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Micro Generalized Star Semi Closed sets in Micro Topological Spaces

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Abstract The main objective of this paper is to introduce a new class of sets namely micro generalized star semi closed set (briefly Mic-g*s closed set) and micro genralized star semi open set (briefly Mic-g*s open set) in micro topological spaces. Few characteristics of these sets are explored. In addition the notions of micro generalized star semi closure (briefly Mic-g*s Clr.) and micro generalized star semi interior (briefly Mic-g*s Intr.) are outlined.

Keywords Micro topology, Mic-g*s closed set, Mic-g*s open set, Mic-g*s interior, Mic-g*s closure.

AMS 2010 subject classifications 54A05, 54A10, 54A99

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1. Introduction

The concept of semi open and generalized closed set was introduced by Levine [9, 8] in 1963 and 1970 respectively. Veerakumar [18] defined g* closed set in 1994. In 2011, Pushpalatha and Anitha [10] studied the properties of g*s closed sets in topological spaces. Lellis Thivagar and Carmel Richard [7] refined general topology and identified a new form of topological space called nano topological space where he defined nano semi open set in 2013.

Bhuvaneshwari and Mythili Gnanapriya [2] introduced nano generalized closed sets in 2014. The concept of nano genralized star closed set and nano genralized star semi closed set was defined by Rajendran et.al., [12, 11] in 2015. Chandrasekar [3] extended the concepts of nano topology to micro topology and defined micro semi-open and micro pre-open in 2019. Jasim et.al., [6] defined micro generalized closed sets in 2021. In 2022, Sandhiya and Balamani [13] introduced and studied the properties of micro g* closed set. In 2024, Sathishmohan et. all., [15, 16] studied the properties of micro semi-open, micro pre-open sets, micro α-open sets and micro β-open sets in micro topological spaces respectively. In this paper we defined a new set namely micro generalized star semi closed set (briefly Mic-g*s closed set) and micro genralized star semi open set (briefly Mic-g*s open set) in micro topological spaces.

2. Preliminaries

Definition 2.1
[3] Let U be the Universe. R be an equivalence relation on U and τ_R(X) = {U, $\emptyset$, $L_R(X)$, $U_R(X)$, $B_R(X)$}, where X $\subseteq$ U. τ_R(X) satisfies the following axioms:

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1. U and $\emptyset \in \tau_R$(X).
2. The union of elements of any sub collection of τ_R(X) is in τ_R(X).
3. The intersection of the elements of any finite sub collection of τ_R(X) is in τ_R(X).

That is, τ_R(X) forms a topology on U is called the nano topology on U with respect to X. (U, τ_R(X)) is called the nano topological space.

Definition 2.2
[3] Let (U, τ_R(X)) is a nano topological space here μ_R(X) = {N ∪ (N' ∩ μ): N, N' $\in \tau_R$(X)} and called it Micro topology of τ_R(X) by μ where $\mu \notin \tau_R$(X).

Definition 2.3
[3] The Micro topology μ_R(X) satisfies the following axioms.

1. U and $\emptyset \in \mu_R$(X).
2. The union of elements of any sub collection of μ_R(X) is in μ_R(X).
3. The intersection of the elements of any finite sub collection of μ_R(X) is in μ_R(X).

The triplet (U, τ_R(X), μ_R(X)) is called Micro topological spaces and The elements of μ_R(X) are called micro-open sets and the complement of a micro-open set is called a micro-closed set.

Definition 2.4
[3] The micro closure of a set A is denoted by Mic-cl(A) and is defined as Mic-cl(A) = ∩{B : B is micro-closed and A⊆B}.

Definition 2.5
[3] The micro interior of a set A is denoted by Mic-int(A) and is defined as Mic-int(A) = ∪{B : B is micro-open and A⊇B}.

Definition 2.6
[3] Let (U, τ_R(X), μ_R(X)) be a micro topological space and A ⊆ U. Then A is said to be micro semi-open if A ⊆ Mic-cl(Mic-int(A)) and micro semi-closed if Mic-int(Mic-cl(A)) ⊆ A.

