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# Sharia Stock Return Volatility Model through Bayesian MSGARCH with Asymmetric Effects and Data Structure Changes

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**Abstract** The Indonesian Sharia stock market has experienced significant growth over the past year, accompanied by a rise in market capitalization. However, high volatility remains a critical challenge for investors when deciding to invest in Sharia-compliant stocks. Accurately modeling return volatility is essential to support effective risk management and investment decision-making. This study conducts a comprehensive comparative analysis of several volatility models to identify the most effective approach for modeling Sharia stock return volatility. The models evaluated include classical Generalized Autoregressive Conditional Heteroscedasticity (GARCH), asymmetric variants such as Exponential GARCH (EGARCH), Threshold GARCH (TGARCH), and Asymmetric Power GARCH (APGARCH), as well as the regime-switching Markov Switching GARCH (MSGARCH) and a Bayesian extension of MSGARCH that incorporates structural shifts and parameter uncertainty. The comparison is based on information criteria and predictive accuracy metrics to assess both in-sample fit and out-of-sample performance. Results indicate that the Bayesian MSGARCH model consistently outperforms the other models in capturing the complex volatility dynamics of Jakarta Islamic Index (JII) returns, particularly during structural breaks and market regime changes. Furthermore, the analysis reveals that investment activity significantly affects volatility behavior during periods of market appreciation and depreciation. These findings provide valuable guidance for selecting appropriate volatility models in Sharia markets and inform the design of more robust investment and risk mitigation strategies.

**Keywords** APGARCH, Bayesian MSGARCH, EGARCH, GARCH, Indonesian Sharia Stock, MSGARCH, TGARCH, Volatility

**AMS 2010 subject classifications** 62M10, 62P20

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## 1. Introduction

The development of the stock market in Indonesia is not limited to the conventional sector, as the Sharia stock market has experienced significant growth in recent years. According to the Financial Services Authority (FSA), the capitalization of Indonesia's Sharia stock market reached IDR 6,759.54 trillion as of 27 December 2024, marking a 9.98% year-to-date increase, while the Indonesia Sharia Stock Index (ISSI) closed at 213.86 points, reflecting strong market performance. The growing prominence of Sharia-compliant stocks highlights their strengthening role in the national financial system, driven by robust investor interest, proactive regulatory oversight, and the increasing participation of younger retail investors, who now constitute 79% of individual market participants. However, high volatility remains a defining characteristic, with stock fluctuations influenced by macroeconomic conditions, government policies, and global uncertainties, necessitating advanced risk management techniques [18].

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Time series modeling has been extensively developed to measure volatility and help investors predict stock return volatility levels. The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model is among the most commonly used models. GARCH effectively captures volatility components such as time variation, volatility clustering, and leverage effects, which describe increased volatility resulting from data fluctuations [18]. While GARCH effectively models volatility, it struggles to address asymmetric responses to positive and negative shocks, particularly in financial data. This limitation has led to the development of asymmetric models such as Exponential GARCH (EGARCH), Threshold GARCH (TGARCH), and Asymmetric Power GARCH (APGARCH), which are designed to capture leverage effects better and provide more accurate predictions [11].

In particular, the Sharia stock market faces unique challenges, such as liquidity fluctuations and structural changes that can cause volatility to shift unpredictably over time. The challenge variations may lead to inaccurate results when using traditional GARCH models, as they fail to account for sudden shifts in market conditions. To address the inaccurate results, the Markov Switching GARCH (MSGARCH) model has been introduced, which accounts for structural changes by dividing the model into multiple regimes. The incorporation of Time-Varying Transition Probabilities (TVTP) into MSGARCH has shown improved goodness-of-fit and predictive accuracy, as it allows regime changes to be driven by market liquidity conditions rather than remaining constant. Empirical findings suggest that deteriorating liquidity can increase the probability of entering a high-volatility state, making TVTP-MSGARCH a more effective tool for forecasting market uncertainty [41]. Additionally, Bayesian methods enhance parameter estimation, particularly in small sample conditions, leading to more robust volatility predictions in Sharia stock markets. The optimal volatility model for Sharia stocks should integrate liquidity effects, leverage asymmetry, and structural regime shifts, ensuring more accurate risk assessments and investment strategies.

Volatility plays a crucial role in risk management within financial markets, including the Sharia stock market, where dynamic fluctuations require advanced modeling techniques. Markov Switching GARCH (MSGARCH) with Bayesian estimation enhances predictive accuracy by integrating time-varying transition probabilities (TVTP), allowing for adaptive regime switching based on liquidity shifts and structural changes. Asymmetric models such as EGARCH, TGARCH, and APGARCH further refine volatility estimates by capturing leverage effects, ensuring a more accurate representation of risk dynamics in Sharia-compliant markets. In incorporating Bayesian MSGARCH with multi-chain Markov-switching structures, investors can optimize portfolio allocation, strengthen hedging strategies, and mitigate risk during uncertain market periods [4].

Stock market data exhibit varying characteristics, with Sharia stock data differing from conventional stock data, particularly regarding volatility. For instance, the volatility of Sharia stocks is generally lower than that of conventional stocks, making Sharia stocks an attractive option for investors seeking lower-risk investments [24]. The GARCH volatility model, introduced by Bollerslev as an extension of ARCH, addresses heteroscedasticity in time series data. It has been widely used to measure stock volatility, market risk, and hedging in both Sharia and conventional stock markets [26].

Despite its strengths, GARCH struggles to handle asymmetric data, where volatility responds more strongly to negative shocks than positive ones. Asymmetric models such as EGARCH, TGARCH, and APGARCH were developed to address this issue. The asymmetric models effectively estimate volatility in the Indonesian financial sector, emphasizing their ability to capture leverage effects better than standard GARCH models [25].

MSGARCH, an advanced version of GARCH, accounts for regime changes in stock market data by capturing shifts between high-volatility and low-volatility market environments. This model also allows for more robust parameter estimation by considering market uncertainty. Bayesian estimation in MSGARCH leads to more accurate volatility predictions, particularly during periods of instability [32]. The Bayesian estimation that is combined with Markov Switching GARCH also improves volatility predictions over traditional GARCH models [34].

In a similar case, the MSGARCH method is applied to measure the volatility of the Composite Stock Price Index, and it effectively captured changes in market volatility [20, 21]. The study specifically demonstrated how the Bayesian MSGARCH model performs well in modeling volatility during structural changes and provides valuable insights for risk assessment using Value at Risk (VaR). Furthermore, the combination of MSGARCH and Bayesian estimation can also measure the Value at Risk (VaR) in the ASEAN market, confirming that this model is highly effective in managing volatility risk in Sharia stocks [28].

The use of asymmetric models for volatility analysis with Semi-Markov Switching GARCH emphasizes the importance of these models in assessing the stability of financial systems, especially in markets with asymmetric volatility [40]. It underscores the relevance of incorporating both asymmetric and switching models to measure volatility in the Sharia stock market during unstable conditions.

Despite its modeling advantages, Bayesian MSGARCH also involves considerable computational costs. The reliance on MCMC techniques, such as reversible jump and delayed rejection algorithms, often requires long pilot runs and extensive iterations to achieve convergence—particularly in models with high parameter correlations or complex posterior structures [31]. This introduces a trade-off between flexibility and efficiency, especially when working with high-dimensional or multi-regime specifications. The high dimensionality and latent structure inherent in Bayesian MSGARCH models can complicate inference, making results sensitive to prior distributions and risking model misfit due to overfitting or underfitting [9]. To mitigate these challenges, more efficient samplers such as multi-move Gibbs techniques have been developed to enhance convergence by jointly updating latent states in regime-switching models [22]. These developments contribute to stabilizing parameter estimation in real financial environments.

In light of these considerations, this study aims to evaluate the volatility dynamics of Indonesia's Sharia stock index by employing a range of GARCH-type models, with particular emphasis on the Bayesian MSGARCH model under a Markov-switching framework. By incorporating asymmetry, regime dependence, and liquidity conditions, this approach seeks to generate accurate, interpretable, and practically relevant insights to support portfolio construction, hedging strategies, and Sharia-compliant risk management. Furthermore, the study addresses a notable gap in the literature, namely the limited comparative analysis of volatility models in Sharia markets, especially those utilizing regime-switching techniques such as Bayesian MSGARCH, which remain underexplored in the context of Islamic financial instruments.

## 2. Methodology

Several time series models and diagnostic tests were used to identify the best model for measuring Sharia stock return data volatility.

### *2.1. Logarithmic Return*

Return describes how price changes over a specific period, whether in asset prices, projects, or investments. Historical data from the Jakarta Islamic Index (JII), such as asset price differences or percentages, are used. Return is based on the principle that profit is directly proportional to risk. It means that when the asset return rate is high, the associated risk will also be high. Conversely, when the asset return rate is low then the risk is getting lower. Therefore, the return can be expressed as follows:

$$R_t = \ln\left(\frac{d_t}{d_{t-1}}\right) \tag{1}$$

where $d_t$ is the data at time $t$, and $d_{t-1}$ is the data at time $t-1$.

