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The Prediction by using Nonlinear Autoregressive Model

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Abstract The study proposed a nonlinear autoregressive model along with its stability conditions. The primary objective was to apply this model to predict the daily number of new COVID-19 cases in the Kingdom of Saudi Arabia during 2022. The model consists of two parts: a linear term and a nonlinear term that incorporates a decreasing function. This structure made it possible to construct a numerical example that meets the theoretical stability criteria, as illustrated in Example 1 of the paper. The model was then applied to real-world data, specifically the time series of daily COVID-19 infections recorded over a continuous three-month period in 2022. Using a Python-based implementation, the model parameters were estimated to fulfill the stability conditions. The residuals were analyzed, and the model was subsequently employed to forecast the expected number of new infections for the following month of 2022.

Keywords Limit cycle; Singular point; coefficient estimation; prediction; stability analysis.

AMS 2010 subject classifications 62M10, 62M20, 62M99, 62P99

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1. Introduction

A nonlinear autoregressive model was proposed that simulates the exponential autoregressive model of the scientist Ozaki and we used the "local linearization technique" to obtain an approximate linear model for the proposed study model. We first determined the single point of the proposed model, then determined the stability criteria of this single point and the stability of the limit cycle. To illustrate this technique, we mentioned examples 1 and 2 ,to illustrate this technique. The research model was used to analyze the real time series data related to COVID-19 infection in the Kingdom of Saudi Arabia during a period of three months in 2022 from the website `https://www.worldometers.info/coronavirus/country/saudi-arabia/`. To ensure the stability of the time series data, we applied the natural logarithm transformation because the original data is increasing exponentially, and then the first difference was taken to stabilize the time series as shown in Figure 7 For the research, Python programs were employed to process the data and estimate the parameters of the proposed first-order nonlinear model that met the stability criteria. Finally, we applied the study model to predict future COVID-19 cases in the Kingdom of Saudi Arabia, which is the goal of the research.

We can see the definition of the ARMA model of order (p, q) in [1]. Definitions of the "autoregressive model of time series of order p in discrete time," the "exponential nonlinear model of order p," the "singular point with its stability," the "limit cycle with its stability," and the technique of "local linearization" were given in [2]. Also, the "exponential nonlinear model with γ of order p" is defined in [3].

Numerous scholars have identified the criteria for ensuring the stability of nonlinear regression models through the utilization of the linear local approximation technique pioneered by Ozaki. The stability conditions for a

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limit cycle in the Gompertz autoregressive model were investigated in [4]. The stationary condition for the GJR-GARCH(Q, P) model was determined in [5]. Stability conditions for SATER models were found in [6], and for nonlinear time series models in [7]. Stability conditions for exponential GARCH models were studied in [8]. The stability of boiling water reactor oscillation through nonlinear time series modelling was studied in [9]. The stability analysis of RBF network-based state-dependent autoregressive models for nonlinear time series was conducted in [10].

Many scholars have employed regression models to forecast future values of analyzed data. Maobin Li, Shouwen Ji, and Gang Liu used three forecast models for ECS in [11]. Models were developed to forecast wind speeds over timescales ranging from 10 minutes to one hour in [12]. Forecasts for COVID-19 using a simple frequency method based on daily confirmed cases were generated in [13]. The future of the COVID-19 pandemic in Europe in 2021 was studied in [14]. Data from cohorts vaccinated against COVID-19 and BCG were examined to analyze how both COVID-19 and TB spread using a mathematical partitioned model in [15].

A new nonlinear autoregressive neural network time series model (NAR-NNTS) was introduced for predicting COVID-19 cases in [16]. A univariate time series model was used to predict the number of COVID-19 cases likely to occur in the coming days in India in [17]. A time series model using a nonlinear regression extrinsic neural network (NARX) was proposed for predicting recovered and deceased COVID-19 cases in [18]. The nonlinear smoothed autoregressive transition (STAR) model was demonstrated to improve the prediction of COVID-19 incidence rate in [19]. Linear and nonlinear forecasting models were confirmed to accurately capture the COVID-19 trend in Nigeria in [20].

Models were used to predict COVID-19 data using improved LSTM-ARIMA algorithms in [21]. A hybrid approach combining ARIMA and a single hidden layer autoregressive neural network (NNAR) was used to forecast daily COVID-19 cases in [22]. The accuracy of the best-fit ARIMA model forecasts was evaluated in [23]. A comparison between ARIMA and XGBoost models for predicting COVID-19 in the United States was conducted in [24]. COVID-19 confirmed cases and deaths in Chile were predicted using time series techniques in [25].

A deep belief network-based state-dependent autoregressive (DBN-AR) model was investigated in [26]. A newly developed nonlinear autoregressive model was applied to forecast daily new COVID-19 cases in Iraq in [27]. The NARNN framework employs the sigmoid activation function under varying settings of previous values and hidden neuron counts in [28]. A nonlinear autoregressive exogenous (NARX) model was constructed to forecast the spread of COVID-19 in Jordan in [29]. The study investigates the appropriateness of applying the K-means clustering algorithm to categorize five separate datasets that undergo feature selection through the Binary Harris Hawks Optimization Algorithm (BHHOA) in [30].

2. THE PROPOSED STUDY FRAMEWORK

The proposed model for studying nonlinear time series is

$$y_t = \sum_{i=1}^{p} \left[a_i + b_i \left(\frac{1}{2y_{t-1}} \right) \right] y_{t-i} + \varepsilon_t, \quad y_{t-1} \neq 0 \tag{1}$$

where,$\{\varepsilon_t\}$ is white noise $a_1, ..., a_p$;$b_1, ..., b_p$ are the constants of equation (1).

If $y_{t-1} \longrightarrow \mp\infty$, then

$$\frac{1}{y_{t-1}} \longrightarrow 0,$$

the model of equation (1) is $y_t = \sum_{i=1}^{p} a_i \, y_{t-i} + \varepsilon_t$

3. RESULTS

The technique employed to examine the stability of the suggested model is an approximation method.

3.1. Singular point Z

Replacing with p=1 in equation (1) to get

$$y_t = \left[a_1 + b_1\left(\frac{1}{2y_{t-1}}\right)\right] y_{t-1} + \varepsilon_t, \quad y_{t-1} \neq 0 \tag{2}$$

Since $\varepsilon_t = 0$, $Z = f(Z)$. Then

$$Z = [a_1 + b_1(\frac{1}{2Z})]Z, Z \neq 0, b_1 \neq 0 \tag{3}$$

The singular point Z

$$Z = \frac{b_1}{2(1-(a_1))} \tag{4}$$

3.2. Stability at a singular point:

The stability conditions for a singular point were formulated using $y_s = Z + Z_s, \forall s = t; t-1$, and for an equation (2), $\varepsilon_t = 0$, and since , $\forall s = t; t-1; Z_s$ is smallest, to reached that Z_s^n convergent to zero , $\forall n \geq 2, \forall s = t;\ t-1;$ $Z_t.Z_{t-1} = 0$.

$$Z + Z_t = [a_1 + b_1(\frac{1}{2(Z + Z_{t-1})})](Z + Z_{t-1}) \tag{5}$$

Then,

$$Z_t = [(\frac{4a_1Z + b_1 - 2Z}{2Z})]Z_{t-1} = m_1 Z_{t-1}, m_1 = [(\frac{4a_1Z + b_1 - 2Z}{2Z})] \tag{6}$$

If the solution to equation (6) is within a unit circle, then (6) represents a stable first-order linear model. Expressed using symbols
$\mid n_1 \mid = \mid m_1 \mid < 1$

3.3. A limit cycle

A period q limit cycle of $y_t = y_t;\ y_{t+1}; \ldots; y_{t+q}$; for the model suggested in Equation (2). When y_s is a points nearly a limit cycle is replaced

$$\forall s \in \{t, t-1\}, \quad y_s = y_s + Z_s$$

$$y_t + Z_t = \left[a_1 + b_1\left(\frac{1}{2(y_{t-1} + Z_{t-1})}\right)\right](y_{t-1} + Z_{t-1}) \tag{7}$$

Then

$$Z_t = \left(\frac{4a_1 y_{t-1} + b_1 - 2y_t}{2y_{t-1}}\right) Z_{t-1} \tag{8}$$