Definition 2.7
[3] Let (U, τ_R(X), μ_R(X)) be a micro topological space and A ⊆ U. Then A is said to be micro pre-open if A ⊆ Mic-int(Mic-cl(A)) and micro pre-closed if Mic-cl(Mic-int(A)) ⊆ A.

Definition 2.8
[4] Let (U, τ_R(X), μ_R(X)) be a Micro topological space. A set A is called an Micro-α open set (briefly, Mic-αOS) if A⊆ Mic-int(Mic-cl(Mic-int(A))). The complement of an Micro-α open set is called an Micro-α closed set.

Definition 2.9
[6] A subset B of (X, τ_R(A), μ_R(A)) is called micro generalized closed set (shortly, Mic g-closed) if Mic-cl(B) ⊆ U for B ⊆ U and U is micro-open set in (X, τ_R(A), μ_R(A)).

Definition 2.10
[13] Let (U, τ_R(X), μ_R(X)) be a micro topological space. A subset A of U is said to be micro g^*-closed if Mic-cl(A) ⊆ L whenever A ⊆ L and L is micro g-open in U.

Definition 2.11
[1] A subset A of a micro topological space (U, τ_R(X), μ_R(X)) is called Mic sg-closed set if Mic-cl(A) ⊆ U whenever A ⊆ U and U is micro semi-open in U.

Definition 2.12
[1] A subset A of a micro topological space (U, τ_R(X), μ_R(X)) is called Mic gs-closed set if Mic-Scl(A) ⊆ U whenever A ⊆ U and U is micro-open in U.

Definition 2.13
[14] In a micro-topological space (U, τ_R(X), μ_R(X)) the sub-set P is said a 'micro-generalized pre-closed' (shortly Mic-g.p-closed) if Mic.p.clo.(p) $\subseteq$ O, where the set O is micro-open sub set of (U, τ_R(X), μ_R(X)).

Definition 2.14
[17] The Micro semi-closure of a subset M of U, denoted by Mscl(M) is defined to be the intersection of all Micro semi-closed sets of (U, τ_R(X), μ_R(X)) containing A.

Definition 2.15
[17] The Micro semi-interior of a subset M of U, denoted by Msint(M) is defined to be the union of all Micro semi-open sets of (U, τ_R(X), μ_R(X)) contained in A.

Remark 1
[5] The concepts of Micro pre-open and Micro semi-open sets are independent.

3. Micro Generalized Star Semi Closed Set

In this section, we introduce a new concept of micro-closed set namely micro generalized star semi closed set (briefly Mic-g*s closed) and its interrelations with existing micro-closed sets are obtained.

Definition 3.1
Let (U, τ_R(X), μ_R(X)) be a micro topological space. A subset A of (U, τ_R(X), μ_R(X)) is said to be micro generalized star semi closed set (briefly Mic-g*s closed) if Mic-Scl(A) $\subseteq$ V, whenever A $\subseteq$ V, V is micro g-open in U.

Example 3.1
Let U = {a, b, c, d}, U/R = {{a, b}, {c, d}}, X = {a}, τ_R(X) = {U, $\emptyset$, {a, b}}, μ = {b}, μ_R(X) = {U, $\emptyset$, {b}, {a, b}}, $(\mu_R(X))^C$ = {U, $\emptyset$, {c, d}, {a, c, d}}. Mic-g*s closed = {U, $\emptyset$, {a}, {c}, {d}, {a, c}, {a, d}, {c, d}, {b, c, d}, {a, c, d}}

Definition 3.2
The intersection of all Mic-g*s closed sets containing A is said to be micro generalized star semi closure of A. (briefly Mic-g*s Clr.(A)).

Example 3.2
Let U = {a, b, c, d}, U/R = {{a, d}, {b, c}}, X = {b}, τ_R(X) = {U, $\emptyset$, {b, c}}, μ = {a}, μ_R(X) = {U, $\emptyset$, {a}, {b, c}, {a, b, c}}, $(\mu_R(X))^C$ = {U, $\emptyset$, {d}, {a, d}, {b, c, d}}. Mic-g*s closed = {U, $\emptyset$, {a}, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}}. Let A = {a, b, c}, Mic-g*s Clr.(A) = U. Let B = {b, c}, Mic-g*s Clr.(B) = {b, c, d}.