### *2.2. ARIMA*

A time series $Y_t$ follows Autoregressive Integrated Moving Average (ARIMA) when the differencing process is applied to non-stationary ARMA [36]. The general form of ARIMA, denoted by ARIMA($p$,$d$,$q$), is as follows:

$$(1-B)^d Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} - (\theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q}) + \varepsilon_t \tag{2}$$

or can be written as follows:

$$\Phi_p(B)(1-B)^d Y_t = \Theta_q(B)\varepsilon_t \tag{3}$$

where $\Phi_p(B) = 1 - \Phi_1 B - \Phi_2 B^2 - \cdots - \Phi_p B^p$, $\Theta_q(B) = 1 - \Theta_1 B - \Theta_2 B^2 - \cdots - \Theta_q B^q$, $(1-B)^d Y_t$ is the stationary time return data after differencing $d$, $\phi_i$ is the Autoregressive parameter for $i = 1, 2, \ldots, p$, $\Theta_j$ is the Moving Average parameter for $j = 1, 2, \ldots, q$, and $B$ is the backward shift operator.

### 2.3. GARCH

Bollerslev and Taylor introduced $p$ lag of conditional variance, where $p$ is referred to as GARCH order [36]. This combined model is called GARCH. GARCH($p$, $q$) equation is presented as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{(t-i)}^2 + \sum_{j=1}^{p} \beta_j \sigma_{(t-j)}^2 \tag{4}$$

where $\omega > 0$.

### 2.4. Asymmetric Models

GARCH is an extension of ARCH, incorporating lags in its conditional variance. However, it is not explicitly designed to handle asymmetric problems, such as nonlinearity in data. Asymmetric models respond differently to adverse shocks than to positive types. Various asymmetric models are explained as follows:

*2.4.1. Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH)* The first model that allows asymmetric responses due to leverage effects is exponential EGARCH, introduced by Nelson in 1991 [5]. EGARCH(p,q) is defined as $r_t = \sigma_t \varepsilon_t$, where:

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^{q} \alpha_i \left( \frac{\epsilon_{(t-i)}}{\sigma_{(t-i)}} - \gamma_i \mathbb{E}\left[ \frac{\epsilon_{(t-i)}}{\sigma_{(t-i)}} \right] \right) + \sum_{j=1}^{p} \beta_j \ln(\sigma_{(t-j)}^2) + \sum_{k=1}^{p} \gamma_k \frac{\epsilon_{(t-k)}}{\sigma_{(t-k)}} \tag{5}$$

*2.4.2. Threshold Generalized Autoregressive Conditional Heteroscedasticity (TGARCH)* TGARCH is based on GARCH developed by Bollerslev [5]. Suppose $\sigma_t^2$ is a TGARCH($p$,$q$) process, defined as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{(t-i)}^2 + \sum_{j=1}^{p} \beta_j \sigma_{(t-j)}^2 + \sum_{k=1}^{p} \gamma_k \epsilon_{(t-k)}^2 \tag{6}$$

where $\lambda_k$ is the coefficient for asymmetric effects.

*2.4.3. Asymmetric Power Generalized Autoregressive Conditional Heteroscedasticity (APGARCH)* APGARCH represents a general class of models that includes ARCH and GARCH [1]. The equation of the model is presented as:

$$\sigma_t^\delta = \omega + \sum_{i=1}^{q} \alpha_i \left( |\epsilon_{t-i}| - \gamma_i \epsilon_{t-i} \right)^\delta + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^\delta \tag{7}$$

where $\delta$ is the power parameter that allows the model to capture the nonlinear nature of volatility, in summary, EGARCH, TGARCH, and APGARCH capture asymmetric effects differently.

### 2.5. MSGARCH

MS-GARCH is a popular model that was first introduced by Hamilton in 1989. It incorporates structural changes from random properties by assuming the system can be in one of a finite number of regimes [38]. The model also assumes that the data formation process follows a consistent model structure across regimes, but each regime applies unique parameters. MS-GARCH(p,q) is defined as the following system:

$$\sigma_t^2 \quad = \quad \omega_{s_t} + \sum_{i=1}^{q} \alpha_{s_t}\, \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_{s_t}\, \sigma_{t-j}^2 \tag{8}$$

where $\omega_{s_t}$, $\alpha_{s_t}$, and $\beta_{s_t}$ are different parameters depending on the regime $s_t$, with $s_t = 1, 2, \ldots, n$.

### *2.6. Bayesian MSGARCH*

Parameter estimation of the MSGARCH model using the Bayesian approach depends on the likelihood function and the selected prior distributions to obtain the posterior distribution. Let $F_{t-1}$ denote the information set containing all data up to time $t-1$, then the joint distribution function of $r_t$ and $s_t$ is given by

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(r_t - \mu_{s_t})^2}{2\sigma_t^2}\right) \eta_{s_t t} \tag{9}$$

and the likelihood function for the MSGARCH model is

$$L(\theta) \propto \prod_{t=1}^{T} \sigma_t^{-1} \exp\left(-\frac{(r_t - \mu_j)^2}{2\sigma_j^2}\right) \eta_{jt} \tag{10}$$

where $L(\theta)$ denotes the likelihood function for $t = 1, \ldots, T$. The parameter $\theta$ represents all model parameters in the MSGARCH framework, including the transition probabilities $\eta = (\eta_{11}, \eta_{21}, \eta_{12}, \eta_{22})$, the regime-dependent means $\mu = (\mu_1, \mu_2)$, and the model parameters $\omega_k, \alpha_k, \beta_k$.

The posterior parameter estimates for each parameter $\theta$ are determined as follows:

1. Assume the prior for the transition probability parameter $\eta$ follows a Beta distribution, i.e., $\eta \sim \text{Beta}(\alpha, \beta)$. The prior density function for $\eta$ is given by: $f(\eta) = \frac{\eta^{\alpha-1}(1-\eta)^{\beta-1}}{B(\alpha,\beta)}$ The posterior distribution for $\eta$ is:

$$f(\eta|r,\theta) \quad \propto \quad \prod_{t=1}^{T} \sigma_t^{-1} \exp\left(-\frac{(r_t - \mu_{s_t})^2}{2\sigma_t^2}\right) \eta_{s_t t} \cdot \frac{\eta^{\alpha-1}(1-\eta)^{\beta-1}}{B(\alpha,\beta)} \tag{11}$$

2. Assume the prior for the parameter $\mu$ follows a normal distribution, i.e., $\mu \sim \mathcal{N}(\mu_\mu, \sigma_\mu^2)$. The prior function is: $f(\mu) = \frac{1}{\sqrt{2\pi\sigma_\mu^2}} \exp\left(-\frac{(\mu-\mu_\mu)^2}{2\sigma_\mu^2}\right)$ The posterior distribution for $\mu$ is:

$$f(\mu|r,\theta) \quad \propto \quad \prod_{t=1}^{T} \sigma_t^{-1} \exp\left(-\frac{(r_t - \mu_{s_t})^2}{2\sigma_t^2}\right) \eta_{s_t t} \cdot \frac{1}{\sqrt{2\pi\sigma_\mu^2}} \exp\left(-\frac{(\mu - \mu_\mu)^2}{2\sigma_\mu^2}\right) \tag{12}$$

3. Assume the prior for the parameter $\omega$ follows a normal distribution, i.e., $\omega \sim \mathcal{N}(\mu_\omega, \sigma_\omega^2)$. The prior function is: $f(\omega) = \frac{1}{\sqrt{2\pi\sigma_\omega^2}} \exp\left(-\frac{(\omega-\mu_\omega)^2}{2\sigma_\omega^2}\right)$ The posterior distribution for $\omega$ is:

$$f(\omega|r,\theta) \propto \prod_{t=1}^{T} \sigma_t^{-1} \exp\left(-\frac{(r_t - \mu_{s_t})^2}{2\sigma_t^2}\right) \eta_{s_t t} \cdot \frac{1}{\sqrt{2\pi\sigma_\omega^2}} \exp\left(-\frac{(\omega - \mu_\omega)^2}{2\sigma_\omega^2}\right) \tag{13}$$