Put, $t = t + q$ in (8) forgetting

$$Z_{t+q} = \left(\frac{4a_1\, y_{t+q-1} + b_1 - 2\, y_{t+q}}{2\, y_{t+q-1}}\right) Z_{t+q-1} \tag{9}$$

Therefore

$$Z_{t+q} = \prod_{i=1}^{q}\left[\frac{4a_1 y_{t+q-i} + b_1 - 2y_{t+q-(i-1)}}{2y_{t+q-i}}\right] Z_t \tag{10}$$

Then

$$\left|\frac{Z_{t+q}}{Z_t}\right| = \left|\prod_{i=1}^{q}\left(\frac{4a_1\, y_{t+q-i} + b_1 - 2\, y_{t+q-(i-1)}}{2\, y_{t+q-i}}\right)\right| < 1 \tag{11}$$

Then, (11) is

$$\left|\frac{Z_{t+q}}{Z_t}\right| = \left|\prod_{i=1}^{q}\left(\frac{4a_1\, y_{t+i-1} + b_1 - 2\, y_{t+i}}{2\, y_{t+i-1}}\right)\right| < 1 \tag{12}$$

4. Examples

This section of the study comprises two examples, denoted as examples (1) and (2).These illustrations serve the purpose of pinpointing the singular point for the proposed model, applying stability constraints, and charting the trajectories of first-order models. Additionally, you can refer to Appendix (A) for MATLAB programs that display the orbits of the study model for both Examples (1) and (2).

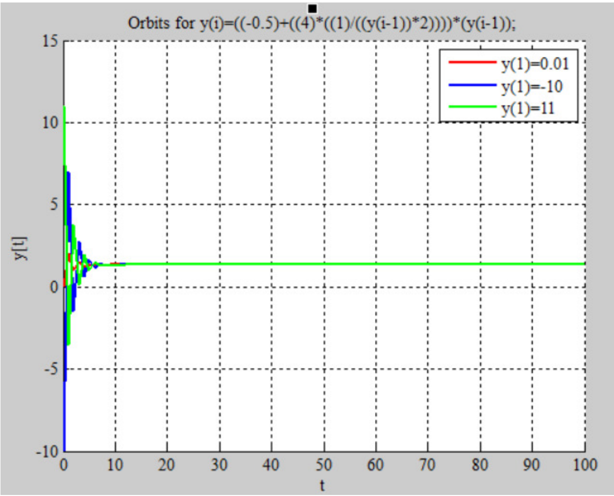


Figure 1. The stability of a singular point with different initial values.

4.1. Example (1)

Let's suppose that we have the given model. y_t=$[-0.5 + 4(\frac{1}{2y_{t-1}})]y_{t-1}$ + ε_t While ,$a_1 = -0.5$,$b_1 = 4$. By using equation (4), we derive that the singular point is such that:

$$Z = \frac{b_1}{2(1-(a_1))} = \frac{4}{2(1+0.5)} = \frac{4}{2(1.5)} = 1.333$$

When $Z = 1.333$, and by using (6),

$$Z_t = -0.8878\, Z_{t-1} \quad (*)$$

Therefore, $Z = 1.333$,is a stable because the root $n_1 = -0.8878$ of equation (*) is in a unit circle.

Therefore, the next figure display the stability of the proposed model with different initial values

$$y(1) = 0.01; y(1) = -10; y(1) = 11.$$

$Z = 1.333$, with constants $a_1 = -0.5$,$b_1 = 4$.

Figure 1, illustrates the trajectory plot for the stable proposed first-order model with a singular point

4.2. *Example (2)*

If , $a_1 = -1.9, b_1 = 4.2,\ y_t = [-1.9 + 4.2(\frac{1}{2y_{t-1}})]y_{t-1} + \varepsilon_t$ By utilizing equation (4), we determine that the singular point is such that: $Z = \frac{b_1}{2(1-(a_1))} = \frac{4.2}{2(1+1.9)} = \frac{4.2}{2(2.9)} = 0.7241$ When $Z = 0.7241$ and using (6), we obtain

$$Z_t = -1.8998\, Z_{t-1}.$$

Therefore, $Z = 0.7241$ is unstable (the root $n_1 = 1.8998$) is outside a unity circle.
Therefore, the figure shows model orbits that unstable in different initial values.

$y(1) = 0.1$;$y(1) = -10$;$y(1) = 11$;

Figure 2 , illustrates the trajectory plot for unstable proposed first-order model with a singular point $Z = 0.7241$, with constants $a_1 = -1.9, b_1 = 4.2$.

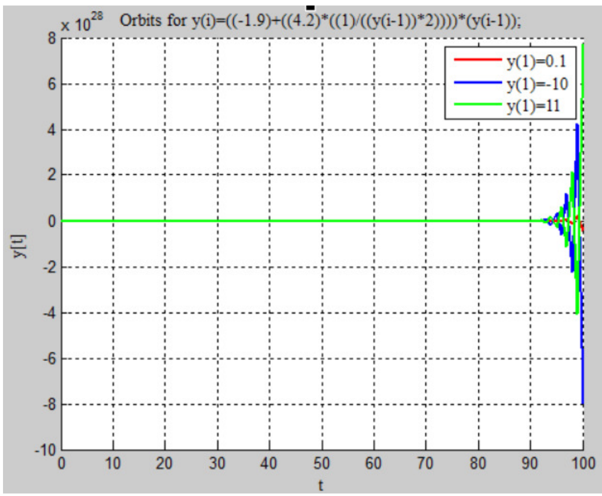


Figure 2. unstable singular point in different initial values.

5. Application

5.1. *Description of the data utilized in the study*

The study utilized information gathered from the website https://www.worldometers.info/coronavirus/country/saudi-arabia/, which provides information on the daily count of new Covid-19 cases in Kingdom of Saudi Arabia between 1st April 2022 and 30th June 2022. The dataset comprises 91 observations, representing the number of new cases reported each day during this specific timeframe. The dataset's lowest value occurred on 04-Apr-2022, with 78 new cases reported, whereas the highest value was recorded on 21-Jun-2022, with 1232 new cases reported, and to access the data used in the study, please refer to Appendix (A). The next figure, figure 3 , display The $COVID = y_t$ series data between 01-Apr-2022 and 30-Jun-22. figure 3 , the daily count of new Covid-19 cases in Kingdom of Saudi Arabia between 1st April 2022 and 30th June 2022.

By utilizing the SPSS program to analyze the graph of the primary COVID data series as shown in Figure 3 ,it becomes evident that the data is exhibiting growth, following an exponential pattern, and displaying instability. Through the utilization of the EViews 9 software, an analysis was conducted on the initial COVID dataset (y_t) using autocorrelation and partial auto correlation functions as depicted in Figure 4. The observation made from this analysis was that the COVID variable lacks stability Figure 4 for the autocorrelation and partial autocorrelation functions for the real data series COVID.

