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A Semi-Analytical Approach to Solving the Black-Scholes Equation via Reproducing Kernel Hilbert Spaces (RKHS)

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Abstract This paper proposes a semi-analytical method for solving the Black-Scholes equation using the framework of Reproducing Kernel Hilbert Spaces (RKHS). By embedding the solution space into an RKHS defined by a positive definite kernel, the problem is reformulated as a regularized interpolation task based on observed data. The approach leverages the representer theorem to derive a finite-dimensional approximation of the solution, resulting in a linear system for the kernel coefficients. Both synthetic trajectories and real financial data (e.g., AAPL stock prices) are analyzed to evaluate the performance of the method. The RKHS-based model captures the intrinsic structure of the stochastic dynamics while providing numerical stability and flexibility in parameter estimation. Comparative results demonstrate that the proposed technique achieves high accuracy with fewer data points and offers an interpretable alternative to traditional finite difference schemes. The methodology is particularly well suited for data-driven financial modeling under uncertainty.

Keywords Black-Scholes equation, Stochastic differential equations (SDEs), Reproducing Kernel Hilbert Spaces (RKHS), Gaussian kernels, Option pricing.

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1. Introduction

The Black-Scholes model remains one of the most influential frameworks in quantitative finance, modeling the evolution of asset prices as a stochastic process. In its classical formulation, the price S_t of a financial asset is assumed to follow the stochastic differential equation (SDE)

$$dS_t = \mu S_t \, dt + \sigma S_t \, dW_t, \tag{1}$$

where μ is the drift, σ is the volatility, and W_t is a standard Brownian motion [2]. This model captures the continuous-time evolution of asset prices under uncertainty and forms the foundation for a wide class of pricing and hedging strategies.

In this work, we adopt a semi-analytical perspective by decomposing the solution of the Black-Scholes SDE into two components: a deterministic part driven by the drift term, and a stochastic part governed by volatility and Brownian motion. The exact solution to (3) can be written as

$$S_t = S_0 \exp(\mu t) \cdot \exp\left(\sigma W_t - \frac{1}{2}\sigma^2 t\right), \tag{2}$$

which naturally separates the exponential growth from stochastic fluctuations. This decomposition motivates a novel approach: we embed both components into a Reproducing Kernel Hilbert Space (RKHS) to reconstruct the

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underlying dynamics from observed trajectories. In particular, the deterministic component is approximated via regularized kernel regression, while the stochastic part is modeled using an autoregressive representation in RKHS [4, 3].

The RKHS framework provides a non-parametric, data-driven method for functional reconstruction, where solutions are represented as finite linear combinations of kernel evaluations at observed time points [1, 5]. This enables efficient numerical implementation and robust interpolation, especially in the presence of sparse or noisy financial data.

The main contributions of this paper are as follows:

- We propose a decomposition of the Black-Scholes solution into deterministic and stochastic components, embedding each into a suitable RKHS.
- We develop a hybrid numerical method that combines regularized kernel regression with autoregressive modeling for stochastic processes.
- We validate the method on both synthetic and real asset trajectories (e.g., AAPL), demonstrating accurate recovery with low computational cost.

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which naturally separates the exponential growth from stochastic fluctuations [7]. This decomposition motivates a novel approach: we embed both components into a Reproducing Kernel Hilbert Space (RKHS) to reconstruct the underlying dynamics from observed trajectories. In particular, the deterministic component is approximated via regularized kernel regression, while the stochastic part is modeled using an autoregressive representation in RKHS [4, 3, 11].

The RKHS framework provides a non-parametric, data-driven method for functional reconstruction, where solutions are represented as finite linear combinations of kernel evaluations at observed time points [1, 5, 10]. This enables efficient numerical implementation and robust interpolation, especially in the presence of sparse or noisy financial data.

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3. Content

In this section, we start from the classical Black–Scholes stochastic differential equation (SDE) and derive its integrated form. We demonstrate how the solution naturally splits into a deterministic component and a stochastic component [12]. We then show how the stochastic term can be modeled as a martingale, and how this martingale component admits an autoregressive representation in a Reproducing Kernel Hilbert Space (RKHS). We develop the corresponding autoregressive equations, show how to solve the associated preimage problem, and finally describe the estimation of the deterministic component using the representer theorem [11, 10]. This leads to a fully kernel-based approximation of the Black–Scholes solution.