4. Assume the prior for the parameter $\alpha$ follows a normal distribution, i.e., $\alpha \sim \mathcal{N}(\mu_\alpha, \sigma_\alpha^2)$. The prior function is: $f(\alpha) = \frac{1}{\sqrt{2\pi\sigma_\alpha^2}} \exp\left(-\frac{(\alpha-\mu_\alpha)^2}{2\sigma_\alpha^2}\right)$ The posterior distribution for $\alpha$ is:

$$f(\alpha|r,\theta) \propto \prod_{t=1}^{T} \sigma_t^{-1} \exp\left(-\frac{(r_t - \mu_{s_t})^2}{2\sigma_t^2}\right) \eta_{s_t t} \cdot \frac{1}{\sqrt{2\pi\sigma_\alpha^2}} \exp\left(-\frac{(\alpha - \mu_\alpha)^2}{2\sigma_\alpha^2}\right) \tag{14}$$

5. Assume the prior for the parameter $\beta$ follows a normal distribution, i.e., $\beta \sim \mathcal{N}(\mu_\beta, \sigma_\beta^2)$. The prior function is: $f(\beta) = \frac{1}{\sqrt{2\pi\sigma_\beta^2}} \exp\left(-\frac{(\beta-\mu_\beta)^2}{2\sigma_\beta^2}\right)$ The posterior distribution for $\beta$ is:

$$f(\beta|r,\theta) \propto \prod_{t=1}^{T} \sigma_t^{-1} \exp\left(-\frac{(r_t - \mu_{s_t})^2}{2\sigma_t^2}\right) \eta_{s_t t} \cdot \frac{1}{\sqrt{2\pi\sigma_\beta^2}} \exp\left(-\frac{(\beta-\mu_\beta)^2}{2\sigma_\beta^2}\right) \tag{15}$$

Due to the complexity of analytically computing the marginal posterior distributions, the Markov Chain Monte Carlo (MCMC) method is employed to obtain effective posterior estimates via sampling techniques.

*2.6.1. Model Interpretability and Process Flow.* While the mathematical formulation of the Bayesian MSGARCH model offers statistical rigor, it can pose challenges for readers unfamiliar with advanced econometric modeling. To address this, a conceptual narrative and flow-based illustration are included to enhance interpretability.

Figure 1 illustrates a conceptual flowchart detailing the estimation procedure of the Bayesian MSGARCH model. The diagram delineates the dynamic process through which the model transitions between volatility regimes via a hidden Markov structure, while simultaneously incorporating prior beliefs to construct posterior distributions of the model parameters. Each procedural component that ranging from prior specification, likelihood evaluation, regime state estimation, to posterior inference is then systematically represented to provide a comprehensive overview of the Bayesian MSGARCH estimation framework.

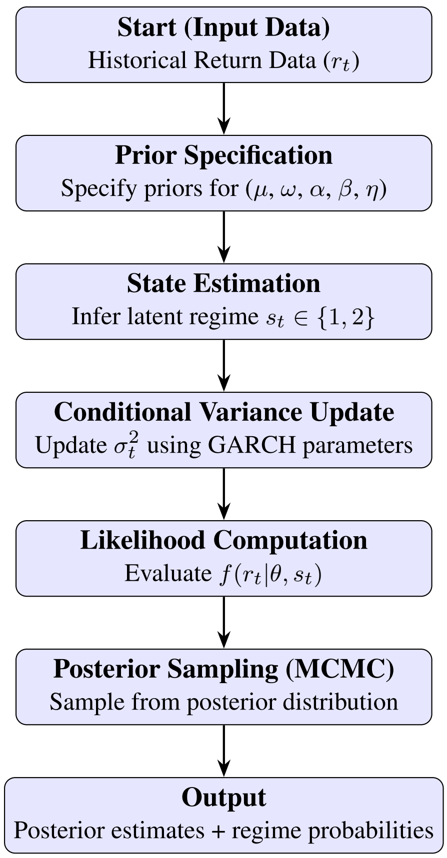


Figure 1. Flowchart of Bayesian MSGARCH Model Estimation Process

The flowchart in Figure 1 illustrates the step-by-step procedure in estimating the Bayesian MSGARCH model. The process begins with the input of historical return data ($r_t$), which serves as the basis for model estimation. Next, prior distributions are specified for each of the model parameters, incorporating initial beliefs before observing the data. State estimation follows, where the hidden regimes ($s_t$) are inferred using the underlying Markov process. The model then updates the conditional variance ($\sigma_t^2$) based on regime-specific GARCH parameters. With this variance and the inferred states, the likelihood function is computed, combining the probability of the observed data given the current parameter set.

To obtain the posterior distribution of the parameters, the model employs Markov Chain Monte Carlo (MCMC) sampling. This step generates samples from the posterior, integrating the prior information and likelihood. Finally, the model outputs posterior estimates of the parameters and the inferred probabilities of being in each regime at each point in time. This structured flow ensures a coherent and interpretable estimation framework for the Bayesian MSGARCH model.

*2.6.2. Pseudocode for Gibbs Sampling in Bayesian MSGARCH* Among the simulation-based approaches proposed in the literature, one notable method is the Bayesian estimation framework that develops a *single-move Gibbs sampling algorithm* for Markov Switching GARCH (MS-GARCH) models with a fixed number of regimes. This approach not only formulates the estimation procedure but also provides theoretical guarantees regarding geometric ergodicity and the existence of moments for the resulting process [3]. Despite its theoretical appeal, the single-move sampler suffers from a key practical limitation of high autocorrelation in the Markov chains, which leads to inefficient mixing and slow convergence. This inefficiency limits the practical applicability of the approach, particularly for high-dimensional parameter spaces or models with pronounced regime persistence.

To address these challenges, alternative simulation-based methods have been proposed. For instance, a sequential Monte Carlo (SMC) approach, also known as particle filtering, has been introduced to estimate GARCH-type models that accommodate structural breaks [27]. While this method is more flexible in capturing abrupt regime changes, it introduces additional computational complexity and is sensitive to the resampling mechanism. The Gibbs sampling algorithm adopted in this study builds upon the foundation laid in [3], while incorporating model-specific adaptations for the Bayesian MSGARCH framework used herein. The overall estimation steps are presented in Algorithm 1.

**Algorithm 1** Gibbs Sampling for Bayesian MSGARCH Estimation

**Input:** Return data $\{r_t\}_{t=1}^T$, prior hyperparameters
**Initialize:** Parameters $\theta^{(0)}$, states $s_{1:T}^{(0)}$
**for** $i = 1$ to $N$ **do**
  Sample latent states $s_{1:T}^{(i)} \mid r_{1:T}, \theta^{(i-1)}$
  Sample transition probabilities $\eta^{(i)} \mid s_{1:T}^{(i)}$
  **for** each regime $j = 1, 2$ **do**
    Extract returns where $s_t = j$
    Sample GARCH parameters $\mu_j, \omega_j, \alpha_j, \beta_j$
  **end for**
  **if** $i >$ burn-in and $i \bmod$ thinning = 0 **then**
    Store $\theta^{(i)}$
  **end if**
**end for**
**Output:** Posterior samples $\{\theta^{(i)}\}$ and smoothed $P(s_t = j \mid r_{1:T})$

## 3. Experimental Results

### *3.1. Description of Sharia Stock Data*

This study used data from 51 Sharia stocks listed on the Jakarta Islamic Index (JII). The data comprised weekly closing prices from 1 January 2024 to 31 December 2024. Subsequently, the data were subjected to a plot observation to identify patterns. The data plot is presented in Figure 2.

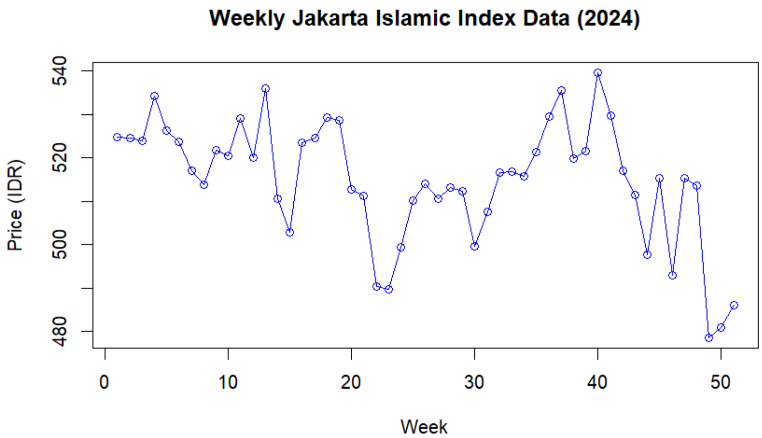


Figure 2. Weekly JII closing data plot.

Figure 2 shows that the Weekly Jakarta Islamic Index (JII) data for 2024 exhibits a cyclical and volatile pattern with no apparent long-term upward or downward trend. In the early part of the year, the index remains relatively stable with slight fluctuations, indicating a consolidation phase without significant directional movement. This stability is followed by a noticeable downward trend, suggesting a possible reaction to market uncertainty or negative external influences. The decline reaches a visible low point, marking one of the more significant dips in the index over the year.