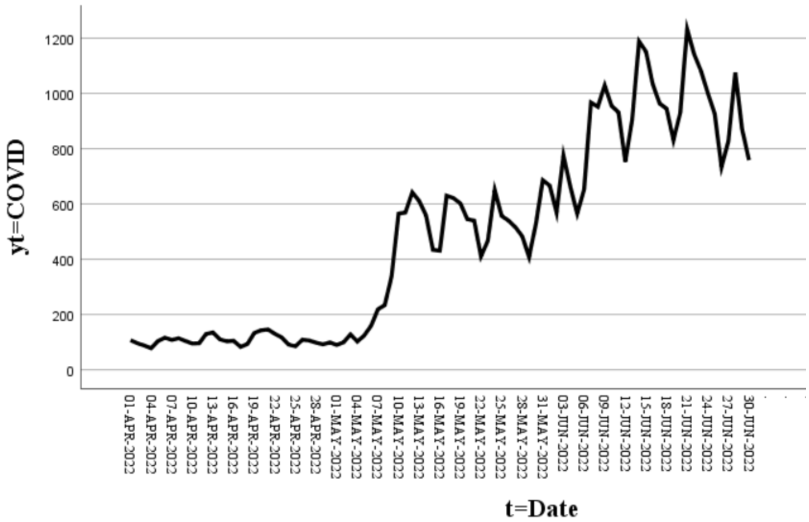
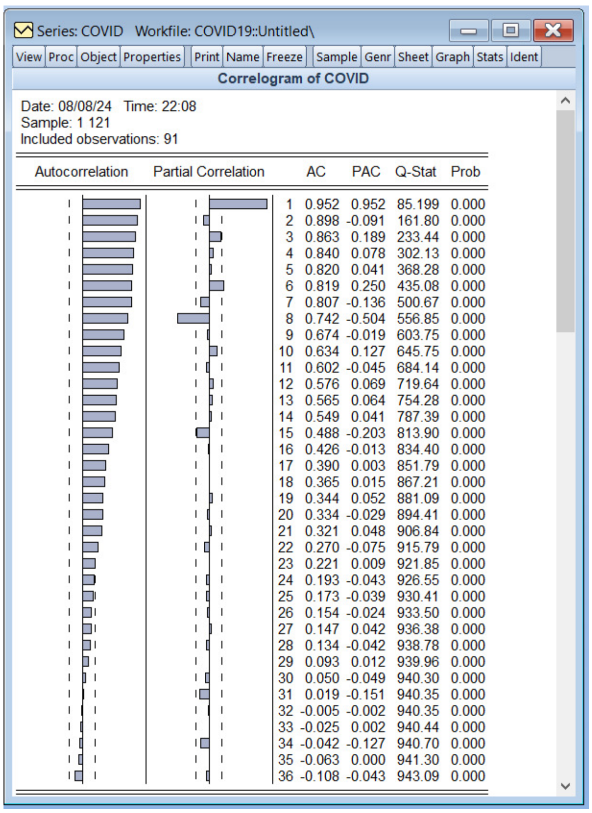


Figure 3. The COVID series data between 01-Apr-2022 and 30-Jun-22.



Series: COVID Workfile: COVID19::Untitled\

View | Proc | Object | Properties | Print | Name | Freeze | Sample | Genr | Sheet | Graph | Stats | Ident

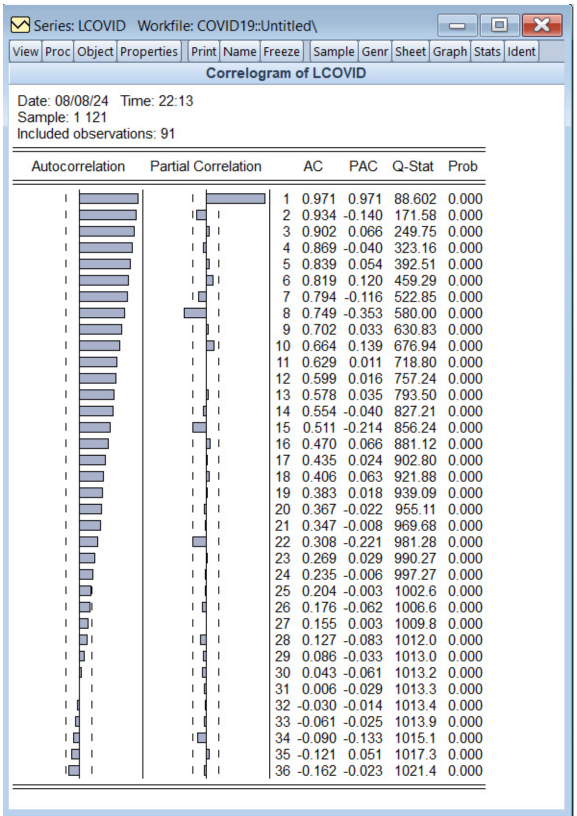
Correlogram of COVID

Date: 08/08/24 Time: 22:08
Sample: 1 121
Included observations: 91

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.952	0.952	85.199	0.000
		2	0.898	-0.091	161.80	0.000
		3	0.863	0.189	233.44	0.000
		4	0.840	0.078	302.13	0.000
		5	0.820	0.041	368.28	0.000
		6	0.819	0.250	435.08	0.000
		7	0.807	-0.136	500.67	0.000
		8	0.742	-0.504	556.85	0.000
		9	0.674	-0.019	603.75	0.000
		10	0.634	0.127	645.75	0.000
		11	0.602	-0.045	684.14	0.000
		12	0.576	0.069	719.64	0.000
		13	0.565	0.064	754.28	0.000
		14	0.549	0.041	787.39	0.000
		15	0.488	-0.203	813.90	0.000
		16	0.426	-0.013	834.40	0.000
		17	0.390	0.003	851.79	0.000
		18	0.365	0.015	867.21	0.000
		19	0.344	0.052	881.09	0.000
		20	0.334	-0.029	894.41	0.000
		21	0.321	0.048	906.84	0.000
		22	0.270	-0.075	915.79	0.000
		23	0.221	0.009	921.85	0.000
		24	0.193	-0.043	926.55	0.000
		25	0.173	-0.039	930.41	0.000
		26	0.154	-0.024	933.50	0.000
		27	0.147	0.042	936.38	0.000
		28	0.134	-0.042	938.78	0.000
		29	0.093	0.012	939.96	0.000
		30	0.050	-0.049	940.30	0.000
		31	0.019	-0.151	940.35	0.000
		32	-0.005	-0.002	940.35	0.000
		33	-0.025	0.002	940.44	0.000
		34	-0.042	-0.127	940.70	0.000
		35	-0.063	0.000	941.30	0.000
		36	-0.108	-0.043	943.09	0.000

Figure 4. COVID's autocorrelation and partial autocorrelation functions.

Due to the exponential nature of the original series, a fresh series named LCOVID was generated by logarithmically transforming the COVID data. Subsequently, autocorrelation and partial autocorrelation functions were graphed for the LCOVID series.Nonetheless, upon observation, the newly derived LCOVID time series data was found to exhibit instability, as illustrated in Figure 5. Figure 5 for the autocorrelation and partial autocorrelation functions for the transformed data series LCOVID.



	AC	PAC	Q-Stat	Prob
1	0.971	0.971	88.602	0.000
2	0.934	-0.140	171.58	0.000
3	0.902	0.066	249.75	0.000
4	0.869	-0.040	323.16	0.000
5	0.839	0.054	392.51	0.000
6	0.819	0.120	459.29	0.000
7	0.794	-0.116	522.85	0.000
8	0.749	-0.353	580.00	0.000
9	0.702	0.033	630.83	0.000
10	0.664	0.139	676.94	0.000
11	0.629	0.011	718.80	0.000
12	0.599	0.016	757.24	0.000
13	0.578	0.035	793.50	0.000
14	0.554	-0.040	827.21	0.000
15	0.511	-0.214	856.24	0.000
16	0.470	0.066	881.12	0.000
17	0.435	0.024	902.80	0.000
18	0.406	0.063	921.88	0.000
19	0.383	0.018	939.09	0.000
20	0.367	-0.022	955.11	0.000
21	0.347	-0.008	969.68	0.000
22	0.308	-0.221	981.28	0.000
23	0.269	0.029	990.27	0.000
24	0.235	-0.006	997.27	0.000
25	0.204	-0.003	1002.6	0.000
26	0.176	-0.062	1006.6	0.000
27	0.155	0.003	1009.8	0.000
28	0.127	-0.083	1012.0	0.000
29	0.086	-0.033	1013.0	0.000
30	0.043	-0.061	1013.2	0.000
31	0.006	-0.029	1013.3	0.000
32	-0.030	-0.014	1013.4	0.000
33	-0.061	-0.025	1013.9	0.000
34	-0.090	-0.133	1015.1	0.000
35	-0.121	0.051	1017.3	0.000
36	-0.162	-0.023	1021.4	0.000

Figure 5. LCOVID Graphs of autocorrelation and partial autocorrelation functions.

After applying the first difference to the original data by taking the natural logarithm, we obtained a new variable,

$$\text{DLCOVID} = \log(y_t) - \log(y_{t-1}) = Z_t$$

The autocorrelation and partial autocorrelation functions were plotted for this new variable DLCOVID, as shown in Figure 6. Figure 6 for the autocorrelation and partial autocorrelation functions for the transformed data series with the first difference D(LCOVID).

