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Fixed Point Results in Neutrosophic n-Controlled Bipolar Metric Spaces

M. Rathivel[1], C. Inbam[2] and M. Jeyaraman[3,*]

[1] *Research Scholar, P.G. and Research Department of Mathematics, Raja Doraisingam Govt. Arts College, Sivagangai, Affiliated to Alagappa University, Karaikudi, Tamilnadu, India. Email: rathiravi52379@gmail.com*
[2] *Associate Professor, Department of Mathematics, Sri Meenakshi Govt. Arts College For Women(A), Affiliated to Madurai Kamaraj University, Madurai, Tamilnadu, India. Email: cinbam1978@gmail.com*
[3] *Associate Professor and Head, P. G. and Research Department of Mathematics, Raja Doraisingam Govt. Arts College, Sivagangai, Affiliated to Alagappa University, Karaikudi, Tamilnadu, India. E-mail: jeya.math@gmail.com.*

Abstract We present the idea of Neutrosophic n-Controlled Bipolar Metric space in this study. We use n-non-comparable functions to establish several fixed point results. Furthermore, we employ a version of the Banach contraction principle to extrapolate the conclusions and provide a few non-trivial examples. Subsequently, we apply the main findings to solve financial modeling problems in fractional differential equations.

Keywords Fixed point, Neutrosophic n-controlled bipolar Metric Space, Differential equation.

AMS 2010 subject classifications 47H10, 54H25.

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1. Introduction

Fretchet introduced the concept of metric space in his dissertation from 1906. Later, in 1922, Banach proved the Banach contraction principle in his doctoral dissertation. Since then, several researchers have been experimenting with this idea that have been under numerous circumstances. It is thought to be the most important tool for non-linear analysis. It explains why there is only one fixed point in all contractive mappings in complete metric spaces. It is an extension and generalisation of many metric space types. Zadeh [22] originated the idea of a fuzzy set in 1965. It is a modified form of a conventional set wherein each element has a membership value that goes within a reasonable range.In 1998, Smarandache coined the phrase "neutrosophic set" and proved it next to Sowndrarajan [21]. As a team, they revealed a number of noteworthy findings from neutrosophic metric space. Simsek and Kirisci [13] moved out in 2019 and proposed neutrosophic metric space (NMS). Remarkable fixed point results in neutrosophic metric space were validated in 2020 by Sowdrarajan and Jeyaraman et al. [21].

Very recently, in 2016, Mutlu and Gurdal [**?**] created their concept of bipolar metric spaces (BMS). They also evaluated several fixed point conclusions on this space. In the present paper, we will keep trying to study fixed points in the Neutrosophic n-Controlled Bipolar Metric space (NnCBMS). Especially some common fixed-point results for an array of contra variants and covariant.Plenty of structures were obtained and the results were standardised across numerous spaces using this topic. It could be achieved to preserve some of the fundamental findings on this topic, such as controlled metric type spaces (CMS) and the associated contraction principle in [20], controlled neutrosophic metric spaces ($CNMS$) and certain associated fixed point results in [4], and, more

*Correspondence to: M. Jeyaraman (Email: jeya.math@gmail.com). Associate Professor and Head, P. G. and Research Department of Mathematics, Raja Doraisingam Govt. Arts College, Sivagangai, Affiliated to Alagappa University, karaikudi, Tamilnadu, India.

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recently, the unique characteristics of metric spaces in [20].

We establish a number of fixed point findings using n-non-comparable functions.Likewise, we provides several non-trivial cases and extend the results using a variant of the Banach contraction principle. We illustrate the validity and value of the hypothesis based on the results achieved. The ensuing findings corroborate and expand upon the concepts outlined in the paper [20] in a number of afterward literatures.

2. Preliminaries

Definition 2.1. [21] An ordered 6-tuple $(Q, Z, K, Y, \circledast, \circledcirc)$ is called NMS if $Q \neq \emptyset$, $\circledast$ $NCTN$, $\circledcirc$ $NCTC$ and Z, K, Y are NS on $Q \times Q \times (0, \infty)$ the following requirements are met : For all $\mathbb{f}, \mathbb{g}, \mathbb{h} \in Q, \Theta, x > 0$.
(a) $0 \leq Z(\mathbb{f}, \mathbb{g}, \Theta) \leq 1; 0 \leq K(\mathbb{f}, \mathbb{g}, \Theta) \leq 1; 0 \leq Y(\mathbb{f}, \mathbb{g}, \Theta) \leq 1$;
(b) $Z(\mathbb{f}, \mathbb{g}, \Theta)) + K(\mathbb{f}, \mathbb{g}, \Theta) + Y(\mathbb{f}, \mathbb{g}, \Theta) \leq 3$;
(c) $Z(\mathbb{f}, \mathbb{g}, \Theta) = 1, \forall\, \Theta > 0, \Leftrightarrow \mathbb{f} = \mathbb{g}$;
(d) $Z(\mathbb{f}, \mathbb{g}, \Theta) = Z(\mathbb{g}, \mathbb{f}, \Theta)$, for $\Theta > 0$;
(e) $Z(\mathbb{f}, \mathbb{g}, \Theta) \circledast Z(\mathbb{g}, \mathbb{h}, x) \leq Z(\mathbb{f}, \mathbb{h}, \Theta + x)\, \forall\, \Theta, x > 0$;
(f) $Z(\mathbb{f}, \mathbb{g}, .) : (0, +\infty) \to [0, 1]$ is $NCTS$ and $\lim_{z \to +\infty} Z(\mathbb{f}, \mathbb{g}, \Theta) = 1$;
(g) $K(\mathbb{f}, \mathbb{g}, \Theta) = 0, \forall\, \Theta > 0, \Leftrightarrow \mathbb{f} = \mathbb{g}$;
(h) $K(\mathbb{f}, \mathbb{g}, \Theta) = K(\mathbb{f}, \mathbb{g}, \Theta)$, for $\Theta > 0$;
(i) $K(\mathbb{f}, \mathbb{g}, \Theta) \circledcirc K(\mathbb{g}, \mathbb{h}, x) \geq K(\mathbb{f}, \mathbb{h}, \Theta + x)\, \forall\, \Theta, x > 0$;
(j) $K(\mathbb{f}, \mathbb{g}, .) : (0, +\infty) \to [0, 1]$ is $NCTS$ and $\lim_{z \to +\infty} K(\mathbb{f}, \mathbb{g}, \Theta) = 0$;
(k) $Y(\mathbb{f}, \mathbb{g}, \Theta) = 0, \forall\, \Theta > 0, \Leftrightarrow \mathbb{f} = \mathbb{g}$;
(l) $Y(\mathbb{f}, \mathbb{g}, \Theta) = Y(\Theta, \epsilon, z)$; for $\Theta > 0$;
(m) $Y(\mathbb{f}, \mathbb{g}, \Theta) \circledcirc D(\mathbb{g}, \mathbb{h}, x) \geq Y(\mathbb{f}, \mathbb{h}, \Theta + x)\, \forall\, \Theta, x > 0$;
(n) $Y(\mathbb{f}, \mathbb{g}, .) : (0, +\infty) \to [0, 1]$ is $NCTS$ and $\lim_{z \to +\infty} Y(\mathbb{f}, \mathbb{g}, \Theta) = 0$;
Then, $(Q, Z, K, Y, \circledast, \circledcirc)$ is called a NMS.