After this decline, the index begins to recover and moves upward, reflecting potential market sentiment or investor confidence improvements. This rising phase continues until the index reaches its highest point of the year. However, the upward momentum is not sustained, as the index subsequently experiences a sharp drop. The final part of the year is marked by increased volatility and irregular fluctuations, ending at a lower level than its peak. Therefore, the visual pattern suggests that JII's performance in 2024 was shaped by short-term market dynamics rather than a sustained trend, possibly influenced by macroeconomic factors, investor sentiment, or sectoral changes within the index.

### *3.2. Identification of ARIMA Model*

Identification of ARIMA is the first step in the model-building process. At this stage, ARIMA parameters are identified using Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots in Figure 3. The vertical dashed blue lines in both plots of ACF and PACF represent the 95% confidence intervals. Any spike that extends beyond these bounds is considered statistically significant, suggesting the presence of serial correlation at that lag. These visual cues serve as a practical guide in determining plausible AR and MA orders.

Based on Figure 3, the ACF plot of JII return shows the most significant autocorrelation coefficient at Lag 2, indicating a positive correlation between values at time $t$, $t-1$, and $t-2$. Other lags mostly fall within the confidence bounds, implying insignificance. Similarly, the PACF plot also exhibits a clear spike at Lag 2, with coefficients gradually decreasing thereafter.

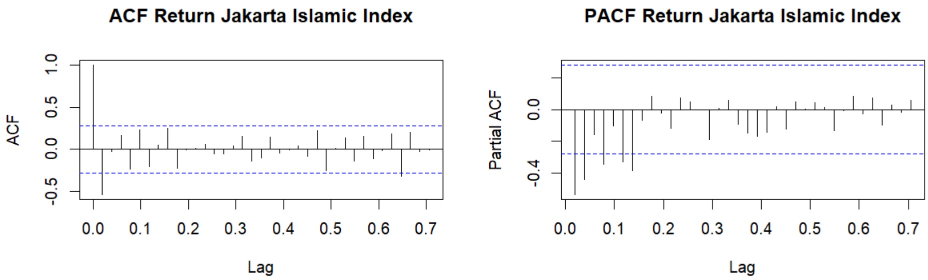


Figure 3. ACF and PACF plots of JII return data.

Based on this preliminary identification, several candidate ARIMA models were considered: ARIMA(0,1,1), ARIMA(1,1,0), ARIMA(1,1,1), ARIMA(2,1,0), and ARIMA(2,1,1). The next step involved parameter estimation and selection of the best model based on the Akaike Information Criterion (AIC), as presented below.

Table 1. AIC Values of ARIMA Models

| Model | AIC |
|---|---|
| ARIMA(0,1,1) | -174.3018 |
| ARIMA(1,1,0) | -152.9824 |
| ARIMA(1,1,1) | -187.3580 |
| ARIMA(2,1,0) | -173.2878 |
| ARIMA(2,1,1) | -196.7124 |

Based on Table 1, ARIMA(2,1,1) was the best ARIMA for measuring JII return data. It occurred when it had the lowest AIC evaluation value of $-196.7124$ compared to others. The parameter estimation for ARIMA(2,1,1) is presented as follows:

Table 2. Parameter Estimation for ARIMA(1,1,2)

| Parameter | Value |
|---|---|
| $\phi_1$ | -0.7879 |
| $\phi_2$ | -0.4852 |
| $\theta_1$ | -0.9999 |
| $\sigma^2$ | 0.0007 |

Based on Table 2, the estimation for Autoregressive (AR) and Moving Average (MA) parameters of ARIMA(2,1,1) was obtained. The equation for ARIMA(2,1,1) based on the results of its parameter estimation is as follows:

$$Y_t = -0.7879Y_{t-1} - 0.4852Y_{t-2} - 0.9999\varepsilon_{t-1} + \varepsilon_t \quad (16)$$

where $Y_t$ is the return data at time $t$, $Y_{t-1}$ is the return data at time $t-1$, $Y_{t-2}$ is the return data at time $t-2$, $\varepsilon_{t-1}$ is the error at $t-1$, and $\varepsilon_t$ is the error at $t$.

### *3.3. Identification of GARCH*

GARCH is useful for capturing changes in variance that are not constant over time. The initial stage in implementing the model is to determine the appropriate GARCH($p$,$q$) specification based on the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots of the residuals from the ARIMA(2,1,1) model in Figure 4. The vertical dashed lines in both plots represent the 95% confidence intervals. Any spike that crosses these bounds indicates statistically significant autocorrelation at that lag. This helps identify whether remaining patterns exist in the conditional heteroscedasticity of the residuals, which is a signal for applying GARCH-type models.

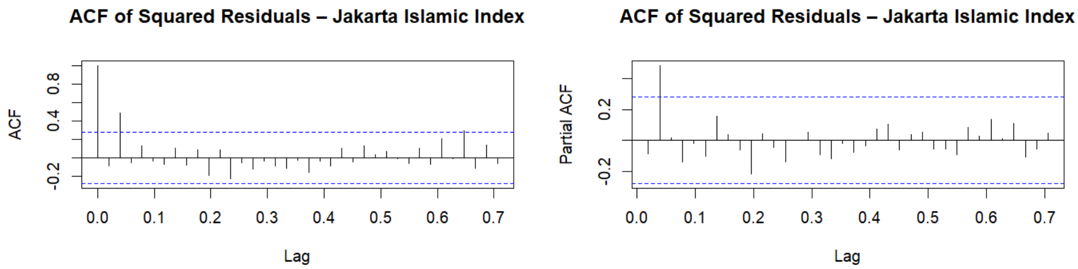


Figure 4. ACF and PACF plots for the residuals of the ARIMA(2,1,1) model.

Based on Figure 4, the ACF plot of the residuals shows a significant spike at Lag 1, while the PACF plot displays significance at Lag 2. This suggests the presence of autocorrelated volatility in the residuals, justifying the application of GARCH modeling. From these observations, several candidate GARCH models were considered: GARCH(1,0), GARCH(1,1), GARCH(2,0), and GARCH(2,1), to be evaluated further in the model selection stage. Subsequently, the parameter estimation was carried out for these possible GARCHs to obtain the coefficient values presented below.

Table 3. Parameter Estimation of GARCH Models

| Model | $\omega$ | $\alpha_1$ | $\alpha_2$ | $\beta_1$ | Log Likelihood |
|---|---|---|---|---|---|
| GARCH(1,0) | 0.0004* | 0.7855* | - | - | 100.5360 |
| GARCH(1,1) | 0.0004* | 0.8251* | - | -0.0328 (ns) | 100.5630 |
| GARCH(2,0) | 0.0004* | 0.8256* | -0.0359 (ns) | - | 100.3490 |
| GARCH(2,1) | 0.0004* | 0.8239* | -0.0305 (ns) | -0.0014 (ns) | 100.3970 |

Significance levels: *** $p < 0.001$, ** $p < 0.01$, * $p < 0.05$, (ns) not significant

Based on Table 3, the GARCH(1,0) model appears to be the most appropriate among the evaluated models, as it produces the highest log likelihood value (100.536) and contains only statistically significant parameters. In contrast, the other models such as GARCH(1,1), GARCH(2,0), and GARCH(2,1) include one or more parameters that are not statistically significant, which may reduce the reliability of these models. For example, the additional $\beta_1$ parameter in GARCH(1,1) and GARCH(2,1), as well as the second $\alpha$ term in GARCH(2,0) and GARCH(2,1), are found to be insignificant, suggesting that their inclusion does not meaningfully improve model performance.

In time series modeling, a more parsimonious model with statistically significant parameters and a higher log likelihood is generally preferred because it provides a better balance between model fit and interpretability. Given its simplicity and strong statistical performance, the GARCH(1,0) model is selected as the best fitting model for capturing the volatility dynamics in the residuals of the ARIMA(2,1,1) model. The fitted variance equation for the selected GARCH(1,0) model is:

$$\sigma_t^2 = 0.0004 + 0.7855\varepsilon_{t-1}^2 \tag{17}$$

This equation indicates that the current conditional variance is influenced solely by the past squared residuals, characteristic of the ARCH-type structure.

The identification of asymmetric models, including Exponential GARCH (EGARCH), Threshold GARCH (TGARCH), and Asymmetric Power GARCH (APGARCH), was subsequently carried out.

### *3.4. Identification of EGARCH*

EGARCH is a time series model used to capture volatility with asymmetric effects. In addition, it is used to measure asymmetric effects through the logarithm of the conditional variance in GARCH(1,0). The parameter estimation and coefficient values of the model are presented as follows.