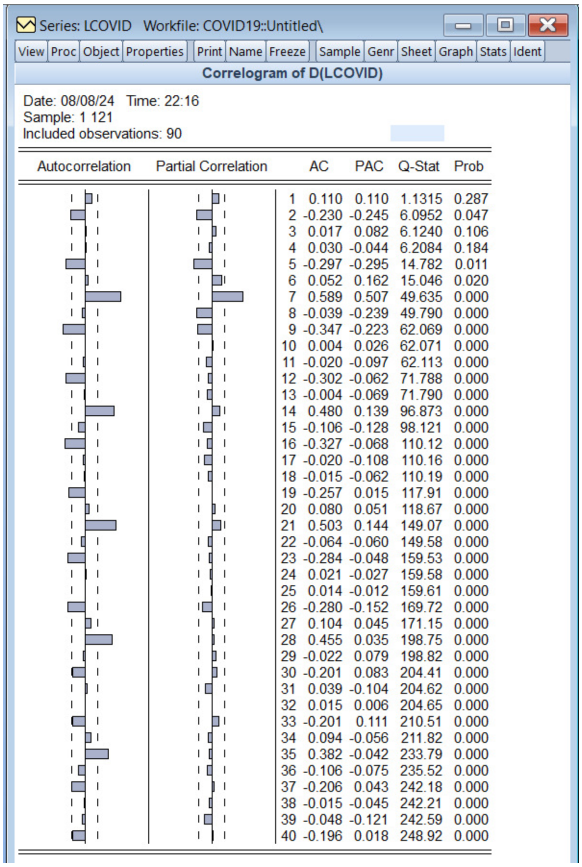
The Figure 7 , displays the resulting stable time series and can be used for predicting future observations. The following figure, figure 7 , shows the stable series of logarithms of the first difference of the original data that DLCOVID = Z_t

5.2. *Estimate the parameters and stability condition , forcasted, prediction for the study model with data*

Programs in the Python language were used to estimate the values of the parameters suitable for the study data and the proposed non-linear model and to achieve the stability conditions of the research model, which were found on the theoretical side in the third paragraph of the research, which is related to the results. We can see the program in the python in the link:

`https://colab.research.google.com/drive/1spI2QiMN0_6HjbZojfLLcwWtps4RNk8B`

5.2.1. Estimate the parameters for data and model with stability condition From the Python program we estimate the parameters for the nonlinear proposed model that suitable for the study data in our search. The program starts



	AC	PAC	Q-Stat	Prob
1	0.110	0.110	1.1315	0.287
2	-0.230	-0.245	6.0952	0.047
3	0.017	0.082	6.1240	0.106
4	0.030	-0.044	6.2084	0.184
5	-0.297	-0.295	14.782	0.011
6	0.052	0.162	15.046	0.020
7	0.589	0.507	49.635	0.000
8	-0.039	-0.239	49.790	0.000
9	-0.347	-0.223	62.069	0.000
10	0.004	0.026	62.071	0.000
11	-0.020	-0.097	62.113	0.000
12	-0.302	-0.062	71.788	0.000
13	-0.004	-0.069	71.790	0.000
14	0.480	0.139	96.873	0.000
15	-0.106	-0.128	98.121	0.000
16	-0.327	-0.068	110.12	0.000
17	-0.020	-0.108	110.16	0.000
18	-0.015	-0.062	110.19	0.000
19	-0.257	0.015	117.91	0.000
20	0.080	0.051	118.67	0.000
21	0.503	0.144	149.07	0.000
22	-0.064	-0.060	149.58	0.000
23	-0.284	-0.048	159.53	0.000
24	0.021	-0.027	159.58	0.000
25	0.014	-0.012	159.61	0.000
26	-0.280	-0.152	169.72	0.000
27	0.104	0.045	171.15	0.000
28	0.455	0.035	198.75	0.000
29	-0.022	0.079	198.82	0.000
30	-0.201	0.083	204.41	0.000
31	0.039	-0.104	204.62	0.000
32	0.015	0.006	204.65	0.000
33	-0.201	0.111	210.51	0.000
34	0.094	-0.056	211.82	0.000
35	0.382	-0.042	233.79	0.000
36	-0.106	-0.075	235.52	0.000
37	-0.206	0.043	242.18	0.000
38	-0.015	-0.045	242.21	0.000
39	-0.048	-0.121	242.59	0.000
40	-0.196	0.018	248.92	0.000

Figure 6. Graphs of autocorrelation and partial autocorrelation functions for LDCOVID=Zt.

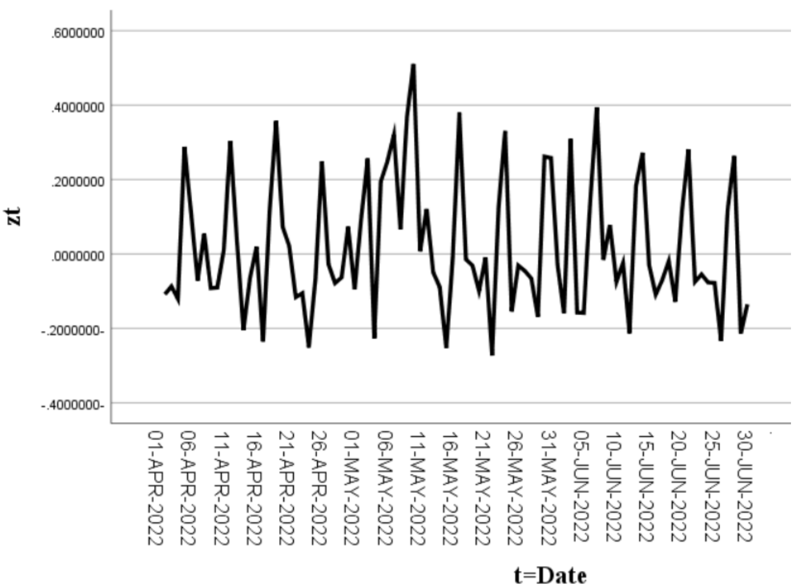


Figure 7. DLCOVID = Zt, plot.

by employing random initial values for the estimated parameters. Then $c = a_1 = 0.97, d = b_1 = 0.27$ and by using equation (2), we get that:

$y_t = [0.97 + 0.27(\frac{1}{2y_{t-1}})]y_{t-1} + \varepsilon_t$

By using equation (4), we get the singular point Z such that:

Z=$\frac{b_1}{2(1-(a_1))} = \frac{0.27}{2(1-0.97)} = \frac{0.27}{2(0.03)} = 4.5$. When $Z = 4.5$, and by using (6),

$$Z_t = 0.97\, Z_{t-1} \quad (**)$$

Therefore , $Z = 4.5$ is stable because the root $n_1 = 0.97$ of equation (**) is in a unit circle.

The next figure, figure 8 explain the stability singular point $Z = 4.5$, with estimate parameters $c = a_1 = 0.97$, $d = b_1 = 0.27$ that suitable the study data and study proposed model.

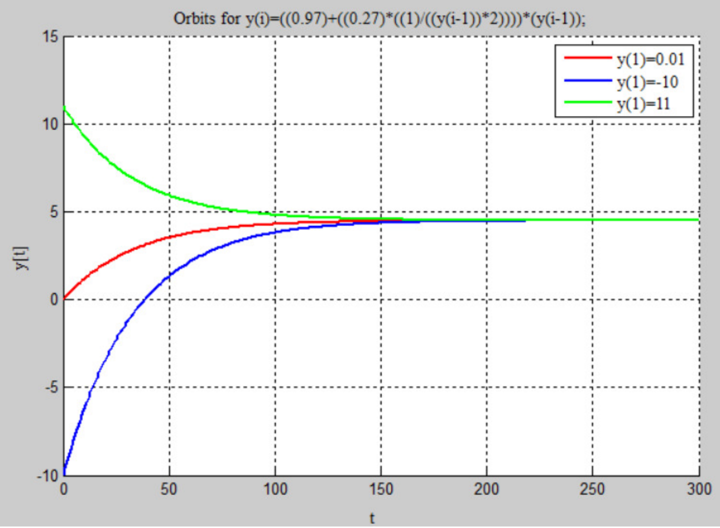


Figure 8. The stable proposed model that suitable the study data with estimated parameters.