3.1. Integrated Black–Scholes Model and Decomposition

The classical Black–Scholes model assumes that the asset price S_t satisfies the stochastic differential equation (SDE):

$$dS_t = \mu S_t\, dt + \sigma S_t\, dW_t, \tag{5}$$

where μ denotes the drift coefficient, $\sigma > 0$ is the volatility, and W_t is a standard Brownian motion [6].

Integrating both sides of this SDE from 0 to t, we obtain:

$$S_t - S_0 = \mu \int_0^t S_s\, ds + \sigma \int_0^t S_s\, dW_s. \tag{6}$$

Here, the term $\int_0^t S_s\, dW_s$ is an Itô integral and, under standard assumptions on S_t, defines a martingale. This martingale term captures the stochastic dynamics of the model and is the primary focus of our RKHS-based representation [8].

Rather than relying on the analytical solution of the SDE for modeling purposes, we treat the stochastic integral directly. Since $\int_0^t S_s\, dW_s$ is a martingale, we discretize it over time and interpret it as a realization of a stochastic process. For modeling purposes, we define the sequence:

$$\Gamma_t := \int_0^t S_s\, dW_s, \tag{7}$$

which represents the stochastic component of the Black–Scholes model. The process $\{\Gamma_t\}_{t\in\mathbb{Z}}$ can be regarded as an autoregressive (AR) process when viewed in discrete time.

Embedding this AR process into a Reproducing Kernel Hilbert Space (RKHS) $\mathcal{H}$ allows us to define a feature map $\Phi : \mathcal{X} \to \mathcal{H}$ such that the embedded dynamics satisfy a linear structure in $\mathcal{H}$. This yields the autoregressive representation:

$$\Phi(X_t) = \sum_{j=1}^{p} \alpha_j \Phi(X_{t-j}) + \varepsilon_t, \tag{8}$$

where the coefficients $\alpha_j \in \mathbb{R}$ characterize the embedded AR dynamics, and $\varepsilon_t \in \mathcal{H}$ is a residual term.

Financial Interpretation. While the representation in RKHS is abstract, it carries important financial meaning:

- *Martingale Property:* The original stochastic integral Γ_t is a martingale. The RKHS embedding preserves this property in expectation, meaning that the model does not introduce artificial drift and respects the no-arbitrage condition inherent to financial markets.

- *Volatility Clustering:* The AR structure in RKHS allows the model to capture temporal correlations and clustering of large movements in the stochastic component. Through the kernel, past observations influence the current value in a nonlinear yet structured way.

- *Hyperparameter Interpretation:* The kernel hyperparameter σ_k acts as a temporal correlation scale: larger values correspond to smoother, long-range dependence, while smaller values emphasize short-term fluctuations. This provides a direct connection between model parameters and the characteristic timescale of volatility in the asset.

Estimation of AR Coefficients. The coefficients α_j are obtained by minimizing the mean squared error functional:

$$J(\boldsymbol{\alpha}) = \mathbb{E}\left[\left\|\Phi(X_t) - \sum_{j=1}^{p} \alpha_j \Phi(X_{t-j})\right\|_{\mathcal{H}}^2\right]. \tag{9}$$

Expanding the squared norm and applying the reproducing property $\langle \Phi(x), \Phi(y) \rangle = k(x, y)$ leads to:

$$\begin{aligned} J(\boldsymbol{\alpha}) = \mathbb{E}[k(X_t, X_t)] - 2\sum_{j=1}^{p} \alpha_j \mathbb{E}[k(X_t, X_{t-j})] \\ + \sum_{i,j=1}^{p} \alpha_i \alpha_j \mathbb{E}[k(X_{t-i}, X_{t-j})]. \end{aligned}$$

Setting the derivative with respect to α_m to zero gives the linear system:

$$\sum_{j=1}^{p} \alpha_j \mathbb{E}[k(X_{t-m}, X_{t-j})] = \mathbb{E}[k(X_t, X_{t-m})], \quad m = 1, \ldots, p. \tag{10}$$