Table 4. Parameter Estimation of EGARCH(1,0) with ARIMA(2,1,1)

| Parameter | Estimate | Std. Error | t value | Pr(¿—t—) |
|---|---|---|---|---|
| $\mu$ | -0.001241 | 0.002542 | -0.48810 | 0.6255 |
| $\omega$ | -4.995307 | 1.058940 | -4.71727 | 0.0000 |
| $\alpha_1$ | 0.036193 | 0.129130 | 0.28029 | 0.7793 |
| $\beta_1$ | -0.166385 | 0.110946 | -1.49970 | 0.1337 |
| $\beta_2$ | 0.524259 | 0.020760 | 25.25387 | 0.0000 |
| $\gamma_1$ | 0.767778 | 0.042365 | 18.12284 | 0.0000 |

Based on the parameter estimation of EGARCH(1,0) in Table 4, the following equation was obtained:

$$\begin{aligned} \ln(\sigma_t^2) &= \omega + \alpha_1 \left( \left| \frac{\epsilon_{t-1}}{\sigma_{t-1}} \right| - \mathbb{E}\left[ \left| \frac{\epsilon_{t-1}}{\sigma_{t-1}} \right| \right] \right) + \gamma_1 \left( \frac{\epsilon_{t-1}}{\sigma_{t-1}} \right) \\ &= -4.9953 + 0.0362 \left( \left| \frac{\epsilon_{t-1}}{\sigma_{t-1}} \right| - \mathbb{E}\left[ \left| \frac{\epsilon_{t-1}}{\sigma_{t-1}} \right| \right] \right) + 0.7678 \left( \frac{\epsilon_{t-1}}{\sigma_{t-1}} \right) \end{aligned} \quad (18)$$

### *3.5. Identification of TGARCH*

TGARCH is a model that captures asymmetric effects in volatility. Positive and negative shocks to the measured variables have different impacts on volatility.

Table 5. Parameter Estimation of TGARCH(1,0) with ARIMA(2,1,1)

| Parameter | Estimate | Std. Error | t value | Pr(¿—t—) |
|---|---|---|---|---|
| $\mu$ | -0.000154 | 0.003741 | -0.041147 | 0.96718 |
| $\omega$ | 0.000004 | 0.000037 | 0.120543 | 0.90405 |
| $\alpha_1$ | 0.078070 | 0.183373 | 0.425747 | 0.67029 |
| $\beta_1$ | 0.000000 | 0.150022 | 0.000000 | 1.00000 |
| $\beta_2$ | 1.000000 | 0.007407 | 135.011334 | 0.00000 |
| $\gamma_1$ | -0.158141 | 0.174059 | -0.908544 | 0.36359 |

Based on the parameter estimation of TGARCH(1,1) in Table 6, the following equation was obtained:

$$\sigma_t^2 = 0.000004 + 0.078070\,\varepsilon_{t-1}^2 - 0.158141\,\varepsilon_{t-1}^2 \cdot \mathbb{I}(\varepsilon_{t-1} < 0) \quad (19)$$

Based on Table 6, the parameter $\lambda_1$ confirmed asymmetric effects on JII return data by showing a negative value. Hence, volatility in TGARCH(1,0) was more responsive to adverse shocks than positive. However, based on the t-value and p-value, $\lambda_1$ was not statistically significant, as TGARCH showed insignificant and asymmetric effects on JII return data. It confirmed no significant bias in the residuals from positive or negative shocks. TGARCH could capture the dynamics of volatility, although asymmetric effects were insignificant. It occurred when the model showed various diagnostic tests fit the data, and there was no remaining autocorrelation or heteroscedasticity. TGARCH also showed good fit results when measuring JII return data.

### *3.6. Identification of APGARCH*

APGARCH is an extension of GARCH that captures asymmetric and leverage effects (leverage effects show that negative shocks have a greater influence than positive) in JII return volatility data. This model includes an additional parameter $\delta$ (power), allowing for differences in volatility response to positive and negative shocks.

Table 6. Parameter Estimation of APGARCH(1,0) with ARIMA(2,1,1)

| Parameter | Estimation | Standard error | t-value | Pr(¿—t—) |
|---|---|---|---|---|
| $\mu$ | 0.000591 | 0.002182 | 0.2707 | 0.7866 |
| $\omega$ | 0.000063 | 0.000049 | 1.2767 | 0.2017 |
| $\alpha_1$ | 0.588402 | 0.170668 | 3.4476 | 0.0006 |
| $\beta_1$ | 0.000000 | 0.078278 | 0.0000 | 1.0000 |
| $\beta_2$ | 0.000000 | 0.084842 | 0.0000 | 1.0000 |
| $\gamma_1$ | 0.146392 | 0.228363 | 0.6411 | 0.5215 |
| $\delta$ | 2.372820 | 0.255811 | 9.2757 | 0.0000 |

Based on the parameter estimation of APGARCH(1,0), the following equation was obtained:

$$\sigma_t^{2.372820} = 0.000063 + 0.588402\left(|\varepsilon_{t-1}| - 0.146392\,\varepsilon_{t-1}\right)^{2.372820} \tag{20}$$

Table 6 indicates that the AR and MA parameters were significant within the ARIMA component at various levels, suggesting that past values and shocks influenced the current return. In contrast, within the APGARCH component, only the $\alpha_1$ and $\delta$ parameters were statistically significant. The results imply that past shocks and the variance transformation's power parameters significantly contributed to volatility modeling.

However, the parameters $\omega$, $\gamma_1$, and both ARIMA lags $\beta_1$ and $\beta_2$ were not statistically significant. It indicates that the constant term, the asymmetric effect, and the AR terms in the ARIMA component did not have a meaningful individual impact on the model. Despite this, the APGARCH(1,0) model demonstrated a strong capability to capture volatility dynamics, as evidenced by good model fit and the absence of residual autocorrelation and heteroscedasticity. It confirms that the model is suitable for modeling the return volatility of the JII.

### 3.7. *Identification of MSGARCH*

MSGARCH combines elements of GARCH with Markov Switching structure to address problems of heteroscedasticity and structural changes. This study used GARCH (1,0) to measure heteroscedasticity and two regimes in the Markov Switching model, namely one for high volatility and another for low volatility. Subsequently, the parameter estimation of MSGARCH was carried out and presented in Table 7. Based on Table 7, the p-value

Table 7. Parameter Estimation of MSGARCH

| Parameter | Estimate | Std. Error | t value | Pr(¿—t—) |
|---|---|---|---|---|
| $\alpha_{01}$ | 0.0004 | 0.0003 | 1.3551 | 0.0877 |
| $\alpha_{11}$ | 0.8256 | 0.8401 | 0.9828 | 0.1629 |
| $\alpha_{21}$ | 0.0330 | 1.5654 | 0.0211 | 0.4916 |
| $\beta_1$ | 0.0002 | 0.0251 | 0.0085 | 0.4966 |
| $\nu_1$ | 99.7961 | 12.5876 | 7.9281 | 1.11e-15 |
| $\alpha_{02}$ | 0.0004 | 0.0003 | 1.3176 | 0.0938 |
| $\alpha_{12}$ | 0.8256 | 0.8654 | 0.9540 | 0.1700 |
| $\alpha_{22}$ | 0.0330 | 1.6181 | 0.0204 | 0.4919 |
| $\beta_2$ | 0.0002 | 0.0256 | 0.0084 | 0.4967 |
| $P_{11}$ | 0.9065 | 17.3005 | 0.0524 | 0.4791 |
| $P_{21}$ | 0.0974 | 17.8856 | 0.0054 | 0.4978 |

was smaller than $\alpha = 0.05$, confirming that all parameters in MSGARCH were significant. Parameter estimation for Bayesian MSGARCH is presented as follows:

$$\sigma_t^2 = \begin{cases} 0.0001 + 0.1424\epsilon_{t-1}^2, & \text{regime 1} \\ 0.0007 + 0.0421\sigma_{t-1}^2, & \text{regime 2} \end{cases} \tag{21}$$

The parameter values $\alpha$ and $\beta$ were positive for each regime, confirming that the conditional variance of JII return at time $t-1$ was influenced by the conditional variance and squared residuals at time $t-1$. The transition probability matrix is presented as follows:

$$P = \begin{pmatrix} 0.9065 & 0.0935 \\ 0.0974 & 0.9026 \end{pmatrix} \tag{22}$$

Based on the transition probability matrix, the system has high persistence in both volatility regimes. Specifically, the probability of remaining in the low volatility state is 0.9065, while the probability of switching from low to high volatility is 0.0935. Conversely, the probability of staying in the high volatility state is 0.9026, and the probability of transitioning from high to low volatility is 0.0974.

These values indicate that low and high-volatility states are highly persistent, with only a slight chance of transitioning between regimes in the short term. This persistence reflects the tendency of financial markets to remain in a given volatility regime for extended periods before switching.

### *3.8. Identification of Bayesian MSGARCH*

*3.8.1. Prior Specification and Sensitivity Analysis.* This study adopts a Bayesian estimation approach and explores two prior settings to assess the sensitivity of parameter estimates and forecasting performance: a *default weakly-informative prior* and an *alternative informative prior*. The prior choices are motivated by both theoretical considerations and common practices in the literature on regime-switching volatility models.