Theorem 3.1
Every micro-closed set is Mic-g*s closed.

Proof
Let A be a micro-closed set of U and A $\subseteq$ V, V is micro g-open in U. Since A is micro-closed set of U, we have A = Mic-cl(A) which implies that Mic-cl(A) $\subseteq$ V. But Mic-Scl(A) $\subseteq$ Mic-cl(A) $\subseteq$ V. This implies that Mic-Scl(A) $\subseteq$ V. Thus A is Mic-g*s closed.

The converse part of the above theorem need not be true which is given by the following example.

Example 3.3
Let U = {a, b, c, d}, U/R = {{a, b}, {c}, {d}}, X = {a}, τ_R(X) = {U, $\emptyset$, {a, b}}, μ = {b, c}, μ_R(X) = {U, $\emptyset$, {b}, {a, b}, {b, c}, {a, b, c}}, $(\mu_R(X))^C$ = {U, $\emptyset$, {d}, {a, d}, {c, d}, {a, c, d}}, Mic-g*s closed = {U, $\emptyset$, {a}, {c}, {d}, {a, c}, {a, d}, {b, d}, {c, d}, {a, b, c}, {b, c, d}, {a, c, d}}. The subsets {a}, {c}, {a, c}, {b, d}, {a, b, c} and {b, c, d} are Mic-g*s closed but not micro-closed.

Theorem 3.2
Every micro semi-closed set is Mic-g*s closed.

Proof
Let A be a micro semi-closed set of U and A $\subseteq$ V, V is micro g-open in U. Since A is micro semi-closed set of U, we have A = Mic-Scl(A). This implies that Mic-Scl(A) $\subseteq$ V. Thus A is Mic-g*s closed.

The converse part of the above theorem need not be true which is given by the following example.

Example 3.4
Let U = {a, b, c, d}, U/R = {{a, c}, {b, d}}, X = {a, c}, τ_R(X) = {U, $\emptyset$, {a, c}}, μ = {c}, μ_R(X) = {U, $\emptyset$, {c}, {a, c}}, $(\mu_R(X))^C$ = {U, $\emptyset$, {b, d}, {a, b, d}}, micro semi-closed = {U, $\emptyset$, {a}, {b}, {d}, {a, b}, {b, d}, {a, d}, {a, b, d}}, Mic-g*s closed = {U, $\emptyset$, {a}, {b}, {d}, {a, b}, {b, d}, {a, d}, {a, b, d}, {b, c, d}}. The subset {b, c, d} is Mic-g*s closed but not micro semi-closed.

Theorem 3.3
Every micro α-closed set is Mic-g*s closed.

Proof
Let A be a micro α-closed set of U and A $\subseteq$ V, V is micro g-open in U. Since A is micro α-closed set of U, we have A = Mic-αcl(A) which implies that Mic-αcl(A) $\subseteq$ V. But Mic-Scl(A) $\subseteq$ Mic-αcl(A) $\subseteq$ V. This implies that Mic-Scl(A) $\subseteq$ V. Thus A is Mic-g*s closed.

The converse part of the above theorem need not be true which is given by the following example.

Example 3.5
Let U = {a, b, c, d}, U/R = {{a}, {b}, {c}, {d}}, X = {b, c}, τ_R(X) = {U, $\emptyset$, {b, c}}, μ = {c, d}, μ_R(X) = {U, $\emptyset$, {c}, {b, c}, {c, d}, {b, c, d}}, $(\mu_R(X))^C$ = {U, $\emptyset$, {a}, {a, b}, {a, d} {a, b, d}}, micro α-closed = {U, $\emptyset$, {a}, {b}, {d}, {a, b}, {a, d}, {b, d}, {a, b, d}}, Mic-g*s closed = {U, $\emptyset$, {a}, {b}, {d}, {a, b}, {a, d}, {b, d}, {a, c}, {a, b, d}, {a, c, d}, {a, b, c}}. The subsets {a, c}, {a, c, d} and {a, b, c} are Mic-g*s closed but not micro α-closed.