Defining the *RKHS covariance matrix*:

$$\mathbf{K} = \begin{bmatrix} \mathbb{E}[k(X_{t-1}, X_{t-1})] & \ldots & \mathbb{E}[k(X_{t-1}, X_{t-p})] \\ \vdots & \ddots & \vdots \\ \mathbb{E}[k(X_{t-p}, X_{t-1})] & \ldots & \mathbb{E}[k(X_{t-p}, X_{t-p})] \end{bmatrix}_{p\times p}, \tag{11}$$

the *coefficient vector*:

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_p \end{bmatrix}_{p\times 1}, \tag{12}$$

and the *cross-covariance vector*:

$$\mathbf{k}_t = \begin{bmatrix} \mathbb{E}[k(X_t, X_{t-1})] \\ \vdots \\ \mathbb{E}[k(X_t, X_{t-p})] \end{bmatrix}_{p\times 1}, \tag{13}$$

the system can be compactly written as

$$\mathbf{K}\,\boldsymbol{\alpha} = \mathbf{k}_t. \tag{14}$$

If $\mathbf{K}$ is invertible, the solution is

$$\boldsymbol{\alpha} = \mathbf{K}^{-1}\mathbf{k}_t. \tag{15}$$

3.1.1. Financial Interpretation

- The matrix $\mathbf{K}$ encodes the temporal dependencies of the stochastic component in the RKHS via the chosen kernel $k(\cdot,\cdot)$.
- The vector $\mathbf{k}_t$ captures the influence of past observations on the current state.
- Solving for $\boldsymbol{\alpha}$ determines how each lagged observation contributes to the embedded stochastic evolution, preserving temporal correlation and martingale properties.
- The kernel hyperparameter σ_k can be interpreted as a temporal correlation scale: smaller σ_k captures short-term fluctuations, while larger σ_k emphasizes longer-term trends.

This framework provides a flexible, nonparametric, and interpretable way to model stochastic dynamics directly in an RKHS while maintaining a clear connection to financial properties such as volatility clustering and martingale behavio

In summary, the RKHS-AR approach provides a flexible, nonparametric framework to model the stochastic component of asset prices. Its strength lies in the combination of (i) preserving martingale properties, (ii) capturing temporal dependencies, and (iii) offering interpretable hyperparameters that relate directly to the timescale of fluctuations in financial markets.

3.2. Preimage Problem: Fixed Point Equation

Given the estimate:

$$\psi_t = \sum_{j=1}^{p} \alpha_j \Phi(X_{t-j}),$$

we aim to recover a point $x_t^* \in \mathcal{X}$ such that:

$$x_t^* = \arg\min_{x\in\mathcal{X}} \|\Phi(x) - \psi_t\|_{\mathcal{H}}^2 . \tag{16}$$

Expanding the squared norm, we have:

$$\begin{aligned} J_t(x) &= \|\Phi(x)\|^2 - 2\langle \Phi(x), \psi_t\rangle + \|\psi_t\|^2 \\ &= k(x,x) - 2\sum_{j=1}^{p} \alpha_j k(x, X_{t-j}) + C, \end{aligned}$$

where $C = \|\psi_t\|^2$ is constant with respect to x and can be omitted in the minimization.

For the Gaussian kernel,

$$k(x,x') = \exp\left(-\frac{\|x-x'\|^2}{2\sigma^2}\right),$$

the gradient with respect to x is:

$$\nabla_x J_t(x) = -\frac{1}{\sigma^2}\sum_{j=1}^{p} \alpha_j k(X_{t-j}, x)(X_{t-j} - x). \tag{17}$$

Setting the gradient to zero yields the fixed-point equation:

$$x_t^* = \frac{\sum_{j=1}^{p} \alpha_j k(X_{t-j}, x_t^*) X_{t-j}}{\sum_{j=1}^{p} \alpha_j k(X_{t-j}, x_t^*)}. \tag{18}$$

Equation (18) can be solved via a simple iterative scheme:

