.$$

A pair (F, E) is known as a *fuzzy soft set* over U.

Definition 2.3. [6] Regarding two fuzzy soft sets (F, A) and (G, B) over U, (F, A) is known as a fuzzy soft subset of. (G, B) if

1. $A \subset B$ and
2. $\forall \varepsilon \in A, F(\varepsilon)$ is fuzzy subset of $G(\varepsilon)$.

The relationship is represented by $(F,\ A) \tilde{\subset} (G, B)$. In this situation, (G, B) is known as a fuzzy, soft superset of. (F, A).

Definition 2.4. [6] The complement of a fuzzy soft set (F, A) is denoted by $(F, A)^c$ And has been defined by $(F, A)^c = (F^c, \rceil A)$ where $F^c : \rceil A \to P(U)$ is a mapping provided by

$$F^c(\alpha) = c(F(\rceil \alpha)), \forall \alpha \in \rceil A.$$

c describes any fuzzy complement.

Definition 2.5. [6] If (F, A) and (G, B) are two fuzzy soft sets then "(F, A) AND (G, B)" denoted by $(F,\ A) \wedge (G, B)$ is defined by

$$(F, A) \wedge (G, B) = (H, A \times B)$$

such that $H(\alpha, \beta) = t(F(\alpha), G(\beta)), \forall (\alpha, \beta) \in A \times B$, where t is any t-norm.

Definition 2.6. [6] If (F, A) and (G, B) are two fuzzy soft sets then "(F, A) OR (G, B)" denoted by $(F, A) \vee (G, B)$ is defined by

$$(F,A) \vee (G,B) = (O, A \times B)$$

such that $O(\alpha,\beta) = s(F(\alpha), G(\beta)), \forall (\alpha,\beta) \in A \times B$, where s is any s-norm.

Definition 2.7. [6] The union of two fuzzy soft sets (F,A) and (G,B) over a common universe U is the fuzzy soft set (H,C) where $C = A \cup B$, and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & if \quad \varepsilon \in A - B, \\ G(\varepsilon), & if \quad \varepsilon \in B - A, \\ s(F(\varepsilon), G(\varepsilon)), & if \quad \varepsilon \in A \cup B. \end{cases}$$

Where s is any s-norm.

Definition 2.8. [6] The intersection of two fuzzy soft sets (F,A) and (G,B) over a common universe U is the fuzzy soft set (H,C) where $C = A \cup B$, and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & if \quad \varepsilon \in A - B, \\ G(\varepsilon), & if \quad \varepsilon \in B - A, \\ s(F(\varepsilon), G(\varepsilon)), & if \quad \varepsilon \in A \cap B. \end{cases}$$

Definition 2.9. [8]. Let $U = \{x_1, x_2, ..., x_n\}$ be the universal set of elements and $E = \{e_1, e_2, ..., e_m\}$ be the universal set of parameters. The pair (U,E) is called a soft universe. Let $F : E \to I^U$and μ be a fuzzy subset of E, i.e. $\mu : E \to I = [0,1]$, where I^U is the collection of all fuzzy subsets of U. Let $F_\mu : E \to I^U \times I$ be a function defined as follows:

$$F_\mu(e) = (F(e)\,,\, \mu(e))\,.$$

Then F_μ is called a *generalized fuzzy soft set* (in short GFSS) over the soft set (U,E). Here for each parameter $e_i\,,\; F_\mu(e_i) = (F(e_i)\,,\, \mu(e_i))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$ but also the degree of possibility of such belongingness which is represented by $\mu(e_i)$. So we can write this as follows:

$$F_\mu(e_i) = \left(\left\{ \frac{x_1}{F(e_i)(x_1)},\; \frac{x_2}{F(e_i)(x_2)}, ...,\; \frac{x_n}{F(e_i)(x_n)} \right\},\; \mu(e_i) \right).$$

where $F(e_i)(x_1),\; F(e_i)(x_2),\; F(e_i)(x_3)$ and $F(e_i)(x_n)$ are the degree of belongingness and $\mu(e_i)$ is the degree of possibility of such belongingness.

Let U be a universe set, E be a set of parameters, X be a set of experts (agents), and $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$. μ be a fuzzy set of Z, i.e. $\mu : Z \to I = [0,1]$

Definition 2.10. [19] A pair (F_μ, A) is called a *generalized fuzzy soft expert set* (GFSES in short) over U, where F_μ is a mapping given by

$$F_\mu : A \to I^U \times I$$

where I^U denotes the collection of all fuzzy subsets of U. Here for each parameter e_i, $F_\mu(e_i) = (F(e_i), \mu(e_i))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$ but also the degree of possibility of such belongingness which is represented by $\mu(e_i)$.

Definition 2.11. [11]. Let U be an initial universe, E the set of all parameters and X a fuzzy set over E with membership function

$$\mu_X : E \to [0,1]\,,$$

and let γ_X be a fuzzy set over U for all $x \in E$. Then an fpfs-set Γ_X over U is a set defined by a function $\gamma_X(x)$ representing a mapping $\gamma_X : E \to F(U)$ such that

$$\gamma_X(x) = \emptyset \text{ if } \mu_X(x) = 0.$$

Here, γ_X is called a fuzzy approximate function of the fpfs-set Γ_X, and the value $\gamma_X(x)$ is a set called x-element of the fpfs-set for all $x \in E$. Thus, an fpfs-set Γ_X over U can be represented by the set of ordered pairs

$$\Gamma_X = \{(\mu_X(x)/x, \gamma_X(x)) : x \in E,\; \gamma_X(x) \in F(U), \mu_X(x) \in [0,1]\}\,.$$

Definition 2.12. [13] Let U be an initial universal set and let E be a set of parameters. Let I^U denote the power set of all fuzzy subsets of U, let $A \subseteq E$ and T be a set of time where $T = \{t_1, t_2, ..., t_n\}$. A collection of pairs $(F, E)^{Gt}\,\forall\, t \in T$ is called a *time-fuzzy soft set* $\{T - FSS\}$ over U where F is a mapping given by

$$F^{Gt} : A \to I^U.$$

3. Generlized Time Fuzzy Soft Expert Set

3.1. Main Definition

In this part, we define a Generlized time fuzzy soft expert set ($\overline{GT - FESS}$) and discuss its fundamental characteristics.

Let U be a universe set, E be a set of parameters, I^E represent all fuzzy subsets of E, X be a set of experts, and $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ a set of two opinions and T be a set of time where $T = \{t_1, t_2, ..., t_n\}$.. Let $Z = \Psi \times X \times O$ and $A \subseteq Z$ where $\Psi \subset I^E$.

Definition 3.1. Let F be mapping given by

$$F_\Psi^{Gt} : A \to I^U,$$

I^U represent all fuzzy subset of U. A pair $(F, A)_\Psi^{Gt}$ is known as *generalized time fuzzy soft expert set* ($\overline{GT - FSES's}$) over U.

Example 3.2. Assume that a manufacturer distributes its products to three major regions of the country. Because of the geographical separation between both demand and supply areas, as well as the aim to meet consumer demand and ensure the availability of their goods, the factory's management has opted to open outlets near their current client bases. Now, let $W = \{u_1,\ u_2,\ u_3, u_4\}$ be a set of alternatives for stores location , $E = \{e_1, e_2, e_3\}$ be a set of decision parameters where $e_i\,(i = 1, 2, 3)$ represents the following parameters: land or building expenses associated with the location, the cost of traveling between the demand and supply areas, and "the continuing safety of the area," respectively. Let $\Psi = \{e_1, e_2, e_3\}$ be a fuzzy subset of I^P and $Y = \{m, n, r\}$ be a set of experts.These insights allow us to choose the best location for the new factory stores. Now suppose that

$$F_1\left(e_1, m, 1\right) = \left\{\frac{{u_1}^{Gt_1}}{0.8}, \frac{{u_2}^{Gt_1}}{0.5}, \frac{{u_3}^{Gt_1}}{0.4}, \frac{{u_4}^{Gt_1}}{0.3}, 0.2\right\}, F_1\left(e_1, n, 1\right) = \left\{\frac{{u_1}^{Gt_1}}{0.6}, \frac{{u_2}^{Gt_1}}{0.9}, \frac{{u_3}^{Gt_1}}{0.6}, \frac{{u_4}^{Gt_1}}{0.5}, 0.4\right\},$$
$$F_1\left(e_1, r, 1\right) = \left\{\frac{{u_1}^{Gt_1}}{0.7}, \frac{{u_2}^{Gt_1}}{0.6}, \frac{{u_3}^{Gt_1}}{0.3}, \frac{{u_4}^{Gt_1}}{0.7}, 0.6\right\}, F_1\left(e_2, m, 1\right) = \left\{\frac{{u_1}^{Gt_1}}{0.6}, \frac{{u_2}^{Gt_1}}{0.1}, \frac{{u_3}^{Gt_1}}{0.4}, \frac{{u_4}^{Gt_1}}{0.8}, 0.3\right\},$$

$$F_1\left(e_2, n, 1\right) = \left\{\frac{{u_1}^{Gt_1}}{0.5}, \frac{{u_2}^{Gt_1}}{0.7}, \frac{{u_3}^{Gt_1}}{0.2}, \frac{{u_4}^{Gt_1}}{0.5}, 0.5\right\}, F_1\left(e_2, r, 1\right) = \left\{\frac{{u_1}^{Gt_1}}{0.2}, \frac{{u_2}^{Gt_1}}{0.6}, \frac{{u_3}^{Gt_1}}{0.7}, \frac{{u_4}^{Gt_1}}{0.3}, 0.7\right\},$$

$$F_1\left(e_3, m, 1\right) = \left\{\frac{{u_1}^{Gt_1}}{0.2}, \frac{{u_2}^{Gt_1}}{0.4}, \frac{{u_3}^{Gt_1}}{0.9}, \frac{{u_4}^{Gt_1}}{0.6}, 0.9\right\}, F_1\left(e_3, n, 1\right) = \left\{\frac{{u_1}^{Gt_1}}{0.6}, \frac{{u_2}^{Gt_1}}{0.5}, \frac{{u_3}^{Gt_1}}{0.6}, \frac{{u_4}^{Gt_1}}{0.7}, 0.8\right\},$$

$$F_1\left(e_3, r, 1\right) = \left\{\frac{{u_1}^{Gt_1}}{0.4}, \frac{{u_2}^{Gt_1}}{0.7}, \frac{{u_3}^{Gt_1}}{0.1}, \frac{{u_4}^{Gt_1}}{0.5}, 0.1\right\}, F_2\left(e_1, m, 1\right) = \left\{\frac{{u_1}^{Gt_2}}{0.4}, \frac{{u_2}^{Gt_2}}{0.8}, \frac{{u_3}^{Gt_2}}{0.2}, \frac{{u_4}^{Gt_2}}{0.4}, 0.9\right\},$$

$$F_2\left(e_1, n, 1\right) = \left\{\frac{{u_1}^{Gt_2}}{0.6}, \frac{{u_2}^{Gt_2}}{0.9}, \frac{{u_3}^{Gt_2}}{0.4}, \frac{{u_4}^{Gt_2}}{0.8}, 0.3\right\}, F_2\left(e_1, r, 1\right) = \left\{\frac{{u_1}^{Gt_2}}{0.9}, \frac{{u_2}^{Gt_2}}{0.2}, \frac{{u_3}^{Gt_2}}{0.3}, \frac{{u_4}^{Gt_2}}{0.3}, 0.2\right\},$$

$$F_2\left(e_2, m, 1\right) = \left\{\frac{{u_1}^{Gt_2}}{0.2}, \frac{{u_2}^{Gt_2}}{0.9}, \frac{{u_3}^{Gt_2}}{0.5}, \frac{{u_4}^{Gt_2}}{0.5}, 0.8\right\}, F_2\left(e_2, n, 1\right) = \left\{\frac{{u_1}^{Gt_2}}{0.7}, \frac{{u_2}^{Gt_2}}{0.6}, \frac{{u_3}^{Gt_2}}{0.2}, \frac{{u_4}^{Gt_2}}{0.6}, 0.5\right\},$$

$$F_2\left(e_2, r, 1\right) = \left\{\frac{{u_1}^{Gt_2}}{0.5}, \frac{{u_2}^{Gt_2}}{0.4}, \frac{{u_3}^{Gt_2}}{0.7}, \frac{{u_4}^{Gt_2}}{0.4}, 0.4\right\}, F_2\left(e_3, m, 1\right) = \left\{\frac{{u_1}^{Gt_2}}{0.9}, \frac{{u_2}^{Gt_2}}{0.4}, \frac{{u_3}^{Gt_2}}{0.1}, \frac{{u_4}^{Gt_2}}{0.7}, 0.1\right\},$$

$$F_2\left(e_3, n, 1\right) = \left\{\frac{{u_1}^{Gt_2}}{0.8}, \frac{{u_2}^{Gt_2}}{0.6}, \frac{{u_3}^{Gt_2}}{0.1}, \frac{{u_4}^{Gt_2}}{0.5}, 0.7\right\}, F_2\left(e_3, r, 1\right) = \left\{\frac{{u_1}^{Gt_2}}{0.2}, \frac{{u_2}^{Gt_2}}{0.9}, \frac{{u_3}^{Gt_2}}{0.6}, \frac{{u_4}^{Gt_2}}{0.7}, 0.6\right\},$$

$$F_3\left(e_1, m, 1\right) = \left\{\frac{{u_1}^{Gt_3}}{0.2}, \frac{{u_2}^{Gt_3}}{0.7}, \frac{{u_3}^{Gt_3}}{0.8}, \frac{{u_4}^{Gt_3}}{0.3}, 0.5\right\}, F_3\left(e_1, n, 1\right) = \left\{\frac{{u_1}^{Gt_3}}{0.4}, \frac{{u_2}^{Gt_3}}{0.9}, \frac{{u_3}^{Gt_3}}{0.6}, \frac{{u_4}^{Gt_3}}{0.5}, 0.4\right\},$$

$$F_3\left(e_1, r, 1\right) = \left\{\frac{{u_1}^{Gt_3}}{0.6}, \frac{{u_2}^{Gt_3}}{0.5}, \frac{{u_3}^{Gt_3}}{0.7}, \frac{{u_4}^{Gt_3}}{0.8}, 0.8\right\}, F_3\left(e_2, m, 1\right) = \left\{\frac{{u_1}^{Gt_3}}{0.5}, \frac{{u_2}^{Gt_3}}{0.4}, \frac{{u_3}^{Gt_3}}{0.7}, \frac{{u_4}^{Gt_3}}{0.9}, 0.5\right\},$$

$F_3(e_2,n,1)=\left\{\frac{u_1{}^{Gt_3}}{0.8},\frac{u_2{}^{Gt_3}}{0.6},\frac{u_3{}^{Gt_3}}{0.2},\frac{u_4{}^{Gt_3}}{0.7},0.3\right\}, F_3(e_2,r,1)=\left\{\frac{u_1{}^{Gt_3}}{0.1},\frac{u_2{}^{Gt_3}}{0.8},\frac{u_3{}^{Gt_3}}{0.5},\frac{u_4{}^{Gt_3}}{0.8},0.7\right\},$

$F_3(e_3,m,1)=\left\{\frac{u_1{}^{Gt_3}}{0.7},\frac{u_2{}^{Gt_3}}{0.4},\frac{u_3{}^{Gt_3}}{0.1},\frac{u_4{}^{Gt_3}}{0.8},0.4\right\}, F_3(e_3,n,1)=\left\{\frac{u_1{}^{Gt_3}}{0.6},\frac{u_2{}^{Gt_3}}{0.5},\frac{u_3{}^{Gt_3}}{0.3},\frac{u_4{}^{Gt_3}}{0.9},0.2\right\},$

$F_3(e_3,r,1)=\left\{\frac{u_1{}^{Gt_3}}{0.4},\frac{u_2{}^{Gt_3}}{0.7},\frac{u_3{}^{Gt_3}}{0.5},\frac{u_4{}^{Gt_3}}{0.6},0.6\right\}, F_1(e_1,m,0)=\left\{\frac{u_1{}^{Gt_1}}{0.3},\frac{u_2{}^{Gt_1}}{0.6},\frac{u_3{}^{Gt_1}}{0.7},\frac{u_4{}^{Gt_1}}{0.8},0.8\right\},$

$F_1(e_1,n,0)=\left\{\frac{u_1{}^{Gt_1}}{0.5},\frac{u_2{}^{Gt_1}}{0.3},\frac{u_3{}^{Gt_1}}{0.5},\frac{u_4{}^{Gt_1}}{0.4},0.4\right\}, F_1(e_1,r,0)=\left\{\frac{u_1{}^{Gt_1}}{0.4},\frac{u_2{}^{Gt_1}}{0.5},\frac{u_3{}^{Gt_1}}{0.8},\frac{u_4{}^{Gt_1}}{0.3},0.5\right\},$

$F_1(e_2,m,0)=\left\{\frac{u_1{}^{Gt_1}}{0.5},\frac{u_2{}^{Gt_1}}{0.7},\frac{u_3{}^{Gt_1}}{0.5},\frac{u_4{}^{Gt_1}}{0.3},0.7\right\}, F_1(e_2,n,0)=\left\{\frac{u_1{}^{Gt_1}}{0.4},\frac{u_2{}^{Gt_1}}{0.4},\frac{u_3{}^{Gt_1}}{0.9},\frac{u_4{}^{Gt_1}}{0.6},0.3\right\},$

$F_1(e_2,r,0)=\left\{\frac{u_1{}^{Gt_1}}{0.6},\frac{u_2{}^{Gt_1}}{0.3},\frac{u_3{}^{Gt_1}}{0.4},\frac{u_4{}^{Gt_1}}{0.1},0.5\right\}, F_1(e_3,m,0)=\left\{\frac{u_1{}^{Gt_1}}{0.7},\frac{u_2{}^{Gt_1}}{0.5},\frac{u_3{}^{Gt_1}}{0.2},\frac{u_4{}^{Gt_1}}{0.3},0.6\right\},$

$F_1(e_3,n,0)=\left\{\frac{u_1{}^{Gt_1}}{0.5},\frac{u_2{}^{Gt_1}}{0.4},\frac{u_3{}^{Gt_1}}{0.5},\frac{u_4{}^{Gt_1}}{0.2},0.2\right\}, F_1(e_3,r,0)=\left\{\frac{u_1{}^{Gt_1}}{0.8},\frac{u_2{}^{Gt_1}}{0.2},\frac{u_3{}^{Gt_1}}{0.8},\frac{u_4{}^{Gt_1}}{0.4},0.4\right\},$

$F_2(e_1,m,0)=\left\{\frac{u_1{}^{Gt_2}}{0.7},\frac{u_2{}^{Gt_2}}{0.4},\frac{u_3{}^{Gt_2}}{0.7},\frac{u_4{}^{Gt_2}}{0.5},0.2\right\}, F_2(e_1,n,0)=\left\{\frac{u_1{}^{Gt_2}}{0.5},\frac{u_2{}^{Gt_2}}{0.2},\frac{u_3{}^{Gt_2}}{0.5},\frac{u_4{}^{Gt_2}}{0.4},0.3\right\},$