In the default specification, the transition probabilities $P_{ij}$ are assigned Beta distributions, consistent with their support on the $[0, 1]$ interval. The volatility-related parameters ($\omega$, $\alpha$, and $\beta$) follow Gamma distributions, ensuring positivity and reflecting common prior choices in volatility modeling. For the conditional mean parameter $\mu$, a diffuse Normal prior centered at zero with a large variance is used, providing minimal influence on the posterior while allowing flexibility.

The alternative prior is constructed to be more informative, with tighter distributions centered around prior expectations. For example, the parameters governing volatility dynamics are still assigned Gamma distributions, but with smaller variance and higher shape parameters, concentrating prior mass near central values. Similarly, the Normal prior on $\mu$ is given a smaller variance, and Beta priors on $P_{ij}$ are adjusted to favor more persistent regimes.

Table 8 reports the predictive performance of the model under both specifications, measured via out-of-sample Mean Absolute Percentage Error (MAPE) and Mean Squared Error (MSE).

Table 8. Predictive Performance under Different Prior Specifications

| **Prior Specification** | **MAPE** | **MSE** |
|---|---|---|
| Default Prior | 2.6825 | 0.1977 |
| Alternative Prior | 3.5735 | 0.3543 |

The results reveal that the default prior delivers superior forecasting accuracy. Although the alternative prior incorporates more prior knowledge, it reduces the model's flexibility to learn from the data, particularly in the presence of latent regime-switching behavior. Informative priors can overly constrain the posterior distribution and bias the estimates, especially in cases with weak identification or persistent state dynamics. Conversely, the weakly informative priors allow for greater posterior adaptability, leading to more accurate and robust inferences. This sensitivity analysis illustrates that even within standard prior families (e.g., Gamma for volatility, Beta for transition probabilities), the informativeness of the prior plays a crucial role. The default weakly informative setting is therefore preferred and adopted in the final model estimation.

These findings underscore the importance of careful prior selection in Bayesian regime-switching models, especially in financial time series characterized by high volatility persistence and nonlinear dynamics. In particular, overly informative priors may lead to posterior shrinkage, masking regime transitions or underestimating tail risks that are critical for risk management and forecasting in Sharia-compliant markets. The tendency of informative priors to dominate the likelihood is especially problematic in models with latent states, where transition probabilities and volatility parameters exhibit path dependence and are weakly identified in short samples. Moreover, the superior performance of the weakly informative (default) prior confirms that a more data-driven approach allows the model to dynamically capture the asymmetries and regime shifts inherent in financial returns. This reinforces the need for prior calibration not only based on theoretical intuition but also empirical validation through predictive performance metrics like MAPE and MSE.

*3.8.2. MCMC Settings and Convergence Diagnostics.* Parameter estimation was performed via Markov Chain Monte Carlo (MCMC) using a Gibbs sampling algorithm tailored for latent-variable models. Due to the path dependence and unobserved regime sequences in MS-GARCH models, classical likelihood-based inference becomes computationally infeasible [3]. MCMC offers a viable alternative, allowing posterior sampling by increasing the latent states and iteratively drawing from conditional distributions.

Two MCMC configurations were considered: 30,000 and 100,000 iterations. The initial choice of 30,000 iterations reflects common practice in the Bayesian GARCH literature. However, given the complexity introduced by regime switching and high persistence in volatility dynamics, longer chains may be necessary to ensure convergence. To evaluate convergence adequacy, standard diagnostics were applied: the Gelman-Rubin Potential Scale Reduction Factor (PSRF) and trace plots.

**Gelman-Rubin Convergence Diagnostics.** Table 9 presents the Potential Scale Reduction Factor (PSRF) values for both iteration settings.

Table 9. Gelman-Rubin PSRF under Default Prior for Different Iteration Settings

| Parameter | 30,000 Iterations | | 100,000 Iterations | |
|---|---|---|---|---|
| | Point | UCI | Point | UCI |
| $\alpha_{01}$ | 1.29 | 1.37 | 1.00 | 1.01 |
| $\alpha_{11}$ | 1.08 | 1.12 | 1.01 | 1.06 |
| $\beta_1$ | 1.06 | 1.23 | 1.01 | 1.05 |
| $\alpha_{02}$ | 3.13 | 15.86 | 1.00 | 1.01 |
| $\alpha_{12}$ | 1.60 | 2.75 | 1.02 | 1.08 |
| $\beta_2$ | 2.19 | 5.48 | 1.00 | 1.02 |
| $P_{11}$ | 2.37 | 12.24 | 1.00 | 1.00 |
| $P_{21}$ | 1.09 | 1.10 | 1.01 | 1.03 |
| **Multivariate PSRF** | **2.91** | – | **1.03** | – |

Based on Table 9, under the 30,000-iteration setting, several parameters exceed the critical threshold of 1.1 for the Upper Confidence Interval (UCI), and the multivariate Potential Scale Reduction Factor (PSRF) reaches 2.91, indicating poor convergence and potential non-stationarity of the Markov chains. Such values suggest that the chains have not yet stabilized across the parameter space and may be trapped in local modes, leading to unreliable posterior estimates. Particularly for regime-switching models with high-dimensional latent structures like MSGARCH, shorter iteration lengths often fail to capture the full dynamics of state transitions, especially when parameters are highly correlated or exhibit multimodality. This can result in biased estimates, inflated uncertainty measures, and poor model fit—ultimately undermining the credibility of subsequent inferences and predictions.

In contrast, with 100,000 iterations, all UCI values fall below the 1.1 convergence threshold, and the multivariate PSRF decreases substantially to 1.03, indicating strong convergence and consistent mixing behavior across chains. This improvement reflects the effectiveness of longer MCMC runs in overcoming initial burn-in effects and achieving stationarity in posterior draws. The tighter convergence also enhances the stability of parameter estimation, particularly for transition probabilities and volatility persistence terms that are highly sensitive to model specification and initial values. These results underscore the importance of diagnostic-based iteration tuning in

Bayesian estimation routines. Therefore, the 100,000-iteration configuration is not only justified statistically but is also crucial for producing robust and interpretable results in regime-switching volatility models like Bayesian MSGARCH.

**Traceplot Interpretation.** To complement the quantitative PSRF results presented earlier, a visual inspection of the Markov Chain Monte Carlo (MCMC) simulation is conducted through trace plots, as illustrated in Figure 5. This figure showcases the trace plots of four key parameters from one regime—specifically $\omega$ (long-run average volatility), $\alpha_1$ (short-run shock response), $\beta_1$ (volatility persistence), and $\mu$ (unconditional expected return). These parameters were chosen because they represent the core dynamics of volatility behavior within regime 1 and are critical for understanding the structural characteristics of the GARCH process in the Bayesian MSGARCH framework. Notably, the $\alpha_1$ parameter in the figure corresponds to $\alpha_{11}$ in the formal model specification, indicating its role in capturing the autoregressive response to past shocks within regime 1.

While the trace plots offer a detailed look into the behavior of select parameters, they serve as a qualitative diagnostic tool and should be interpreted in tandem with the broader convergence diagnostics presented in Table 9. That table covers the full parameter space, including parameters from both regimes—such as $\alpha_{01}$ and $\alpha_{11}$ for regime 1, and $\alpha_{02}$ and $\alpha_{12}$ for regime 2—as well as regime-specific volatility parameters $\beta_1$ and $\beta_2$, and the Markov transition probabilities $P_{11}$ and $P_{21}$. The inclusion of only regime 1 parameters in the trace plots is intentional, aiming to provide a focused and interpretable visual overview without overwhelming the reader with the full complexity of the model. This selective approach helps highlight the mixing behavior, chain stability, and convergence tendencies in a representative subset of the parameter space. Together, the trace plots and PSRF statistics offer a comprehensive convergence assessment, combining both visual intuition and formal statistical evidence to validate the robustness of the MCMC estimation process.