Definition 2.2. [20] Given Υ, let $Q \neq \emptyset$ and $\Upsilon : Q \times Q \to [1, +\infty)$ are incompetent mapping, if $Z : Q \times Q \to (0, +\infty)$ is said to be a CMS if
(a) $Z(\epsilon, \Theta) = 0$ iff $\epsilon = \Theta$;
(b) $Z(\epsilon, \Theta) = Z(\Theta, \epsilon)$;
(c) $Z(\epsilon, \Theta) \leq \Upsilon(\epsilon, \eta) Z(\epsilon, \eta) + \Upsilon(\eta, \Theta) Z(\eta, \Theta)$; for every $\epsilon, \Theta, \eta \in Q$.

Definition 2.3. [4] Let $Q \neq \emptyset$ and $\Upsilon : Q \times Q \to [1, +\infty), \circledast$ $NCTN$, $\circledcirc$ $NCTC$ and Z, K, Y are NS on $Q \times Q \times (0, \infty)$ the following requirements are met: For all $\mathbb{f}, \mathbb{g}, \mathbb{h} \in Q, \Theta, x > 0$
(a) $0 \leq Z(\mathbb{f}, \mathbb{g}, \Theta) \leq 1; 0 \leq K(\mathbb{f}, \mathbb{g}, \Theta) \leq 1; 0 \leq Y(\mathbb{f}, \mathbb{g}, \Theta) \leq 1$;
(b) $Z(\mathbb{f}, \mathbb{g}, \Theta) + K(\mathbb{f}, \mathbb{g}, \Theta) + Y(\mathbb{f}, \mathbb{g}, \Theta) \leq 3$;
(c) $Z(\mathbb{f}, \mathbb{g}, 0) = 0$;
(d) $Z(\mathbb{f}, \mathbb{g}, \Theta) = 1, \forall\, \Theta > 0, \Leftrightarrow \mathbb{f} = \mathbb{g}$;
(e) $Z(\mathbb{f}, \mathbb{g}, \Theta) = Z(\mathbb{g}, \mathbb{f}, \Theta)$;
(f) $Z(\mathbb{f}, \mathbb{h}, \Theta + x) \geq Z\left(\mathbb{f}, \mathbb{g}, \frac{\Theta}{\Upsilon(\mathbb{f}, \mathbb{g})}\right) \circledast Z\left(\mathbb{g}, \mathbb{h}, \frac{x}{\Upsilon(g, h)}\right)$;
(g) $Z(\mathbb{f}, \mathbb{g}, .) : (0, +\infty) \to [0, 1]$ is CTS and $\lim_{z \to +\infty} Z(\mathbb{f}, \mathbb{g}, \Theta) = 1$;
(h) $K(\mathbb{f}, \mathbb{g}, 0) = 1$;
(i) $K(\mathbb{f}, \mathbb{g}, \Theta) = 0, \forall\, \Theta > 0 \Leftrightarrow \mathbb{f} = \mathbb{g}$;
(j) $K(\mathbb{f}, \mathbb{g}, \Theta) = K(\mathbb{g}, \mathbb{f}, \Theta)$;
(k) $K(\mathbb{f}, \mathbb{h}, \Theta + x) \leq K\left(\mathbb{f}, \mathbb{g}, \frac{\Theta}{\Upsilon(\mathbb{f}, \mathbb{g})}\right) \circledcirc K\left(\mathbb{g}, \mathbb{h}, \frac{x}{\Upsilon(g, h)}\right)$;
(l) $K(\mathbb{f}, \mathbb{g}, .) : (0, +\infty) \to [0, 1]$ is CTS and $\lim_{z \to +\infty} K(\mathbb{f}, \mathbb{g}, \Theta) = 0$;
(m) $Y(\mathbb{f}, \mathbb{g}, 0) = 1$;

(n) $Y(\mathbb{f},\mathbb{g},\Theta)=0$, $\forall\,\Theta>0 \Leftrightarrow \mathbb{f}=\mathbb{g}$;
(o) $Y(\mathbb{f},\mathbb{g},\Theta)=Y(\mathbb{g},\mathbb{f},\Theta)$;
(p) $Y(\mathbb{f},\mathbb{h},\Theta+x)\leq Y\left(\mathbb{f},\mathbb{g},\frac{\Theta}{\Upsilon(\mathbb{f},\mathbb{g})}\right)\circledcirc Y\left(\mathbb{g},\mathbb{h},\frac{x}{\Upsilon(g,h)}\right)$;
(q) $Y(\mathbb{f},\mathbb{g},.):(0,+\infty)\rightarrow[0,1]$ is CTS and $\lim\limits_{z\rightarrow+\infty}Y(\mathbb{f},\mathbb{g},\Theta)=0$;
Then, $(Q,Z,K,Y,\circledast,\circledcirc)$ is called a $NCMS$.

Definition 2.4. [9] Suppose $Q,L\neq\emptyset$ and $\Upsilon:Q\times Q\rightarrow[1,+\infty)$ are considered as a incompetant mappings, $\circledast$ as CTN defined as $r\circledast s=\min\{r,s\}$ and $\circledcirc$ as CTC defined as $r\circledcirc s=\max\{r,s\}$ and Z,K,Y are NS on $Q\times L\times(0,+\infty)$ is characterized $NBMS$, if for each one $(Q,L,Z,K,Y,\circledast,\circledcirc)$ fulfills all $\mathbb{f}\in Q, l\in L$ and $\Theta,x,z>0$ holds the following:
(a) $0\leq Z(\mathbb{f},\mathbb{g},\Theta)\leq 1; 0\leq K(\mathbb{f},\mathbb{g},\Theta)\leq 1; 0\leq Y(\mathbb{f},\mathbb{g},\Theta)\leq 1$;
(b) $Z(\mathbb{f},\mathbb{g},\Theta)+K(\mathbb{f},\mathbb{g},\Theta)+Y(\mathbb{f},\mathbb{g},\Theta)\leq 3$;
(c) $Z(\mathbb{f},\mathbb{g},0)>0$, for every value $\mathbb{f},\mathbb{g}\in Q\times L$;
(d) $Z(\mathbb{f},\mathbb{g},\Theta)=1$ iff $\mathbb{f}=l$ for $\mathbb{f}\in Q,\mathbb{g}\in L$;
(e) $Z(\mathbb{f},\mathbb{g},\Theta)=Z(\mathbb{g},\mathbb{f},\Theta)$, for every value $\mathbb{f},\mathbb{g}\in Q\cap L$;
(f) $Z(\mathbb{f}_1,\mathbb{g}_2,\Theta+x+z)\geq Z(\mathbb{f}_1,\mathbb{g}_1,\Theta)\circledast Z(\mathbb{f}_2,\mathbb{g}_1,x)\circledast Z(\mathbb{f}_2,\mathbb{g}_2,z)$ for $\mathbb{f}_1,\mathbb{f}_2\in Q$ and $\mathbb{g}_1,\mathbb{g}_2\in L\,\forall\,\Theta,x,z>0$;
(g) $Z(\mathbb{f},\mathbb{g},.):[0,+\infty)\rightarrow[0,1]$ is CTS;
(h) $Z(\mathbb{f},\mathbb{g},.)$ is non decreasing for every value $\mathbb{f}\in Q,\mathbb{g}\in L$;
(i) $K(\mathbb{f},\mathbb{g},0)<1$, for every value $\mathbb{f},\mathbb{g}\in Q\times L$;
(j) $K(\mathbb{f},\mathbb{g},\Theta)=1$ iff $\mathbb{f}=l$ for $\mathbb{f}\in Q,\mathbb{g}\in L$;
(k) $K(\mathbb{f},\mathbb{g},\Theta)=K(\mathbb{g},\mathbb{f},\Theta)$, for every value $\mathbb{f},\mathbb{g}\in Q\cap L$;
(l) $K(\mathbb{f}_1,\mathbb{g}_2,\Theta+x+z)\geq K(\mathbb{f}_1,\mathbb{g}_1,\Theta)\circledcirc K(\mathbb{f}_2,\mathbb{g}_1,x)\circledcirc K(\mathbb{f}_2,\mathbb{g}_2,z)$ for $\mathbb{f}_1,\mathbb{f}_2\in Q$ and $\mathbb{g}_1,\mathbb{g}_2\in L\,\forall\Theta,x,z>0$;
(m) $K(\mathbb{f},\mathbb{g},.):[0,+\infty)\rightarrow[0,1]$ is CTS;
(n) $K(\mathbb{f},\mathbb{g},.)$ is non increasing for every value $\mathbb{f}\in Q,\mathbb{g}\in L$;
(o) $Y(\mathbb{f},\mathbb{g},0)<1$, for every value $\mathbb{f},\mathbb{g}\in Q\times L$;
(p) $Y(\mathbb{f},\mathbb{g},\Theta)=1$ iff $\mathbb{f}=l$ for $\mathbb{f}\in Q,\mathbb{g}\in L$;
(q) $Y(\mathbb{f},\mathbb{g},\Theta)=Y(\mathbb{g},\mathbb{f},\Theta)$, for every value $\mathbb{f},\mathbb{g}\in Q\cap L$;
(r) $Y(\mathbb{f}_1,\mathbb{g}_2,\Theta+x+z)\geq Y(\mathbb{f}_1,\mathbb{g}_1,\Theta)\circledcirc D(\mathbb{f}_2,\mathbb{g}_1,x)\circledcirc D(\mathbb{f}_2,\mathbb{g}_2,z)$ for $\mathbb{f}_1,\mathbb{f}_2\in Q$ and $\mathbb{g}_1,\mathbb{g}_2\in L\,\forall\,\Theta,x,z>0$;
(s) $Y(\mathbb{f},\mathbb{g},.):[0,+\infty)\rightarrow[0,1]$ is CTS;
(t) $Y(\mathbb{f},\mathbb{g},.)$ is non increasing for every value $\mathbb{f}\in Q,\mathbb{g}\in L$.
Then, $(Q,L,Z,K,Y,\circledast,\circledcirc)$ is called a $NBMS$.