$F_2(e_1,r,0)=\left\{\frac{u_1{}^{Gt_2}}{0.3},\frac{u_2{}^{Gt_2}}{0.7},\frac{u_3{}^{Gt_2}}{0.8},\frac{u_4{}^{Gt_2}}{0.8},0.9\right\}, F_2(e_2,m,0)=\left\{\frac{u_1{}^{Gt_2}}{0.9},\frac{u_2{}^{Gt_2}}{0.3},\frac{u_3{}^{Gt_2}}{0.6},\frac{u_4{}^{Gt_2}}{0.6},0.4\right\},$

$F_2(e_2,n,0)=\left\{\frac{u_1{}^{Gt_2}}{0.4},\frac{u_2{}^{Gt_2}}{0.5},\frac{u_3{}^{Gt_2}}{0.9},\frac{u_4{}^{Gt_2}}{0.4},0.5\right\}, F_2(e_2,r,0)=\left\{\frac{u_1{}^{Gt_2}}{0.6},\frac{u_2{}^{Gt_2}}{0.7},\frac{u_3{}^{Gt_2}}{0.4},\frac{u_4{}^{Gt_2}}{0.1},0.8\right\},$

$F_2(e_3,m,0)=\left\{\frac{u_1{}^{Gt_2}}{0.2},\frac{u_2{}^{Gt_2}}{0.7},\frac{u_3{}^{Gt_2}}{0.8},\frac{u_4{}^{Gt_2}}{0.2},0.6\right\}, F_2(e_3,n,0)=\left\{\frac{u_1{}^{Gt_2}}{0.4},\frac{u_2{}^{Gt_2}}{0.5},\frac{u_3{}^{Gt_2}}{0.7},\frac{u_4{}^{Gt_2}}{0.6},0.7\right\},$

$F_2(e_3,r,0)=\left\{\frac{u_1{}^{Gt_2}}{0.7},\frac{u_2{}^{Gt_2}}{0.1},\frac{u_3{}^{Gt_2}}{0.3},\frac{u_4{}^{Gt_2}}{0.3},0.1\right\}, F_3(e_1,m,0)=\left\{\frac{u_1{}^{Gt_3}}{0.6},\frac{u_2{}^{Gt_3}}{0.8},\frac{u_3{}^{Gt_3}}{0.2},\frac{u_4{}^{Gt_3}}{0.4},0.6\right\},$

$F_3(e_1,n,0)=\left\{\frac{u_1{}^{Gt_3}}{0.7},\frac{u_2{}^{Gt_3}}{0.5},\frac{u_3{}^{Gt_3}}{0.4},\frac{u_4{}^{Gt_3}}{0.6},0.4\right\}, F_3(e_1,r,0)=\left\{\frac{u_1{}^{Gt_3}}{0.5},\frac{u_2{}^{Gt_3}}{0.7},\frac{u_3{}^{Gt_3}}{0.2},\frac{u_4{}^{Gt_3}}{0.5},0.2\right\},$

$F_3(e_2,m,0)=\left\{\frac{u_1{}^{Gt_3}}{0.3},\frac{u_2{}^{Gt_3}}{0.7},\frac{u_3{}^{Gt_3}}{0.4},\frac{u_4{}^{Gt_3}}{0.1},0.7\right\}, F_3(e_2,n,0)=\left\{\frac{u_1{}^{Gt_3}}{0.1},\frac{u_2{}^{Gt_3}}{0.5},\frac{u_3{}^{Gt_3}}{0.6},\frac{u_4{}^{Gt_3}}{0.4},0.5\right\},$

$F_3(e_2,r,0)=\left\{\frac{u_1{}^{Gt_3}}{0.7},\frac{u_2{}^{Gt_3}}{0.3},\frac{u_3{}^{Gt_3}}{0.3},\frac{u_4{}^{Gt_3}}{0.2},0.3\right\}, F_3(e_3,m,0)=\left\{\frac{u_1{}^{Gt_3}}{0.4},\frac{u_2{}^{Gt_3}}{0.6},\frac{u_3{}^{Gt_3}}{0.8},\frac{u_4{}^{Gt_3}}{0.3},0.1\right\},$

$F_3(e_3,n,0)=\left\{\frac{u_1{}^{Gt_3}}{0.3},\frac{u_2{}^{Gt_3}}{0.4},\frac{u_3{}^{Gt_3}}{0.7},\frac{u_4{}^{Gt_3}}{0.2},0.8\right\}, F_3(e_3,r,0)=\left\{\frac{u_1{}^{Gt_3}}{0.7},\frac{u_2{}^{Gt_3}}{0.2},\frac{u_3{}^{Gt_3}}{0.6},\frac{u_4{}^{Gt_3}}{0.5},0.9\right\}.$

Then we can find the Generalized time fuzzy soft expert sets $(F,E)^{Gt}$ as consisting of the following collection of approximations:

$$(F,E)^{Gt}_{\Psi}=\Bigg\{\left((e_1,m,1),\left\{\frac{u_1{}^{Gt_1}}{0.8},\frac{u_2{}^{Gt_1}}{0.5},\frac{u_3{}^{Gt_1}}{0.4},\frac{u_4{}^{Gt_1}}{0.3},0.2\right\}\right),\left((e_1,n,1),\left\{\frac{u_1{}^{Gt_1}}{0.6},\frac{u_2{}^{Gt_1}}{0.9},\frac{u_3{}^{Gt_1}}{0.6},\frac{u_4{}^{Gt_1}}{0.5},0.4\right\}\right)$$
$$\vdots \qquad\qquad \ddots$$
$$\left((e_3,n,0),\left\{\frac{u_1{}^{Gt_3}}{0.3},\frac{u_2{}^{Gt_3}}{0.4},\frac{u_3{}^{Gt_3}}{0.7},\frac{u_4{}^{Gt_3}}{0.2},0.8\right\}\right),\left((e_3,r,0),\left\{\frac{u_1{}^{Gt_3}}{0.7},\frac{u_2{}^{Gt_3}}{0.2},\frac{u_3{}^{Gt_3}}{0.6},\frac{u_4{}^{Gt_3}}{0.5},0.9\right\}\right)\Bigg\}.$$

Definition 3.3. For two $(\overline{Z}GT-FSES's)$ $(F,A)^{Gt}_{\Psi}$ and $(G,B)^{Gt}_{\Upsilon}$ over U, $(F,A)^{Gt}_{\Psi}$ is called a$(\overline{GT-FSES's})$subset of $(G,B)^{Gt}_{\Upsilon}$ if

1. $\Upsilon\subseteq\Psi$,
2. $\forall\varepsilon\in B, G(\varepsilon)$ is generlized time fuzzy soft expert subset of $J(\varepsilon)$.

Definition 3.4. Two $(\overline{GT-FSES's})$ $(F,A)^{Gt}_{\Psi}$ and $(G,B)^{Gt}_{\Upsilon}$ over U, are said to be *equal* if $(F,A)^{Gt}_{\Psi}$ is a$(\overline{GT-FSES's})$subset of $(G,N)^{Gt}_{\Upsilon}$ and $(G,N)^{Gt}_{\Upsilon}$ is a$(\overline{GT-FSES's})$subset of $(F,A)^{Gt}_{\Psi}$.

Example 3.5. Consider Example 3.2, where

$$A=\Big\{(e_1,m,1)^{t_1},(e_2,m,1)^{t_1},(e_2,n,1)^{t_3},(e_2,r,1)^{t_3},(e_2,n,0)^{t_2},(e_2,r,0)^{t_2},(e_2,r,0)^{t_3},(e_3,m,0)^{t_3}\Big\},$$
$$B=\Big\{(e_1,m,1)^{t_1},(e_2,m,1)^{t_1},(e_2,n,1)^{t_3},(e_2,r,1)^{t_3},(e_2,n,0)^{t_2}\Big\}.$$

Clearly $B\subset A$. Now, let $(G,B)^{Gt}_{\Upsilon}$ and $(F,A)^{Gt}_{\Psi}$ be defined as follows:

$$(F,E)^{Gt}_{\Psi}=\Bigg\{\Big((e_1,m,1),\Big\{\frac{u_1{}^{Gt_1}}{0.8},\frac{u_2{}^{Gt_1}}{0.5},\frac{u_3{}^{Gt_1}}{0.4},\frac{u_4{}^{Gt_1}}{0.3}\Big\},0.2\Big),\Big((e_2,m,1),\Big\{\frac{u_1{}^{Gt_1}}{0.6},\frac{u_2{}^{Gt_1}}{0.1},\frac{u_3{}^{Gt_1}}{0.4},\frac{u_4{}^{Gt_1}}{0.8}\Big\},0.3\Big),$$
$$\Big((e_2,n,1),\Big\{\frac{u_1{}^{Gt_3}}{0.8},\frac{u_2{}^{Gt_3}}{0.6},\frac{u_3{}^{Gt_3}}{0.2},\frac{u_4{}^{Gt_3}}{0.7}\Big\},0.3\Big),\Big((e_2,r,1),\Big\{\frac{u_1{}^{Gt_3}}{0.1},\frac{u_2{}^{Gt_3}}{0.8},\frac{u_3{}^{Gt_3}}{0.5},\frac{u_4{}^{Gt_3}}{0.8}\Big\},0.7\Big),$$
$$\Big((e_2,n,0),\Big\{\frac{u_1{}^{Gt_2}}{0.4},\frac{u_2{}^{Gt_2}}{0.5},\frac{u_3{}^{Gt_2}}{0.9},\frac{u_4{}^{Gt_2}}{0.4}\Big\},0.5\Big),\Big((e_2,r,0)\Big\{\frac{u_1{}^{Gt_2}}{0.6},\frac{u_2{}^{Gt_2}}{0.7},\frac{u_3{}^{Gt_2}}{0.4},\frac{u_4{}^{Gt_2}}{0.1}\Big\},0.8\Big),$$
$$\Big((e_2,r,0),\Big\{\frac{u_1{}^{Gt_3}}{0.7},\frac{u_2{}^{Gt_3}}{0.3},\frac{u_3{}^{Gt_3}}{0.3},\frac{u_4{}^{Gt_3}}{0.2}\Big\},0.3\Big),\Big((e_3,m,0),\Big\{\frac{u_1{}^{Gt_3}}{0.4},\frac{u_2{}^{Gt_3}}{0.6},\frac{u_3{}^{Gt_3}}{0.8},\frac{u_4{}^{Gt_3}}{0.3}\Big\},0.1\Big)\Bigg\},$$

$$(G,E)^{Gt}_{\Upsilon}=\Bigg\{\Big((e_1,m,1),\Big\{\frac{u_1{}^{Gt_1}}{0.6},\frac{u_2{}^{Gt_1}}{0.2},\frac{u_3{}^{Gt_1}}{0.1},\frac{u_4{}^{Gt_1}}{0.0}\Big\},0.2\Big),\Big((e_2,m,1),\Big\{\frac{u_1{}^{Gt_1}}{0.5},\frac{u_2{}^{Gt_1}}{0.1},\frac{u_3{}^{Gt_1}}{0.4},\frac{u_4{}^{Gt_1}}{0.7}\Big\},0.1\Big),$$
$$\Big((e_2,n,1),\Big\{\frac{u_1{}^{Gt_3}}{0.8},\frac{u_2{}^{Gt_3}}{0.5},\frac{u_3{}^{Gt_3}}{0.2},\frac{u_4{}^{Gt_3}}{0.6}\Big\},0.2\Big),\Big((e_2,r,1),\Big\{\frac{u_1{}^{Gt_3}}{0.0},\frac{u_2{}^{Gt_3}}{0.6},\frac{u_3{}^{Gt_3}}{0.5},\frac{u_4{}^{Gt_3}}{0.3}\Big\},0.5\Big),$$
$$\Big((e_2,n,0),\Big\{\frac{u_1{}^{Gt_2}}{0.4},\frac{u_2{}^{Gt_2}}{0.3},\frac{u_3{}^{Gt_2}}{0.7},\frac{u_4{}^{Gt_2}}{0.2}\Big\},0.4\Big)\Bigg\}.$$

We can easily verify that $(G,E)^{Gt}_{\Upsilon}\subseteq(F,E)^{Gt}_{\Psi}$.

Definition 3.6. An *agree-GT-FSES's* $(F,A)^{Gt}_{\Psi_1}$ over U is a$(\overline{GT-FSES's})$subset of $(F,A)^{Gt}_{\Psi}$ be defined as outlined below::

$$(F,A)^{Gt}_{\Psi_1}=\left\{F^{Gt}_{\Psi_1}(\alpha):\alpha\in\Psi\times X\times\{1\}\right\}.$$

Definition 3.7. A *disagree-GT-FSES's* $(F,A)^{Gt}_{\Psi_0}$ over U is a$(\overline{GT-FSES's})$subset of $(F,A)^{Gt}_{\Psi}$ be defined as outlined below:

$$(F,A)^{Gt}_{\Psi_0}=\{J_{\Psi_0}(\alpha):\alpha\in\Psi\times X\times\{0\}\}.$$

Example 3.8. Think about this example: 3.2. Then the agree-generalized time fuzzy soft expert set $\left((F,A)^{Gt}_{\Psi}\right)_1$ over U is

$$\left((F,A)^{Gt}_{\Psi}\right)_1=$$
$$\Bigg\{\Big((e_1,m,1),\Big\{\frac{u_1{}^{Gt_1}}{0.8},\frac{u_2{}^{Gt_1}}{0.5},\frac{u_3{}^{Gt_1}}{0.4},\frac{u_4{}^{Gt_1}}{0.3},0.2\Big\}\Big),\Big((e_1,n,1),\Big\{\frac{u_1{}^{Gt_1}}{0.6},\frac{u_2{}^{Gt_1}}{0.9},\frac{u_3{}^{Gt_1}}{0.6},\frac{u_4{}^{Gt_1}}{0.5},.04\Big\}\Big),$$
$$\Big((e_1,r,1),\Big\{\frac{u_1{}^{Gt_1}}{0.7},\frac{u_2{}^{Gt_1}}{0.6},\frac{u_3{}^{Gt_1}}{0.3},\frac{u_4{}^{Gt_1}}{0.7},0.6\Big\}\Big),\Big((e_2,m,1),\Big\{\frac{u_1{}^{Gt_1}}{0.6},\frac{u_2{}^{Gt_1}}{0.1},\frac{u_3{}^{Gt_1}}{0.4},\frac{u_4{}^{Gt_1}}{0.8},0.3\Big\}\Big),$$

$$\left((e_2,n,1),\left\{\frac{u_1{}^{Gt_1}}{0.5},\frac{u_2{}^{Gt_1}}{0.7},\frac{u_3{}^{Gt_1}}{0.2},\frac{u_4{}^{Gt_1}}{0.5},0.5\right\}\right),\left((e_2,r,1),\left\{\frac{u_1{}^{Gt_1}}{0.2},\frac{u_2{}^{Gt_1}}{0.6},\frac{u_3{}^{Gt_1}}{0.7},\frac{u_4{}^{Gt_1}}{0.3},o.7\right\}\right),$$

$$\left((e_3,m,1),\left\{\frac{u_1{}^{Gt_1}}{0.2},\frac{u_2{}^{Gt_1}}{0.4},\frac{u_3{}^{Gt_1}}{0.9},\frac{u_4{}^{Gt_1}}{0.6},0.9\right\}\right),\left((e_3,n,1),\left\{\frac{u_1{}^{Gt_1}}{0.6},\frac{u_2{}^{Gt_1}}{0.5},\frac{u_3{}^{Gt_1}}{0.6},\frac{u_4{}^{Gt_1}}{0.7},0.8\right\}\right),$$

$$\left((e_3,r,1),\left\{\frac{u_1{}^{Gt_1}}{0.4},\frac{u_2{}^{Gt_1}}{0.7},\frac{u_3{}^{Gt_1}}{0.1},\frac{u_4{}^{Gt_1}}{0.5},0.1\right\}\right),\left((e_1,m,1),\left\{\frac{u_1{}^{Gt_2}}{0.4},\frac{u_2{}^{Gt_2}}{0.8},\frac{u_3{}^{Gt_2}}{0.2},\frac{u_4{}^{Gt_2}}{0.4},0.9\right\}\right),$$

$$\left((e_1,n,1),\left\{\frac{u_1{}^{Gt_2}}{0.6},\frac{u_2{}^{Gt_2}}{0.9},\frac{u_3{}^{Gt_2}}{0.4},\frac{u_4{}^{Gt_2}}{0.8},0.3\right\}\right),\left((e_1,r,1),\left\{\frac{u_1{}^{Gt_2}}{0.9},\frac{u_2{}^{Gt_2}}{0.2},\frac{u_3{}^{Gt_2}}{0.3},\frac{u_4{}^{Gt_2}}{0.3},0.2\right\}\right),$$

$$\left((e_2,m,1),\left\{\frac{u_1{}^{Gt_2}}{0.2},\frac{u_2{}^{Gt_2}}{0.9},\frac{u_3{}^{Gt_2}}{0.5},\frac{u_4{}^{Gt_2}}{0.5},0.8\right\}\right),\left((e_2,n,1),\left\{\frac{u_1{}^{Gt_2}}{0.7},\frac{u_2{}^{Gt_2}}{0.6},\frac{u_3{}^{Gt_2}}{0.2},\frac{u_4{}^{Gt_2}}{0.6},0.5\right\}\right),$$

$$\left((e_2,r,1),\left\{\frac{u_1{}^{Gt_2}}{0.5},\frac{u_2{}^{Gt_2}}{0.4},\frac{u_3{}^{Gt_2}}{0.7},\frac{u_4{}^{Gt_2}}{0.4},0.4\right\}\right),\left((e_3,m,1),\left\{\frac{u_1{}^{Gt_2}}{0.9},\frac{u_2{}^{Gt_2}}{0.4},\frac{u_3{}^{Gt_2}}{0.1},\frac{u_4{}^{Gt_2}}{0.7},0.1\right\}\right),$$

$$\left((e_3,n,1),\left\{\frac{u_1{}^{Gt_2}}{0.8},\frac{u_2{}^{Gt_2}}{0.6},\frac{u_3{}^{Gt_2}}{0.1},\frac{u_4{}^{Gt_2}}{0.5},0.7\right\}\right),\left((e_3,r,1),\left\{\frac{u_1{}^{Gt_2}}{0.2},\frac{u_2{}^{Gt_2}}{0.9},\frac{u_3{}^{Gt_2}}{0.6},\frac{u_4{}^{Gt_2}}{0.7},0.6\right\}\right),$$

$$\left((e_1,m,1),\left\{\frac{u_1{}^{Gt_3}}{0.2},\frac{u_2{}^{Gt_3}}{0.7},\frac{u_3{}^{Gt_3}}{0.8},\frac{u_4{}^{Gt_3}}{0.3}0.5\right\}\right),\left((e_1,n,1),\left\{\frac{u_1{}^{Gt_3}}{0.4},\frac{u_2{}^{Gt_3}}{0.9},\frac{u_3{}^{Gt_3}}{0.6},\frac{u_4{}^{Gt_3}}{0.5},0.4\right\}\right),$$