5.2.2. *The forecasting for study model with transform data* We were forecasting by using the proposed model with the estimated parameters that we found in the Python program; we can see Appendix (B) to find the forecasting and residuals for the transformed data by using the study model and the statistical criteria were that:

The Mean Squared Error $(MSE) = 0.0311197180760586 15$, $ResidualVariance = 0.03181903758338577$, $AIC = 10.895381004361774$, $BIC = -304.718116685427$, $NBIC : -3.34855073280689$, and the next figure, figure 9 show that the transform original data and forcasted data by using the study proposed model.

5.2.3. *The residuals test for the study model* The autocorrelation function and the partial autocorrelation function were found for the residuals of the model proposed in the first-order study for the transferred data, most of which fall within the limits of confidence, which indicates that the model agrees with the data used and the possibility of using it to predict future values. The next figures, figure 10 and figure 11 explained the autocorrelation and partial autocorrelation functions graphs for the study model residuals The residual is clearly white noise, indicating that the model is correct.

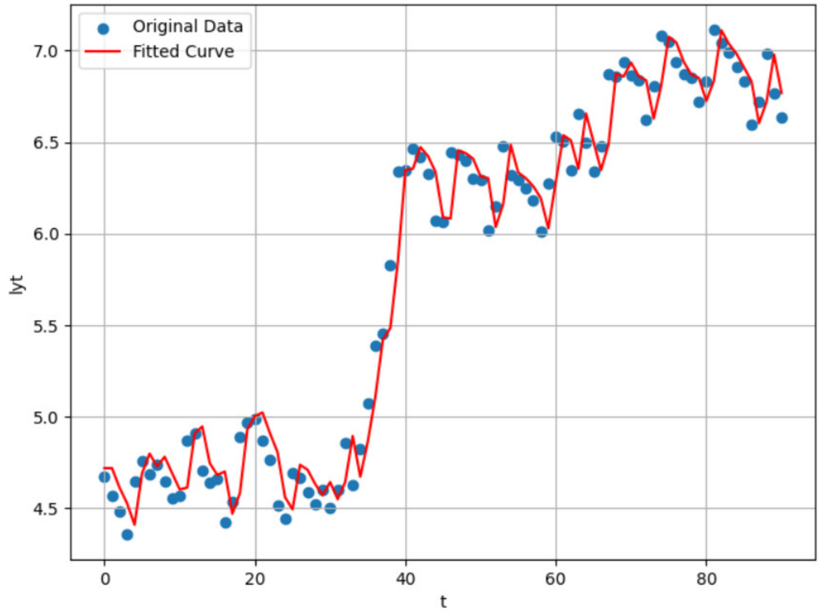


Figure 9. The forecasted data and transformed real data plot.

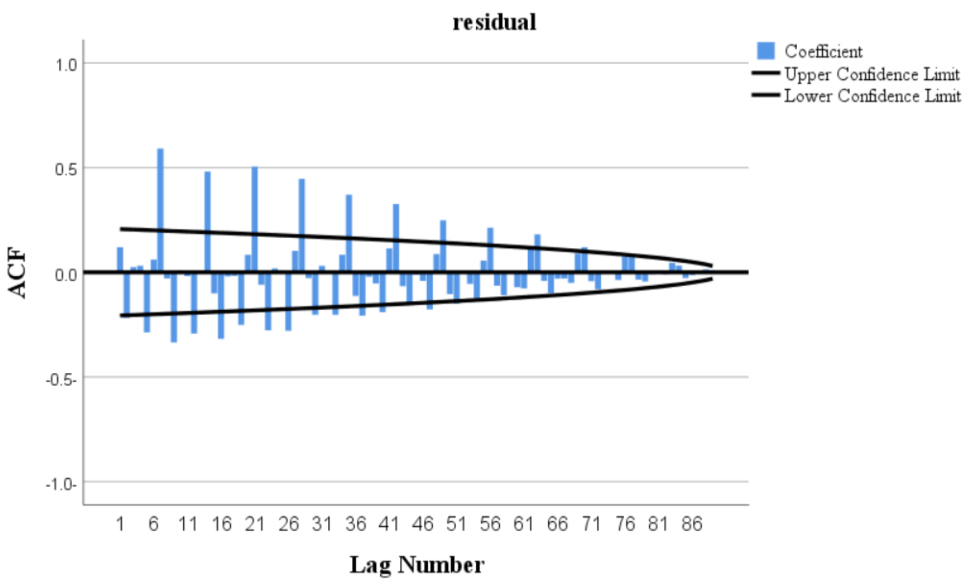


Figure 10. The autocorrelation function for the study model residuals.

5.2.4. We forecasted COVID data for the next month that we used by using study model We can see the program in the python in the link:

https://colab.research.google.com/drive/1udVKbUFF-0TjJ_Aycuq_yazRD1LZkzTf

From the Python program we estimate the parameters for the nonlinear proposed model that suitable for the study data of Covid in our search. The program starts by employing random initial values for the estimated parameters. Then, $c = a_1 = 0.959108457269065$,$d = b_1 = 53.09035963635171$ we can see in Appendix (C) that we found the

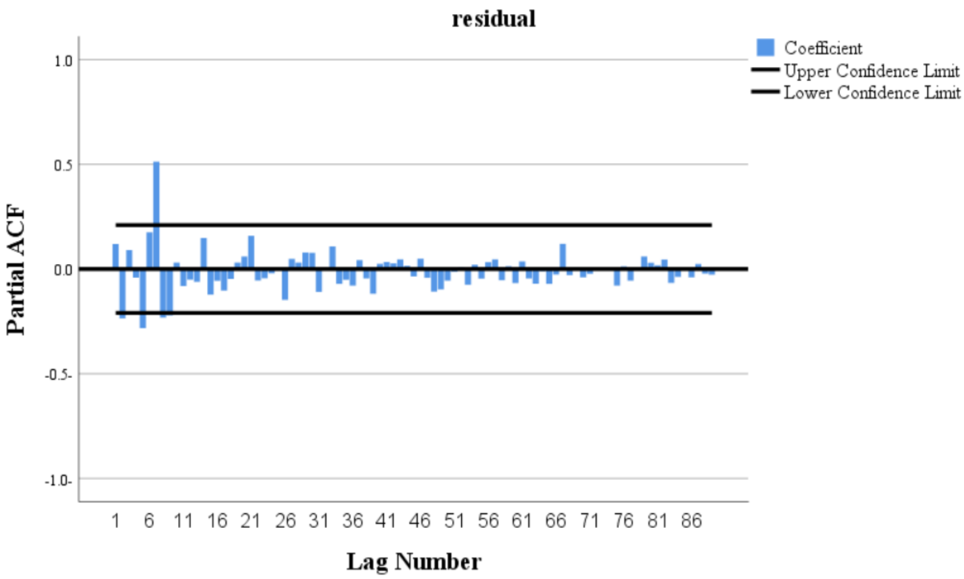


Figure 11. The partial autocorrelation function for the study model residuals.

forecasting COVID data by using the study model with the original data. The next figure, figure 12, showsthe forecasting of COVID data by using the study model.

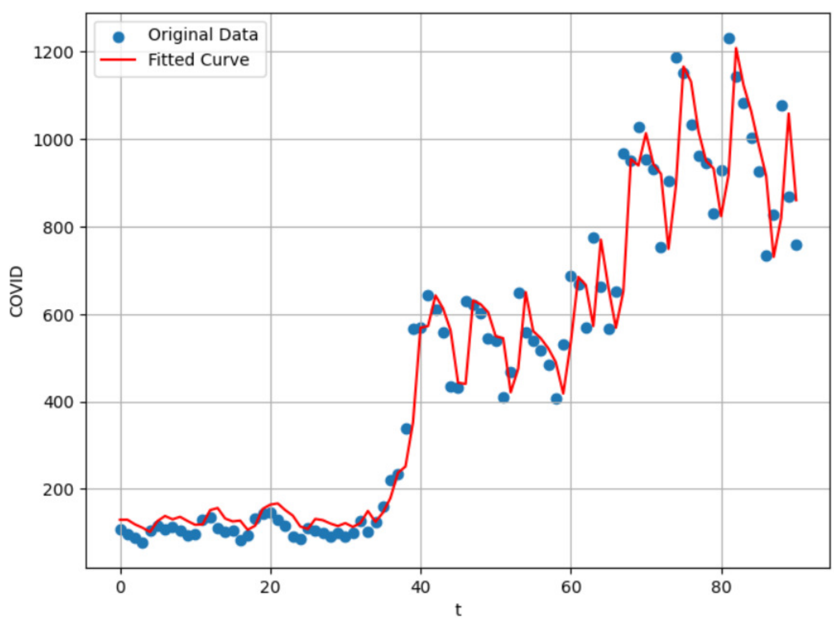


Figure 12. Forecasting COVID data by using the study model.