Definition 2.5. [19] Suppose $Q,L\neq\emptyset$ and $\Upsilon:Q\times L\rightarrow[1,+\infty)$ are considered as an incompetant mappings, $\circledast$ as CTN and $\circledcirc$ as CTC and Z,K,Y are NS on $Q\times L\times(0,+\infty)$ is characterized $NCBMS$, if for each one $(Q,L,Z,K,Y,\circledast,\circledcirc)$ fulfills all $\mathbb{f}\in Q,\mathbb{g}\in L$ and $x,\Theta,z>0$ holds the following:
(a) $0\leq Z(\mathbb{f},\mathbb{g},\Theta)\leq 1; 0\leq K(\mathbb{f},\mathbb{g},\Theta)\leq 1; 0\leq Y(\mathbb{f},\mathbb{g},\Theta)\leq 1$, for every value $(\mathbb{f},\mathbb{g})\in Q\times L$;
(b) $Z(\mathbb{f},\mathbb{g},\Theta)+K(\mathbb{f},\mathbb{g},\Theta)+Y(\mathbb{f},\mathbb{g},\Theta)\leq 3$;
(c) $Z(\mathbb{f},\mathbb{g},0)=0$, for every value $(\mathbb{f},\mathbb{g})\in Q\times L$;
(d) $Z(\mathbb{f},\mathbb{g},\Theta)=1$ iff $\mathbb{f}=\mathbb{g}$ for $\mathbb{f}\in Q,\mathbb{g}\in L$;
(e) $Z(\mathbb{f},\mathbb{g},\Theta)=Z(\mathbb{g},\mathbb{f},\Theta)$, for every value $\mathbb{f},\mathbb{g}\in Q\cap L$;
(f) $Z(\mathbb{f}_1,\mathbb{g}_2,\Theta+x+z)\geq Z\left(\mathbb{f}_1,\mathbb{g}_1,\frac{\Theta}{\Upsilon(\mathbb{f}_1,l_1)}\right)\circledast Z\left(\mathbb{f}_2,\mathbb{g}_1,\frac{x}{\Upsilon(\mathbb{f}_2,l_1)}\right)\circledast Z\left(\mathbb{f}_2,\mathbb{g}_2,\frac{z}{\Upsilon(\mathbb{f}_2,l_2)}\right)$ for $\mathbb{f}_1,\mathbb{f}_2\in Q$ and $\mathbb{g}_1,\mathbb{g}_2\in L$;
(g) $Z(\mathbb{f},\mathbb{g}.):[0,+\infty)\rightarrow[0,1]$ is CTS;
(h) $Z(\mathbb{f},\mathbb{g},.)$ is non decreasing for every value $\mathbb{f}\in Q,\mathbb{g}\in L$;
(i) $K(\mathbb{f},\mathbb{g},0)=1$;
(j) $K(\mathbb{f},\mathbb{g},\Theta)=0$ iff $\mathbb{f}=\mathbb{g}$ for $\mathbb{f}\in Q,\mathbb{g}\in L$;
(k) $K(\mathbb{f},\mathbb{g},\Theta)=K(\mathbb{g},\mathbb{f},\Theta)$, for every value $\mathbb{f},\mathbb{g}\in Q\cap L$;
(l) $K(\mathbb{f}_1,\mathbb{g}_2,\Theta+x+z)\leq K\left(\mathbb{f}_1,\mathbb{g}_1,\frac{\Theta}{\Upsilon(\mathbb{f}_1,l_1)}\right)\circledcirc K\left(\mathbb{f}_2,\mathbb{g}_1,\frac{x}{\Upsilon(\mathbb{f}_2,l_1)}\right)\circledcirc K\left(\mathbb{f}_2,\mathbb{g}_2,\frac{z}{\Upsilon(\mathbb{f}_2,l_2)}\right)$ for $\mathbb{f}_1,\mathbb{f}_2\in Q$ and $\mathbb{g}_1,\mathbb{g}_2\in L$;

(m) $K(\mathbb{f}, \mathbb{g}, .) : [0, +\infty) \to [0, 1]$ is CTS;
(n) $K(\mathbb{f}, \mathbb{g}, .)$ is non increasing for every value $\mathbb{f} \in Q, \mathbb{g} \in L$;
(o) $Y(\mathbb{f}, \mathbb{g}, 0) = 1$;
(p) $Y(\mathbb{f}, \mathbb{g}, \Theta) = 0$ iff $\mathbb{f} = \mathbb{g}$ for $\mathbb{f} \in Q, \mathbb{g} \in L$;
(q) $Y(\mathbb{f}, \mathbb{g}, \Theta) = Y(\mathbb{g}, \mathbb{f}, \Theta)$, for every value $\mathbb{f}, \mathbb{g} \in Q \cap L$;
(r) $Y(\mathbb{f}_1, \mathbb{g}_2, \Theta + x + z) \leq Y\left(\mathbb{f}_1, \mathbb{g}_1, \frac{\Theta}{\Upsilon(\mathbb{f}_1,\mathbb{g}_1)}\right) \circledcirc D\left(\mathbb{f}_2, \mathbb{g}_1, \frac{x}{\Upsilon(\mathbb{f}_2,\mathbb{g}_1)}\right) \circledcirc D\left(\mathbb{f}_2, \mathbb{g}_2, \frac{z}{\Upsilon(\mathbb{f}_2,\mathbb{g}_2)}\right)$ for $\mathbb{f}_1, \mathbb{f}_2 \in Q$ and $\mathbb{g}_1, \mathbb{g}_2 \in L$;
(s) $Y(\mathbb{f}, \mathbb{g}, .) : [0, +\infty) \to [0, 1]$ is CTS;
(t) $Y(\mathbb{f}, \mathbb{g}, .)$ is non increasing for every value $\mathbb{f} \in Q, \mathbb{g} \in L$.
Then, $(Q, L, Z, K, Y, \circledast, \circledcirc)$ is called a $NCBMS$.