Theorem 3.4
Every micro g-closed set is Mic-g*s closed.

Proof
Let A be a micro g-closed set of U and A $\subseteq$ V, V is micro g-open in U. Since Every micro-open is micro g-open and A is micro g-closed set of U, we have Mic-cl(A) $\subseteq$ V. But Mic-Scl(A) $\subseteq$ Mic-cl(A) $\subseteq$ V. This implies that Mic-Scl(A) $\subseteq$ V. Thus A is Mic-g*s closed.

The converse part of the above theorem need not be true which is given by the following example.

Example 3.6
Let U = {a, b, c, d}, U/R = {{a, b, c}, {d}}, X = {a, c}, τ_R(X) = {U, $\emptyset$, {a, b, c}}, μ = {a, d}, μ_R(X) = {U, $\emptyset$, {a}, {a, d}, {a, b, c}}, $(\mu_R(X))^C$ = {U, $\emptyset$, {d}, {b, c}, {b, c, d}}, micro g-closed = {U, $\emptyset$, {d}, {b, c}, {c, d}, {b, d}, {a, c, d}, {a, b, d}, {b, c, d}}, Mic-g*s closed = {U, $\emptyset$, {b}, {c}, {d}, {b, c}, {c, d}, {b, d}, {a, c, d}, {a, b, d}, {b, c, d}}. The subsets {b} and {c} are Mic-g*s closed but not micro g-closed.

Theorem 3.5
Every micro g*-closed set is Mic-g*s closed.

Proof
Let A be a micro g*-closed set of U and A $\subseteq$ V, V is micro g-open in U. Since A is micro g*-closed set of U, we have Mic-cl(A) $\subseteq$ V. But Mic-Scl(A) $\subseteq$ Mic-cl(A) $\subseteq$ V. This implies that Mic-Scl(A) $\subseteq$ V. Thus A is Mic-g*s closed.

The converse part of the above theorem need not be true which is given by the following example.

Example 3.7
Let U = {a, b, c, d}, U/R = {{a}, {b, c, d}}, X = {b}, τ_R(X) = {U, ∅, {b, c, d}}, μ = {b}, μ_R(X) = {U, ∅, {b}, {b, c, d}}, $(\mu_R(X))^C$ = {U, ∅, {a}, {a, c, d}}, micro g*-closed = {U, ∅, {a}, {a, b}, {a, d}, {a, c}, {a, b, c}, {a, b, d}, {a, c, d}}, Mic-g*s closed = {U, ∅, {a}, {c}, {d}, {a, c}, {a, d}, {c, d}, {a, b}, {a, c, d}, {a, b, d}, {a, b, c}}. The subsets {c}, {d} and {c, d} are Mic-g*s closed but not micro g*-closed.

Theorem 3.6
Every Mic-g*s closed set is micro gs-closed.

Proof
Let A be a Mic-g*s closed set of U and A ⊆ V, V is micro g-open in U. Since every micro-open set is micro g-open and A is micro g*s-closed set of U, we have Mic-Scl(A) ⊆ V. Thus A is micro gs-closed.

The converse part of the above theorem need not be true which is given by the following example.

Example 3.8
Let U = {a, b, c, d}, U/R = {{a, b}, {c, d}}, X = {a}, τ_R(X) = {U, ∅, {a, b}}, μ = {a, c, d}, μ_R(X) = {U, ∅, {a}, {a, b}, {a, c, d}}, $(\mu_R(X))^C$ = {U, ∅, {b}, {c, d}, {b, c, d}}, Mic-g*s closed = {U, ∅, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}}, micro gs-closed = {U, ∅, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}, {a, c, d}, {a, b, d}}. The subsets {a, c, d} and {a, b, d} are micro gs-closed but not Mic-g*s closed.

Theorem 3.7
Every Mic-g*s closed set is micro gsp-closed.

Proof
Let A be a Mic-g*s closed set of U and A ⊆ V, V is micro g-open in U. Since Every micro-open is micro g-open and A is Mic-g*s closed set, we have Mic-Scl(A) ⊆ V. But Mic-βcl(A) ⊆ Mic-Scl(A) ⊆ V. This implies that Mic-βcl(A) ⊆ V. Thus A is micro gsp-closed.