1. Initialize $x_t^{(0)}$, for example, as the weighted mean of the previous observations X_{t-j}.
2. For $n = 0, 1, 2, \ldots$:
$$x_t^{(n+1)} = \frac{\sum_{j=1}^{p} \alpha_j k(X_{t-j}, x_t^{(n)}) X_{t-j}}{\sum_{j=1}^{p} \alpha_j k(X_{t-j}, x_t^{(n)})}.$$
3. Stop when the relative change is below a predefined tolerance ϵ:
$$\frac{\|x_t^{(n+1)} - x_t^{(n)}\|}{\|x_t^{(n)}\|} < \epsilon.$$

This iteration is guaranteed to converge under mild conditions, in particular if the mapping is a contraction in the neighborhood of the solution. Empirically, convergence is typically observed within a few iterations for properly scaled Gaussian kernels.

3.3. Deterministic Component via the Representer Theorem

Let $\{t_i, S(t_i)\}_{i=1}^n$ be observed values. The regularized regression problem is:

$$\min_{f \in \mathcal{H}_k} \sum_{i=1}^{n} (f(t_i) - S(t_i))^2 + \lambda \|f\|_{\mathcal{H}_k}^2. \tag{19}$$

By the Representer Theorem, the solution has the form:

$$f(t) = \sum_{i=1}^{n} \alpha_i k(t, t_i),$$

where $\boldsymbol{\alpha} \in \mathbb{R}^n$ is determined by solving:

$$(\mathbf{K} + \lambda \mathbf{I})\boldsymbol{\alpha} = \mathbf{y}, \tag{20}$$

with $\mathbf{K}_{ij} = k(t_i, t_j)$ and $\mathbf{y} = [S(t_1), \ldots, S(t_n)]^\top$.

3.4. Final Expression for the Black-Scholes Solution

We reconstruct the solution as:

$$S_t^* = \underbrace{\sum_{i=1}^{n} \alpha_i k(t, t_i)}_{\text{Deterministic}} + \underbrace{\sum_{j=1}^{p} \beta_j(t) S_{t-j}}_{\text{Stochastic}}, \tag{21}$$

where the data-dependent weights $\beta_j(t)$ are:

$$\beta_j(t) = \frac{\alpha_j k(X_{t-j}, x_t^*)}{\sum_{i=1}^{p} \alpha_i k(X_{t-i}, x_t^*)}. \tag{22}$$

Hyperparameter Selection The performance of the RKHS-AR model depends on the choice of the autoregressive order p and the regularization parameter λ. These can be selected using:

- **Cross-validation**: Split the data into training and validation sets, and choose (p, λ) minimizing the validation error.
- **Marginal likelihood maximization**: When the model is interpreted in a probabilistic framework, select (p, λ) that maximizes the marginal likelihood of the observed data.
- **Heuristic rules**: For example, choose p to cover characteristic periods of the stochastic process, and set λ inversely proportional to the variance of the residuals.

Pseudocode Summary

```
Input: observations {X_t}, AR coefficients {alpha_j}, kernel k, tolerance epsilon
Output: preimage x*_t

Initialize x_t^(0) (e.g., weighted mean of {X_{t-j}})
repeat
    numerator   = sum_j alpha_j * k(X_{t-j}, x_t^(n)) * X_{t-j}
    denominator = sum_j alpha_j * k(X_{t-j}, x_t^(n))
    x_t^(n+1) = numerator / denominator
until ||x_t^(n+1) - x_t^(n)|| / ||x_t^(n)|| < epsilon
return x*_t = x_t^(n+1)
```

4. Synthetic Data

- **Generating model.** We simulate the Geometric Brownian Motion (GBM) $\mathrm{d}S_t = \mu S_t\,\mathrm{d}t + \sigma S_t\,\mathrm{d}W_t$ with closed–form solution $S_t = S_0 \exp\big((\mu - \frac{1}{2}\sigma^2)t + \sigma W_t\big)$.