Example 2.6. Let $Q = [0, 1), L = [1, \infty)$. Define Z, K, D are NS on $Q \times Q \times (0, +\infty)$ as
$Z(\mathbb{f}, \mathbb{g}, \Theta) = \frac{\Theta}{\Theta + d(\mathbb{f},\mathbb{g})}, K(\mathbb{f}, \mathbb{g}, \Theta) = \frac{d(\mathbb{f},\mathbb{g})}{\Theta + d(\mathbb{f},\mathbb{g})}, Y(\mathbb{f}, l, \Theta) = \frac{d(\mathbb{f},\mathbb{g})}{\Theta}$ with the CTN $\circledast$ such that
$\Theta_1 \circledast \Theta_2 = \min\{\Theta_1, \Theta_2\}$ and $\circledcirc$ as CTC defined as $\Theta_1 \circledcirc \Theta_2 = \max\{\Theta_1, \Theta_2\}$. Define $\Upsilon : Q \times Q \to [1, +\infty)$ as
$\Upsilon(\mathbb{f}, \mathbb{g}) = \begin{cases} 1 & \text{if } \mathbb{f} \in Q \text{ and } \mathbb{g} \in L \\ \max\{\mathbb{f}, \mathbb{g}\} & \text{otherwise} \end{cases}.$
Then $(Q, L, Z, K, Y, \circledast, \circledcirc)$ be a $NCBMS$.

Definition 2.7. [16] Suppose $Q, L \neq \emptyset$ and $\Upsilon, \varrho, \chi : Q \times L \to [1, +\infty)$ are considered as an incompetant mappings, $\circledast$ as CTN and Z are NS on $Q \times L \times (0, +\infty)$ is characterized $FTCBMS$ on Q, if for each one $(Q, L, Z, \circledast)$ fulfills all $\mathbb{f} \in Q, \mathbb{g} \in L$ and $x, \Theta, z > 0$ holds the following:
(a) $Z(\mathbb{f}, \mathbb{g}, \Theta) > 0$, for every value $(\mathbb{f}, \mathbb{g}) \in Q \times L$;
(b) $Z(\mathbb{f}, \mathbb{g}, \Theta) = 1$ iff $\mathbb{f} = \mathbb{g}$ for $\mathbb{f} \in Q, \mathbb{g} \in L$;
(c) $Z(\mathbb{f}, \mathbb{g}, \Theta) = Z(\mathbb{g}, \mathbb{f}, \Theta)$, for every value $\mathbb{f}, \mathbb{g} \in Q \cap L$;
(d) $Z(\mathbb{f}_1, \mathbb{g}_2, \Theta + x + z) \geq Z\left(\mathbb{f}_1, \mathbb{g}_1, \frac{\Theta}{\Upsilon(\mathbb{f}_1,\mathbb{g}_1)}\right) \circledast Z\left(\mathbb{f}_2, \mathbb{g}_1, \frac{x}{\Upsilon(\mathbb{f}_2,\mathbb{g}_1)}\right) \circledast Z\left(\mathbb{f}_2, \mathbb{g}_2, \frac{z}{\Upsilon(\mathbb{f}_2,\mathbb{g}_2)}\right)$ for $\mathbb{f}_1, \mathbb{f}_2 \in Q$ and $\mathbb{g}_1, \mathbb{g}_2 \in L$;
(e) $Z(\mathbb{f}, \mathbb{g}, .) : [0, +\infty) \to [0, 1]$ is CTS $Z(\mathbb{f}, \mathbb{g}, .)$ is non decreasing for every value $\mathbb{f} \in Q, \mathbb{g} \in L$.
Then, $(Q, L, Z, K, Y, \circledast, \circledcirc)$ is called a $FTCBMS$.

3. Main results

Definition 3.1. Suppose $Q, L \neq \emptyset$ and $\Upsilon, \varrho, \chi : Q \times L \to [1, +\infty)$ are considered as a incompetant mappings, $\circledast$ as CTN and $\circledcirc$ as CTC and Z, K, Y are NS on $Q \times L \times (0, +\infty)$ is characterized $NTCBMS$, if for each one $(Q, L, Z, K, Y, \circledast, \circledcirc)$ fulfills all $\mathbb{f} \in Q, \mathbb{g} \in L$ and $x, \Theta, z > 0$ holds the following:
(a) $0 \leq Z(\mathbb{f}, \mathbb{g}, \Theta) \leq 1; 0 \leq K(\mathbb{f}, \mathbb{g}, \Theta) \leq 1; 0 \leq Y(\mathbb{f}, \mathbb{g}, \Theta) \leq 1$, for every value $(\mathbb{f}, \mathbb{g}) \in Q \times L$;
(b) $Z(\mathbb{f}, \mathbb{g}, \Theta) + K(\mathbb{f}, \mathbb{g}, \Theta) + Y(\mathbb{f}, \mathbb{g}, \Theta) \leq 3$;
(c) $Z(\mathbb{f}, \mathbb{g}, \Theta) > 0$, for every value $(\mathbb{f}, \mathbb{g}) \in Q \times L$;
(d) $Z(\mathbb{f}, \mathbb{g}, \Theta) = 1$ iff $\mathbb{f} = \mathbb{g}$ for $\mathbb{f} \in Q, \mathbb{g} \in L$;
(e) $Z(\mathbb{f}, \mathbb{g}, \Theta) = Z(\mathbb{g}, \mathbb{f}, \Theta)$, for every value $\mathbb{f}, \mathbb{g} \in Q \cap L$;
(f) $Z(\mathbb{f}_1, \mathbb{g}_2, \Theta + x + z) \geq Z\left(\mathbb{f}_1, \mathbb{g}_1, \frac{\Theta}{\Upsilon(\mathbb{f}_1,\mathbb{g}_1)}\right) \circledast Z\left(\mathbb{f}_2, \mathbb{g}_1, \frac{x}{\Upsilon(\mathbb{f}_2,\mathbb{g}_1)}\right) \circledast Z\left(\mathbb{f}_2, \mathbb{g}_2, \frac{z}{\Upsilon(\mathbb{f}_2,\mathbb{g}_2)}\right)$ for $\mathbb{f}_1, \mathbb{f}_2 \in Q$ and $\mathbb{g}_1, \mathbb{g}_2 \in L$;
(g) $Z(\mathbb{f}, \mathbb{g}, .) : [0, +\infty) \to [0, 1]$ is CTS;
(h) $Z(\mathbb{f}, \mathbb{g}, .)$ is non decreasing $\forall \mathbb{f} \in Q, \mathbb{g} \in L$;
(i) $K(\mathbb{f}, \mathbb{g}, \Theta) < 1$;
(j) $K(\mathbb{f}, \mathbb{g}, \Theta) = 0$ iff $\mathbb{f} = \mathbb{g}$ for $\mathbb{f} \in Q, \mathbb{g} \in L$;
(k) $K(\mathbb{f}, \mathbb{g}, \Theta) = K(\mathbb{g}, \mathbb{f}, \Theta)$, for every value $\mathbb{f}, \mathbb{g} \in Q \cap L$;
(l) $K(\mathbb{f}_1, \mathbb{g}_2, \Theta + x + z) \leq K\left(\mathbb{f}_1, \mathbb{g}_1, \frac{\Theta}{\Upsilon(\mathbb{f}_1,\mathbb{g}_1)}\right) \circledcirc K\left(\mathbb{f}_2, \mathbb{g}_1, \frac{x}{\Upsilon(\mathbb{f}_2,\mathbb{g}_1)}\right) \circledcirc K\left(\mathbb{f}_2, \mathbb{g}_2, \frac{z}{\Upsilon(\mathbb{f}_2,\mathbb{g}_2)}\right)$ for $\mathbb{f}_1, \mathbb{f}_2 \in Q$ and $\mathbb{g}_1, \mathbb{g}_2 \in L$;