The converse part of the above theorem need not be true which is given by the following example.

Example 3.9
Let U = {a, b, c, d}, U/R = {{a, b}, {c, d}}, X = {a}, τ_R(X) = {U, ∅, {a, b}}, μ = {b, c, d}, μ_R(X) = {U, ∅, {b}, {a, b}, {b, c, d}}, $(\mu_R(X))^C$ = {U, ∅, {a}, {c, d}, {a, c, d}}, Mic-g*s closed = {U, ∅, {a}, {c}, {d}, {a, c}, {a, d}, {c, d}, {a, c, d}}, micro gsp-closed = {U, ∅, {a}, {c}, {d}, {a, c}, {a, d}, {c, d}, {a, c, d}, {a, b, d}, {a, b, c}}. The subsets {a, b, c} and {a, b, d} are micro gsp-closed but not Mic-g*s closed.

Remark 2
The subsets micro pre-closed and Mic-g*s closed are independent to each other.

Proof
The proof of the remark follows from the Definitions 2.7, 3.1 and the Remark 1

Example 3.10
Let U = {a, b, c, d}, U/R = {{a, c}, {b, d}}, X = {a, c}, τ_R(X) = {U, ∅, {a, c}}, μ = {a, b, c}, μ_R(X) = {U, ∅, {a, c}, {a, b, c}}, $(\mu_R(X))^C$ = {U, ∅, {d}, {b, d}}, micro pre-closed = {U, ∅, {a}, {b}, {c}, {d}, {a, b}, {b, c}, {a, d}, {b, d}, {c, d}, {b, c, d}, {a, b, d}}, Mic-g*s closed = {U, ∅, {b}, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {b, c, d}, {a, c, d}}. The subsets {a}, {c}, {a, b} and {b, c} are micro pre-closed but not Mic-g*s closed and the subset {a, c, d} is Mic-g*s closed but not micro-pre closed.

Remark 3
The subsets Mic-g*s closed and micro sg-closed are independent to each other.

Proof
The proof of the remark follows from the Definitions 2.11, 3.1.

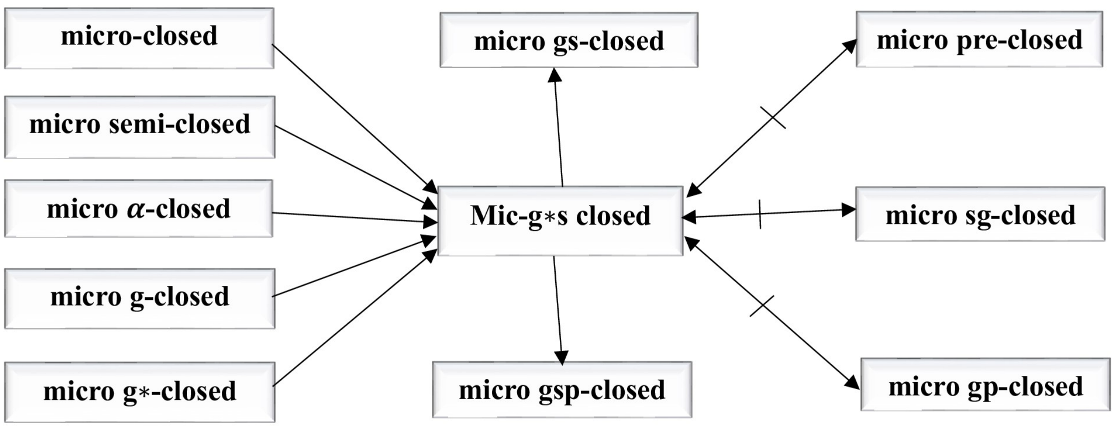
Example 3.11
Let U = {a, b, c, d}, U/R = {{a, d}, {c, b}}, X = {b}, τ_R(X) = {U, ∅, {b, c}}, μ = {a}, μ_R(X) = {U, ∅, {a}, {b, c}, {a, b, c}}, $(\mu_R(X))^C$ = {U, ∅, {d}, {a, d}, {b, c, d}}, Mic-g*s closed = {U, ∅, {a}, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {b, c, d}, {a, c, d}}, micro sg-closed = {U, ∅, {a}, {b}, {c}, {d}, {a, b, d}, {b, c, d}, {a, c, d}}. The subsets {a, d}, {b, d} and {c, d} are Mic-g*s closed but not micro sg-closed and the subsets {b} and {c} are micro sg-closed but not Mic-g*s closed.