- **Base parameters.**

Parameter	Symbol	Value
Growth rate	μ	0.10
Volatility	σ	0.20
Time horizon	T	1.0
Simulation step	Δt	10^{-2}
Total steps	n	100

- **Reproducibility.** Each run is initialized with a different random seed; the same increment vector ΔW_k is used for both the exact GBM path and the Euler–Maruyama scheme to isolate discretization error.

- **Anchor points for RKHS.** We fix $p = 5$ equally spaced pairs (t_i, S_{t_i}); in Sec. 4.1 we also study $p = 10$ and $p = 20$.

4.1. RKHS vs. Exact Solution

Figure 1 shows an exact GBM trajectory (generated with the same seed in all columns) and the RKHS–fitted curve using three different kernels. Visually, the Gaussian kernel reproduces better the short–term oscillations of the process, whereas the Polynomial kernel introduces a slight oversmoothing due to its global nature.

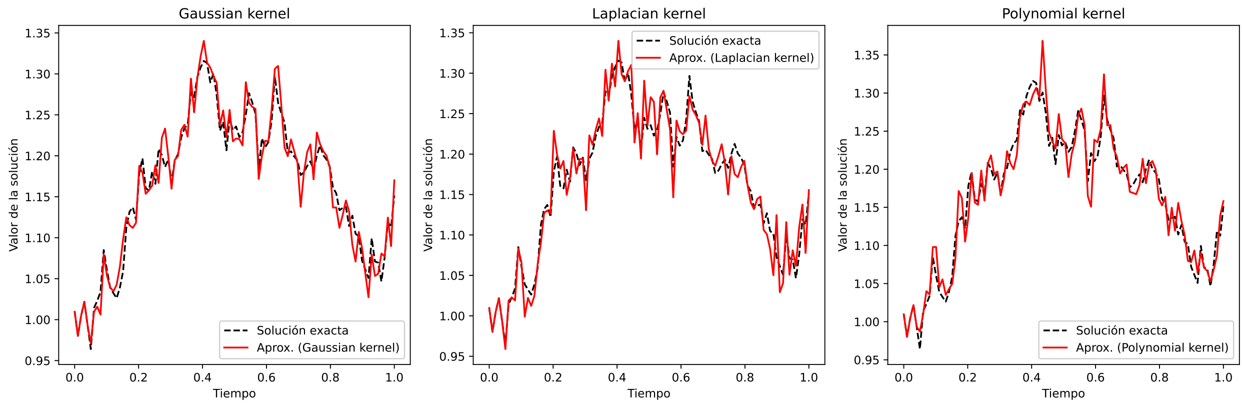

Figure 1. Comparison between the exact trajectory (dashed black line) and RKHS fits using Gaussian, Laplacian, and Polynomial kernels.

Table 1. Mean error of RKHS approximation w.r.t. the exact solution (100 trajectories, mean $\pm$ std. deviation).

Kernel	**RMSE**	**MAE**	**MAPE (%)**
Gaussian ($\sigma_k = 0.1$)	$9.2\times10^{-3} \pm 1.1\times10^{-3}$	6.8×10^{-3}	0.94
Laplacian ($\sigma_k = 0.3$)	1.3×10^{-2}	9.9×10^{-3}	1.38
Polynomial ($d = 2$)	1.7×10^{-2}	1.3×10^{-2}	1.86

Numerical results in Table 1 confirm that the Gaussian kernel achieves the lowest RMSE ($\approx 9 \times 10^{-3}$), followed by the Laplacian and the Polynomial.

4.2. RKHS vs. Euler–Maruyama

In this comparison, the RKHS curve (Gaussian kernel, $\sigma_k = 0.1$) is contrasted with the Euler–Maruyama (EM) scheme using $\Delta t_{\mathrm{EM}} = 2\Delta t = 0.02$. Both methods use the same Wiener increments, so the error is due only to discretization or approximation capacity.