(m) $K(\mathbb{f},\mathbb{g},.):[0,+\infty)\to[0,1]$ is CTS;
(n) $K(\mathbb{f},\mathbb{g},.)$ is non increasing for every value $\mathbb{f}\in Q,\mathbb{g}\in L$;
(o) $Y(\mathbb{f},\mathbb{g},\Theta)<1$;
(p) $Y(\mathbb{f},\mathbb{g},\Theta)=0$ iff $\mathbb{f}=\mathbb{g}\,\forall\,\mathbb{f}\in Q,\mathbb{g}\in L$;
(q) $Y(\mathbb{f},\mathbb{g},\Theta)=Y(\mathbb{g},\mathbb{f},\Theta)\,\forall\,\mathbb{f},\mathbb{g}\in Q\cap L$;
(r) $Y(\mathbb{f}_1,\mathbb{g}_2,\Theta+x+z)\leq Y\left(\mathbb{f}_1,\mathbb{g}_1,\frac{\Theta}{\Upsilon(\mathbb{f}_1,\mathbb{g}_1)}\right)\circledcirc D\left(\mathbb{f}_2,\mathbb{g}_1,\frac{x}{\Upsilon(\mathbb{f}_2,\mathbb{g}_1)}\right)\circledcirc D\left(\mathbb{f}_2,\mathbb{g}_2,\frac{z}{\Upsilon(\mathbb{f}_2,\mathbb{g}_2)}\right)$ for $\mathbb{f}_1,\mathbb{f}_2\in Q$ and $\mathbb{g}_1,\mathbb{g}_2\in L$;
(s) $Y(\mathbb{f},\mathbb{g},.):[0,+\infty)\to[0,1]$ is CTS;
(t) $Y(\mathbb{f},\mathbb{g},.)$ is non increasing $\forall\,\mathbb{f}\in Q,\mathbb{g}\in L$;
Then, $(Q,L,Z,K,Y,\circledast,\circledcirc)$ is called a $NTCBMS$.

Definition 3.2. Suppose $Q,L\neq\emptyset$ and $\Upsilon_i:Q\times L\to[1,+\infty)(1\leq i\leq n)$ are considered as a incompetant mappings, $\circledast$ as CTN and $\circledcirc$ as CTC and Z,K,Y are NS on $Q\times L\times(0,+\infty)$ is characterized $NnCBMS$, if for each one $(Q,L,Z,K,Y,\circledast,\circledcirc)$ fulfills all $\mathbb{f}\in Q,\mathbb{g}\in L$ and $\Theta_i>0$ holds the following:
(a) $0\leq Z(\mathbb{f},\mathbb{g},\Theta)\leq 1; 0\leq K(\mathbb{f},\mathbb{g},\Theta)\leq 1; 0\leq Y(\mathbb{f},\mathbb{g},\Theta)\leq 1,$ for every value $(\mathbb{f},\mathbb{g})\in Q\times L$;
(b) $Z(\mathbb{f},\mathbb{g},\Theta)+K(\mathbb{f},\mathbb{g},\Theta)+Y(\mathbb{f},\mathbb{g},\Theta)\leq 3$;
(c) $Z(\mathbb{f},\mathbb{g},\Theta)>0,$ for every value $(\mathbb{f},\mathbb{g})\in Q\times L$;
(d) $Z(\mathbb{f},\mathbb{g},\Theta)=1$ iff $\mathbb{f}=\mathbb{g}\,\forall\,\mathbb{f}\in Q,\mathbb{g}\in L$;
(e) $Z(\mathbb{f},\mathbb{g},\Theta)=Z(\mathbb{g},\mathbb{f},\Theta),$ for every value $\mathbb{f},\mathbb{g}\in Q\cap L$;
(f) $Z(\mathbb{f}_1,\mathbb{g}_\theta,\Theta_1+\Theta_2+\Theta_3+\Theta_4\cdots+\Theta_n)$
$\geq Z\left(\mathbb{f}_1,\mathbb{g}_1,\frac{\Theta_1}{\Upsilon_1(\mathbb{f}_1,\mathbb{g}_1)}\right)\circledast Z\left(\mathbb{f}_2,\mathbb{g}_1,\frac{\Theta_2}{\Upsilon_2(\mathbb{f}_2,\mathbb{g}_1)}\right)\circledast Z\left(\mathbb{f}_2,\mathbb{g}_2,\frac{\Theta_3}{\Upsilon_3(\mathbb{f}_2,\mathbb{g}_2)}\right)\cdots\circledast Z\left(\mathbb{f}_\theta,\mathbb{g}_{\theta-1},\frac{\Theta_{n-1}}{\Upsilon_{n-1}(\mathbb{f}_\theta,\mathbb{g}_{\theta-1})}\right)\circledast$
$Z\left(\mathbb{f}_\theta,\mathbb{g}_\theta,\frac{\Theta_n}{\Upsilon_n(\mathbb{f}_\theta,\mathbb{g}_\theta)}\right)\forall\,\mathbb{f}_1,\mathbb{f}_2\ldots\mathbb{f}_\theta\in Q$ and $\mathbb{g}_1,\mathbb{g}_2\ldots\mathbb{g}_\theta\in L$;
(g) $Z(\mathbb{f},\mathbb{g},.):[0,+\infty)\to[0,1]$ is CTS;
(h) $Z(\mathbb{f},\mathbb{g},.)$ is non decreasing for every value $\mathbb{f}\in Q,\mathbb{g}\in L$;
(i) $K(\mathbb{f},\mathbb{g},\Theta)<1,$ for every value $(\mathbb{f},\mathbb{g})\in Q\times L$;
(j) $K(\mathbb{f},\mathbb{g},\Theta)=0$ iff $\mathbb{f}=\mathbb{g}\,\forall\,\mathbb{f}\in Q,\mathbb{g}\in L$;
(k) $K(\mathbb{f},\mathbb{g},\Theta)=K(\mathbb{g},\mathbb{f},\Theta)\,\forall\,\mathbb{f},\mathbb{g}\in Q\cap L$;
(l) $K(\mathbb{f}_1,\mathbb{g}_\theta,\Theta_1+\Theta_2+\Theta_3+\Theta_4\cdots+\Theta_n)$
$\leq K\left(\mathbb{f}_1,\mathbb{g}_1,\frac{\Theta_1}{\Upsilon_i(\mathbb{f}_1,\mathbb{g}_1)}\right)\circledcirc K\left(\mathbb{f}_2,\mathbb{g}_1,\frac{\Theta_2}{\Upsilon_i(\mathbb{f}_2,\mathbb{g}_1)}\right)\circledcirc K\left(\mathbb{f}_2,\mathbb{g}_2,\frac{\Theta_3}{\Upsilon_i(\mathbb{f}_2,\mathbb{g}_2)}\right)\cdots\circledcirc K\left(\mathbb{f}_\theta,\mathbb{g}_{\theta-1},\frac{\Theta_{n-1}}{\Upsilon_{n-1}(\mathbb{f}_\theta,\mathbb{g}_{\theta-1})}\right)\circledcirc$
$K\left(\mathbb{f}_\theta,\mathbb{g}_\theta,\frac{\Theta_n}{\Upsilon_n(\mathbb{f}_\theta,\mathbb{g}_\theta)}\right)\forall\,\mathbb{f}_1,\mathbb{f}_2\ldots\mathbb{f}_\theta\in Q$ and $\mathbb{g}_1,\mathbb{g}_2\ldots\mathbb{g}_\theta\in L$;
(m) $K(\mathbb{f},\mathbb{g},.):[0,+\infty)\to[0,1]$ is CTS;
(n) $K(\mathbb{f},\mathbb{g},.)$ is non increasing for every value $\mathbb{f}\in Q,\mathbb{g}\in L$;
(o) $Y(\mathbb{f},\mathbb{g},\Theta)<1\,\forall\,(\mathbb{f},\mathbb{g})\in Q\times L$;
(p) $Y(\mathbb{f},\mathbb{g},\Theta)=0$ iff $\mathbb{f}=\mathbb{g}\,\forall\,\mathbb{f}\in Q,\mathbb{g}\in L$;
(q) $Y(\mathbb{f},\mathbb{g},\Theta)=Y(\mathbb{g},\mathbb{f},\Theta)\,\forall\,\mathbb{f},\mathbb{g}\in Q\cap L$;
(r) $Y(\mathbb{f}_1,\mathbb{g}_\theta,\Theta_1+\Theta_2+\Theta_3+\Theta_4\cdots+\Theta_n)\leq\left(\mathbb{f}_1,\mathbb{g}_1,\frac{\Theta_1}{\Upsilon_i(\mathbb{f}_1,\mathbb{g}_1)}\right)\circledcirc\left(\mathbb{f}_2,\mathbb{g}_1,\frac{\Theta_2}{\Upsilon_i(\mathbb{f}_2,\mathbb{g}_1)}\right)\circledcirc Y\left(\mathbb{f}_2,\mathbb{g}_2,\frac{\Theta_3}{\Upsilon_i(\mathbb{f}_2,\mathbb{g}_2)}\right)\cdots$
$\circledcirc D\left(\mathbb{f}_\theta,\mathbb{g}_{\theta-1},\frac{\Theta_{n-1}}{\Upsilon_{n-1}(\mathbb{f}_\theta,\mathbb{g}_{\theta-1})}\right)\circledcirc Y\left(\mathbb{f}_\theta,\mathbb{g}_\theta,\frac{\Theta_n}{\Upsilon_n(\mathbb{f}_\theta,\mathbb{g}_\theta)}\right)$ for $\mathbb{f}_1,\mathbb{f}_2\ldots\mathbb{f}_\theta\in Q$ and $\mathbb{g}_1,\mathbb{g}_2\ldots\mathbb{g}_\theta\in L$;
(s) $Y(\mathbb{f},\mathbb{g},.):[0,+\infty)\to[0,1]$ is CTS;
(t) $Y(\mathbb{f},\mathbb{g},.)$ is non increasing for every value $\mathbb{f}\in Q,\mathbb{g}\in L$;
Then, $(Q,L,Z,K,Y,\circledast,\circledcirc)$ is called a $NnCBMS$.