$$\left((e_1,r,1),\left\{\frac{u_1{}^{Gt_3}}{0.6},\frac{u_2{}^{Gt_3}}{0.5},\frac{u_3{}^{Gt_3}}{0.7},\frac{u_4{}^{Gt_3}}{0.8},0.8\right\}\right),\left((e_2,m,1),\left\{\frac{u_1{}^{Gt_3}}{0.5},\frac{u_2{}^{Gt_3}}{0.4},\frac{u_3{}^{Gt_3}}{0.7},\frac{u_4{}^{Gt_3}}{0.9},0.5\right\}\right),$$

$$\left((e_2,n,1),\left\{\frac{u_1{}^{Gt_3}}{0.8},\frac{u_2{}^{Gt_3}}{0.6},\frac{u_3{}^{Gt_3}}{0.2},\frac{u_4{}^{Gt_3}}{0.7},0.3\right\}\right),\left((e_2,r,1),\left\{\frac{u_1{}^{Gt_3}}{0.1},\frac{u_2{}^{Gt_3}}{0.8},\frac{u_3{}^{Gt_3}}{0.5},\frac{u_4{}^{Gt_3}}{0.8},0.7\right\}\right),$$

$$\left((e_3,m,1),\left\{\frac{u_1{}^{Gt_3}}{0.7},\frac{u_2{}^{Gt_3}}{0.4},\frac{u_3{}^{Gt_3}}{0.1},\frac{u_4{}^{Gt_3}}{0.8},0.4\right\}\right),\left((e_3,n,1),\left\{\frac{u_1{}^{Gt_3}}{0.6},\frac{u_2{}^{Gt_3}}{0.5},\frac{u_3{}^{Gt_3}}{0.3},\frac{u_4{}^{Gt_3}}{0.9},0.2\right\}\right),$$

$$\left((e_3,r,1),\left\{\frac{u_1{}^{Gt_3}}{0.4},\frac{u_2{}^{Gt_3}}{0.7},\frac{u_3{}^{Gt_3}}{0.5},\frac{u_4{}^{Gt_3}}{0.6},0.6\right\}\right)\Bigg\}.$$

and the disagree- generalized time fuzzy soft expert set $\left((F,A)_{\Psi}^{Gt}\right)_0$ over U is

$$\left((F,A)_{\Psi}^{Gt}\right)_0 =$$

$$\Bigg\{\left((e_1,m,0),\left\{\frac{u_1{}^{Gt_1}}{0.3},\frac{u_2{}^{Gt_1}}{0.6},\frac{u_3{}^{Gt_1}}{0.7},\frac{u_4{}^{Gt_1}}{0.8},0.8\right\}\right),\left((e_1,n,0),\left\{\frac{u_1{}^{Gt_1}}{0.5},\frac{u_2{}^{Gt_1}}{0.3},\frac{u_3{}^{Gt_1}}{0.5},\frac{u_4{}^{Gt_1}}{0.4},0.4\right\}\right),$$

$$\left((e_1,r,0),\left\{\frac{u_1{}^{Gt_1}}{0.4},\frac{u_2{}^{Gt_1}}{0.5},\frac{u_3{}^{Gt_1}}{0.8},\frac{u_4{}^{Gt_1}}{0.3},0.5\right\}\right),\left((e_2,m,0),\left\{\frac{u_1{}^{Gt_1}}{0.5},\frac{u_2{}^{Gt_1}}{0.7},\frac{u_3{}^{Gt_1}}{0.5},\frac{u_4{}^{Gt_1}}{0.3},0.7\right\}\right),$$

$$\left((e_2,n,0),\left\{\frac{u_1{}^{Gt_1}}{0.4},\frac{u_2{}^{Gt_1}}{0.4},\frac{u_3{}^{Gt_1}}{0.9},\frac{u_4{}^{Gt_1}}{0.6},0.3\right\}\right),\left((e_2,r,0),\left\{\frac{u_1{}^{Gt_1}}{0.6},\frac{u_2{}^{Gt_1}}{0.3},\frac{u_3{}^{Gt_1}}{0.4},\frac{u_4{}^{Gt_1}}{0.1},0.5\right\}\right),$$

$$\left((e_3,m,0),\left\{\frac{u_1{}^{Gt_1}}{0.7},\frac{u_2{}^{Gt_1}}{0.5},\frac{u_3{}^{Gt_1}}{0.2},\frac{u_4{}^{Gt_1}}{0.3},0.6\right\}\right),\left((e_3,n,0),\left\{\frac{u_1{}^{Gt_1}}{0.5},\frac{u_2{}^{Gt_1}}{0.4},\frac{u_3{}^{Gt_1}}{0.5},\frac{u_4{}^{Gt_1}}{0.2},0.2\right\}\right),$$

$$\left((e_3,r,0),\left\{\frac{u_1{}^{Gt_1}}{0.8},\frac{u_2{}^{Gt_1}}{0.2},\frac{u_3{}^{Gt_1}}{0.8},\frac{u_4{}^{Gt_1}}{0.4},0.4\right\}\right),\left((e_1,m,0),\left\{\frac{u_1{}^{Gt_2}}{0.7},\frac{u_2{}^{Gt_2}}{0.4},\frac{u_3{}^{Gt_2}}{0.7},\frac{u_4{}^{Gt_2}}{0.5},0.2\right\}\right),$$

$$\left((e_1,n,0),\left\{\frac{u_1{}^{Gt_2}}{0.5},\frac{u_2{}^{Gt_2}}{0.2},\frac{u_3{}^{Gt_2}}{0.5},\frac{u_4{}^{Gt_2}}{0.4},0.3\right\}\right),\left((e_1,r,0),\left\{\frac{u_1{}^{Gt_2}}{0.3},\frac{u_2{}^{Gt_2}}{0.7},\frac{u_3{}^{Gt_2}}{0.8},\frac{u_4{}^{Gt_2}}{0.8},0.9\right\}\right),$$

$$\left((e_2,m,0),\left\{\frac{u_1{}^{Gt_2}}{0.9},\frac{u_2{}^{Gt_2}}{0.3},\frac{u_3{}^{Gt_2}}{0.6},\frac{u_4{}^{Gt_2}}{0.6},0.4\right\}\right),\left((e_2,n,0),\left\{\frac{u_1{}^{Gt_2}}{0.4},\frac{u_2{}^{Gt_2}}{0.5},\frac{u_3{}^{Gt_2}}{0.9},\frac{u_4{}^{Gt_2}}{0.4},0.5\right\}\right),$$

$$\left((e_2,r,0)\left\{\frac{u_1{}^{Gt_2}}{0.6},\frac{u_2{}^{Gt_2}}{0.7},\frac{u_3{}^{Gt_2}}{0.4},\frac{u_4{}^{Gt_2}}{0.1},0.8\right\}\right),\left((e_3,m,0),\left\{\frac{u_1{}^{Gt_2}}{0.2},\frac{u_2{}^{Gt_2}}{0.7},\frac{u_3{}^{Gt_2}}{0.8},\frac{u_4{}^{Gt_2}}{0.2},0.6\right\}\right),$$

$$\left((e_3,n,0),\left\{\frac{{u_1}^{Gt_2}}{0.4},\frac{{u_2}^{Gt_2}}{0.5},\frac{{u_3}^{Gt_2}}{0.7},\frac{{u_4}^{Gt_2}}{0.6},0.7\right\}\right),\left((e_3,r,0),\left\{\frac{{u_1}^{Gt_2}}{0.7},\frac{{u_2}^{Gt_2}}{0.1},\frac{{u_3}^{Gt_2}}{0.3},\frac{{u_4}^{Gt_2}}{0.3},0.1\right\}\right),$$

$$\left((e_1,m,0),\left\{\frac{{u_1}^{Gt_3}}{0.6},\frac{{u_2}^{Gt_3}}{0.8},\frac{{u_3}^{Gt_3}}{0.2},\frac{{u_4}^{Gt_3}}{0.4},0.6\right\}\right),\left((e_1,n,0),\left\{\frac{{u_1}^{Gt_3}}{0.7},\frac{{u_2}^{Gt_3}}{0.5},\frac{{u_3}^{Gt_3}}{0.4},\frac{{u_4}^{Gt_3}}{0.6},0.4\right\}\right),$$

$$\left((e_1,r,0),\left\{\frac{{u_1}^{Gt_3}}{0.5},\frac{{u_2}^{Gt_3}}{0.7},\frac{{u_3}^{Gt_3}}{0.2},\frac{{u_4}^{Gt_3}}{0.5},0.2\right\}\right),\left((e_2,m,0),\left\{\frac{{u_1}^{Gt_3}}{0.3},\frac{{u_2}^{Gt_3}}{0.7},\frac{{u_3}^{Gt_3}}{0.4},\frac{{u_4}^{Gt_3}}{0.1},0.7\right\}\right),$$

$$\left((e_2,n,0),\left\{\frac{{u_1}^{Gt_3}}{0.1},\frac{{u_2}^{Gt_3}}{0.5},\frac{{u_3}^{Gt_3}}{0.6},\frac{{u_4}^{Gt_3}}{0.4},0.5\right\}\right),\left((e_2,r,0),\left\{\frac{{u_1}^{Gt_3}}{0.7},\frac{{u_2}^{Gt_3}}{0.3},\frac{{u_3}^{Gt_3}}{0.3},\frac{{u_4}^{Gt_3}}{0.2},0.3\right\}\right),$$

$$\left((e_3,m,0),\left\{\frac{{u_1}^{Gt_3}}{0.4},\frac{{u_2}^{Gt_3}}{0.6},\frac{{u_3}^{Gt_3}}{0.8},\frac{{u_4}^{Gt_3}}{0.3},0.1\right\}\right),\left((e_3,n,0),\left\{\frac{{u_1}^{Gt_3}}{0.3},\frac{{u_2}^{Gt_3}}{0.4},\frac{{u_3}^{Gt_3}}{0.7},\frac{{u_4}^{Gt_3}}{0.2},0.8\right\}\right),$$

$$\left((e_3,r,0),\left\{\frac{{u_1}^{Gt_3}}{0.7},\frac{{u_2}^{Gt_3}}{0.2},\frac{{u_3}^{Gt_3}}{0.6},\frac{{u_4}^{Gt_3}}{0.5},0.9\right\}\right)\Bigg\}.$$

4. Basic Operations

In this Section with a discussion on the necessity of extending classical set-theoretic operations to the context of time-fuzzy soft expert sets. In particular, information frequently changes over time and includes a certain amount of fuzziness and expert doubt in real-world decision-making systems, such as market analysis, environmental monitoring, and medical diagnosis. Thus, logical operations like *AND* and *OR*, as well as basic operations like *complement*, *union*, and *intersection*, are essential for combining and working with time-varying fuzzy data. In this part, we define the complement, union, and intersection of FPFSES, deduce several features, and provide illustrative instances.

4.1. The Operations' Benefits and Motivation

Operations like *complement*, *union*, and *intersection*, together with logical operations like *AND* and *OR*, are essential for facilitating dynamic and uncertain decision-making processes in the context of time-fuzzy soft expert sets. These processes are crucial for merging and modifying several expert judgments that change over time and may have different levels of dependability and fuzziness. Such procedures are crucial for simulating real-world decision contexts when information is ambiguous, time-sensitive, or incomplete, as demonstrated by traditional fuzzy set theory and soft set extensions. However, these operations must be extended into the time-fuzzy soft expert set environment in order to use expert systems in temporal domains—like risk assessment, investment forecasting, and medical diagnosis across time—effectively.

Every process offers a distinct advantage:

Each operation provides a unique benefit:

- The **complement** operation allows decision-makers to assess negated scenarios or opposing expert views over time, which is critical in sensitivity analysis and conflict resolution.
- The **union (OR)** operation helps in aggregating multiple possibilities or expert suggestions, reflecting inclusive or optimistic strategies where the satisfaction of any condition may be sufficient.
- The **intersection (AND)** operation is useful for identifying commonalities among expert opinions, ensuring that only those decisions which satisfy all criteria are considered—thus supporting more conservative or cautious decision policies.
- The **logical AND/OR** operations further enhance decision modeling by enabling rule-based constructions based on logical combinations of time-varying fuzzy conditions.

Our goal in proposing and formalizing these operations is to offer a versatile and mathematically sound toolset for real-world applications that need to handle expert-based, fuzzy, and temporal uncertainty. These additions not only expand on earlier soft set theory techniques, but they also provide the model greater flexibility and intelligence when dealing with challenging decision-making situations.

4.2. Complement

Definition 4.1. Let $(F, A)^{Gt}_{\Psi}$ be a Generalized Time-dependent Fuzzy Soft Expert Set (GT-FSES), where:

- A is a non-empty set of parameters;
- $\Psi \subseteq I^E$ is a set of expert-time pairs (or indexed time-expert combinations);
- $F^{Gt}_{\Psi} : A \to \tilde{\mathcal{P}}(U)$ is a mapping that assigns to each parameter $a \in A$ a fuzzy subset of the universe U, depending on both expert opinions and time (captured by Gt and Ψ).

Then, the *complement* of the GT-FSES $(F, A)^{Gt}_{\Psi}$, denoted by $\widetilde{c}((F, A))^{Gt}_{\Psi}$, is defined as:

$$\widetilde{c}((F, A))^{Gt}_{\Psi} = (F^c, A)^{Gt}_{\Psi^c}$$

where:

- F^c is the fuzzy soft expert complement of F;
- Ψ^c represents the complement (or dual) configuration of the expert-time set Ψ.

Example 4.2. Let us examine Example 3.2. Utilizing the fundamental fuzzy complement, we have

$$\begin{aligned}
\widetilde{c}(F, A)^{Gt}_{\Psi} = \Bigg\{ &\left((e_1, m, 1), \left\{\tfrac{u_1{}^{Gt_1}}{0.2}, \tfrac{u_2{}^{Gt_1}}{0.5}, \tfrac{u_3{}^{Gt_1}}{0.6}, \tfrac{u_4{}^{Gt_1}}{0.7}, 0.8\right\}\right), \left((e_1, n, 1), \left\{\tfrac{u_1{}^{Gt_1}}{0.4}, \tfrac{u_2{}^{Gt_1}}{0.1}, \tfrac{u_3{}^{Gt_1}}{0.4}, \tfrac{u_4{}^{Gt_1}}{0.5}, 0.6\right\}\right) \\
&\left((e_1, r, 1), \left\{\tfrac{u_1{}^{Gt_1}}{0.3}, \tfrac{u_2{}^{Gt_1}}{0.4}, \tfrac{u_3{}^{Gt_1}}{0.7}, \tfrac{u_4{}^{Gt_1}}{0.3}, 0.4\right\}\right), \left((e_2, m, 1), \left\{\tfrac{u_1{}^{Gt_1}}{0.4}, \tfrac{u_2{}^{Gt_1}}{0.9}, \tfrac{u_3{}^{Gt_1}}{0.6}, \tfrac{u_4{}^{Gt_1}}{0.2}, 0.7\right\}\right), \\
&\left((e_2, n, 1), \left\{\tfrac{u_1{}^{Gt_1}}{0.5}, \tfrac{u_2{}^{Gt_1}}{0.3}, \tfrac{u_3{}^{Gt_1}}{0.8}, \tfrac{u_4{}^{Gt_1}}{0.5}, 0.5\right\}\right), \left((e_2, r, 1), \left\{\tfrac{u_1{}^{Gt_1}}{0.8}, \tfrac{u_2{}^{Gt_1}}{0.4}, \tfrac{u_3{}^{Gt_1}}{0.3}, \tfrac{u_4{}^{Gt_1}}{0.7}, 0.3\right\}\right), \\
&\left((e_3, m, 1), \left\{\tfrac{u_1{}^{Gt_1}}{0.8}, \tfrac{u_2{}^{Gt_1}}{0.6}, \tfrac{u_3{}^{Gt_1}}{0.1}, \tfrac{u_4{}^{Gt_1}}{0.4}, 0.1\right\}\right), \left((e_3, n, 1), \left\{\tfrac{u_1{}^{Gt_1}}{0.4}, \tfrac{u_2{}^{Gt_1}}{0.5}, \tfrac{u_3{}^{Gt_1}}{0.4}, \tfrac{u_4{}^{Gt_1}}{0.3}, 0.2\right\}\right), \\
&\left((e_3, r, 1), \left\{\tfrac{u_1{}^{Gt_1}}{0.6}, \tfrac{u_2{}^{Gt_1}}{0.3}, \tfrac{u_3{}^{Gt_1}}{0.9}, \tfrac{u_4{}^{Gt_1}}{0.5}, 0.9\right\}\right), \left((e_1, m, 1), \left\{\tfrac{u_1{}^{Gt_2}}{0.6}, \tfrac{u_2{}^{Gt_2}}{0.2}, \tfrac{u_3{}^{Gt_2}}{0.8}, \tfrac{u_4{}^{Gt_2}}{0.6}, 0.1\right\}\right), \\
&\left((e_1, n, 1), \left\{\tfrac{u_1{}^{Gt_2}}{0.4}, \tfrac{u_2{}^{Gt_2}}{0.1}, \tfrac{u_3{}^{Gt_2}}{0.6}, \tfrac{u_4{}^{Gt_2}}{0.2}, 0.7\right\}\right), \left((e_1, r, 1), \left\{\tfrac{u_1{}^{Gt_2}}{0.1}, \tfrac{u_2{}^{Gt_2}}{0.8}, \tfrac{u_3{}^{Gt_2}}{0.7}, \tfrac{u_4{}^{Gt_2}}{0.7}, 0.8\right\}\right), \\
&\left((e_2, m, 1), \left\{\tfrac{u_1{}^{Gt_2}}{0.8}, \tfrac{u_2{}^{Gt_2}}{0.1}, \tfrac{u_3{}^{Gt_2}}{0.5}, \tfrac{u_4{}^{Gt_2}}{0.5}, 0.2\right\}\right), \left((e_2, n, 1), \left\{\tfrac{u_1{}^{Gt_2}}{0.3}, \tfrac{u_2{}^{Gt_2}}{0.4}, \tfrac{u_3{}^{Gt_2}}{0.8}, \tfrac{u_4{}^{Gt_2}}{0.4}, o.5\right\}\right), \\
&\left((e_2, r, 1), \left\{\tfrac{u_1{}^{Gt_2}}{0.5}, \tfrac{u_2{}^{Gt_2}}{0.6}, \tfrac{u_3{}^{Gt_2}}{0.3}, \tfrac{u_4{}^{Gt_2}}{0.6}, 0.6\right\}\right), \left((e_3, m, 1), \left\{\tfrac{u_1{}^{Gt_2}}{0.1}, \tfrac{u_2{}^{Gt_2}}{0.6}, \tfrac{u_3{}^{Gt_2}}{0.9}, \tfrac{u_4{}^{Gt_2}}{0.3}, 0.9\right\}\right), \\
&\left((e_3, n, 1), \left\{\tfrac{u_1{}^{Gt_2}}{0.2}, \tfrac{u_2{}^{Gt_2}}{0.4}, \tfrac{u_3{}^{Gt_2}}{0.9}, \tfrac{u_4{}^{Gt_2}}{0.5}, o.3\right\}\right), \left((e_3, r, 1), \left\{\tfrac{u_1{}^{Gt_2}}{0.8}, \tfrac{u_2{}^{Gt_2}}{0.1}, \tfrac{u_3{}^{Gt_2}}{0.4}, \tfrac{u_4{}^{Gt_2}}{0.3}, 0.4\right\}\right), \\
&\left((e_1, m, 1), \left\{\tfrac{u_1{}^{Gt_3}}{0.8}, \tfrac{u_2{}^{Gt_3}}{0.3}, \tfrac{u_3{}^{Gt_3}}{0.2}, \tfrac{u_4{}^{Gt_3}}{0.7}, 0.5\right\}\right), \left((e_1, n, 1), \left\{\tfrac{u_1{}^{Gt_3}}{0.6}, \tfrac{u_2{}^{Gt_3}}{0.1}, \tfrac{u_3{}^{Gt_3}}{0.4}, \tfrac{u_4{}^{Gt_3}}{0.5}, 0.6\right\}\right), \\
&\left((e_1, r, 1), \left\{\tfrac{u_1{}^{Gt_3}}{0.4}, \tfrac{u_2{}^{Gt_3}}{0.5}, \tfrac{u_3{}^{Gt_3}}{0.3}, \tfrac{u_4{}^{Gt_3}}{0.2}, 0.2\right\}\right), \left((e_2, m, 1), \left\{\tfrac{u_1{}^{Gt_3}}{0.5}, \tfrac{u_2{}^{Gt_3}}{0.6}, \tfrac{u_3{}^{Gt_3}}{0.3}, \tfrac{u_4{}^{Gt_3}}{0.1}, 0.5\right\}\right), \\
&\left((e_2, n, 1), \left\{\tfrac{u_1{}^{Gt_3}}{0.2}, \tfrac{u_2{}^{Gt_3}}{0.4}, \tfrac{u_3{}^{Gt_3}}{0.8}, \tfrac{u_4{}^{Gt_3}}{0.3}, 0.7\right\}\right), \left((e_2, r, 1), \left\{\tfrac{u_1{}^{Gt_3}}{0.9}, \tfrac{u_2{}^{Gt_3}}{0.2}, \tfrac{u_3{}^{Gt_3}}{0.5}, \tfrac{u_4{}^{Gt_3}}{0.2}, 0.3\right\}\right), \\
&\left((e_3, m, 1), \left\{\tfrac{u_1{}^{Gt_3}}{0.3}, \tfrac{u_2{}^{Gt_3}}{0.6}, \tfrac{u_3{}^{Gt_3}}{0.9}, \tfrac{u_4{}^{Gt_3}}{0.2}, 0.6\right\}\right), \left((e_3, n, 1), \left\{\tfrac{u_1{}^{Gt_3}}{0.4}, \tfrac{u_2{}^{Gt_3}}{0.5}, \tfrac{u_3{}^{Gt_3}}{0.7}, \tfrac{u_4{}^{Gt_3}}{0.1}, 0.8\right\}\right), \\
&\left((e_3, r, 1), \left\{\tfrac{u_1{}^{Gt_3}}{0.6}, \tfrac{u_2{}^{Gt_3}}{0.3}, \tfrac{u_3{}^{Gt_3}}{0.5}, \tfrac{u_4{}^{Gt_3}}{0.4}, 0.4\right\}\right), \left((e_1, m, 0), \left\{\tfrac{u_1{}^{Gt_1}}{0.7}, \tfrac{u_2{}^{Gt_1}}{0.4}, \tfrac{u_3{}^{Gt_1}}{0.3}, \tfrac{u_4{}^{Gt_1}}{0.2}, 0.2\right\}\right), \\
&\left((e_1, n, 0), \left\{\tfrac{u_1{}^{Gt_1}}{0.5}, \tfrac{u_2{}^{Gt_1}}{0.7}, \tfrac{u_3{}^{Gt_1}}{0.5}, \tfrac{u_4{}^{Gt_1}}{0.6}, 0.6\right\}\right), \left((e_1, r, 0), \left\{\tfrac{u_1{}^{Gt_1}}{0.6}, \tfrac{u_2{}^{Gt_1}}{0.5}, \tfrac{u_3{}^{Gt_1}}{0.2}, \tfrac{u_4{}^{Gt_1}}{0.7}, 0.5\right\}\right),
\end{aligned}$$