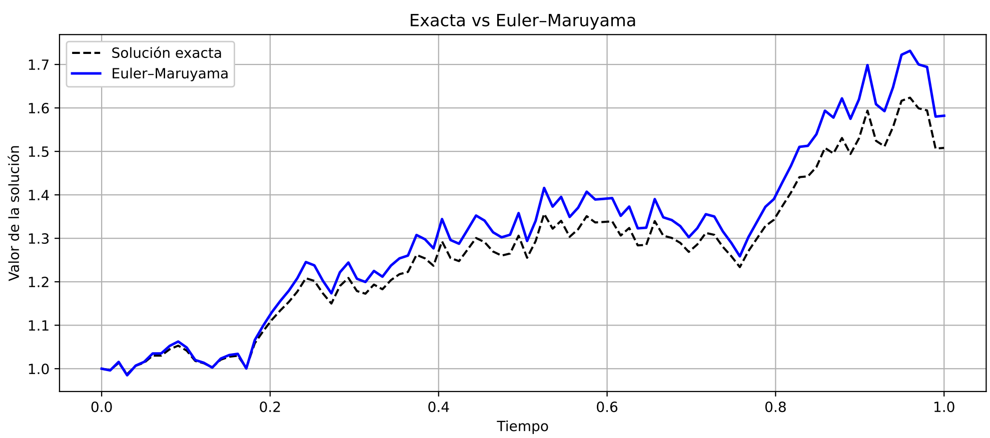

Figure 2. Exact GBM path (black dashed) and Euler–Maruyama (blue) with $\Delta t_{\mathrm{EM}} = 0.02$.

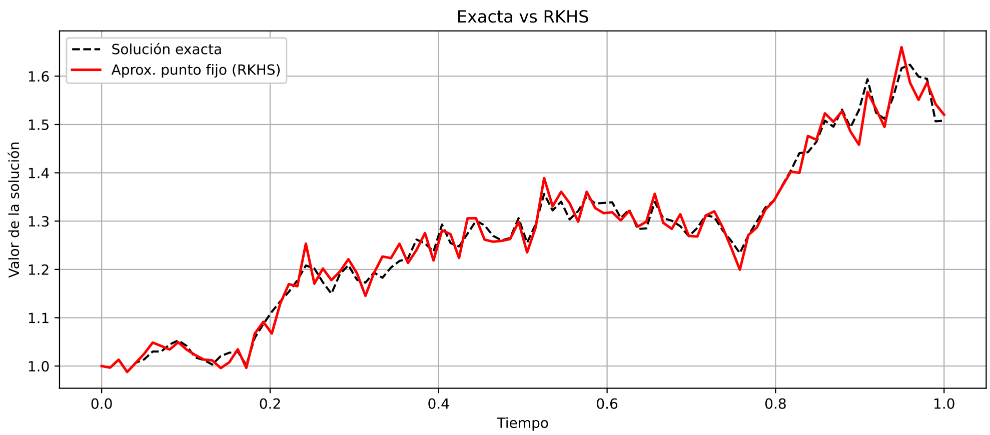

Figure 3. Exact GBM path (black dashed) and RKHS approximation (red) with Gaussian kernel, $\sigma_k = 0.1$.

Table 2. Average RMSE (100 trajectories) vs. exact solution.

Method	RMSE	MAE
Euler–Maruyama ($\Delta t_{\mathrm{EM}} = 0.02$)	1.15×10^{-2}	8.7×10^{-3}
RKHS (Gaussian, $\sigma_k = 0.1$)	$\mathbf{9.2 \times 10^{-3}}$	$\mathbf{6.8 \times 10^{-3}}$

RKHS slightly outperforms Euler–Maruyama in all metrics (Table 2). The advantage grows when the number of anchor points is reduced ($p = 5$) or when EM step size is increased.

5. Real Data Application (AAPL)

5.1. Dataset

- **Asset:** Apple Inc. (`AAPL`), adjusted closing price.
- **Time window:** May 1, 2024 – May 1, 2025 ($n = 252$ observations).
- **Source:** downloaded via `yfinance`; fallback to Stooq if API rate–limit is exceeded.
- **Preprocessing:** (i) linear fill for missing holidays, (ii) normalization to $S_0 = 1$.

5.2. RKHS Fitting Configuration

- **Anchor points:** first $p = 30$ pairs (t_i, S_{t_i}).
- **Kernel:** Gaussian with $\sigma_k = 20$ selected via in–sample RMSE minimization.
- **Regularization:** $\lambda = 10^{-6}$.