Example 3.3. Let $Q=\left[0,\frac{1}{2^n}\right],L=\left[\frac{1}{2^{n+1}},1\right]$ and $\Upsilon_i:Q\times L\to[1,\infty),(1\leq i\leq 9)$ be nine non-comparable mappings defined as
$\Upsilon_1(\mathbb{f}_1,\mathbb{g}_1)=\mathbb{f}_1+\mathbb{g}_1+1,\Upsilon_2(\mathbb{f}_2,\mathbb{g}_1)=\mathbb{f}_2+\mathbb{g}_1+1,\Upsilon_3(\mathbb{f}_2,\mathbb{g}_2)=\mathbb{f}_2+\mathbb{g}_2+1,$
$\Upsilon_4(\mathbb{f}_3,\mathbb{g}_2)=\mathbb{f}_3+\mathbb{g}_2+1,\Upsilon_5(\mathbb{f}_3,\mathbb{g}_3)=\mathbb{f}_3+\mathbb{g}_3+1,\Upsilon_6(\mathbb{f}_4,\mathbb{g}_3)=\mathbb{f}_4+\mathbb{g}_3+1,$
$\Upsilon_7(\mathbb{f}_4,\mathbb{g}_4)=\mathbb{f}_4+\mathbb{g}_4+1,\Upsilon_8(\mathbb{f}_5,\mathbb{g}_4)=\mathbb{f}_5+\mathbb{g}_4+1,\Upsilon_9(\mathbb{f}_5,\mathbb{g}_5)=\mathbb{f}_5+\mathbb{g}_5+1.$
Define $Z(\mathbb{f}_1,\mathbb{g}_5,\Theta_i)=\frac{\min(\mathbb{f},\mathbb{g})+\Theta}{\max(\mathbb{f},\mathbb{g})+\Theta},$ for every value $\Theta_i>0,K(\mathbb{f}_1,\mathbb{g}_5,\Theta_i)=\frac{\max(\mathbb{f},\mathbb{g})-\min(\mathbb{f},\mathbb{g})}{\max(\mathbb{f},\mathbb{g})+\Theta},$ for every value

$\Theta_i > 0$ and $Y(\mathbb{f}_1, \mathbb{g}_5, \Theta_i) = \frac{\max(\mathbb{f},\mathbb{g})-\min(\mathbb{f},\mathbb{g})}{\min(\mathbb{f},\mathbb{g})+\Theta}$, for every value $\Theta_i > 0$, $\mathbb{f} \in Q$ and $\mathbb{g} \in L$. Clearly, $(Q, L, Z, K, Y, \circledast, \circledcirc)$ is a $NnCBMS$, where $\circledast$ is a CTC defined as $\Theta_1 \circledast \Theta_2 = \Theta_1\Theta_2$ and $\circledcirc$ is a CTN defined as $\Theta_1 \circledcirc \Theta_2 = \max\{\Theta_1, \Theta_2\}$.

Proof
The first three conditions are met easily so we go for fourth condition Z, K, Y as follows:
For $\mathbb{f}_i \neq \mathbb{g}_i$ and $\Theta_i \geq 0$.
By assuming $\mathbb{f}_1 = 0, \mathbb{f}_2 = \frac{1}{6}, \mathbb{f}_3 = \frac{1}{5}, \mathbb{f}_4 = \frac{2}{5}, \mathbb{f}_5 = \frac{1}{2}, \mathbb{g}_1 = \frac{1}{4}, \mathbb{g}_2 = \frac{2}{5}, \mathbb{g}_3 = \frac{1}{2}, \mathbb{g}_4 = \frac{3}{4}, \mathbb{g}_5 = 1$ we obtain a non-trivial sequence as
$(\mathbb{f}_1, \mathbb{g}_5) = \{(\mathbb{f}_1, \mathbb{g}_1), (\mathbb{f}_2, \mathbb{g}_1), (\mathbb{f}_2, \mathbb{g}_2), (\mathbb{f}_3, \mathbb{g}_2), (\mathbb{f}_3, \mathbb{g}_3), (\mathbb{f}_4, \mathbb{g}_3), (\mathbb{f}_4, \mathbb{g}_4), (\mathbb{f}_5, \mathbb{g}_4), (\mathbb{f}_5, \mathbb{g}_5)\}$,
$(0,1) = \left\{\left(0, \frac{1}{4}\right), \left(\frac{1}{6}, \frac{1}{4}\right), \left(\frac{1}{6}, \frac{2}{5}\right), \left(\frac{1}{5}, \frac{2}{5}\right), \left(\frac{1}{5}, \frac{1}{2}\right), \left(\frac{2}{5}, \frac{1}{2}\right), \left(\frac{2}{5}, \frac{3}{4}\right), \left(\frac{1}{2}, \frac{3}{4}\right), \left(\frac{1}{2}, 1\right)\right\}$ and taking $\Theta_i = 1$.