$$\left((e_2,m,0),\left\{\frac{u_1{}^{Gt_1}}{0.5},\frac{u_2{}^{Gt_1}}{0.3},\frac{u_3{}^{Gt_1}}{0.5},\frac{u_4{}^{Gt_1}}{0.7},0.3\right\}\right),\left((e_2,n,0),\left\{\frac{u_1{}^{Gt_1}}{0.6},\frac{u_2{}^{Gt_1}}{0.6},\frac{u_3{}^{Gt_1}}{0.1},\frac{u_4{}^{Gt_1}}{0.4},0.7\right\}\right),$$
$$\left((e_2,r,0),\left\{\frac{u_1{}^{Gt_1}}{0.4},\frac{u_2{}^{Gt_1}}{0.7},\frac{u_3{}^{Gt_1}}{0.6},\frac{u_4{}^{Gt_1}}{0.9},0.5\right\}\right),\left((e_3,m,0),\left\{\frac{u_1{}^{Gt_1}}{0.3},\frac{u_2{}^{Gt_1}}{0.5},\frac{u_3{}^{Gt_1}}{0.8},\frac{u_4{}^{Gt_1}}{0.7},0.4\right\}\right),$$
$$\left((e_3,n,0),\left\{\frac{u_1{}^{Gt_1}}{0.5},\frac{u_2{}^{Gt_1}}{0.6},\frac{u_3{}^{Gt_1}}{0.5},\frac{u_4{}^{Gt_1}}{0.8},0.8\right\}\right),\left((e_3,r,0),\left\{\frac{u_1{}^{Gt_1}}{0.2},\frac{u_2{}^{Gt_1}}{0.8},\frac{u_3{}^{Gt_1}}{0.2},\frac{u_4{}^{Gt_1}}{0.6},0.6\right\}\right),$$
$$\left((e_1,m,0),\left\{\frac{u_1{}^{Gt_2}}{0.3},\frac{u_2{}^{Gt_2}}{0.6},\frac{u_3{}^{Gt_2}}{0.3},\frac{u_4{}^{Gt_2}}{0.5},0.8\right\}\right),\left((e_1,n,0),\left\{\frac{u_1{}^{Gt_2}}{0.5},\frac{u_2{}^{Gt_2}}{0.8},\frac{u_3{}^{Gt_2}}{0.5},\frac{u_4{}^{Gt_2}}{0.6},0.7\right\}\right),$$
$$\left((e_1,r,0),\left\{\frac{u_1{}^{Gt_2}}{0.7},\frac{u_2{}^{Gt_2}}{0.3},\frac{u_3{}^{Gt_2}}{0.2},\frac{u_4{}^{Gt_2}}{0.2},0.1\right\}\right),\left((e_2,m,0),\left\{\frac{u_1{}^{Gt_2}}{0.1},\frac{u_2{}^{Gt_2}}{0.7},\frac{u_3{}^{Gt_2}}{0.4},\frac{u_4{}^{Gt_2}}{0.4},0.6\right\}\right),$$
$$\left((e_2,n,0),\left\{\frac{u_1{}^{Gt_2}}{0.6},\frac{u_2{}^{Gt_2}}{0.5},\frac{u_3{}^{Gt_2}}{0.1},\frac{u_4{}^{Gt_2}}{0.6},0.5\right\}\right),\left((e_2,r,0)\left\{\frac{u_1{}^{Gt_2}}{0.4},\frac{u_2{}^{Gt_2}}{0.3},\frac{u_3{}^{Gt_2}}{0.6},\frac{u_4{}^{Gt_2}}{0.9}0.2\right\}\right),$$
$$\left((e_3,m,0),\left\{\frac{u_1{}^{Gt_2}}{0.8},\frac{u_2{}^{Gt_2}}{0.3},\frac{u_3{}^{Gt_2}}{0.2},\frac{u_4{}^{Gt_2}}{0.8},0.4\right\}\right),\left((e_3,n,0),\left\{\frac{u_1{}^{Gt_2}}{0.6},\frac{u_2{}^{Gt_2}}{0.5},\frac{u_3{}^{Gt_2}}{0.3},\frac{u_4{}^{Gt_2}}{0.4},0.3\right\}\right),$$
$$\left((e_3,r,0),\left\{\frac{u_1{}^{Gt_2}}{0.3},\frac{u_2{}^{Gt_2}}{0.9},\frac{u_3{}^{Gt_2}}{0.7},\frac{u_4{}^{Gt_2}}{0.7},0.9\right\}\right),\left((e_1,m,0),\left\{\frac{u_1{}^{Gt_3}}{0.4},\frac{u_2{}^{Gt_3}}{0.2},\frac{u_3{}^{Gt_3}}{0.8},\frac{u_4{}^{Gt_3}}{0.6},0.4\right\}\right),$$
$$\left((e_1,n,0),\left\{\frac{u_1{}^{Gt_3}}{0.3},\frac{u_2{}^{Gt_3}}{0.5},\frac{u_3{}^{Gt_3}}{0.6},\frac{u_4{}^{Gt_3}}{0.4},0.6\right\}\right),\left((e_1,r,0),\left\{\frac{u_1{}^{Gt_3}}{0.5},\frac{u_2{}^{Gt_3}}{0.3},\frac{u_3{}^{Gt_3}}{0.8},\frac{u_4{}^{Gt_3}}{0.5},0.8\right\}\right),$$
$$\left((e_2,m,0),\left\{\frac{u_1{}^{Gt_3}}{0.7},\frac{u_2{}^{Gt_3}}{0.3},\frac{u_3{}^{Gt_3}}{0.6},\frac{u_4{}^{Gt_3}}{0.9},0.3\right\}\right),\left((e_2,n,0),\left\{\frac{u_1{}^{Gt_3}}{0.9},\frac{u_2{}^{Gt_3}}{0.5},\frac{u_3{}^{Gt_3}}{0.4},\frac{u_4{}^{Gt_3}}{0.6},0.5\right\}\right),$$
$$\left((e_2,r,0),\left\{\frac{u_1{}^{Gt_3}}{0.3},\frac{u_2{}^{Gt_3}}{0.7},\frac{u_3{}^{Gt_3}}{0.7},\frac{u_4{}^{Gt_3}}{0.8},0.7\right\}\right),\left((e_3,m,0),\left\{\frac{u_1{}^{Gt_3}}{0.6},\frac{u_2{}^{Gt_3}}{0.4},\frac{u_3{}^{Gt_3}}{0.2},\frac{u_4{}^{Gt_3}}{0.7},0.9\right\}\right),$$
$$\left.\left((e_3,n,0),\left\{\frac{u_1{}^{Gt_3}}{0.7},\frac{u_2{}^{Gt_3}}{0.6},\frac{u_3{}^{Gt_3}}{0.3},\frac{u_4{}^{Gt_3}}{0.8},0.2\right\}\right),\left((e_3,r,0),\left\{\frac{u_1{}^{Gt_3}}{0.3},\frac{u_2{}^{Gt_3}}{0.8},\frac{u_3{}^{Gt_3}}{0.4},\frac{u_4{}^{Gt_3}}{0.5},0.1\right\}\right)\right\}.$$

Proposition 4.3
Let $(F,A)^{Gt}_{\Psi}$ be a Generalized Time-Fuzzy Soft Expert Set (GT-FSES) over the universe U, with parameter set A and expert opinion function Ψ. Then, the following identity holds:

1. $\left(\left((F,A)^{Gt}_{\Psi^c}\right)^c\right)=(F,A)^{Gt}_{\Psi}$,

where:

- Ψ^c denotes the complement of the expert opinion function.
- The outer superscript ^{c} denotes the complement operation on the GT-FSES structure.

Proof

By employing Definition 4.1 we have $(F,A)^{Gt^c}_{\Psi}$ is defined by
$c\left(\mu_{\Psi}(x)\right)$ and $\tilde{c}\left(F(x)\right)$, $\forall x\in Z$.
Then $c\left(\mu_{\Psi}(x)\right)^c$ is defined by $c\left(c\left(\mu_{\Psi}(x)\right)\right)$ and $\tilde{c}\left(\tilde{c}\left(F(x)\right)\right)$, $\forall x\in Z$.
But $c\left(c\left(\mu_{\Psi}(x)\right)\right)=\left(\mu_{\Psi}(x)\right)$, and $\tilde{c}\left(\tilde{c}\left(F(x)\right)\right)=\left(F(x)\right)$.

□

where c is generalized time fuzzy soft expert set complement.

4.3. Union

Definition 4.4. The *union* of two $(\overline{GT-FSES's})$ $(F,A)_{\Psi}^{Gt}$ and $(G,B)_{\Upsilon}^{Gt}$ over U, denoted by $(F,A)_{\Psi}^{Gt}\,\widetilde{\cup}\,(G,B)_{\Upsilon}^{Gt}$, is the$(\overline{GT-FSES's})(H,C)_{\Omega}^{Gt}$ such that $R=M\cup N$, $\Omega=max\,(\Psi\cup\Upsilon)$,$\forall\,\varepsilon\in R$,
$H\left(\varepsilon\right)=\left(F\left(\varepsilon\right)\widetilde{\cup}G\left(\varepsilon\right)\right)$ where $\widetilde{\cup}$ is an generalized time fuzzy soft set union.

Example 4.5. Think about this example: 3.2. Let Suppose $(F,A)_{\Psi}^{Gt}$ and $(G,B)_{\Upsilon}^{Gt}$ are two generalized time fuzzy soft set over U such that

$$(F,A)_{\Psi}^{Gt}=\Bigg\{\left((e_2,n,1),\left\{\frac{u_1{}^{Gt_1}}{0.5},\frac{u_2{}^{Gt_1}}{0.7},\frac{u_3{}^{Gt_1}}{0.2},\frac{u_4{}^{Gt_1}}{0.5},0,5\right\}\right),\left((e_2,r,1),\left\{\frac{u_1{}^{Gt_1}}{0.2},\frac{u_2{}^{Gt_1}}{0.6},\frac{u_3{}^{Gt_1}}{0.7},\frac{u_4{}^{Gt_1}}{0.3},0.7\right\}\right),$$

$$\left((e_2,r,1),\left\{\frac{u_1{}^{Gt_2}}{0.5},\frac{u_2{}^{Gt_2}}{0.4},\frac{u_3{}^{Gt_2}}{0.7},\frac{u_4{}^{Gt_2}}{0.4},0.4\right\}\right),\left((e_3,n,1),\left\{\frac{u_1{}^{Gt_2}}{0.8},\frac{u_2{}^{Gt_2}}{0.6},\frac{u_3{}^{Gt_2}}{0.1},\frac{u_4{}^{Gt_2}}{0.5},0.7\right\}\right),$$

$$\left((e_3,m,1),\left\{\frac{u_1{}^{Gt_3}}{0.7},\frac{u_2{}^{Gt_3}}{0.4},\frac{u_3{}^{Gt_3}}{0.1},\frac{u_4{}^{Gt_3}}{0.8},0.4\right\}\right),\left((e_1,m,0),\left\{\frac{u_1{}^{Gt_1}}{0.3},\frac{u_2{}^{Gt_1}}{0.6},\frac{u_3{}^{Gt_1}}{0.7},\frac{u_4{}^{Gt_1}}{0.8},0.8\right\}\right),$$

$$\left((e_3,n,0),\left\{\frac{u_1{}^{Gt_1}}{0.5},\frac{u_2{}^{Gt_1}}{0.4},\frac{u_3{}^{Gt_1}}{0.5},\frac{u_4{}^{Gt_1}}{0.2},0.2\right\}\right),\left((e_3,r,0),\left\{\frac{u_1{}^{Gt_1}}{0.8},\frac{u_2{}^{Gt_1}}{0.2},\frac{u_3{}^{Gt_1}}{0.8},\frac{u_4{}^{Gt_1}}{0.4},0.4\right\}\right),$$

$$\left((e_1,m,0),\left\{\frac{u_1{}^{Gt_2}}{0.7},\frac{u_2{}^{Gt_2}}{0.4},\frac{u_3{}^{Gt_2}}{0.7},\frac{u_4{}^{Gt_2}}{0.5},0.2\right\}\right),\left((e_3,m,0),\left\{\frac{u_1{}^{Gt_3}}{0.4},\frac{u_2{}^{Gt_3}}{0.6},\frac{u_3{}^{Gt_3}}{0.8},\frac{u_4{}^{Gt_3}}{0.3},0.6\right\}\right)\Bigg\}.$$

$$(G,B)_{\Upsilon}^{Gt}=\Bigg\{\left((e_2,n,1),\left\{\frac{u_1{}^{Gt_1}}{0.7},\frac{u_2{}^{Gt_1}}{0.9},\frac{u_3{}^{Gt_1}}{0.1},\frac{u_4{}^{Gt_1}}{0.3},0.7\right\}\right),\left((e_2,r,1),\left\{\frac{u_1{}^{Gt_1}}{0.6},\frac{u_2{}^{Gt_1}}{0.4},\frac{u_3{}^{Gt_1}}{0.8},\frac{u_4{}^{Gt_1}}{0.1},0.6\right\}\right),$$

$$\left((e_2,r,1),\left\{\frac{u_1{}^{Gt_2}}{0.7},\frac{u_2{}^{Gt_2}}{0.3},\frac{u_3{}^{Gt_2}}{0.2},\frac{u_4{}^{Gt_2}}{0.6},0.4\right\}\right),\left((e_3,n,1),\left\{\frac{u_1{}^{Gt_2}}{0.5},\frac{u_2{}^{Gt_2}}{0.8},\frac{u_3{}^{Gt_2}}{0.7},\frac{u_4{}^{Gt_2}}{0.3},0.9\right\}\right),$$

$$\left((e_1,m,0),\left\{\frac{u_1{}^{Gt_2}}{0.5},\frac{u_2{}^{Gt_2}}{0.6},\frac{u_3{}^{Gt_2}}{0.8},\frac{u_4{}^{Gt_2}}{0.4},0.6\right\}\right),\left((e_1,r,0),\left\{\frac{u_1{}^{Gt_2}}{0.7},\frac{u_2{}^{Gt_2}}{0.5},\frac{u_3{}^{Gt_2}}{0.6},\frac{u_4{}^{Gt_2}}{0.7},0.5\right\}\right),$$

$$\left((e_2,m,0),\left\{\frac{u_1{}^{Gt_2}}{0.6},\frac{u_2{}^{Gt_2}}{0.7},\frac{u_3{}^{Gt_2}}{0.4},\frac{u_4{}^{Gt_2}}{0.9},0.7\right\}\right),\left((e_3,m,0),\left\{\frac{u_1{}^{Gt_3}}{0.2},\frac{u_2{}^{Gt_3}}{0.8},\frac{u_3{}^{Gt_3}}{0.9},\frac{u_4{}^{Gt_3}}{0.1},0.2\right\}\right)\Bigg\}.$$