Remark 4
The subsets Mic-g*s closed and micro gp-closed are independent to each other.

Proof
The proof of the remark follows from the Definitions 2.13, 3.1 and the Remark 1

Example 3.12
Let U = {a, b, c, d}, U/R = {{a, b}, {c, d}}, X = {c}, τ_R(X) = {U, ∅, {c, d}}, μ = {b}, μ_R(X) = {U, ∅, {b}, {c, d}, {b, c, d}}, $(\mu_R(X))^C$ = {U, ∅, {a}, {a, b}, {a, c, d}}, micro gp-closed = {U, ∅, {a}, {c}, {d}, {a, b}, {a, c}, {a, d}, {a, c, d}, {a, b, d}, {a, b, c}}, Mic-g*s closed = {U, ∅, {a}, {b}, {a, b}, {a, c}, {a, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}}. The subsets {b} and {c, d} are Mic-g*s closed but not micro gp-closed and the subsets {c} and {d} are micro gp-closed but not Mic-g*s closed.

The following diagram shows the relationship between Mic-g*s closed set with other existing sets where A $\longrightarrow$ B represents A implies B but not conversely and A $\not\longleftrightarrow$ B represents that both A and B are independent to each other.



Theorem 3.8
The union of two Mic-g*s closed subset is Mic-g*s closed.

Proof
Let A and B be two Mic-g*s closed subsets of (U, τ_R(X), μ_R(X)) and V be a micro g-open set of U containing A and B. A ⊆ V, B ⊆ V implies that A∪B ⊆ V. Since A and B are Mic-g*s closed, we have Mic-Scl(A) ⊆ V and Mic-Scl(B) ⊆ V respectively. But Mic-Scl(A∪B) = Mic-Scl(A) ∪ Mic-Scl(B) ⊆ V. This implies that Mic-Scl(A∪B) ⊆ V. Therefore A∪B is Mic-g*s closed.

Example 3.13
Let U = {a, b, c, d}, U/R = {{a, b}, {c, d}}, X = {c}, τ_R(X) = {U, ∅, {c, d}}, μ = {b, c}, μ_R(X) = {U, ∅, {c}, {c, d}, {b, c}, {b, c, d}}, Mic-g*s closed = {U, ∅, {a}, {b}, {d}, {a, b}, {a, c}, {a, d}, {b, d}, {a, b, c}, {a, b, d}, {a, c, d}}. Let A = {a, c} is Mic-g*s closed and B = {d} is Mic-g*s closed then A∪B = {a, c, d} is also Mic-g*s closed.

Theorem 3.9
Let A be a Mic-g*s closed subset of (U, τ_R(X), μ_R(X)) and if A $\subseteq$ B $\subseteq$ Mic-Scl(A), then B is Mic-g*s closed subset of (U, τ_R(X), μ_R(X)).

Proof
Let V be a micro g-open set containing B. Since A is Mic-g*s closed, we have Mic-Scl(A) $\subseteq$ V whenever A $\subseteq$ V. Given that A $\subseteq$ B $\subseteq$ Mic-Scl(A), it then follows that, A $\subseteq$ B $\subseteq$ Mic-Scl(A) $\subseteq$ V. B $\subseteq$ Mic-Scl(A) $\subseteq$ V implies that Mic-Scl(B) $\subseteq$ Mic-Scl(Mic-Scl(A))=Mic-Scl(A) $\subseteq$ V. Therefore, Mic-Scl(B) $\subseteq$ V. Thus B is Mic-g*s closed.

Theorem 3.10
The subset A is Mic-g*s closed set of (U, τ_R(X), μ_R(X)) iff Mic-Scl(A)$-$A has no non empty micro g-closed set.