5.2.5. The predictions for study model with real data We can see the predictions for the study model with real data for the next thirty days. The next figure, figure 13, explain the prediction of the real COVID data and the real data that we used in our search by using the study model for the next thirty days. Figure 13 the predictions for COVID data by using the study model for the next month.

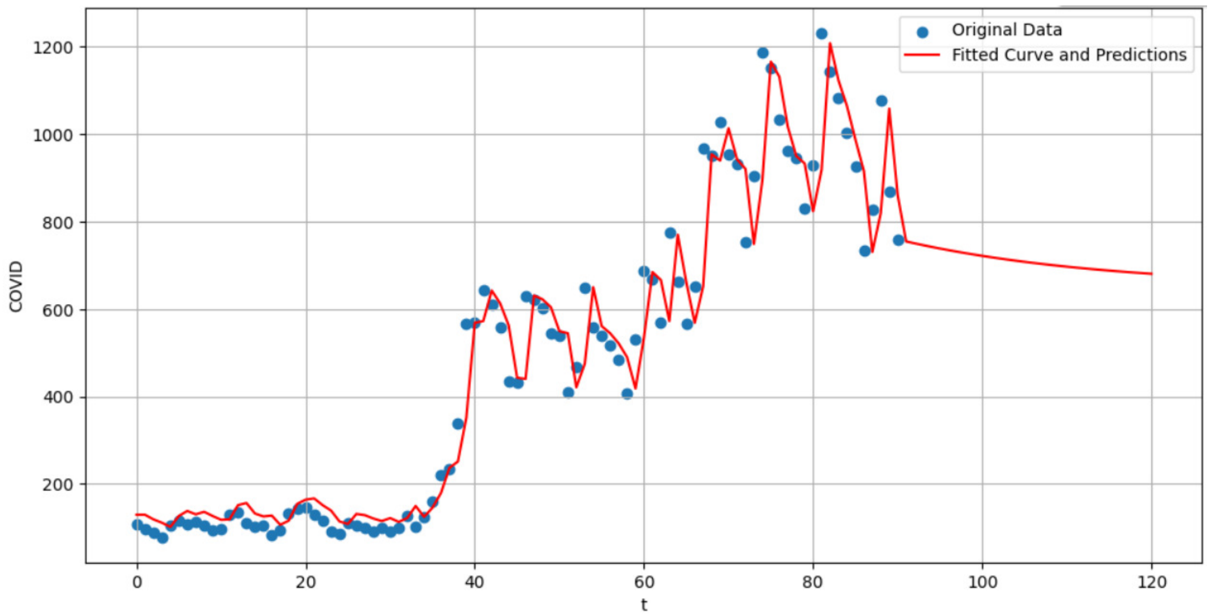


Figure 13. predictions for COVID data by using the study model for the next thirty days.

The next table, table(1) show that the predictions for COVID data by using the proposed study model for the next thirty days.

t	prediction of COVID	t	prediction of COVID	t	prediction of COVID
1-jul-22	755	11-jul-22	719	21-jul-22	695
2-jul-22	750	12-jul-22	716	22-jul-22	693
3-jul-22	746	13-jul-22	713	23-jul-22	691
4-jul-22	742	14-jul-22	710	24-jul-22	689
5-jul-22	738	15-jul-22	708	25-jul-22	688
6-jul-22	735	16-jul-22	705	26-jul-22	686
7-jul-22	731	17-jul-22	703	27-jul-22	685
8-jul-22	728	18-jul-22	701	28-jul-22	683
9-jul-22	725	19-jul-22	699	29-jul-22	682
10-jul-22	722	20-jul-22	697	30-jul-22	681

Table 1. Prediction of COVID by using study model

6. Conclusions

The study investigates the determination of stability conditions for the proposed model using the local linear approximation method, which focuses on identifying the unique non-zero point of the model and its associated stability constraints, supported by a numerical example. Structurally, the proposed model bears a strong resemblance to the exponential autoregressive model developed by the Japanese researcher Ozaki, as it integrates both a linear component and a nonlinear component, the latter characterized by a decreasing function. The model was validated with real-world data, specifically the daily counts of new COVID-19 cases in Saudi Arabia over a continuous three-month span in 2022. Parameter estimation of the first-order model was carried out in Python to ensure compliance with the stability condition at the singular point. The forecasts within the study period demonstrated high accuracy, reflected in minimal residuals (Appendix B) and robust statistical performance. A key finding of this research is that the proposed nonlinear time series model exhibits strong predictive capabilities for anticipating the future dynamics of critical phenomena, such as daily COVID-19 infections, thereby supporting the design and implementation of timely public health interventions to curb their spread and mitigate their consequences.

Appendix (A)

The COVID series data between 01-Apr-2022 and 30-Jun-2022 in the Kingdom of Saudi Arabia.

t	COVID	t	COVID	t	COVID
01-Apr-22	107	01-May-22	90	31-May-22	686
02-Apr-22	96	02-May-22	99	01-Jun-22	667
03-Apr-22	88	03-May-22	128	02-Jun-22	569
04-Apr-22	78	04-May-22	102	03-Jun-22	775
05-Apr-22	104	05-May-22	124	04-Jun-22	662
06-Apr-22	116	06-May-22	159	05-Jun-22	565
07-Apr-22	108	07-May-22	219	06-Jun-22	652
08-Apr-22	114	08-May-22	234	07-Jun-22	967
09-Apr-22	104	09-May-22	339	08-Jun-22	952
10-Apr-22	95	10-May-22	565	09-Jun-22	1029
11-Apr-22	96	11-May-22	569	10-Jun-22	955
12-Apr-22	130	12-May-22	642	11-Jun-22	932
13-Apr-22	135	13-May-22	611	12-Jun-22	753
14-Apr-22	110	14-May-22	559	13-Jun-22	905
15-Apr-22	103	15-May-22	434	14-Jun-22	1188
16-Apr-22	105	16-May-22	431	15-Jun-22	1152
17-Apr-22	83	17-May-22	630	16-Jun-22	1033
18-Apr-22	93	18-May-22	621	17-Jun-22	963
19-Apr-22	133	19-May-22	602	18-Jun-22	945
20-Apr-22	143	20-May-22	545	19-Jun-22	831
21-Apr-22	146	21-May-22	540	20-Jun-22	930
22-Apr-22	130	22-May-22	411	21-Jun-22	1232
23-Apr-22	117	23-May-22	467	22-Jun-22	1143
24-Apr-22	91	24-May-22	650	23-Jun-22	1082
25-Apr-22	85	25-May-22	557	24-Jun-22	1002
26-Apr-22	109	26-May-22	540	25-Jun-22	927
27-Apr-22	106	27-May-22	516	26-Jun-22	734

t	COVID	t	COVID	t	COVID
28-Apr-22	98	28-May-22	483	27-Jun-22	827
29-Apr-22	92	29-May-22	408	28-Jun-22	1076
30-Apr-22	99	30-May-22	530	29-Jun-22	869
				30-Jun-22	759

Appendix (B)

Log(COVID) values, forecasting values, and residuals using the nonlinear study model.