Then $(F,A)_{\Upsilon}^{Gt}\,\widetilde{\cup}\,(G,B)_{\Upsilon}^{Gt}=(H,C)_{\Omega}^{Gt}$ where

$$(H,C)_{\Omega}^{Gt}=\Bigg\{\left((e_2,n,1),\left\{\frac{u_1{}^{Gt_1}}{0.7},\frac{u_2{}^{Gt_1}}{0.9},\frac{u_3{}^{Gt_1}}{0.2},\frac{u_4{}^{Gt_1}}{0.5},0.7\right\}\right),\left((e_2,r,1),\left\{\frac{u_1{}^{Gt_1}}{0.6},\frac{u_2{}^{Gt_1}}{0.6},\frac{u_3{}^{Gt_1}}{0.8},\frac{u_4{}^{Gt_1}}{0.3},0.7\right\}\right),$$

$$\left((e_2,r,1),\left\{\frac{u_1{}^{Gt_2}}{0.7},\frac{u_2{}^{Gt_2}}{0.4},\frac{u_3{}^{Gt_2}}{0.7},\frac{u_4{}^{Gt_2}}{0.6}.0.4\right\}\right),\left((e_3,n,1),\left\{\frac{u_1{}^{Gt_2}}{0.8},\frac{u_2{}^{Gt_2}}{0.8},\frac{u_3{}^{Gt_2}}{0.7},\frac{u_4{}^{Gt_2}}{0.5},0.9\right\}\right),$$

$$\left((e_3,m,1),\left\{\frac{u_1{}^{Gt_3}}{0.7},\frac{u_2{}^{Gt_3}}{0.4},\frac{u_3{}^{Gt_3}}{0.1},\frac{u_4{}^{Gt_3}}{0.8},0.4\right\}\right),\left((e_1,m,0),\left\{\frac{u_1{}^{Gt_1}}{0.3},\frac{u_2{}^{Gt_1}}{0.6},\frac{u_3{}^{Gt_1}}{0.7},\frac{u_4{}^{Gt_1}}{0.8},0.8\right\}\right),$$

$$\left((e_3,n,0),\left\{\frac{u_1{}^{Gt_1}}{0.5},\frac{u_2{}^{Gt_1}}{0.4},\frac{u_3{}^{Gt_1}}{0.5},\frac{u_4{}^{Gt_1}}{0.2},0.2\right\}\right),\left((e_3,r,0),\left\{\frac{u_1{}^{Gt_1}}{0.8},\frac{u_2{}^{Gt_1}}{0.2},\frac{u_3{}^{Gt_1}}{0.8},\frac{u_4{}^{Gt_1}}{0.4},0.4\right\}\right),$$

$$\left((e_1,m,0),\left\{\frac{u_1{}^{Gt_2}}{0.7},\frac{u_2{}^{Gt_2}}{0.6},\frac{u_3{}^{Gt_2}}{0.8},\frac{u_4{}^{Gt_2}}{0.5},0.2\right\}\right),\left((e_1,r,0),\left\{\frac{u_1{}^{Gt_2}}{0.7},\frac{u_2{}^{Gt_2}}{0.5},\frac{u_3{}^{Gt_2}}{0.6},\frac{u_4{}^{Gt_2}}{0.7},0.5\right\}\right),$$

$$\left((e_2,m,0),\left\{\frac{u_1{}^{Gt_2}}{0.6},\frac{u_2{}^{Gt_2}}{0.7},\frac{u_3{}^{Gt_2}}{0.4},\frac{u_4{}^{Gt_2}}{0.9},0.7\right\}\right),\left((e_3,m,0),\left\{\frac{u_1{}^{Gt_3}}{0.4},\frac{u_2{}^{Gt_3}}{0.8},\frac{u_3{}^{Gt_3}}{0.9},\frac{u_4{}^{Gt_3}}{0.3},0.6\right\}\right)\Bigg\}.$$

Proposition 4.6
If $(F,A)_{\Psi}^{Gt}$, $(G,B)_{\Upsilon}^{Gt}$ and $(H,C)_{\Omega}^{Gt}$ are three $(\overline{GT-FSES's})$ over U, then

1. $\left(F,A\right)_{\Psi}^{Gt}\tilde{\cup}\left(\left(G,B\right)_{\Upsilon}^{Gt}\tilde{\cup}\left(H,C\right)_{\Omega}^{Gt}\right)\right)=\left(\left(F,A\right)_{\Psi}^{Gt}\tilde{\cup}\left(G,B\right)\right)_{\Upsilon}\tilde{\cup}\left(H,C\right)_{\Omega}^{Gt}\ \right),$
2. $(F,A)_{\Psi}^{Gt}\tilde{\cup}(F,A)_{\Psi}^{Gt}=(F,A)_{\Psi}^{Gt}.$

Proof

1. By using definition 4.4 we want to prove that
$(F,A)_{\Psi}^{Gt}\tilde{\cup}\left((G,B)_{\Upsilon}^{Gt}\tilde{\cup}(H,C)_{\Omega}^{Gt}\right)=\left((F,A)_{\Psi}^{Gt}\tilde{\cup}(G,B)_{\Upsilon}^{Gt}\right)\tilde{\cup}(H,C)_{\Omega}^{Gt}.$

   But we know that if M, N and R are three fuzzy sets then $(A\cup N)\cup C=A\cup(B\cup C)$ and this fact also hold if M, N and R are three fuzzy soft expert sets. By using these facts we have
$(F,A)_{\Psi}^{Gt}\tilde{\cup}\left((G,B)_{\Upsilon}^{Gt}\tilde{\cup}(H,C)_{\Omega}^{Gt}\right)=\left(F_{\Psi}^{Gt}(\varepsilon)\tilde{\cup}\left(G_{\Upsilon}^{Gt}(\varepsilon)\tilde{\cup}H_{\Omega}(\varepsilon)\right),A\tilde{\cup}(B\tilde{\cup}C)\right).$

   $$=\left(\left(F_{\Psi}^{Gt}(\varepsilon)\tilde{\cup}G_{\Upsilon}^{Gt}(\varepsilon)\right)\tilde{\cup}H_{\Omega}(\varepsilon),(A\tilde{\cup}B)\tilde{\cup}C\right).$$

   $$=\left((F,A)_{\Psi}^{Gt}\tilde{\cup}(G,B)_{\Upsilon}^{Gt}\right)\tilde{\cup}(H,C,)_{\Omega}.$$

   $=\left((F,A)_{\Psi}^{Gt}\tilde{\cup}(G,B)_{\Upsilon}^{Gt}\right)\tilde{\cup}(H,C)_{\Omega}^{Gt}.$
2. Likewise, the proof of 2 can be shown in similar way.

□

4.4. Intersection

Definition 4.7. The *intersection* of two $(\overline{GT-FSES's})$ $(F,A)_{\Psi}^{Gt}$ and $(G,B)_{\Upsilon}^{Gt}$ over U, denoted by $(F,A)_{\Psi}^{Gt}\tilde{\cap}(G,B)_{\Upsilon}^{Gt}$, is the$(\overline{GT-FSES's})(H,C)_{\Omega}^{Gt}$ such that $C=M\cap N$, $\Omega=max\,(\Psi\cap\Upsilon)$,$\forall\,\varepsilon\in C$,
$H(\varepsilon)=\left(F(\varepsilon)\tilde{\cap}G(\varepsilon)\right)$ where $\tilde{\cap}$ is an generalized time fuzzy soft set intersection.

Example 4.8. Consider Example 4.5. By using basic fuzzy intersection (minimum) we have $(F,A)_{\Psi}^{Gt}\tilde{\cap}(G,B)_{\Upsilon}^{Gt}=(H,C)_{\Omega}^{Gt}$ Where

$$(H,C)_{\Omega}^{Gt}=\Bigg\{\left((e_2,n,1),\left\{\tfrac{u_1{}^{Gt_1}}{0.5},\tfrac{u_2{}^{Gt_1}}{0.7},\tfrac{u_3{}^{Gt_1}}{0.1},\tfrac{u_4{}^{Gt_1}}{0.3},0.5\right\}\right),\left((e_2,r,1),\left\{\tfrac{u_1{}^{Gt_1}}{0.2},\tfrac{u_2{}^{Gt_1}}{0.4},\tfrac{u_3{}^{Gt_1}}{0.7},\tfrac{u_4{}^{Gt_1}}{0.1},0.6\right\}\right),$$
$$\left((e_2,r,1),\left\{\tfrac{u_1{}^{Gt_2}}{0.5},\tfrac{u_2{}^{Gt_2}}{0.3},\tfrac{u_3{}^{Gt_2}}{0.2},\tfrac{u_4{}^{Gt_2}}{0.4},0.4\right\}\right),\left((e_3,n,1),\left\{\tfrac{u_1{}^{Gt_2}}{0.5},\tfrac{u_2{}^{Gt_2}}{0.6},\tfrac{u_3{}^{Gt_2}}{0.1},\tfrac{u_4{}^{Gt_2}}{0.3},0.7\right\}\right),$$
$$\left((e_3,m,1),\left\{\tfrac{u_1{}^{Gt_3}}{0.7},\tfrac{u_2{}^{Gt_3}}{0.4},\tfrac{u_3{}^{Gt_3}}{0.1},\tfrac{u_4{}^{Gt_3}}{0.8},0.4\right\}\right),\left((e_1,m,0),\left\{\tfrac{u_1{}^{Gt_1}}{0.3},\tfrac{u_2{}^{Gt_1}}{0.6},\tfrac{u_3{}^{Gt_1}}{0.7},\tfrac{u_4{}^{Gt_1}}{0.8},0.8\right\}\right),$$
$$\left((e_3,n,0),\left\{\tfrac{u_1{}^{Gt_1}}{0.5},\tfrac{u_2{}^{Gt_1}}{0.4},\tfrac{u_3{}^{Gt_1}}{0.5},\tfrac{u_4{}^{Gt_1}}{0.2},0.2\right\}\right),\left((e_3,r,0),\left\{\tfrac{u_1{}^{Gt_1}}{0.8},\tfrac{u_2{}^{Gt_1}}{0.2},\tfrac{u_3{}^{Gt_1}}{0.8},\tfrac{u_4{}^{Gt_1}}{0.4},0.4\right\}\right),$$
$$\left((e_1,m,0),\left\{\tfrac{u_1{}^{Gt_2}}{0.5},\tfrac{u_2{}^{Gt_2}}{0.4},\tfrac{u_3{}^{Gt_2}}{0.7},\tfrac{u_4{}^{Gt_2}}{0.4},0.2\right\}\right),\left((e_1,r,0),\left\{\tfrac{u_1{}^{Gt_2}}{0.7},\tfrac{u_2{}^{Gt_2}}{0.5},\tfrac{u_3{}^{Gt_2}}{0.6},\tfrac{u_4{}^{Gt_2}}{0.7},0.5\right\}\right),$$
$$\left((e_2,m,0),\left\{\tfrac{u_1{}^{Gt_2}}{0.6},\tfrac{u_2{}^{Gt_2}}{0.7},\tfrac{u_3{}^{Gt_2}}{0.4},\tfrac{u_4{}^{Gt_2}}{0.9},0.7\right\}\right),\left((e_3,m,0),\left\{\tfrac{u_1{}^{Gt_3}}{0.2},\tfrac{u_2{}^{Gt_3}}{0.6},\tfrac{u_3{}^{Gt_3}}{0.8},\tfrac{u_4{}^{Gt_3}}{0.1},0.2\right\}\right)\Bigg\}.$$

Proposition 4.9
If $(F,A)_{\Psi}^{Gt}$, $(G,B)_{\Upsilon}^{Gt}$ and $(H,C)_{\Omega}^{Gt}$ are three $(\overline{GT-FSES's})$ over U, then

1. $(F,A)_{\Psi}^{Gt}\tilde{\cap}((G,B)_{\Upsilon}^{Gt}\tilde{\cap}(H,C)_{\Omega}^{Gt}=((F,A)_{\Psi}^{Gt}\tilde{\cap}(G,B))_{\Upsilon}^{Gt}\tilde{\cap}(H,C)_{\Omega}^{Gt},$

2. $(F,A)^{Gt}_{\Psi} \widetilde{\cap} (F,A)^{Gt}_{\Psi} = (F,A)^{Gt}_{\Psi}$.

Proof

1. By using definition 4.7 we want to prove that
$(F,A)^{Gt}_{\Psi} \widetilde{\cap} \left((G,B)^{Gt}_{\Upsilon} \widetilde{\cap} (H,C)^{Gt}_{\Omega}\right) = \left((F,A)^{Gt}_{\Psi} \widetilde{\cap} (G,B)^{Gt}_{\Upsilon}\right) \widetilde{\cap} (H,C)^{Gt}_{\Omega}$.

   But we know that if M, N and R are three fuzzy sets then $(A \cap N) \cap C = A \cap (B \cap C)$ and this fact also hold if M, N and R are three generalized time fuzzy soft set. By using these facts we have
$$(F,A)^{Gt}_{\Psi} \widetilde{\cap} \left((G,B)^{Gt}_{\Upsilon} \widetilde{\cap} (H,C)^{Gt}_{\Omega}\right) = \left(F^{Gt}_{\Psi}(\varepsilon) \widetilde{\cap} \left(G^{Gt}_{\Upsilon}(\varepsilon) \widetilde{\cap} H_{\Omega}(\varepsilon)\right), A \widetilde{\cap} (B \widetilde{\cap} C)\right).$$
$$= \left(\left(F^{Gt}_{\Psi}(\varepsilon) \widetilde{\cap} G^{Gt}_{\Upsilon}(\varepsilon)\right) \widetilde{\cap} H_{\Omega}(\varepsilon), (A \widetilde{\cap} B) \widetilde{\cap} C\right).$$
$$= \left((F,A)^{Gt}_{\Psi} \widetilde{\cap} (G,B)^{Gt}_{\Upsilon}\right) \widetilde{\cap} (H,C,)_{\Omega}.$$
$$= \left((F,A)^{Gt}_{\Psi} \widetilde{\cap} (G,B)^{Gt}_{\Upsilon}\right) \widetilde{\cap} (H,C)^{Gt}_{\Omega}.$$
2. Likewise, the proof of b can be shown in similar way.

□

Proposition 4.10
If $(F,A)^{Gt}_{\Psi}$, $(G,B)^{Gt}_{\Upsilon}$ and $(H,C)^{Gt}_{\Omega}$ are three $(\overline{GT - FSES's})$ over U, then

1. $(F,A)^{Gt}_{\Psi} \widetilde{\cup} \left((G,B)^{Gt}_{\Upsilon} \widetilde{\cap} (H,C)^{Gt}_{\Omega}\right) = \left((F,A)^{Gt}_{\Psi} \widetilde{\cup} (G,B)^{Gt}_{\Upsilon}\right) \widetilde{\cap} \left((F,A)^{Gt}_{\Psi} \widetilde{\cup} (H,C)^{Gt}_{\Omega}\right)$,
2. $(F,A)^{Gt}_{\Psi} \widetilde{\cap} \left((G,B)^{Gt}_{\Upsilon} \widetilde{\cup} (H,C)^{Gt}_{\Omega}\right) = \left((F,A)^{Gt}_{\Psi} \widetilde{\cap} (G,B)^{Gt}_{\Upsilon}\right) \widetilde{\cup} \left((F,A)^{Gt}_{\Psi} \widetilde{\cap} (H,C)^{Gt}_{\Omega}\right)$.

Proof

1. We want to prove that
$(F,A)^{Gt}_{\Psi} \widetilde{\cap} \left((G,B)^{Gt}_{\Upsilon} \widetilde{\cup} (H,C)^{Gt}_{\Omega}\right) = \left((F,A)^{Gt}_{\Psi} \widetilde{\cap} (G,B)^{Gt}_{\Upsilon}\right) \widetilde{\cup} \left((F,A)^{Gt}_{\Psi} \widetilde{\cap} (H,C)^{Gt}_{\Omega}\right)$,

   But we know that if M, N and R are three fuzzy sets then $(A \cap N) \cup C = (A \cap N) \cup (A \cap C)$ and this fact also hold if M, N and R are three generalized time fuzzy soft set. By using these facts we have
$$(F,A)^{Gt}_{\Psi} \widetilde{\cap} \left((G,B)^{Gt}_{\Upsilon} \widetilde{\cup} (H,C)^{Gt}_{\Omega}\right) = \left(F^{Gt}_{\Psi}(\varepsilon) \cup \left(G^{Gt}_{\Upsilon}(\varepsilon) \cap H_{\Omega}(\varepsilon)\right), A \cup (B \cup C)\right).$$
$$= \left(\left(F^{Gt}_{\Psi}(\varepsilon) \cup G^{Gt}_{\Upsilon}(\varepsilon)\right), A \cup N\right) \cap (F(\varepsilon) \cup H_{\Omega}(\varepsilon), A \cup C).$$
$$= \left((F,A)^{Gt}_{\Psi} \widetilde{\cup} (G,B)^{Gt}_{\Upsilon}\right) \widetilde{\cap} \left((F,A) \widetilde{\cup} (H,C)^{Gt}_{\Omega}\right).$$
$$= \left((F,A)^{Gt}_{\Psi} \widetilde{\cap} (G,B)^{Gt}_{\Upsilon}\right) \widetilde{\cup} \left((F,A)^{Gt}_{\Psi} \widetilde{\cap} (H,C)^{Gt}_{\Omega}\right).$$
2. Likewise, the proof of b can be shown in similar way.

□

5. AND and OR Operations

This section defines the AND and OR operations for GT-FSES's, explains how they work, and provides some examples.

Definition 5.1. If $(F,A)_{\Psi}^{Gt}$ and $(G,B)_{\Upsilon}^{Gt}$ are two $(\overline{GT-FSES's})$ over U then "$(F,A)_{\Psi}^{Gt}$ AND $(G,B)_{\Upsilon}^{Gt}$" represent by $(F,A)_{\Psi}^{Gt} \wedge (G,B)_{\Upsilon}^{Gt}$ is defined by

$$(F,A)_{\Psi}^{Gt} \wedge (G,B)_{\Upsilon}^{Gt} = (H, M \times N)_{\Omega}$$

$\Omega = \Psi \times \Upsilon$, and $H(\alpha,\beta) = F(\alpha) \widetilde{\cap} G(\beta), \forall (\alpha,\beta) \in M \times N$.