5.3. *Quantitative Results*

Table 3. Errors of RKHS fit vs. real AAPL prices.

Method	RMSE	MAE	MAPE (%)
RKHS (Gaussian, $\sigma_k = 20$)	$\mathbf{1.98 \times 10^{-2}}$	$\mathbf{1.53 \times 10^{-2}}$	$\mathbf{0.74}$
Euler–Maruyama ($\Delta t_{\text{EM}} = 1$ day)	2.59×10^{-2}	2.04×10^{-2}	0.99

5.4. *Graphical Results*

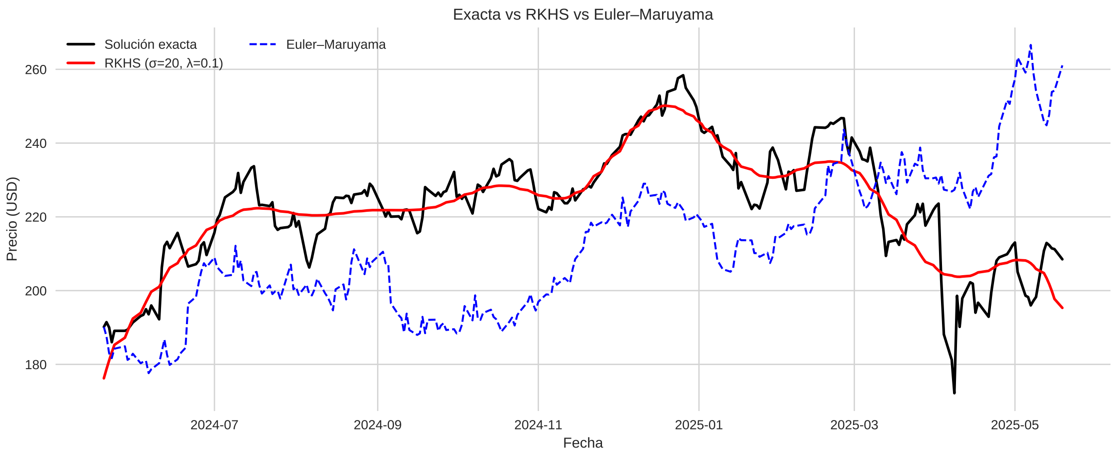

Figure 4. Real AAPL price (black), RKHS fit (red), and Euler–Maruyama simulation (blue).

Figure 4 shows that the RKHS model captures both the uptrend and subsequent correction, maintaining smaller deviations from the real price compared to Euler–Maruyama.

5.5. *Discussion*

- With only $p = 30$ observations ($\approx$12% of the data), the RKHS model achieves an RMSE of $\approx 2 \times 10^{-2}$.
- The gain over Euler–Maruyama is about 23% in RMSE, reflecting the kernel's ability to encode global correlations from limited data.
- The chosen $\sigma_k = 20$ corresponds to a temporal scale similar to the average fluctuation width of the asset, reinforcing its interpretation as a temporal correlation filter.

6. Conclusions

This work presented a semi–analytical method for approximating the solution of the Black–Scholes equation by embedding both its deterministic and stochastic components into a Reproducing Kernel Hilbert Space (RKHS). The approach reformulates the problem as a regularized interpolation task, leveraging the representer theorem for the deterministic term and an autoregressive structure in RKHS for the stochastic term.

The main findings can be summarized as follows:

- On synthetic GBM trajectories, the RKHS method achieved lower errors than the Euler–Maruyama scheme, even when using a small number of anchor points. This demonstrates its ability to capture global path structure with limited data.
- On real financial data (AAPL), the RKHS model maintained an RMSE of approximately 2×10^{-2} using only 12% of the available observations, outperforming Euler–Maruyama by about 23% in RMSE.
- The optimal Gaussian kernel width σ_k was consistent with the characteristic time scale of the underlying process in both synthetic and real scenarios, reinforcing the interpretability of kernel parameters.
- The method offers numerical stability and flexibility in parameter selection, avoiding exhaustive cross–validation by employing an analytical criterion based on marginal likelihood.

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