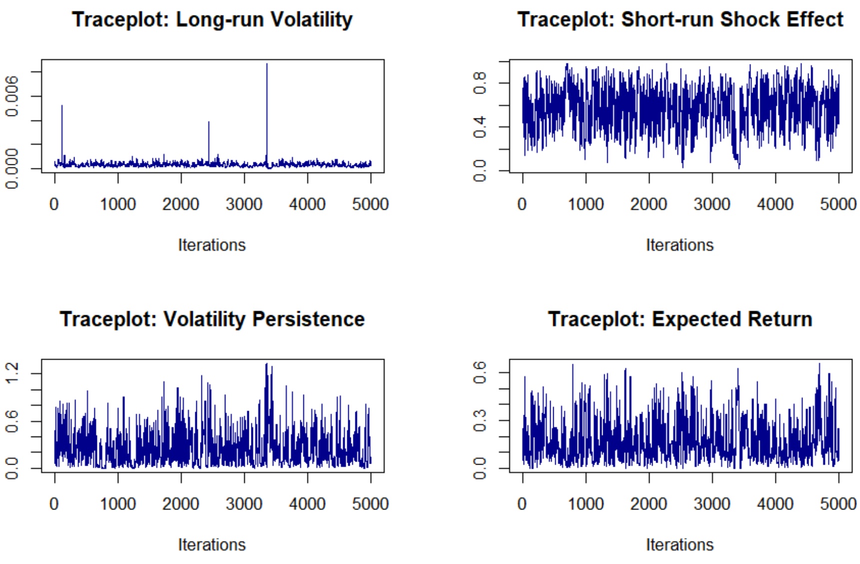


Figure 5. Traceplots of selected parameters ($\omega$, $\alpha_1$, $\beta_1$, $\mu$) under the default prior.

Figure 5 presents the MCMC trace plots for four key parameters—long-run volatility ($\omega$), short-run shock effect ($\alpha_1$), volatility persistence ($\beta_1$), and expected return ($\mu$)—based on the iteration configuration under the default prior setting. The traceplot for *long-run volatility* ($\omega$) exhibits a mostly flat trajectory interspersed with a few significant spikes. While the bulk of the samples are concentrated around a stable region, the occasional jumps to extreme values may signal poor mixing or sensitivity to latent regime changes. This pattern suggests that additional iterations or reparameterization may be necessary to ensure robust inference for this parameter. The *short-run* *shock effect* ($\alpha_1$) shows a well-mixed chain, fluctuating stably around a central mean without discernible trends or autocorrelation. This indicates effective sampling from the posterior distribution and supports convergence for this parameter. The trace for *volatility persistence*, as captured by the $\beta_1$ parameter, spans a wide value range but maintains free movement in the parameter space. The absence of drift or stickiness, despite the parameter's high persistence nature, suggests acceptable mixing and stationarity. Lastly, the *expected return* ($\mu$) traceplot also reflects stable behavior, with samples oscillating around a mean level without evidence of autocorrelation or nonstationarity. The chain appears to have traversed the posterior space adequately.

In summary, the convergence diagnostics, both numerical (PSRF) and graphical (traceplots), strongly indicate that the 100,000 iteration MCMC setting provides stable, well-mixed chains suitable for reliable posterior inference. These findings underscore the importance of both prior specification and sufficient sampling length in the Bayesian estimation of regime-switching models.

*3.8.3. Parameter Estimation of Bayesian MSGARCH.* MSGARCH integrated with Bayesian is a model estimated using Bayesian. This estimation is carried out using the Gibbs Sampling algorithm. Parameter Estimation of Bayesian MSGARCH can be shown as follows:

Table 10. Parameter Estimation of Bayesian MSGARCH

| Parameter | Mean | SD | SE | TSSE | RNE |
|---|---|---|---|---|---|
| $\alpha_{01}$ | 0.0004 | 0.0002 | 0.0000 | 0.0000 | 0.1267 |
| $\alpha_{11}$ | 0.3872 | 0.1864 | 0.0037 | 0.0117 | 0.1021 |
| $\alpha_{21}$ | 0.5832 | 0.3262 | 0.0065 | 0.0268 | 0.0594 |
| $\beta_1$ | 0.0801 | 0.0893 | 0.0018 | 0.0054 | 0.1081 |
| $\nu_1$ | 78.5190 | 22.1034 | 0.4421 | 1.1577 | 0.1458 |
| $\alpha_{02}$ | 49.5149 | 28.0817 | 0.5616 | 1.7238 | 0.1062 |
| $\alpha_{12}$ | 0.2527 | 0.1513 | 0.0030 | 0.0070 | 0.1858 |
| $\alpha_{22}$ | 1.0395 | 0.4267 | 0.0085 | 0.0292 | 0.0856 |
| $\beta_2$ | 0.0045 | 0.0106 | 0.0002 | 0.0007 | 0.1030 |
| $P_{11}$ | 0.9672 | 0.0237 | 0.0005 | 0.0011 | 0.1732 |
| $P_{21}$ | 0.5970 | 0.2449 | 0.0049 | 0.0124 | 0.1558 |

Based on Table 10, the estimated parameter of Bayesian MSGARCH can be presented as:

$$\sigma_t^2 = \begin{cases} 0.0004 + 0.3872, \epsilon_{t-1}^2 + 0.5832, \epsilon_{t-2}^2 + 0.0801, \sigma_{t-1}^2, & \text{if regime 1} \\ 49.5149 + 0.2527, \epsilon_{t-1}^2 + 1.0395, \epsilon_{t-2}^2 + 0.0045, \sigma_{t-1}^2, & \text{if regime 2} \end{cases} \tag{23}$$

The parameter values $\alpha$, and $\beta$ were positive for each regime, meaning that the conditional variance value of JII return at time $t-1$ was influenced by the conditional variance and squared residuals at time $t-1$. The transition probability matrix is presented as follows:

$$P = \begin{pmatrix} 0.9672 & 0.0328 \\ 0.5970 & 0.4030 \end{pmatrix} \tag{24}$$

Based on the transition probability matrix, the probability of staying in the low volatility regime (Regime 1 to Regime 1) is 96.72%, while the probability of switching from low to high volatility (Regime 1 to Regime 2) is 3.28%. Conversely, the probability of remaining in the high volatility regime (Regime 2 to Regime 2) is 40.30%, and the probability of transitioning from high to low volatility (Regime 2 to Regime 1) is 59.70%. These results indicate that the low volatility regime is more persistent, while the high volatility regime tends to revert more frequently to the low volatility state.

### *3.9. Evaluation Models*

In evaluating the performance of volatility models applied to Jakarta Islamic Index (JII) stock return data, two primary evaluation approaches were utilized: information criteria and predictive accuracy metrics. The Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) assess model fit while penalizing complexity, balancing goodness-of-fit and parsimony. The Deviance Information Criterion (DIC) evaluates Bayesian models by accounting for fit and complexity based on posterior parameter distributions. Predictive accuracy was measured using Mean Absolute Percentage Error (MAPE) and Mean Squared Error (MSE), which quantify forecasting capability. Together, these criteria furnish a comprehensive assessment of model performance from statistical, Bayesian, and practical forecasting perspectives.

Table 11. Comparison of AIC, BIC, and DIC Values for Various GARCH Models

| **Model** | **AIC** | **BIC** | **DIC** |
|---|---|---|---|
| GARCH | -4.0219 | -3.9446 | NA |
| EGARCH | -3.8701 | -3.5998 | NA |
| TGARCH | -4.9150 | -4.7256 | NA |
| APGARCH | -3.8819 | -3.6503 | NA |
| MSGARCH | -174.2089 | -153.3989 | NA |
| Bayesian MSGARCH | NA | NA | -188.4558 |

Table 11 presents the comparative evaluation of volatility models estimated via maximum likelihood methods, including GARCH(1,0), EGARCH(1,1), TGARCH(1,1), APGARCH(1,1), and the Markov Switching GARCH (MSGARCH). Notably, the MSGARCH model exhibits the lowest AIC and BIC values, recorded at -174.2089 and -153.3989, respectively, signifying its superior balance between parsimony and model fit in capturing the inherent regime-switching dynamics of the Sharia stock market. These results indicate that MSGARCH provides the most parsimonious yet best-fitting representation of the underlying volatility dynamics, effectively capturing regime-switching behavior inherent in the Sharia stock market. While the TGARCH model demonstrates competitive performance, it remains inferior to MSGARCH based on these information criteria.

The superiority of the MSGARCH model in terms of AIC and BIC underscores its ability to accommodate structural breaks and time-varying volatility regimes that simpler GARCH-type models fail to capture. This capacity is particularly relevant in financial time series exhibiting abrupt changes in market conditions, such as those influenced by economic cycles or regulatory shifts affecting Sharia-compliant stocks. Consequently, the MSGARCH model's flexibility offers a more nuanced understanding of volatility clustering and persistence in the JII returns.

Unlike classical maximum likelihood-based models, the Bayesian MSGARCH framework employs Markov Chain Monte Carlo (MCMC) techniques for parameter estimation. Consequently, standard information criteria such as AIC and BIC, which rely on likelihood-based penalization, are not directly applicable to Bayesian inference. Instead, model adequacy within the Bayesian paradigm is assessed using the Deviance Information Criterion (DIC), a generalized extension of AIC tailored for posterior distributions. With a DIC value of -188.4558, the Bayesian MSGARCH model demonstrates a robust capacity to account for structural breaks and stochastic volatility regimes while incorporating Bayesian regularization.