t	Log(y_t)	Forecast(Log(y_t))	Residual	t	Log(y_t)	Forecast(Log(y_t))	Residual
01-Apr-22	4.672829	4.716431	-0.0436	17-May-22	6.44572	6.081646	0.364074
02-Apr-22	4.564348	4.716431	-0.15208	18-May-22	6.431331	6.453612	-0.02228
03-Apr-22	4.477337	4.610135	-0.1328	19-May-22	6.400257	6.439513	-0.03926
04-Apr-22	4.356709	4.524876	-0.16817	20-May-22	6.300786	6.409065	-0.10828
05-Apr-22	4.644391	4.406678	0.237713	21-May-22	6.291569	6.311597	-0.02003
06-Apr-22	4.753590	4.688566	0.065024	22-May-22	6.018593	6.302566	-0.28397
07-Apr-22	4.682131	4.795565	-0.11343	23-May-22	6.146329	6.035088	0.111241
08-Apr-22	4.736198	4.725546	0.010653	24-May-22	6.476972	6.160252	0.316721
09-Apr-22	4.644391	4.778524	-0.13413	25-May-22	6.322565	6.484235	-0.16167
10-Apr-22	4.553877	4.688566	-0.13469	26-May-22	6.291569	6.332938	-0.04137
11-Apr-22	4.564348	4.599875	-0.03553	27-May-22	6.246107	6.302566	-0.05646
12-Apr-22	4.867534	4.610135	0.257399	28-May-22	6.180017	6.258019	-0.078
13-Apr-22	4.905275	4.907215	-0.00194	29-May-22	6.011267	6.19326	-0.18199
14-Apr-22	4.700480	4.944195	-0.24371	30-May-22	6.272877	6.02791	0.244967
15-Apr-22	4.634729	4.743525	-0.1088	31-May-22	6.530878	6.28425	0.246627
16-Apr-22	4.653960	4.679098	-0.02514	01-Jun-22	6.50279	6.537054	-0.03426
17-Apr-22	4.418841	4.697942	-0.2791	02-Jun-22	6.34388	6.509532	-0.16565
18-Apr-22	4.532599	4.467559	0.065041	03-Jun-22	6.652863	6.353824	0.299039
19-Apr-22	4.890349	4.579026	0.311323	04-Jun-22	6.495266	6.656583	-0.16132
20-Apr-22	4.962845	4.929570	0.033275	05-Jun-22	6.336826	6.502159	-0.16533
21-Apr-22	4.983607	5.000605	-0.017	06-Jun-22	6.480045	6.346911	0.133134
22-Apr-22	4.867534	5.020949	-0.15341	07-Jun-22	6.874198	6.487245	0.386953
23-Apr-22	4.762174	4.907215	-0.14504	08-Jun-22	6.858565	6.873460	-0.01489
24-Apr-22	4.510860	4.803976	-0.29312	09-Jun-22	6.936343	6.858141	0.078202
25-Apr-22	4.442651	4.557724	-0.11507	10-Jun-22	6.861711	6.934352	-0.07264
26-Apr-22	4.691348	4.490890	0.200458	11-Jun-22	6.837333	6.861224	-0.02389
27-Apr-22	4.663439	4.734577	-0.07114	12-Jun-22	6.624065	6.837337	-0.21327
28-Apr-22	4.584967	4.707230	-0.12226	13-Jun-22	6.807935	6.628365	0.17957
29-Apr-22	4.521789	4.630339	-0.10855	14-Jun-22	7.080027	6.808531	0.271496
30-Apr-22	4.595120	4.568433	0.026687	15-Jun-22	7.049255	7.075142	-0.02589
01-May-22	4.499810	4.640287	-0.14048	16-Jun-22	6.940222	7.044990	-0.10477
02-May-22	4.595120	4.546897	0.048223	17-Jun-22	6.870053	6.938154	-0.06810
03-May-22	4.852030	4.640287	0.211743	18-Jun-22	6.851185	6.869398	-0.01821
04-May-22	4.624973	4.892023	-0.26705	19-Jun-22	6.722630	6.850910	-0.12828
05-May-22	4.820282	4.669539	0.150743	20-Jun-22	6.835185	6.724944	0.110241
06-May-22	5.068904	4.860914	0.207991	21-Jun-22	7.116394	6.835232	0.281162

t	Log(y_t)	Forecast(Log(y_t))	Residual	t	Log(y_t)	Forecast(Log(y_t))	Residual
07-May-22	5.389072	5.104528	0.284543	22-Jun-22	7.041412	7.110777	-0.06937
08-May-22	5.455321	5.418247	0.037074	23-Jun-22	6.986566	7.037305	-0.05074
09-May-22	5.826000	5.483162	0.342838	24-Jun-22	6.909753	6.983564	-0.07381
10-May-22	6.336826	5.846375	0.490451	25-Jun-22	6.831954	6.908298	-0.07634
11-May-22	6.343880	6.346911	-0.00303	26-Jun-22	6.598509	6.832066	-0.23356
12-May-22	6.464588	6.353824	0.110765	27-Jun-22	6.717805	6.603323	0.114481
13-May-22	6.415097	6.472100	-0.05700	28-Jun-22	6.981006	6.720216	0.26079
14-May-22	6.326149	6.423606	-0.09746	29-Jun-22	6.767343	6.978116	-0.21077
15-May-22	6.073045	6.336450	-0.26341	30-Jun-22	6.632002	6.768757	-0.13675
16-May-22	6.066108	6.088443	-0.02233				

Appendix (C)

COVID values, forecasting COVID values, and residuals by using the nonlinear study model.

t	COVID	Forecast(COVID)	Residual	t	COVID	Forecast(COVID)	Residual
01-Apr-22	107	129	-22	17-May-22	630	440	190
02-Apr-22	96	129	-33	18-May-22	621	631	-10
03-Apr-22	88	119	-31	19-May-22	602	622	-20
04-Apr-22	78	111	-33	20-May-22	545	604	-59
05-Apr-22	104	101	3	21-May-22	540	549	-9
06-Apr-22	116	126	-10	22-May-22	411	544	-133
07-Apr-22	108	138	-30	23-May-22	467	421	46
08-Apr-22	114	130	-16	24-May-22	650	474	176
09-Apr-22	104	136	-32	25-May-22	557	650	-93
10-Apr-22	95	126	-31	26-May-22	540	561	-21
11-Apr-22	96	118	-22	27-May-22	516	544	-28
12-Apr-22	130	119	11	28-May-22	483	521	-38
13-Apr-22	135	151	-16	29-May-22	408	490	-82
14-Apr-22	110	156	-46	30-May-22	530	418	112
15-Apr-22	103	132	-29	31-May-22	686	535	151
16-Apr-22	105	125	-20	01-Jun-22	667	684	-17
17-Apr-22	83	127	-44	02-Jun-22	569	666	-97
18-Apr-22	93	106	-13	03-Jun-22	775	572	203
19-Apr-22	133	116	17	04-Jun-22	662	770	-108
20-Apr-22	143	154	-11	05-Jun-22	565	661	-96
21-Apr-22	146	164	-18	06-Jun-22	652	568	84
22-Apr-22	130	167	-37	07-Jun-22	967	652	315
23-Apr-22	117	151	-34	08-Jun-22	952	954	-2
24-Apr-22	91	139	-48	09-Jun-22	1029	940	89
25-Apr-22	85	114	-29	10-Jun-22	955	1013	-58
26-Apr-22	109	108	1	11-Jun-22	932	942	-10
27-Apr-22	106	131	-25	12-Jun-22	753	920	-167
28-Apr-22	98	128	-30	13-Jun-22	905	749	156
29-Apr-22	92	121	-29	14-Jun-22	1188	895	293
30-Apr-22	99	115	-16	15-Jun-22	1152	1166	-14

t	COVID	Forecast(COVID)	Residual	t	COVID	Forecast(COVID)	Residual
01-May-22	90	121	-31	16-Jun-22	1033	1131	-98
02-May-22	99	113	-14	17-Jun-22	963	1017	-54
03-May-22	128	121	7	18-Jun-22	945	950	-5
04-May-22	102	149	-47	19-Jun-22	831	933	-102
05-May-22	124	124	0	20-Jun-22	930	824	106
06-May-22	159	145	14	21-Jun-22	1232	919	313
07-May-22	219	179	40	22-Jun-22	1143	1208	-65
08-May-22	234	237	-3	23-Jun-22	1082	1123	-41
09-May-22	339	251	88	24-Jun-22	1002	1064	-62
10-May-22	565	352	213	25-Jun-22	927	988	-61
11-May-22	569	568	1	26-Jun-22	734	916	-182
12-May-22	642	572	70	27-Jun-22	827	731	96
13-May-22	611	642	-31	28-Jun-22	1076	820	256
14-May-22	559	613	-54	29-Jun-22	869	1059	-190
15-May-22	434	563	-129	30-Jun-22	759	860	-101
16-May-22	431	443	-12				