Example 5.2. Consider Example 3.2. Let $(F,A)_{\Psi}^{Gt}$ and $(G,B)_{\Upsilon}^{Gt}$ are generalized time fuzzy soft set over U such that

$$(F,A)_{\Psi}^{Gt} = \left\{ \left((e_2,n,1), \left\{\frac{u_1{}^{Gt_1}}{0.5}, \frac{u_2{}^{Gt_1}}{0.7}, \frac{u_3{}^{Gt_1}}{0.2}, \frac{u_4{}^{Gt_1}}{0.5}, 0.5\right\}\right), \left((e_1,m,1), \left\{\frac{u_1{}^{Gt_2}}{0.4}, \frac{u_2{}^{Gt_2}}{0.8}, \frac{u_3{}^{Gt_2}}{0.2}, \frac{u_4{}^{Gt_2}}{0.4}, 0.9\right\}\right), \left((e_3,r,1), \left\{\frac{u_1{}^{Gt_3}}{0.4}, \frac{u_2{}^{Gt_3}}{0.7}, \frac{u_3{}^{Gt_3}}{0.5}, \frac{u_4{}^{Gt_3}}{0.6}, 0.6\right\}\right), \left((e_1,m,0), \left\{\frac{u_1{}^{Gt_1}}{0.3}, \frac{u_2{}^{Gt_1}}{0.6}, \frac{u_3{}^{Gt_1}}{0.7}, \frac{u_4{}^{Gt_1}}{0.8}, o.8\right\}\right) \right\}.$$

$$(G,B)_{\Upsilon}^{Gt} = \left\{ \left((e_3,n,1), \left\{\frac{u_1{}^{Gt_2}}{0.4}, \frac{u_2{}^{Gt_2}}{0.9}, \frac{u_3{}^{Gt_2}}{0.3}, \frac{u_4{}^{Gt_2}}{0.1}, 0.7\right\}\right), \left((e_2,r,0), \left\{\frac{u_1{}^{Gt_3}}{0.5}, \frac{u_2{}^{Gt_3}}{0.8}, \frac{u_3{}^{Gt_3}}{0.2}, \frac{u_4{}^{Gt_3}}{0.6}, 0.3\right\}\right) \right\}.$$

Then $(F,A)_{\Psi}^{Gt} \wedge (G,B)_{\Upsilon}^{Gt} = (H, A \times B)_{\Omega}^{Gt}$ where

$$\begin{aligned}(H, A\times B)_{\Omega}^{Gt} = \Bigg\{ & \left(\left((e_2,n,1)^{Gt_1}, (e_3,n,1)^{Gt_2}\right), \left\{\frac{u_1{}^{Gt_{1,2}}}{0.4}, \frac{u_2{}^{Gt_{1,2}}}{0.7}, \frac{u_3{}^{Gt_{1,2}}}{0.2}, \frac{u_4{}^{Gt_{1,2}}}{0.1}, 0.5\right\}\right), \\ & \left(\left((e_2,n,1)^{Gt_1}, (e_2,r,0)^{Gt_3}\right), \left\{\frac{u_1{}^{Gt_{1,3}}}{0.5}, \frac{u_2{}^{Gt_{1,3}}}{0.7}, \frac{u_3{}^{Gt_{1,3}}}{0.2}, \frac{u_4{}^{Gt_{1,3}}}{0.5}, 0.3\right\}\right), \\ & \left(\left((e_1,m,1)^{Gt_2}, (e_3,n,1)^{Gt_2}\right), \left\{\frac{u_1{}^{Gt_{2,2}}}{0.4}, \frac{u_2{}^{Gt_{2,2}}}{0.8}, \frac{u_3{}^{Gt_{2,2}}}{0.2}, \frac{u_4{}^{Gt_{2,2}}}{0.1}, 0.7\right\}\right), \\ & \left(\left((e_1,m,1)^{Gt_2}, (e_2,r,0)^{Gt_3}\right), \left\{\frac{u_1{}^{Gt_{2,3}}}{0.4}, \frac{u_2{}^{Gt_{2,3}}}{0.8}, \frac{u_3{}^{Gt_{2,3}}}{0.2}, \frac{u_4{}^{Gt_{2,3}}}{0.4}, 0.3\right\}\right), \\ & \left(\left((e_3,r,1)^{Gt_3}, (e_3,n,1)^{Gt_2}\right), \left\{\frac{u_1{}^{Gt_{3,2}}}{0.4}, \frac{u_2{}^{Gt_{3,2}}}{0.7}, \frac{u_3{}^{Gt_{3,2}}}{0.3}, \frac{u_4{}^{Gt_{3,2}}}{0.1}, 0.6\right\}\right), \\ & \left(\left((e_3,r,1)^{Gt_3}, (e_2,r,0)^{Gt_3}\right), \left\{\frac{u_1{}^{Gt_{3,3}}}{0.4}, \frac{u_2{}^{Gt_{3,3}}}{0.7}, \frac{u_3{}^{Gt_{3,3}}}{0.2}, \frac{u_4{}^{Gt_{3,3}}}{0.6}, 0.3\right\}\right), \\ & \left(\left((e_1,m,0)^{Gt_1}, (e_3,n,1)^{Gt_2}\right), \left\{\frac{u_1{}^{Gt_{1,2}}}{0.3}, \frac{u_2{}^{Gt_{1,2}}}{0.6}, \frac{u_3{}^{Gt_{1,2}}}{0.3}, \frac{u_4{}^{Gt_{1,2}}}{0.1}, 0.7\right\}\right), \\ & \left(\left((e_1,m,0)^{Gt_1}, (e_2,r,0)^{Gt_3}\right), \left\{\frac{u_1{}^{Gt_{1,3}}}{0.3}, \frac{u_2{}^{Gt_{1,3}}}{0.6}, \frac{u_3{}^{Gt_{1,3}}}{0.2}, \frac{u_4{}^{Gt_{1,3}}}{0.6}, 0.3\right\}\right) \Bigg\}.\end{aligned}$$

Definition 5.3. If $(F,A)_{\Psi}^{Gt}$ and $(G,B)_{\Upsilon}^{Gt}$ are two $(\overline{GT-FSES's})$ over U then "$(F,A)_{\Psi}^{Gt}$ OR $(G,B)_{\Upsilon}^{Gt}$" represent by $(F,A)_{\Psi}^{Gt} \vee (G,B)_{\Upsilon}^{Gt}$ is defined by

$$(F,A)_{\Psi}^{Gt} \vee (G,B)_{\Upsilon}^{Gt} = (H, M \times N)_{\Omega}$$

$\Omega = \Psi \times \Upsilon$, and $H(\alpha,\beta) = F(\alpha) \widetilde{\cup} G(\beta), \forall (\alpha,\beta) \in M \times N$.

Example 5.4. Think about the example 5.2. Using the maximum basic fuzzy union, we have $(F,A)_{\Psi}^{Gt} \vee (G,B)_{\Upsilon}^{Gt} = (H, M \times N)_{\Omega}$ where

$(G,B)^{Gt}_{\Upsilon}$" denoted by $(H,C)^{Gt}_{\Omega} = (F,A)^{Gt}_{\Psi} \bigvee (G,B)^{Gt}_{\Upsilon}$ where

$$(H,C)^{Gt}_{\Omega} = \left\{ \left(\left((e_2,n,1)^{Gt_1}, (e_3,n,1)^{Gt_2} \right), \left\{ \frac{u_1{}^{Gt_{1,2}}}{0.5}, \frac{u_2{}^{Gt_{1,2}}}{0.9}, \frac{u_3{}^{Gt_{1,2}}}{0.3}, \frac{u_4{}^{Gt_{1,2}}}{0.5}, 0.7 \right\} \right), \right.$$

$$\left(\left((e_2,n,1)^{Gt_1}, (e_2,r,0)^{Gt_3} \right), \left\{ \frac{u_1{}^{Gt_{1,3}}}{0.5}, \frac{u_2{}^{Gt_{1,3}}}{0.8}, \frac{u_3{}^{Gt_{1,3}}}{0.2}, \frac{u_4{}^{Gt_{1,3}}}{0.6}, 0.5 \right\} \right),$$

$$\left(\left((e_1,m,1)^{Gt_2}, (e_3,n,1)^{Gt_2} \right), \left\{ \frac{u_1{}^{Gt_{2,2}}}{0.4}, \frac{u_2{}^{Gt_{2,2}}}{0.9}, \frac{u_3{}^{Gt_{2,2}}}{0.3}, \frac{u_4{}^{Gt_{2,2}}}{0.4}, 0.9 \right\} \right),$$

$$\left(\left((e_1,m,1)^{Gt_2}, (e_2,r,0)^{Gt_3} \right), \left\{ \frac{u_1{}^{Gt_{2,3}}}{0.5}, \frac{u_2{}^{Gt_{2,3}}}{0.8}, \frac{u_3{}^{Gt_{2,3}}}{0.2}, \frac{u_4{}^{Gt_{2,3}}}{0.6}, 0.9 \right\} \right),$$

$$\left(\left((e_3,r,1)^{Gt_3}, (e_3,n,1)^{Gt_2} \right), \left\{ \frac{u_1{}^{Gt_{3,2}}}{0.4}, \frac{u_2{}^{Gt_{3,2}}}{0.9}, \frac{u_3{}^{Gt_{3,2}}}{0.5}, \frac{u_4{}^{Gt_{3,2}}}{0.6}, 0.7 \right\} \right),$$

$$\left(\left((e_3,r,1)^{Gt_3}, (e_2,r,0)^{Gt_3} \right), \left\{ \frac{u_1{}^{Gt_{3,3}}}{0.5}, \frac{u_2{}^{Gt_{3,3}}}{0.8}, \frac{u_3{}^{Gt_{3,3}}}{0.5}, \frac{u_4{}^{Gt_{3,3}}}{0.6}, 0.6 \right\} \right),$$

$$\left(\left((e_1,m,0)^{Gt_1}, (e_3,n,1)^{Gt_2} \right), \left\{ \frac{u_1{}^{Gt_{1,2}}}{0.4}, \frac{u_2{}^{Gt_{1,2}}}{0.9}, \frac{u_3{}^{Gt_{1,2}}}{0.7}, \frac{u_4{}^{Gt_{1,2}}}{0.8}, 0.8 \right\} \right),$$

$$\left. \left(\left((e_1,m,0)^{Gt_1}, (e_2,r,0)^{Gt_3} \right), \left\{ \frac{u_1{}^{Gt_{1,3}}}{0.5}, \frac{u_2{}^{Gt_{1,3}}}{0.8}, \frac{u_3{}^{Gt_{1,3}}}{0.7}, \frac{u_4{}^{Gt_{1,3}}}{0.8}, 0.8 \right\} \right) \right\}.$$

Proposition 5.5
If $(F,A)^{Gt}_{\Psi}$ and $(G,B)^{Gt}_{\Upsilon}$ are two fuzzy parameterized fuzzy soft expert sets over W, then

1. $\left((F,A)^{Gt}_{\Psi} \wedge (G,B)^{Gt}_{\Upsilon} \right)^c = (F,A)^{Gt}_{\Psi^c} \vee (G,B)^{Gt}_{\Upsilon^c}$
2. $\left((F,A)^{Gt}_{\Psi} \vee (G,B)^{Gt}_{\Upsilon} \right)^c = (F,A)^{Gt}_{\Psi^c} \wedge (G,B)^{Gt}_{\Upsilon^c}$

Proof

1. We want to prove that $\left((F,A)^{Gt}_{\Psi} \wedge (G,B)^{Gt}_{\Upsilon} \right)^c = (F,A)^{Gt}_{\Psi^c} \vee (G,B)^{Gt}_{\Upsilon^c}$

   But we know that if M and N are two generalized time fuzzy soft setthen $(M \wedge N)^c = A^c \vee B^c$ and this fact also hold if M and N are two generalized time fuzzy soft set. By using these facts we have:
   Suppose that $(F,A)^{Gt}_{\Psi} \wedge (G,B)^{Gt}_{\Upsilon} = (O, M \times N)$.
   Therefore, $\left((F,A)^{Gt}_{\Psi} \wedge (G,B)^{Gt}_{\Upsilon} \right)^c = (O, M \times N)^c = (O^c, (M \times N))$. Now,
   $\left((F,A)^{Gt}_{\Psi} \vee (G,B)^{Gt}_{\Upsilon} \right)^c = \left((F^{Gt}{}_{\Psi^c}, M) \vee (G^{Gt}{}_{\Upsilon^c}, N) \right)$
   $= (J, (M \times N))$, where $J(x,y) = t\left(c(F^{Gt}_{\Psi}(\alpha)), c(G^{Gt}_{\Upsilon}(\beta)) \right)$.
   Now, take $(\alpha, \beta) \in (M \times N)$.
   Then,
   $O^c(\alpha,\beta) = \bar{1} - O(\alpha,\beta)$,
   $= \bar{1} - [F^{Gt}_{\Psi}(\alpha) \bigcup G^{Gt}_{\Upsilon}(\beta)]$
   $= [\bar{1} - F^{Gt}_{\Psi}(\alpha)] \bigcap [\bar{1} - G^{Gt}_{\Upsilon}(\beta)]$
   $=t\left(c(F^{Gt}_{\Psi}(\alpha)), c(G^{Gt}_{\Upsilon}(\beta)) \right)$
   $= J(\alpha,\beta)$
   Therefore O^c and F are the same. Hence, proved.

2. The proof is uncomplicated and straightforward .

□

Proposition 5.6
If $(F,A)_{\Psi}^{Gt}$, $(G,N)_{\Upsilon}^{Gt}$ and $(H,C)_{\Omega}^{Gt}$ are three generalized time fuzzy soft set over W, then

1. $(F,A)_{\Psi}^{Gt} \wedge \left((G,B)_{\Upsilon}^{Gt} \wedge (H,C)_{\Omega}^{Gt}\right) = \left((F,A)_{\Psi}^{Gt} \wedge (G,B)_{\Upsilon}^{Gt}\right) \wedge (H,C)_{\Omega}^{Gt}$),
2. $(F,A)_{\Psi}^{Gt} \vee \left((G,B)_{\Upsilon}^{Gt} \vee (H,C)_{\Omega}^{Gt}\right) = \left((F,A)_{\Psi}^{Gt} \vee (G,B)_{\Upsilon}^{Gt}\right) \vee (H,C)_{\Omega}^{Gt}$),
3. $(F,A)_{\Psi}^{Gt} \vee \left((G,B)_{\Upsilon}^{Gt} \wedge (H,C)_{\Omega}^{Gt}\right) = \left((F,A)_{\Psi}^{Gt} \vee (G,B)_{\Upsilon}^{Gt}\right) \wedge \left((F,A)_{\Psi}^{Gt} \vee (H,C)_{\Omega}^{Gt}\right)$,
4. $(F,A)_{\Psi}^{Gt} \wedge \left((G,B)_{\Upsilon}^{Gt} \vee (H,C)_{\Omega}^{Gt}\right) = \left((F,A)_{\Psi}^{Gt} \wedge (G,B)_{\Upsilon}^{Gt}\right) \vee \left((F,A)_{\Psi}^{Gt} \wedge (H,C)_{\Omega}^{Gt}\right)$.

Proof

1. Our goal is showing that $(F,A)_{\Psi}^{Gt} \wedge \left((G,B)_{\Upsilon}^{Gt} \wedge (H,C)_{\Omega}^{Gt}\right) = \left((F,A)_{\Psi}^{Gt} \wedge (G,B)_{\Upsilon}^{Gt}\right) \wedge (H,C)_{\Omega}^{Gt}$,

   But we know that if M, N and R are three fuzzy sets then $(M \wedge N) \wedge C = M \wedge (B \wedge C)$ and this fact also hold if M, N and R are three generalized time fuzzy soft set. By using these facts we have
   Suppose that $(G,N)_{\Upsilon}^{Gt} \wedge (H,C)_{\Omega}^{Gt} = t\left(G_{\Upsilon}^{Gt}(\alpha), H_{\Omega}(\beta)\right), \forall (\alpha,\beta) \in B \times C$.

   Then
   $(F,A)_{\Psi}^{Gt} \wedge \left((G,B)_{\Upsilon}^{Gt} \wedge (H,C)_{\Omega}^{Gt}\right) = t\left(F_{\Psi}^{Gt}(\gamma), t\left(G_{\Upsilon}^{Gt}(\alpha), H_{\Omega}(\beta)\right)\right), \forall (\gamma,(\alpha,\beta)) \in A \times (B \times C)$.

   $$= t\left(t\left(F_{\Psi}^{Gt}(\gamma), G_{\Upsilon}^{Gt}(\alpha)\right), H_{\Omega}(\beta)\right), \forall((\gamma,\alpha),\beta) \in (A \times N) \times C.$$

   $$= \left((F,A)_{\Psi}^{Gt} \wedge (G,B)_{\Upsilon}^{Gt}\right) \wedge (H,C)_{\Omega}^{Gt}).$$
2. The proof is uncomplicated and straightforward .
3. The proof is uncomplicated and straightforward .
4. The proof is uncomplicated and straightforward .

□

6. A Decision-Making Application of Generalized Time Fuzzy Soft Expert Sets With Two Viewpoints

In this section, we provide hypothetical application of the generalized time fuzzy soft expert set theory in a decision making problem which demonstrate that this method can be successfully work and it can be applied to problems of many fields that contain uncertainty. The application consists set of two opinions $\{\text{agree}, \text{disagree}\}$.