Due to the fundamental differences between likelihood-based estimation methods used for GARCH, EGARCH, TGARCH, APGARCH, and MSGARCH models evaluated by AIC or BIC and the Bayesian estimation approach applied to the Bayesian MSGARCH model assessed by DIC, direct comparisons are not strictly equivalent. Therefore, an empirical evaluation based on out-of-sample forecasting accuracy was performed, as presented in Table 12. The findings reveal that the Bayesian MSGARCH model surpasses its classical counterparts in Mean Absolute Percentage Error (MAPE) and Mean Squared Error (MSE), highlighting its superior ability to capture volatility clustering and persistence in JII return dynamics.

Table 12 depicts the comparative predictive accuracy of six volatility models applied to weekly JII returns: GARCH(1,0), EGARCH(1,1), TGARCH(1,1), APGARCH(1,1), MSGARCH, and Bayesian MSGARCH. The

Table 12. Comparison of Volatility Model Performance Based on MAPE and MSE

| Model | MAPE | MSE |
|---|---|---|
| GARCH | 0.8185 | 0.0232 |
| EGARCH | 0.8716 | 0.0273 |
| TGARCH | 0.8634 | 0.0269 |
| APGARCH | 0.8702 | 0.0272 |
| MSGARCH | 0.8161 | 0.0231 |
| Bayesian MSGARCH | 0.8106 | 0.0224 |

Bayesian MSGARCH model achieves the lowest forecasting errors, with a MAPE of 0.8106 and an MSE of 0.0224, outperforming both classical MSGARCH and GARCH-type models. This superior predictive capability can be attributed to the Bayesian framework's flexibility in parameter estimation and ability to incorporate parameter uncertainty effectively. Conversely, models such as APGARCH and EGARCH exhibit relatively higher prediction errors, indicating less reliable forecasting performance. Hence, while the classical MSGARCH model excels in model fit criteria under maximum likelihood estimation, the Bayesian MSGARCH model demonstrates the best overall performance regarding out-of-sample predictive accuracy.

The findings suggest that volatility models accounting for asymmetric effects and structural regime shifts provide robust frameworks for modeling the complex dynamics of JII Sharia stock returns. The asymmetric nature of Sharia stock volatility, characterized by nonlinear responses and threshold effects, necessitates advanced modeling approaches. From an investment perspective, recognizing these characteristics is crucial for designing and implementing effective trading strategies such as long-short, market-neutral, and event-driven approaches. Specifically, the long-short strategy capitalizes on anticipated price movements by taking simultaneous long and short positions; the market-neutral strategy seeks to hedge systemic market risk; and the event-driven strategy exploits volatility induced by specific economic or corporate events.

Despite its strengths, Bayesian MSGARCH involves considerable computational costs. The reliance on MCMC techniques, such as reversible jump and delayed rejection algorithms, often requires long pilot runs and extensive iterations to achieve convergence—particularly in models with high parameter correlations or complex posterior structures [31]. This introduces a trade-off between flexibility and efficiency, especially when working with high-dimensional or multi-regime specifications. The high dimensionality and latent structure inherent in Bayesian MSGARCH models can also complicate inference. The outcomes are sensitive to prior distributions, and when the number of regimes is inferred from data, the model risks overfitting or underfitting depending on prior assumptions. Moreover, interpretability may diminish as model complexity increases, potentially hindering communication of results and their practical application [9].

To address some of these issues, advanced sampling methods such as multi-move Gibbs samplers have been developed to improve computational efficiency. By updating blocks of latent states simultaneously, these methods enhance mixing and reduce autocorrelation in the posterior chain, particularly in settings with multiple regimes [22]. Such techniques are crucial for stabilizing estimation in real-world financial applications. The ability of Bayesian MSGARCH models to track regime changes over time offers valuable insights for portfolio management and risk control. Integrating modern tools like machine learning or sentiment analysis can further augment these models, helping investors navigate the nuanced behavior of Sharia-compliant stocks. Recognizing asymmetric volatility patterns and regime-dependent dynamics is essential for strategy development, whether through long-short positioning, market-neutral hedging, or event-driven trading.

Volatility in JII Sharia stocks exhibits structured asymmetry and regime-dependent dynamics, shaped by macroeconomic shifts, regulatory frameworks, firm-specific fundamentals, and investor sentiment. These characteristics present both risks and opportunities for market participants, necessitating robust modeling techniques such as Bayesian MSGARCH that can adapt to dynamic and nonlinear environments. Notably, the risk profile of Sharia-compliant investments diverges from conventional equities, underscoring the need for tailored risk management strategies. External economic and political developments, along with company-specific financial performance, substantially influence asymmetric volatility and price movements in the Sharia stock market.

Market sentiment likewise plays a pivotal role in shaping these dynamics. In response, investors commonly employ hedging strategies, including long-term buy-and-hold positions and short-term volatility strategies—such as straddle, strangle, ratio writing, and iron condor options—to manage downside risk. These approaches are particularly effective in navigating the evolving structure of volatility regimes. Ultimately, disciplined strategy execution supported by empirical modeling—rather than reactive decision-making—remains crucial in mitigating risk and leveraging the unique features of Islamic financial markets [33].

The practical applications of this study are multifaceted, particularly in the domains of portfolio optimization, Value-at-Risk (VaR) forecasting, and policy design tailored to Sharia-compliant markets. The Bayesian MSGARCH model's ability to detect structural breaks and latent regime transitions enables investors to adjust portfolio exposure proactively, especially during high-volatility periods. This regime-sensitive structure enhances risk estimation accuracy—particularly in the context of sudden shocks or structural change—thereby supporting more informed capital allocation decisions [14]. Additionally, asymmetric volatility patterns revealed through the model inform dynamic hedging and long-short strategies, aligning investment behavior with underlying market regimes. From a regulatory perspective, the identification of volatility clustering and regime durations supports macroprudential oversight and the development of more resilient Islamic financial instruments. Thus, the integration of advanced regime-switching models provides both theoretical insights and practical utility in real-world decision-making within Islamic finance.

## 4. Conclusion

In conclusion, the Indonesian Sharia stock market, as reflected by JII returns, experienced substantial growth accompanied by increased market capitalization throughout the year. Despite this positive trend, high volatility remains a persistent challenge for investors, highlighting the necessity for reliable and sophisticated volatility modeling to support effective investment decisions. In this study, classical volatility models such as GARCH and asymmetric models including EGARCH, TGARCH, APGARCH, and MSGARCH were estimated using the Maximum Likelihood method, while the Bayesian MSGARCH model was estimated using the Markov Chain Monte Carlo method.

The performance evaluation employed multiple approaches, including information criteria (AIC or BIC) for classical models and the Deviance Information Criterion (DIC) for the Bayesian model, alongside predictive accuracy measures such as Mean Absolute Percentage Error (MAPE) and Mean Squared Error (MSE). Results showed that the MSGARCH model achieved the best balance of parsimony and fit among classical models, effectively capturing regime-switching volatility dynamics inherent to the Sharia stock market. Meanwhile, the Bayesian MSGARCH model demonstrated superior predictive accuracy and flexibility in accommodating structural breaks and rapid volatility shifts, particularly with limited data availability.

The findings indicate that Sharia-compliant stock volatility demonstrates significant asymmetric patterns and regime-dependent characteristics. These insights are essential for understanding market behavior during both bullish and bearish conditions. The Bayesian MSGARCH framework, with its capacity to incorporate parameter uncertainty and adjust to evolving market regimes, offers substantial value for applications in risk management and portfolio strategy formulation. The analysis emphasizes the necessity of adopting advanced volatility models that capture the intricate dynamics of Sharia stock returns. Employing such models facilitates improved forecasting accuracy and more effective risk mitigation, thereby supporting sound and disciplined decision-making within the evolving landscape of the Sharia stock market.

Moreover, this study provides empirical evidence to support the integration of regime-switching and Bayesian approaches in the modeling of Islamic equity markets, reinforcing their relevance in both academic research and professional practice. Future research could explore the incorporation of macroeconomic variables, high-frequency data, or hybrid models that combine regime-switching structures such as MSGARCH with machine learning techniques, including Long Short-Term Memory (LSTM) networks or other deep learning architectures, for improved volatility forecasting and pattern recognition in Sharia markets. As the Sharia financial ecosystem continues to expand, such methodological innovations will be increasingly important in aligning financial stability with ethical investment principles.

Furthermore, these findings offer valuable implications for policymakers and investors. For regulators, the distinct volatility structure of Sharia-compliant stocks highlights the need to develop Islamic financial instruments, such as Islamic derivatives-based hedging tools that align with Sharia principles while enabling effective risk management. For investors, the study reinforces the importance of adopting strategy-specific models tailored to the Islamic market context, which may include long-term holding, sector rotation, or Sharia-compliant options strategies to mitigate downside risk and enhance portfolio resilience.

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