$$
\begin{aligned}
Z(\mathbb{f}_1, \mathbb{g}_5, \Theta_i) &= \frac{\min(\mathbb{f},\mathbb{g}) + \Theta}{\max(\mathbb{f},\mathbb{g}) + \Theta}, \text{ for every value } \Theta_i > 0, \\
Z(0,1,9) &\geq Z\left(0, \frac{1}{4}, \frac{1}{\Upsilon_1(0, \frac{1}{4})}\right) \circledast Z\left(\frac{1}{6}, \frac{1}{4}, \frac{1}{\Upsilon_2(\frac{1}{6}, \frac{1}{4})}\right) \circledast Z\left(\frac{1}{6}, \frac{2}{5}, \frac{1}{\Upsilon_3(\frac{1}{6}, \frac{2}{5})}\right) \circledast Z\left(\frac{1}{5}, \frac{2}{5}, \frac{1}{\Upsilon_4(\frac{1}{5}, \frac{2}{5})}\right) \\
&\quad \circledast Z\left(\frac{1}{5}, \frac{1}{2}, \frac{1}{\Upsilon_5(\frac{1}{5}, \frac{1}{2})}\right) \circledast Z\left(\frac{2}{5}, \frac{1}{2}, \frac{1}{\Upsilon_6(\frac{2}{5}, \frac{1}{2})}\right) \circledast Z\left(\frac{2}{5}, \frac{3}{4}, \frac{1}{\Upsilon_7(\frac{2}{5}, \frac{3}{4})}\right) \circledast \\
&\quad Z\left(\frac{1}{2}, \frac{3}{4}, \frac{1}{\Upsilon_8(\frac{1}{2}, \frac{3}{4})}\right) \circledast Z\left(\frac{1}{2}, 1, \frac{1}{\Upsilon_9(\frac{1}{2}, 1)}\right) \\
&= Z\left(\mathbb{f}_1, \mathbb{g}_1, \frac{\Theta_1}{\Upsilon_1(\mathbb{f}_1, l_1)}\right) \circledast Z\left(\mathbb{f}_2, \mathbb{g}_1, \frac{\Theta_2}{\Upsilon_2(\mathbb{f}_2, l_1)}\right) \circledast Z\left(\mathbb{f}_2, \mathbb{g}_2, \frac{\Theta_3}{\Upsilon_3(\mathbb{f}_2, l_2)}\right) \cdots \circledast \\
&\quad Z\left(\mathbb{f}_\theta, \mathbb{g}_{\theta-1}, \frac{\Theta_{n-1}}{\Upsilon_{n-1}(\mathbb{f}_\theta, l_{\theta-1})}\right) \circledast Z\left(\mathbb{f}_\theta, \mathbb{g}_\theta, \frac{\Theta_n}{\Upsilon_n(\mathbb{f}_\theta, l_\theta)}\right) \\
K(\mathbb{f}_1, \mathbb{g}_5, \Theta_i) &= \frac{\max(\mathbb{f}, l) - \min(\mathbb{f}, l)}{\max(\mathbb{f}, l) + \Theta}, \text{ for every value } \Theta_i > 0 \\
K(0,1,9) &\leq K\left(0, \frac{1}{4}, \frac{1}{\Upsilon_1(0, \frac{1}{4})}\right) \circledcirc K\left(\frac{1}{6}, \frac{1}{4}, \frac{1}{\Upsilon_2(\frac{1}{6}, \frac{1}{4})}\right) \circledcirc K\left(\frac{1}{6}, \frac{2}{5}, \frac{1}{\Upsilon_3(\frac{1}{6}, \frac{2}{5})}\right) \circledcirc K\left(\frac{1}{5}, \frac{2}{5}, \frac{1}{\Upsilon_4(\frac{1}{5}, \frac{2}{5})}\right) \\
&\quad \circledcirc K\left(\frac{1}{5}, \frac{1}{2}, \frac{1}{\Upsilon_5(\frac{1}{5}, \frac{1}{2})}\right) \circledcirc K\left(\frac{2}{5}, \frac{1}{2}, \frac{1}{\Upsilon_6(\frac{2}{5}, \frac{1}{2})}\right) \\
&= K\left(\mathbb{f}_1, \mathbb{g}_1, \frac{\Theta_1}{\Upsilon_1(\mathbb{f}_1, \mathbb{g}_1)}\right) \circledcirc K\left(\mathbb{f}_2, \mathbb{g}_1, \frac{\Theta_2}{\Upsilon_2(\mathbb{f}_2, \mathbb{g}_1)}\right) \circledcirc K\left(\mathbb{f}_2, \mathbb{g}_2, \frac{\Theta_3}{\Upsilon_3(\mathbb{f}_2, \mathbb{g}_2)}\right) \cdots \circledcirc \\
&\quad K\left(\mathbb{f}_\theta, \mathbb{g}_{\theta-1}, \frac{\Theta_{n-1}}{\Upsilon_{n-1}(\mathbb{f}_\theta, \mathbb{g}_{\theta-1})}\right) \circledcirc K\left(\mathbb{f}_\theta, \mathbb{g}_\theta, \frac{\Theta_n}{\Upsilon_n(\mathbb{f}_\theta, \mathbb{g}_\theta)}\right) \\
Y(\mathbb{f}_1, \mathbb{g}_5, \Theta_i) &= \frac{\max(\mathbb{f}, l) - \min(\mathbb{f}, l)}{\max(\mathbb{f}, l) + \Theta}, \text{ for every value } \Theta_i > 0 \\
Y(0,1,9) &\leq Y\left(0, \frac{1}{4}, \frac{1}{\Upsilon_1(0, \frac{1}{4})}\right) \circledcirc Y\left(\frac{1}{6}, \frac{1}{4}, \frac{1}{\Upsilon_2(\frac{1}{6}, \frac{1}{4})}\right) \circledcirc Y\left(\frac{1}{6}, \frac{2}{5}, \frac{1}{\Upsilon_3(\frac{1}{6}, \frac{2}{5})}\right) \circledcirc \\
&\quad Y\left(\frac{1}{5}, \frac{2}{5}, \frac{1}{\Upsilon_4(\frac{1}{5}, \frac{2}{5})}\right) \circledcirc Y\left(\frac{1}{5}, \frac{1}{2}, \frac{1}{\Upsilon_5(\frac{1}{5}, \frac{1}{2})}\right) \circledcirc Y\left(\frac{2}{5}, \frac{1}{2}, \frac{1}{\Upsilon_6(\frac{2}{5}, \frac{1}{2})}\right) \\
&= Y\left(\mathbb{f}_1, \mathbb{g}_1, \frac{\Theta_1}{\Upsilon_1(\mathbb{f}_1, \mathbb{g}_1)}\right) \circledcirc Y\left(\mathbb{f}_2, \mathbb{g}_1, \frac{\Theta_2}{\Upsilon_2(\mathbb{f}_2, \mathbb{g}_1)}\right) \circledcirc Y\left(\mathbb{f}_2, \mathbb{g}_2, \frac{\Theta_3}{\Upsilon_3(\mathbb{f}_2, \mathbb{g}_2)}\right) \cdots \circledcirc \\
&\quad Y\left(\mathbb{f}_\theta, \mathbb{g}_{\theta-1}, \frac{\Theta_{n-1}}{\Upsilon_{n-1}(\mathbb{f}_\theta, \mathbb{g}_{\theta-1})}\right) \circledcirc Y\left(\mathbb{f}_\theta, \mathbb{g}_\theta, \frac{\Theta_n}{\Upsilon_n(\mathbb{f}_\theta, \mathbb{g}_\theta)}\right), \text{ for every value } \Theta_i > 0.
\end{aligned}
$$