Example 6.1. Suppose that one of the broadcasting channels want to invite experts to evaluate their show through the discussion of a controversial issue and obtain their opinion of the situation. The producers of the show used the following criteria to determine how to evaluate their findings. Their four alternatives are as follows: $U = \{u_1,\ u_2,\ u_3, u_4\}$,. suppose there are three parameters $E = \{e_1, e_2, e_3\}$, choose is by the experts for the programs. For $.i$ = 1, 2, 3, the parameters e_i $(i = 1, 2, 3, 4, 5)$ stand for "this criteria to discriminate and this criteria is independent of the other criteria ", "this criteria measures one thing and the universal criteria ", "the criteria that is important to some of the stakeholders". $T = \{t_1, t_2, t_3, t_4\}$ be a set of previous time periods. Let $X = \{m, n\}$ be a set of committee members. From those findings we can find the most suitable choice for the decision. After a serious discussion the committee constructs the following generalized fuzzy soft expert set.

$$(F,Z)^{Gt} = \left\{ \left((e_1,m,1), \left\{\frac{{u_1}^{Gt_1}}{0.6}, \frac{{u_2}^{Gt_1}}{0.3}, \frac{{u_3}^{Gt_1}}{0.5}, \frac{{u_4}^{Gt_1}}{0.6}, 0.2\right\}\right), \left((e_1,n,1), \left\{\frac{{u_1}^{Gt_1}}{0.7}, \frac{{u_2}^{Gt_1}}{0.5}, \frac{{u_3}^{Gt_1}}{0.4}, \frac{{u_4}^{Gt_1}}{0.7}, 0.6\right\}\right), \right.$$

$$\left((e_2,m,1), \left\{\frac{{u_1}^{Gt_1}}{0.7}, \frac{{u_2}^{Gt_1}}{0.4}, \frac{{u_3}^{Gt_1}}{0.2}, \frac{{u_4}^{Gt_1}}{0.7}, 0.5\right\}\right), \left((e_2,n,1), \left\{\frac{{u_1}^{Gt_1}}{0.5}, \frac{{u_2}^{Gt_1}}{0.6}, \frac{{u_3}^{Gt_1}}{0.3}, \frac{{u_4}^{Gt_1}}{0.5}, 0.2\right\}\right),$$

$$\Big((e_3,m,1),\Big\{\tfrac{u_1{}^{Gt_1}}{0.4},\tfrac{u_2{}^{Gt_1}}{0.6},\tfrac{u_3{}^{Gt_1}}{0.7},\tfrac{u_4{}^{Gt_1}}{0.3},0.6\Big\}\Big),\Big((e_3,n,1),\Big\{\tfrac{u_1{}^{Gt_1}}{0.6},\tfrac{u_2{}^{Gt_1}}{0.5},\tfrac{u_3{}^{Gt_1}}{0.9},\tfrac{u_4{}^{Gt_1}}{0.5},0.5\Big\}\Big),$$

$$\Big((e_1,m,1),\Big\{\tfrac{u_1{}^{Gt_2}}{0.4},\tfrac{u_2{}^{Gt_2}}{0.8},\tfrac{u_3{}^{Gt_2}}{0.2},\tfrac{u_4{}^{Gt_2}}{0.4},0.7\Big\}\Big),\Big((e_1,n,1),\Big\{\tfrac{u_1{}^{Gt_2}}{0.6},\tfrac{u_2{}^{Gt_2}}{0.9},\tfrac{u_3{}^{Gt_2}}{0.4},\tfrac{u_4{}^{Gt_2}}{0.8},0.5\Big\}\Big),$$

$$\Big((e_2,m,1),\Big\{\tfrac{u_1{}^{Gt_2}}{0.4},\tfrac{u_2{}^{Gt_2}}{0.9},\tfrac{u_3{}^{Gt_2}}{0.7},\tfrac{u_4{}^{Gt_2}}{0.5},0.1\Big\}\Big),\Big((e_2,n,1),\Big\{\tfrac{u_1{}^{Gt_2}}{0.6},\tfrac{u_2{}^{Gt_2}}{0.8},\tfrac{u_3{}^{Gt_2}}{0.5},\tfrac{u_4{}^{Gt_2}}{0.6},0.5\Big\}\Big),$$

$$\Big((e_3,m,1),\Big\{\tfrac{u_1{}^{Gt_2}}{0.6},\tfrac{u_2{}^{Gt_2}}{0.4},\tfrac{u_3{}^{Gt_2}}{0.1},\tfrac{u_4{}^{Gt_2}}{0.8},0.5\Big\}\Big),\Big((e_3,n,1),\Big\{\tfrac{u_1{}^{Gt_2}}{0.8},\tfrac{u_2{}^{Gt_2}}{0.6},\tfrac{u_3{}^{Gt_2}}{0.3},\tfrac{u_4{}^{Gt_2}}{0.7},0.7\Big\}\Big),$$

$$\Big((e_1,m,1),\Big\{\tfrac{u_1{}^{Gt_3}}{0.2},\tfrac{u_2{}^{Gt_3}}{0.7},\tfrac{u_3{}^{Gt_3}}{0.8},\tfrac{u_4{}^{Gt_3}}{0.3},0.6\Big\}\Big),\Big((e_1,n,1),\Big\{\tfrac{u_1{}^{Gt_3}}{0.4},\tfrac{u_2{}^{Gt_3}}{0.9},\tfrac{u_3{}^{Gt_3}}{0.6},\tfrac{u_4{}^{Gt_3}}{0.5},0.6\Big\}\Big),$$

$$\Big((e_2,m,1),\Big\{\tfrac{u_1{}^{Gt_3}}{0.5},\tfrac{u_2{}^{Gt_3}}{0.4},\tfrac{u_3{}^{Gt_3}}{0.7},\tfrac{u_4{}^{Gt_3}}{0.9},0.4\Big\}\Big),\Big((e_2,n,1),\Big\{\tfrac{u_1{}^{Gt_3}}{0.8},\tfrac{u_2{}^{Gt_3}}{0.6},\tfrac{u_3{}^{Gt_3}}{0.2},\tfrac{u_4{}^{Gt_3}}{0.7},0.2\Big\}\Big),$$

$$\Big((e_3,m,1),\Big\{\tfrac{u_1{}^{Gt_3}}{0.7},\tfrac{u_2{}^{Gt_3}}{0.6},\tfrac{u_3{}^{Gt_3}}{0.3},\tfrac{u_4{}^{Gt_3}}{0.7},0.8\Big\}\Big),\Big((e_3,n,1),\Big\{\tfrac{u_1{}^{Gt_3}}{0.6},\tfrac{u_2{}^{Gt_3}}{0.5},\tfrac{u_3{}^{Gt_3}}{0.3},\tfrac{u_4{}^{Gt_3}}{0.9},0.7\Big\}\Big),$$

$$\Big((e_1,m,1),\Big\{\tfrac{u_1{}^{t_4}}{0.2},\tfrac{u_2{}^{t_4}}{0.9},\tfrac{u_3{}^{t_4}}{0.6},\tfrac{u_4{}^{t_4}}{0.7},0.1\Big\}\Big),\Big((e_1,n,1),\Big\{\tfrac{u_1{}^{t_4}}{0.7},\tfrac{u_2{}^{t_4}}{0.6},\tfrac{u_3{}^{t_4}}{0.8},\tfrac{u_4{}^{t_4}}{0.4},0.1\Big\}\Big),$$

$$\Big((e_2,m,1),\Big\{\tfrac{u_1{}^{t_4}}{0.8},\tfrac{u_2{}^{t_4}}{0.6},\tfrac{u_3{}^{t_4}}{0.2},\tfrac{u_4{}^{t_4}}{0.7},0.5\Big\}\Big),\Big((e_2,n,1),\Big\{\tfrac{u_1{}^{t_4}}{0.7},\tfrac{u_2{}^{t_4}}{0.4},\tfrac{u_3{}^{t_4}}{0.5},\tfrac{u_4{}^{t_4}}{0.8},0.4\Big\}\Big),$$

$$\Big((e_3,m,1),\Big\{\tfrac{u_1{}^{t_4}}{0.7},\tfrac{u_2{}^{t_4}}{0.5},\tfrac{u_3{}^{t_4}}{0.3},\tfrac{u_4{}^{t_4}}{0.9},0.4\Big\}\Big),\Big((e_3,n,1),\Big\{\tfrac{u_1{}^{t_4}}{0.6},\tfrac{u_2{}^{t_4}}{0.8},\tfrac{u_3{}^{t_4}}{0.4},\tfrac{u_4{}^{t_4}}{0.5},0.4\Big\}\Big),$$

$$\Big((e_1,m,0),\Big\{\tfrac{u_1{}^{Gt_1}}{0.5},\tfrac{u_2{}^{Gt_1}}{0.6},\tfrac{u_3{}^{Gt_1}}{0.4},\tfrac{u_4{}^{Gt_1}}{0.3},0.4\Big\}\Big),\Big((e_1,n,0),\Big\{\tfrac{u_1{}^{Gt_1}}{0.4},\tfrac{u_2{}^{Gt_1}}{0.5},\tfrac{u_3{}^{Gt_1}}{0.7},\tfrac{u_4{}^{Gt_1}}{0.2},0.5\Big\}\Big),$$

$$\Big((e_2,m,0),\Big\{\tfrac{u_1{}^{Gt_1}}{0.4},\tfrac{u_2{}^{Gt_1}}{0.6},\tfrac{u_3{}^{Gt_1}}{0.7},\tfrac{u_4{}^{Gt_1}}{0.4},0.8\Big\}\Big),\Big((e_2,n,0),\Big\{\tfrac{u_1{}^{Gt_1}}{0.4},\tfrac{u_2{}^{Gt_1}}{0.3},\tfrac{u_3{}^{Gt_1}}{0.6},\tfrac{u_4{}^{Gt_1}}{0.6},0.7\Big\}\Big),$$

$$\Big((e_3,m,0),\Big\{\tfrac{u_1{}^{Gt_1}}{0.7},\tfrac{u_2{}^{Gt_1}}{0.5},\tfrac{u_3{}^{Gt_1}}{0.3},\tfrac{u_4{}^{Gt_1}}{0.4},0.5\Big\}\Big),\Big((e_3,n,0),\Big\{\tfrac{u_1{}^{Gt_1}}{0.4},\tfrac{u_2{}^{Gt_1}}{0.6},\tfrac{u_3{}^{Gt_1}}{0.2},\tfrac{u_4{}^{Gt_1}}{0.6},0.7\Big\}\Big),$$

$$\Big((e_1,m,0),\Big\{\tfrac{u_1{}^{Gt_2}}{0.7},\tfrac{u_2{}^{Gt_2}}{0.4},\tfrac{u_3{}^{Gt_2}}{0.7},\tfrac{u_4{}^{Gt_2}}{0.5},0.6\Big\}\Big),\Big((e_1,n,0),\Big\{\tfrac{u_1{}^{Gt_2}}{0.5},\tfrac{u_2{}^{Gt_2}}{0.2},\tfrac{u_3{}^{Gt_2}}{0.5},\tfrac{u_4{}^{Gt_2}}{0.4},0.5\Big\}\Big),$$

$$\Big((e_2,m,0),\Big\{\tfrac{u_1{}^{Gt_2}}{0.2},\tfrac{u_2{}^{Gt_2}}{0.3},\tfrac{u_3{}^{Gt_2}}{0.4},\tfrac{u_4{}^{Gt_2}}{0.6},0.2\Big\}\Big),\Big((e_2,n,0),\Big\{\tfrac{u_1{}^{Gt_2}}{0.5},\tfrac{u_2{}^{Gt_2}}{0.2},\tfrac{u_3{}^{Gt_2}}{0.7},\tfrac{u_4{}^{Gt_2}}{0.3},0.3\Big\}\Big),$$

$$\Big((e_3,m,0),\Big\{\tfrac{u_1{}^{Gt_2}}{0.3},\tfrac{u_2{}^{Gt_2}}{0.5},\tfrac{u_3{}^{Gt_2}}{0.8},\tfrac{u_4{}^{Gt_2}}{0.3},0.5\Big\}\Big),\Big((e_3,n,0),\Big\{\tfrac{u_1{}^{Gt_2}}{0.5},\tfrac{u_2{}^{Gt_2}}{0.4},\tfrac{u_3{}^{Gt_2}}{0.7},\tfrac{u_4{}^{Gt_2}}{0.2},0.4\Big\}\Big),$$

$$\Big((e_1,m,0),\Big\{\tfrac{u_1{}^{Gt_3}}{0.7},\tfrac{u_2{}^{Gt_3}}{0.2},\tfrac{u_3{}^{Gt_3}}{0.1},\tfrac{u_4{}^{Gt_3}}{0.6},0.4\Big\}\Big),\Big((e_1,n,0),\Big\{\tfrac{u_1{}^{Gt_3}}{0.3},\tfrac{u_2{}^{Gt_3}}{0.4},\tfrac{u_3{}^{Gt_3}}{0.6},\tfrac{u_4{}^{Gt_3}}{0.4},0.2\Big\}\Big),$$

$$\Big((e_2,m,0),\Big\{\tfrac{u_1{}^{Gt_3}}{0.3},\tfrac{u_2{}^{Gt_3}}{0.7},\tfrac{u_3{}^{Gt_3}}{0.4},\tfrac{u_4{}^{Gt_3}}{0.1},0.1\Big\}\Big),\Big((e_2,n,0),\Big\{\tfrac{u_1{}^{Gt_3}}{0.1},\tfrac{u_2{}^{Gt_3}}{0.5},\tfrac{u_3{}^{Gt_3}}{0.6},\tfrac{u_4{}^{Gt_3}}{0.5},0.5\Big\}\Big),$$

$$\Big((e_3,m,0),\Big\{\tfrac{u_1{}^{Gt_3}}{0.4},\tfrac{u_2{}^{Gt_3}}{0.6},\tfrac{u_3{}^{Gt_3}}{0.8},\tfrac{u_4{}^{Gt_3}}{0.3},0.5\Big\}\Big),\Big((e_3,n,0),\Big\{\tfrac{u_1{}^{Gt_3}}{0.3},\tfrac{u_2{}^{Gt_3}}{0.4},\tfrac{u_3{}^{Gt_3}}{0.7},\tfrac{u_4{}^{Gt_3}}{0.2},0.2\Big\}\Big),$$

$$\Big((e_1,m,0),\Big\{\tfrac{u_1{}^{t_4}}{0.6},\tfrac{u_2{}^{t_4}}{0.1},\tfrac{u_3{}^{t_4}}{0.3},\tfrac{u_4{}^{t_4}}{0.4},0.1\Big\}\Big),\Big((e_1,n,0),\Big\{\tfrac{u_1{}^{t_4}}{0.4},\tfrac{u_2{}^{t_4}}{0.4},\tfrac{u_3{}^{t_4}}{0.3},\tfrac{u_4{}^{t_4}}{0.7},0.1\Big\}\Big),$$

$$\Big((e_2,m,0),\Big\{\tfrac{u_1{}^{t_4}}{0.3},\tfrac{u_2{}^{t_4}}{0.3},\tfrac{u_3{}^{t_4}}{0.7},\tfrac{u_4{}^{t_4}}{0.2},0.6\Big\}\Big),\Big((e_2,n,0),\Big\{\tfrac{u_1{}^{t_4}}{0.4},\tfrac{u_2{}^{t_4}}{0.6},\tfrac{u_3{}^{t_4}}{0.6},\tfrac{u_4{}^{t_4}}{0.3},0.6\Big\}\Big),$$

$$\Big((e_3,m,0),\Big\{\tfrac{u_1{}^{t_4}}{0.6},\tfrac{u_2{}^{t_4}}{0.4},\tfrac{u_3{}^{t_4}}{0.7},\tfrac{u_4{}^{t_4}}{0.2},0.7\Big\}\Big),\Big((e_3,n,0),\Big\{\tfrac{u_1{}^{t_4}}{0.5},\tfrac{u_2{}^{t_4}}{0.3},\tfrac{u_3{}^{t_4}}{0.7},\tfrac{u_4{}^{t_4}}{0.5},0.5\Big\}\Big)\Bigg\}.$$

6.1. Algorithm

[8] presented an algorithm to solve fuzzy decision making problems based on generalized fuzzy soft sets. Here some modifications on notations and technical terms of the algorithm have been made to fit the context of our

discussion. The following algorithm may be followed by committee to evaluate their choice to find the most suitable choice for the decision.

1. Input the GT-FSES $(F, Z)^{Gt}$
2. Find an agree- generalized time fuzzy soft expert set and a disagree- generalized time fuzzy soft expert set, Tables (1, 2).
3. Find the tabular representation of $F\,(Z)$ as in Tables (3, 4), where $F\,(Z)$ defined as follows:

$$F\,(z) = \left\{ \frac{u}{\sum_{i=1}^{n} t_i F_{t_i}(z) \big/ n \sum_{i=1}^{n} F_{t_i}(z)} : u \in U, z \in Z \right\} \tag{1}$$

   Also we use the following formula to get Λ_{e_i} score:

$$Score\,(\Lambda_{x_i})^{t_i} = \left\{ \frac{\lambda}{\sum_{i=1}^{n} \lambda t_i \big/ n \sum_{i=1}^{n} \lambda t_i} : \lambda \in \Lambda, t_i \in T, x_i \in X \right\} \tag{2}$$

   where $n = |T|$
4. Calculate the score of each of such expert in agree-GTFSES and disagree-GTFSES.
5. Calculate the final score by subtracting the scores of expert in the agree-GTFSES from the scores of expert in disagree-GTFSES.
   The expert with the highest score is the desired expert.

Table 1. Agreement on Generalized Time Fuzzy Soft Expert Set (Agree-GT-FSES)

U	u_1	u_2	u_3	u_4	Λ
$(e_1, m)^{Gt_1}$	0.6	0.3	0.5	0.6	0.2
$(e_1, m)^{Gt_2}$	0.4	0.8	0.2	0.4	0.7
$(e_1, m)^{Gt_3}$	0.2	0.7	0.8	0.3	0.6
$(e_1, m)^{Gt_4}$	0.2	0.9	0.6	0.7	0.1
$(e_1, n)^{Gt_1}$	0.7	0.5	0.4	0.7	0.6
$(e_1, n)^{Gt_2}$	0.6	0.9	0.4	0.8	0.5
$(e_1, n)^{Gt_3}$	0.4	0.9	0.6	0.5	0.6
$(e_1, n)^{Gt_4}$	0.7	0.6	0.8	0.4	0.1
$(e_2, m)^{Gt_1}$	0.7	0.4	0.2	0.7	0.5
$(e_2, m)^{Gt_2}$	0.4	0.9	0.7	0.5	0.1
$(e_2, m)^{Gt_3}$	0.5	0.4	0.7	0.9	0.4
$(e_2, m)^{Gt_4}$	0.8	0.6	0.2	0.7	0.5
$(e_2, n)^{Gt_1}$	0.5	0.6	0.3	0.5	0.2
$(e_2, n)^{Gt_2}$	0.6	0.8	0.5	0.6	0.5
$(e_2, n)^{Gt_3}$	0.8	0.6	0.2	0.7	0.2
$(e_2, n)^{Gt_4}$	0.7	0.4	0.5	0.8	0.4

Continued on next page

Table 1 – continued from previous page

U	u_1	u_2	u_3	u_4	Λ
$(e_3, m)^{Gt_1}$	0.4	0.6	0.7	0.3	0.6
$(e_3, m)^{Gt_2}$	0.6	0.4	0.1	0.8	0.5
$(e_3, m)^{Gt_3}$	0.7	0.6	0.3	0.7	0.8
$(e_3, m)^{Gt_4}$	0.7	0.5	0.3	0.9	0.4
$(e_3, n)^{Gt_1}$	0.6	0.5	0.9	0.5	0.5
$(e_3, n)^{Gt_2}$	0.8	0.6	0.3	0.7	0.7
$(e_3, n)^{Gt_3}$	0.6	0.5	0.3	0.9	0.7
$(e_3, n)^{Gt_4}$	0.6	0.8	0.4	0.8	0.5

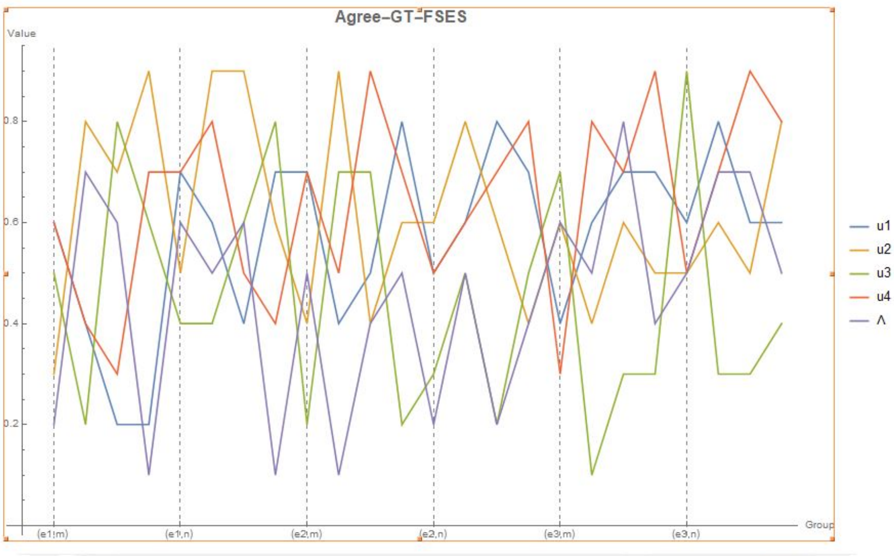

Figure 1. Grouped line plot of features u_1–u_4 and target Λ over 24 samples for agree opinions.