REFERENCES

1. K. Benmouiza and A. Cheknane, *Small-scale solar radiation forecasting using ARMA and nonlinear autoregressive neural network models*, Theoretical and Applied Climatology, vol. 124, no. 3, pp. 945–958, 2016.
2. T. Ozaki, *The statistical analysis of perturbed limit cycle processes using nonlinear time series models*, Journal of Time Series Analysis, vol. 3, no. 1, pp. 29–41, 1982.
3. V. Haggan and T. Ozaki, *Modelling nonlinear random vibrations using an amplitude-dependent autoregressive time series model*, Biometrika, vol. 68, no. 1, pp. 189–196, 1981.
4. N. E. Ali and A. A. Mohammad, *Stability conditions of limit cycle for Gompertz Autoregressive model*, Tikrit Journal of Pure Science, vol. 28, no. 2, 2023.
5. N. A. Noori and A. A. Mohammad, *Dynamical approach in studying GJR-GARCH (Q, P) models with application*, Tikrit Journal of Pure Science, vol. 26, no. 2, pp. 145-156, 2021.
6. K. F. Hameed, N. S. K. Alazawey, *et al.*, *Stability Study of SATER Models in a Dynamic Approach with Application*, Mathematical Statistician and Engineering Applications, vol. 72, no. 1, pp. 773–790, 2023.
7. A. S. Youns and S. M. Ahmad, *Stability Conditions for a Nonlinear Time Series Model*, International Journal of Mathematics and Mathematical Sciences, vol. 2023, no. 1, p. 5575771, 2023.
8. A. A. Mohammad and A. A. Mudhir, *Dynamical approach in studying stability condition of exponential (GARCH) models*, Journal of King Saud University-Science, vol. 32, no. 1, pp. 272–278, 2020.
9. Z. Shi, Y. Tamura, and T. Ozaki, *Monitoring the stability of BWR oscillation by nonlinear time series modeling*, Annals of Nuclear Energy, vol. 28, no. 10, pp. 953–966, 2001.
10. M. Gan and H. Peng, *Stability analysis of RBF network-based state-dependent autoregressive model for nonlinear time series*, Applied Soft Computing, vol. 12, no. 1, pp. 174–181, 2012.
11. Li, Maobin and Ji, Shouwen and Liu, Gang, *Forecasting of Chinese E-Commerce Sales: An Empirical Comparison of ARIMA, Nonlinear Autoregressive Neural Network, and a Combined ARIMA-NARNN Model* , Mathematical Problems in Engineering, vol. 2018, no. 1, pp. 6924960, 2018.
12. Lydia, Mau and Kumar, S Suresh and Selvakumar, A Immanuel and Kumar, G Edwin Prem, *Linear and non-linear autoregressive models for short-term wind speed forecasting*, Energy conversion and management, vol. 112, pp. 115–124, 2016.
13. Perc, Matjaž and Gorišek Miksić, Nina and Slavinec, Mitja and Stožer, Andraž *Forecasting covid-19*, Frontiers in physics, vol. 8, pp. 127, 2020,
14. Iftekhar, Emil Nafis and Priesemann, Viola and Balling, Rudi and Bauer, Simon and Beutels, Philippe and Valdez, André Calero and Cuschieri, Sarah and Czypionka, Thomas and Dumpis, Uga and Glaab, Enrico and others, *A look into the future of the COVID-19 pandemic in Europe: an expert consultation*, The Lancet Regional Health—Europe, vol. 8, 2021,
15. Ansori, Fandi and others *Mathematical Modeling and Sensitivity Analysis of COVID-19 and Tuberculosis Coinfection with Vaccination.*, Mathematical Modelling of Engineering Problems, vol. 11, no. 1, 2024.
16. Namasudra, Suyel, Dhamodharavadhani, S., and Rathipriya, R *Nonlinear neural network based forecasting model for predicting COVID-19 cases*, Neural processing letters, vol. 55, no. 1, pp. 171–191, 2023.
17. Khan, Farhan Mohammad and Gupta, Rajiv *ARIMA and NAR based prediction model for time series analysis of COVID-19 cases in India*, Journal of safety science and Resilience, vol. 1, no. 1, pp. 12–18, 2020.

18. Almasinejad, Peyman, Golabpour, Amin, Ahouz, Fatemeh, Meybodi, Mohammad Reza Mollakhalili, Mirzaie, Kamal, Khosravi, Ahmad and Bastani, Azadeh *Predicting the status of COVID-19 active cases using a neural network time series*, International Journal of Electrical and Computer Engineering, vol. 12, no. 3, pp. 3104–3117, 2022.
19. Aidoo, Eric N and Ampofo, Richar,and Awashie, Gaston E and Appiah, Simon ,d AdebanjSimon K,uke O, *Modelling COVID-19 incidence in the African sub-region using smooth transition autoregressive model*, Modeling Earth Systems and Environment, vol. 8, no. 1, pp. 961–966, 2022.
20. Olarenwaju, Babatunde Abdulrauph and Harrison, Igboeli Uchenna *Modeling of COVID-19 cases of selected states in Nigeria using linear and non-linear prediction models*, Journal of Computer Sciences Institute, vol. 17, pp. 390–395, 2020.
21. Jin, Yong-Chao, Cao, Qian, Sun, Qian, Lin, Ye, Liu, Dong-Mei, Wang, Chen-Xi, Wang, Xiao-Ling, Wang, Xi-Yin, and others *Models for COVID-19 data prediction based on improved LSTM-ARIMA algorithms*, IEEE Access, vol. 12, pp. 3981–3991, 2023
22. Tajmouati, Samya, El Wahbi, Bouazza, and Dakkon, Mohamed. *Modeling COVID-19 confirmed cases using a hybrid model*, Advances in Decision Sciences, vol. 26, no. 1, pp. 1–34, 2022.
23. Alabdulrazzaq, Haneen, Alenezi, Mohammed N., and Rawajfih, Yasmeen, Alghannam, Bareeq A., Al-Hassan, Abeer A., and Al-Anzi, Fawaz S., *On the accuracy of ARIMA based prediction of COVID-19 spread*, Results in Physics, vol. 27, pp. 104509, 2021.
24. Fang, Zheng-gang, Yang, Shu-qin, Lv, Cai-xia, An, Shu-yi, and Wu, Wei, *Application of a data-driven XGBoost model for the prediction of COVID-19 in the USA: a time-series study*, BMJ open, vol. 12, no. 7, pp. e056685, 2022.
25. Barria-Sandoval, Claudia and Ferreira, Guillermo and Benz-Parra, Katherine and Lopez-Flores, Pablo, *Prediction of confirmed cases of and deaths caused by COVID-19 in Chile through time series techniques: A comparative study*, Plos one, vol. 16, no. 4, pp. e0245414, 2021
26. Xu, Wenquan, and Hu, Hui, *Stability analysis of deep belief network: based SD-AR model for nonlinear time series*, Neural Processing Letters, vol. 56, no. 2, pp. 65, 2024.
27. Ahmad, Salim M, and Youns, Anas S, and Hamdi, Manal S, *Nonlinear Autoregressive Model for Stability and Prediction*, Journal of Applied Mathematics, vol. 2025, no. 1, pp. 6695121, 2025.
28. ABD RAHMAN, NUR HAIZUM, *Nonlinear Autoregressive Neural Network for Forecasting COVID-19 Confirmed Cases in Malaysia*, Journal of Statistical Modeling and Analytics (JOSMA), vol. 5, no. 2, 2023.
29. AlAlaween, Wafa'H, Faouri, Noor M., Al-Omar, Sarah H., Hendaileh, Elias M., Almousa, Aya Sama S., AlAlawin, Abdallah H., Abdallah, Omar H., Albashabsheh, Nibal T., Khorsheed, Bilal, and Abueed, Omar A., *A dynamic nonlinear autoregressive exogenous model for the prediction of COVID-19 cases in Jordan*, Cogent Engineering, vol. 9, no. 1, pp. 2047317, 2022.
30. S. Haleem Ibrahem, A. Saad Abduljabbar, and O. Saber Qasim, "Feature selection using binary Harris Hawks optimization algorithm to improve K-Means clustering," *Statistics, Optimization & Information Computing*, vol. 14, no. 4, pp. 1909–1917, 2025.