Proof
Necessary part: Let A be a Mic-g*s closed subset of (U, τ_R(X), μ_R(X)). Suppose that K be a micro g-closed set of Mic-Scl(A)$-$A implying that K $\subseteq$ Mic-Scl(A)$-$A. Further, K $\subseteq$ Mic-Scl(A) but K $\not\subseteq$ A. Then K $\subseteq$ A^C. By using the fundamental result of set theory, we have A $\subseteq$ K^C where K^C is a micro g-open set containing A. Since A is Mic-g*s closed we have Mic-Scl(A) $\subseteq$ K^C. This implies that K $\subseteq$ Mic-Scl(A)^C. From the above we have K $\subseteq$ Mic-Scl(A) and K $\subseteq$ Mic-Scl(A)^C. Therefore K $\subseteq$ Mic-Scl(A) $\cap$ Mic-Scl(A)^C which concludes that K = $\emptyset$.
Sufficient part: Conversely Assume that Mic-Scl(A)$-$A has no non empty micro g-closed set. Let N be a micro g-open set containing A. Suppose that Mic-Scl(A) $\not\subseteq$ N then Mic-Scl(A) $\subseteq$ N^C where N^C is a micro g-closed set not containing A. Since Mic-Scl(A)$\subseteq$N^C, Mic-Scl(A) $\cap$ N^C is a non empty g-closed subset contained in a set other than A. This implies that, there exits a non empty micro g-closed set of Mic-Scl(A)$-$A which is a contradiction. Therefore Mic-Scl(A) $\subseteq$ N, which implies that the subset A is Mic-g*s closed.

4. Micro Generalized Star Semi Open Set

In this section, we introduce a new concept of micro open set namely micro generalized star semi open set (briefly Mic-g*s open) and obtain its basic properties.

Definition 4.1
Let (U, τ_R(X), μ_R(X)) be a micro topological space. A subset A of (U, τ_R(X), μ_R(X)) is said to be micro generalized star semi open set (briefly Mic-g*s open) if its complement, A^C is micro generalized star semi closed.

Example 4.1
Let U = {a, b, c, d}, U/R = {{a, b}, {c, d}}, X = {c}, τ_R(X) = {U, $\emptyset$, {c, d}}, μ = {d}, μ_R(X) = {U, $\emptyset$, {d}, {c, d}}, $(\mu_R(X))^C$ = {U, $\emptyset$, {a, b}, {a, b, c}}, Mic-g*s closed = {U, $\emptyset$, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}, {a, b, d}}, Mic-g*s open = (Mic-g*s closed)^C = {U, $\emptyset$, {c}, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}}

Definition 4.2
The union of all Mic-g*s open sets contained in A is said to be Micro generalized star semi interior of A. (briefly Mic-g*s Intr.(A)).

Example 4.2
Let U = {a, b, c, d}, U/R = {{a, d}, {b, c}}, X = {b}, τ_R(X) = {U, $\emptyset$, {b, c}}, μ = {c}, μ_R(X) = {U, $\emptyset$, {c}, {b, c}}, $(\mu_R(X))^C$ = {U, $\emptyset$, {a, d}, {a, b, d}}, Mic-g*s closed = {U, $\emptyset$, {a}, {b}, {d}, {a, b}, {a, d}, {b, d}, {a, c, d}, {a, b, d}}, Mic-g*s open = (Mic-g*s closed)^C = {U, $\emptyset$, {c}, {b}, {a, c}, {b, c}, {c, d}, {a, b, c}, {a, c, d}, {b, c, d}}. Let A = {a, b, d}, Mic-g*s Intr.(A) = {b}. Let B = {a, d}, Mic-g*s Intr.(B) = $\emptyset$.

Theorem 4.1

1. Every micro-open set is Mic-g*s open.
2. Every micro semi-open set is Mic-g*s open.
3. Every micro α-open set is Mic-g*s open.

4. Every micro g-open set is Mic-g*s open.
5. Every micro g*-open set is Mic-g*s open.
6. Every Mic-g*s open set is micro gs-open.
7. Every Mic-g*s open set is micro gsp-open.