Line plot of the feature variables u_1, u_2, u_3, u_4 and the target variable Λ across 24 grouped data samples derived from the table. The dataset is organized into six distinct groups based on experiment and condition pairs: $(e1, m)$, $(e1, n)$, $(e2, m)$, $(e2, n)$, $(e3, m)$, and $(e3, n)$, with each group containing four samples. These groups are visually separated using vertical dashed grid lines and labeled along the x-axis for clarity.

Each line represents the trend of a specific variable across all 24 samples:

- u_1 to u_4: Input or feature values potentially derived from different modalities or sensor outputs across time or trial conditions.
- Λ: Target or output variable, potentially representing a learned or inferred quantity (e.g., confidence, agreement score, classification weight).

Table 2. Disagreement on Generalized Time Fuzzy Soft Expert Set (Disagree-GT-FSES)

U	u_1	u_2	u_3	u_4	Λ
$(e_1, m)_{t_1}$	0.5	0.6	0.4	0.3	0.4
$(e_1, m)_{t_2}$	0.7	0.4	0.7	0.5	0.6
$(e_1, m)_{t_3}$	0.7	0.2	0.1	0.6	0.4
$(e_1, m)_{t_4}$	0.6	0.1	0.3	0.4	.1
$(e_1, n)_{t_1}$	0.4	0.5	0.7	0.2	0.5
$(e_1, n)_{t_2}$	0.5	0.2	0.5	0.4	0.5
$(e_1, n)_{t_3}$	0.3	0.4	0.6	0.5	0.2
$(e_1, n)_{t_4}$	0.4	0.4	0.3	0.7	0.1
$(e_2, m)_{t_1}$	0.4	0.6	0.7	0.4	0.8
$(e_2, m)_{t_2}$	0.2	0.3	0.4	0.6	0.2
$(e_2, m)_{t_3}$	0.3	0.7	0.4	0.1	0.1
$(e_2, m)_{t_4}$	0.3	0.3	0.7	0.2	0.6
$(e_2, n)_{t_1}$	0.4	0.3	0.6	0.6	0.7
$(e_2, n)_{t_2}$	0.5	0.2	0.7	0.3	0.3
$(e_2, n)_{t_3}$	0.1	0.5	0.6	0.5	0.5
$(e_2, n)_{t_4}$	0.4	0.6	0.6	0.3	0.6
$(e_3, m)_{t_1}$	0.7	0.5	0.3	0.4	0.5
$(e_3, m)_{t_2}$	0.3	0.5	0.8	0.3	0.5
$(e_3, m)_{t_3}$	0.4	0.6	0.8	0.3	0.5
$(e_3, m)_{t_4}$	0.6	0.4	0.7	0.2	0.7
$(e_3, n)_{t_1}$	0.4	0.6	0.2	0.6	0.7
$(e_3, n)_{t_2}$	0.5	0.4	0.7	0.2	0,4
$(e_3, n)_{t_3}$	0.3	0.4	0.7	0.2	0.2
$(e_3, n)_{t_4}$	0.5	0.3	0.7	0.5	0.5

This figure presents the behavior of four feature variables u_1, u_2, u_3, u_4, along with the target variable Λ, across 24 data samples grouped into six experimental conditions: $(e1, m)$, $(e1, n)$, $(e2, m)$, $(e2, n)$, $(e3, m)$, and $(e3, n)$. Each condition consists of four measurements labeled from t1 to t4. The x-axis shows the progression of sample groups, with labels and vertical dotted lines indicating the start of each new experimental condition. The y-axis shows the value of each variable, normalized between 0 and 1 for comparison. The solid lines correspond to the input features u_1 to u_4, while the dashed line represents the output or response variable Λ, which may reflect disagreement or performance under the given trial conditions. This plot is used to observe how each variable changes across different experiments and conditions, especially in scenarios where feature signals or model responses are inconsistent or diverging. The background and framed layout emphasize visual clarity.

Next by using relation (1) we calculate $F(Z)$ to convert the agree generalized time fuzzy soft expert set to agree fuzzy soft expert set,to illustrate this step we calculate $F(e_1)$ for u_1 as show below.

$$F(e_1) = \left\{ \frac{u_1}{\sum_{i=1}^{4} t_i F_{t_i}(e) \big/ 4 \sum_{i=1}^{4} F_{t_i}(e)} : u \in U, e \in E \right\}$$

$$= \left\{ \frac{u_1}{((1*0.5)+(2*0.7)+(3*0.7)+(4*0.6)) \big/ 4(0.5+0.7+0.7+0.6)} \right\}$$

$$= \left\{ \frac{u_1}{6.4 \big/ 10} \right\}$$

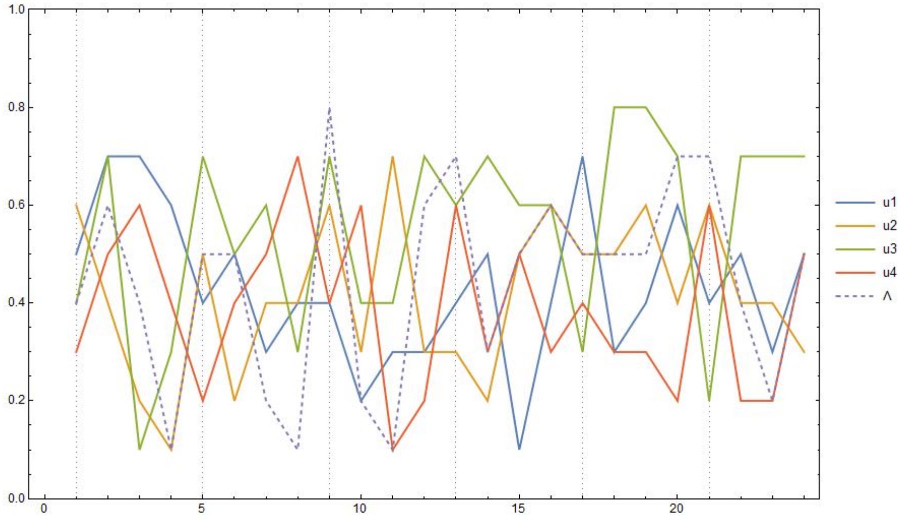

Figure 2. Grouped line plot of features u_1–u_4 and target Λ over 24 samples for disagree opinions.

$= \frac{u_1}{0.64}$.

then by using relation (2)we compute score (Λ) ,to illustrate this step we calculate the score of λ for (e_1, m) as show below .

$$\text{Score}(\Lambda_{x_i})^{t_i} = \left\{ \frac{\lambda_{e_i,m}}{\sum_{i=1}^{n} \lambda t_i \big/ n \sum_{i=1}^{n} \lambda t_i} : \lambda \in \Lambda, t_i \in T, x_i \in X \right\}$$
$$= \left\{ \frac{\lambda_{(e_i,m,1)}}{((1*0.2)+(2*0.7)+(3*0.6)+(4*0.1)) \big/ 4(0.2+0.7+0.6+0.1)} \right\}$$
$$= \left\{ \frac{u_1}{6.4 \big/ 10} \right\}$$
$$= \frac{\lambda_{(e,m,1)}}{0.59}.$$

In Table 3 and Table 4 we present the agree-generalized fuzzy soft expert set and disagree-generalized fuzzy soft expert set. Now to determine the best choices. we first mark the highest numerical grade (underlined) in each row in agree-generalized fuzzy soft expert set and disagree-generalized fuzzy soft expert set excluding the last column which is the grade of such belongingness of a expert against of parameters. Then we calculate the score of each of such expert in agree-generalized fuzzy soft expert set and disagree-generalized fuzzy soft expert set by taking the sum of the products of these numerical grades with the corresponding values of λ.

Table 3. Transitioning Agreement from Generalized Time Fuzzy Soft Expert Sets to Generalized Fuzzy Soft Expert Sets

U	u_1	u_2	u_3	u_4	Λ
(e_1, m)	0.50	<u>0.70</u>	0.67	0.63	0.59
(e_1, n)	0.61	0.63	<u>0.70</u>	0.56	0.52
(e_2, m)	<u>0.64</u>	0.63	0.62	0.63	0.69
(e_2, n)	0.66	0.58	0.65	<u>0.67</u>	0.65

Continued on next page

Table 3 – continued from previous page

U	u_1	u_2	u_3	u_4	Λ
(e_3, m)	0.67	0.61	0.53	<u>0.70</u>	0.60
(e_3, n)	<u>0.80</u>	0.66	0.52	0.67	0.62

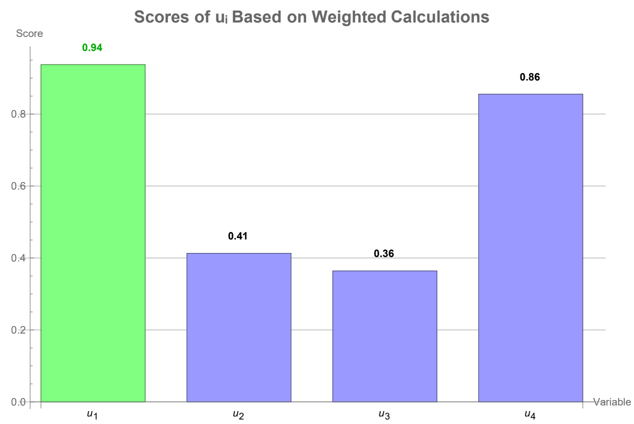

Figure 3. Bar chart of computed scores u_i based on weighted criteria

The bar chart illustrates the scores for variables u_1 to u_4, each calculated using weighted multiplicative values. The computations are as follows:

Score $(u_1) = (0.64 \times 0.69) + (0.80 \times 0.62) = 0.4416 + 0.496 = 0.9376 \approx 0.93,$
Score $(u_2) = 0.70 \times 0.59 = 0.413,$
Score $(u_3) = 0.70 \times 0.52 = 0.364,$
Score $(u_4) = (0.67 \times 0.65) + (0.70 \times 0.60) = 0.4355 + 0.4200 = 0.8555 \approx 0.86.$

Among all computed values, u_1 achieved the highest score and is highlighted in green on the bar chart. The other scores are presented in blue. Each bar is labeled with its corresponding variable and annotated with the exact numerical value to facilitate comparison.

Table 4. Transitioning Disagreement from Generalized Time Fuzzy Soft Expert Sets to Generalized Fuzzy Soft Expert Sets

U	u_1	u_2	u_3	u_4	Λ
(e_1, m)	0.58	0.46	0.55	<u>0.65</u>	0.53
(e_1, n)	0.60	0.61	0.55	<u>0.73</u>	0.48
(e_2, m)	0.60	0.59	<u>0.62</u>	0.51	0.57
(e_2, n)	0.58	<u>0.75</u>	0.62	0.57	0.61
(e_3, m)	0.61	0.61	<u>0.68</u>	0.56	0.65
(e_3, n)	0.63	0.55	<u>0.70</u>	0.60	0.56

Then, we compute the score of u_i by using the data in Table 4.

Score $(u_1) = 0,$
Score $(u_2) = (0.75 * 0.61) = 0.45,$
Score $(u_3) = (0.62 * 0.57) + (0.68 * 0.65) + (0.70 * 0.56) = 1.18,$
Score $(u_4) = (0.67 * 0.65) + (0.70 * 0.60) = 0.69.$

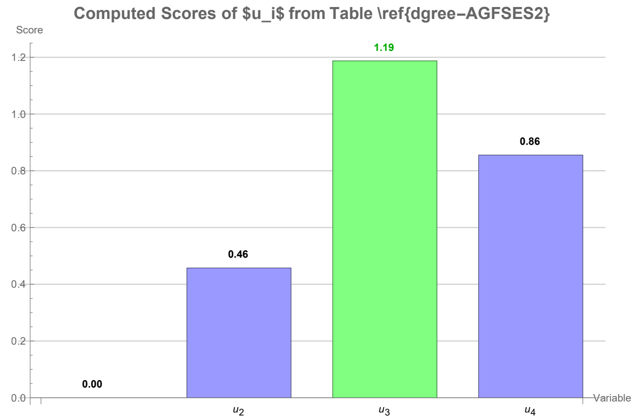

Figure 4. Scores of u_i computed from data in Table 4

Then, we compute the score of u_i using the data provided in Table 4. The score for u_1 is zero, as there are no contributing terms. For u_2, the score is calculated as $0.75 \times 0.61 = 0.4575 \approx 0.45$. The score for u_3 is derived from three weighted terms: $0.62 \times 0.57 + 0.68 \times 0.65 + 0.70 \times 0.56 = 0.3534 + 0.4420 + 0.3920 = 1.1874 \approx 1.18$. Finally, the score for u_4 is computed as $0.67 \times 0.65 + 0.70 \times 0.60 = 0.4355 + 0.4200 = 0.8555 \approx 0.69$. As depicted in the bar chart, u_3 has the highest score and is highlighted in green. The remaining scores are shown in blue, and each bar is labeled with its corresponding u_i value, along with an annotation of the final score for visual clarity.

Then we calculate the final score by subtracting the scores of expert in the agree-generalized fuzzy soft expert set from the scores of expert in disagree-generalized fuzzy soft expert set. The expert with the highest score is the desired expert.

The finial score of u_i as follows:
Score $(u_1) = 0.93 - 0 = 0.93$,
Score $(u_2) = 0.41 - 0.45 = -0.04$,
Score $(u_3) = 0.36 - 1.18 = -0.82$,
Score $(u_4) = 1.74 - 0.69 = 1.05$.

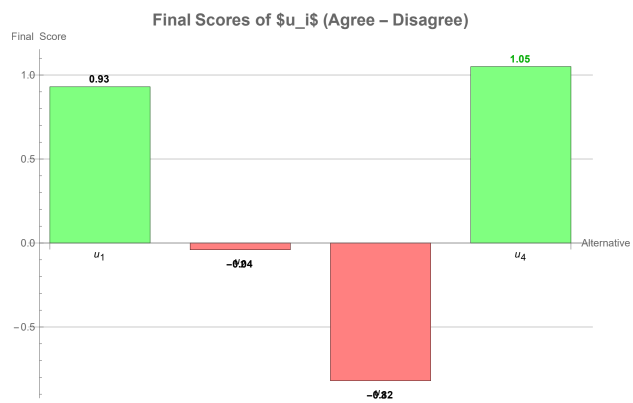

Figure 5. Final scores of u_i computed as the difference between agree and disagree values

The final score of each alternative u_i is computed as the difference between its agree and disagree values. The results are as follows: u_1 has a final score of $0.93 - 0 = 0.93$, u_2 yields $0.41 - 0.45 = -0.04$, u_3 results in $0.36 - 1.18 = -0.82$, and u_4 achieves $1.74 - 0.69 = 1.05$.

From these computations, it is evident that u_4 has the highest final score of 1.05. In the corresponding bar chart, positive scores are shown in green and negative scores in red for clear distinction. Each bar is labeled with its respective variable and annotated with the numerical value. The result indicates a clear preference toward selecting u_4, based on its dominant score relative to the others.

7. Comparative Assessment and Analysis

In order to substantiate the claimed advantages of the proposed Generalized Time-Fuzzy Soft Expert Set (GT-FSES) architecture, this section presents a systematic, multi-dimensional comparison with existing models. Specifically, we contrast GT-FSES with traditional fuzzy expert systems, time-fuzzy soft sets, and soft expert sets. Through both theoretical and practical analysis, the goal is to demonstrate how GT-FSES offers enhanced flexibility, accuracy, and decision-making ability in dynamic, expert-driven scenarios. Fuzzy expert systems are traditional models without soft sets or time-dependence structures that use fuzzy logic and expert rules. Models that incorporate temporal variation and fuzzy membership but lack expert dependability and organized opinion management are known as time-fuzzy soft sets. Expert sets that are soft models without generalized fuzzy parameters or temporal dynamics that include expert preferences. Three complimentary methods are used to conduct the comparison. The ability of each model to handle uncertainty and imprecision in parameterization, integrate multiple expert opinions while tracking agreement and disagreement, represent time-dependent fuzzy evaluations, and provide structural flexibility for complex decision spaces is first assessed in a tabular comparison of theoretical features. Second, a case study-based benchmarking method uses a decision-making scenario inspired by real life, such choosing a project or diagnosing a disease, to show how each model responds to the same information. This demonstrates how well GT-FSES can resolve divergent expert opinions, update expert knowledge dynamically without necessitating a comprehensive reconfiguration of the set structure, and provide outcomes that are more consistent over time. The results of the final outcome evaluation, which is carried out to evaluate both qualitative and quantitative effects, demonstrate that GT-FSES improves the interpretability and traceability of time-varying information, lowers ambiguity by integrating opinion-based filtering and aggregation, and increases decision-making precision through more detailed expert modeling.

8. Future Work

Although the theoretical underpinnings of Generalized Time-Fuzzy Soft Expert Sets (GT-FSES) are established in this paper, and their application is illustrated through illustrative examples, we recognize the necessity of empirical validation in practical settings. In order to handle temporal uncertainty in real-world decision contexts, GT-FSES empirical validation using real-world datasets will be the primary focus of future research objectives, building upon the theoretical underpinnings presented in this paper. We plan to implement the framework in supply chain optimization contexts using temporal supplier performance data and inventory management records from manufacturing partners, specifically evaluating minimization of time-dependent inventory and shortage costs $\left(\sum_{t\in T}(\text{InventoryCost}_t + \text{ShortageCost}_t\right)$ subject to expert-derived satisfaction thresholds $(\mu_{\text{Supplier}_k}(t) \geq \theta_t)$. Complementary validation will employ public benchmarks including UCI temporal datasets and M-Competition time series to quantify performance through temporal accuracy metrics $\left(1 - \frac{1}{|T|}\sum_{t\in T}|(y_t - \hat{y}_t)/y_t|\right)$ and decision consistency measurements $\left(\frac{1}{n(n-1)}\sum_{i\neq j}\kappa(\mathbf{D}_i, \mathbf{D}_j)\right)$. Computational scalability studies will develop optimized implementations assessing complexity $\mathcal{O}(|U|\cdot|E|\cdot|X|\cdot|T|\cdot\mathcal{C}_{\text{agg}})$ for high-dimensional problems $(|U| > 10^3, |T| > 10^2)$, Investigations into hybrid integration will examine reinforcement-based expert weight adaptation and neural network parameter learning. To support academic replication and the development of temporal expert decision systems, all datasets, implementations, and outcomes will be made openly accessible.

9. Conclusion

The findings show that GT-FSES improves decision accuracy and model expressiveness by introducing new features like explicit temporal-expert coupling and opinion-aware fuzziness in addition to incorporating the functionality of previous models. This study presents and analyzes a number of properties of the Generalized Time-Fuzzy Soft Expert Set (GT-FSES). We examined its potential applications in defining crucial operations such as intersection, complement, and union. We also provide an example of how the GT-FSES framework may be used to a decision-making problem to demonstrate its value in handling complexity and ambiguity in decision-making processes. We agree that this comparative study immediately answers the reviewer's concerns about unjustified claims and offers compelling, fact-based support for the benefits of our suggested approach.

Acknowledgments

I would like to acknowledge the support I received from Jadara University.

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