Proof
Proof follows from the above theorems(Theorem 3.1 - Theorem 3.7).

Theorem 4.2
The intersection of any two Mic-g*s open subset is Mic-g*s open.

Proof
Let A and B be two Mic-g*s open subsets of (U, τ_R(X), μ_R(X)) . Let N be a micro g-closed set containing A and B respectively. This implies that A^C and B^C are Mic-g*s closed subsets of (U, τ_R(X), μ_R(X)). By Theorem 3.8, we have A^C $\cup$ B^C is Mic-g*s closed. But (A $\cap$ B)^C = A^C $\cup$ B^C. Therefore (A $\cap$ B)^C is Mic-g*s closed. Thus we conclude that A $\cap$ B is Mic-g*s open.

Example 4.3
Let U = {a, b, c, d}, U/R = {{a, b}, {c, d}}, X = {c}, τ_R(X) = {U, $\emptyset$, {c, d}}, μ = {a, d}, μ_R(X) = {U, $\emptyset$, {d}, {c, d}, {a, d}, {a, c, d}}, Mic-g*s open = {U, $\emptyset$, {a}, {c}, {d}, {c, d}, {a, c}, {a, d}, {b, d}, {b, c, d}, {a, b, d}, {a, c, d}}. Let A = {c, d} is Mic-g*s open and B = {a, d} is Mic-g*s open then A$\cap$B = {d} is also Mic-g*s open.

Theorem 4.3
Let A be a subset of (U, τ_R(X), μ_R(X)). A is Mic-g*s open iff N $\subseteq$ Mic-Sint(A) whenever N $\subseteq$ A, where N is micro g-closed.

Proof
Sufficient part: Suppose that N $\subseteq$ Mic-Sint(A) where N is a micro g-closed set contained in A. N $\subseteq$ A implies that A^C $\subseteq$ N^C. This further implies that N^C is a micro g-open set containing A^C. N $\subseteq$ Mic-Sint(A) implies that, (Mic-Sint(A))^C = Mic-Scl(A^C) $\subseteq$ N^C. Therefore A^C is Mic-g*s closed. Thus we conclude that A is Mic-g*s open.
Necessary part: Conversely, Assume that A is Mic-g*s open. Let N $\subseteq$ A, where N is micro g-closed set. N $\subseteq$ A implies that A^C $\subseteq$ N^C, where N^C is a micro g-open set containing A^C. Since A^C is Mic-g*s closed, we have Mic-Scl(A^C) $\subseteq$ N^C. But (Mic-Sint(A))^C = Mic-Scl(A^C). Therefore (Mic-Sint(A))^C $\subseteq$ N^C implies that N $\subseteq$ Mic-Sint(A) which concludes the proof.

Theorem 4.4
Let A be a Mic-g*s open subset of (U, τ_R(X), μ_R(X)) and if Mic-Sint(A) $\subseteq$ B $\subseteq$ A, then B is Mic-g*s open.

Proof
Given that Mic-Sint(A) $\subseteq$ B $\subseteq$ A and A is Mic-g*s open. The complement of A, A^C is Mic-g*s closed. Also, Mic-Sint(A) $\subseteq$ B $\subseteq$ A implies that A^C $\subseteq$ B^C $\subseteq$ Mic-Scl(A^C). It then follows from Theorem 3.9 that B^C is Mic-g*s closed. This implies that B is Mic-g*s open which concludes the proof.

5. Conclusion

A new class of micro sets namely micro generalized star semi closed set (briefly Mic-g*s closed set) and micro generalized star semi open set (briefly Mic-g*s open set) in Micro topological spaces are introduced. Mic-g*s closed set produces many results when compared to other micro closed sets in micro topological spaces. The main motive behind the introduction of Mic-g*s closed set is to analyze the basic properties of micro topology in a theoretical way which will further help in applying the concepts in Cryptography. The concept of Mic-g*s closed set is defined by using micro g-open set from the micro topology. Also, some of their characteristics are studied and its relationship with some other classes of micro closed sets had been discussed. Further, using Mic-g*s closed sets, many concepts like Continuous, Compactness, Locally Connectedness etc., can be defined which enriches micro topology.

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