Similarly we can prove other cases.
The following diagram shows the graphical behaviour of the above example: By calculating the above, which meets

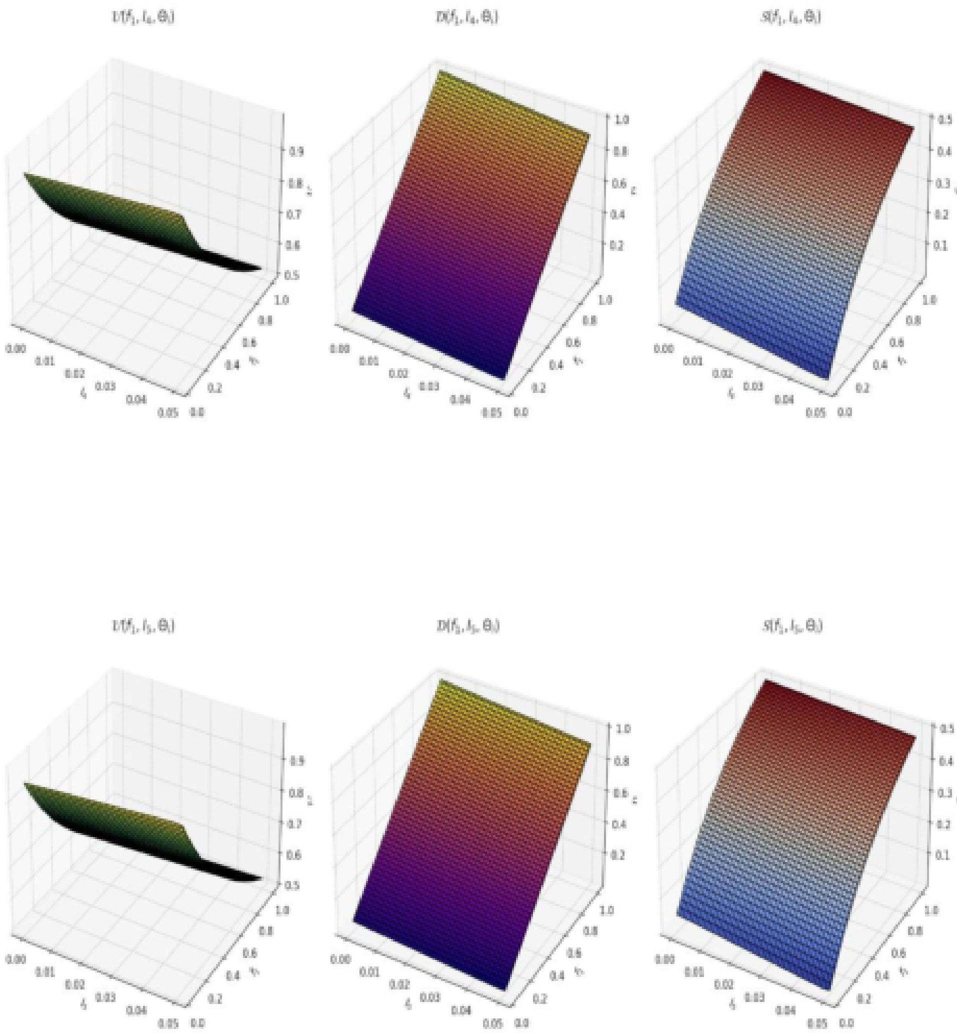

Figure 1. Shows the graphical behaviour of Z, K and$Y$ in the above example

the necessary criterion. We are also able to prove other cases. Similarly, we can demonstrate a greater value of n. Thus, $(Q, L, Z, K, Y, \circledast, \circledcirc)$ is a $NnCBMS$. $\square$

Lemma 3.4. Let $(Q, L, Z, K, Y, \circledast, \circledcirc)$ be a $NnCBMS$ given by $Z(\mathbb{f}, \mathbb{g}, \delta\Theta) \geq Z(\mathbb{f}, \mathbb{g}, \Theta)$ for, $\mathbb{f} \in Q, \mathbb{g} \in L, \Theta \in (0, +\infty)$ and $\delta \in (0, 1)$. Then $\mathbb{f} = \mathbb{g}$.

Proof
Given that,

$$Z(\mathbb{f}, \mathbb{g}, \delta\Theta) \geq Z(\mathbb{f}, \mathbb{g}, \Theta), K(\mathbb{f}, \mathbb{g}, \delta\Theta) \leq K(\mathbb{f}, \mathbb{g}, \Theta), Y(\mathbb{f}, \mathbb{g}, \delta\Theta) \leq Y(\mathbb{f}, \mathbb{g}, \Theta) \text{ for } \Theta > 0. \tag{3.4.1}$$

Since $\delta\Theta < \Theta$ for every value $\Theta > 0$ and $\delta \in (0, 1)$, by (h),(n),(t) of definition (3.2),we have

$$Z(\mathbb{f}, \mathbb{g}, \delta\Theta) \geq Z(\mathbb{f}, \mathbb{g}, \Theta), K(\mathbb{f}, \mathbb{g}, \delta\Theta) \leq K(\mathbb{f}, \mathbb{g}, \Theta), Y(\mathbb{f}, \mathbb{g}, \delta\Theta) \leq Y(\mathbb{f}, \mathbb{g}, \Theta) \text{ for } \Theta > 0 \tag{3.4.2}$$

From the definition of $NnCBMS$ (see (d),(j),(p)), we obtain $\mathbb{f} = \mathbb{g}$. $\square$

Definition 3.5. If every Cauchy bisequence is present, it is considered as complete then $(Q, L, Z, K, Y, \circledast, \circledcirc)$ be a $NnCBMS$ in $Q \times L$ is convergent in it.

Proposition 3.6. In a $NnCBMS$, every convergent has a biconvergence in the Cauchy bisequence .

Proof
Let $(Q, L, Z, K, Y, \circledast, \circledcirc)$ be a $NnCBMS$ and a biconvergence $(\mathbb{f}_n, \mathbb{g}_n) \in Q \times L$ such that $\mathbb{f}_n \to \mathbb{g}$ as $n \to +\infty$ and $\mathbb{g}_n \to \mathbb{f}$ as $n \to +\infty$, where $n \in L$ and $\mathbb{f} \in Q$. Since $(\mathbb{f}_n, \mathbb{g}_n)$ is a convergent Cauchy bisequence , we obtain

$$Z(\mathbb{f}_n, \mathbb{g}_\theta, \Theta) \to 1, K(\mathbb{f}_n, \mathbb{g}_\theta, \Theta) \to 0 \text{ and } Y(\mathbb{f}_n, \mathbb{g}_\theta, \Theta) \to 0 \text{ as } n \to \infty \tag{3.6.1}$$

for every value $\Theta > 0$. Now, from **(3)**, we conclude that $Z(\mathbb{f}, \mathbb{g}, \Theta) = 1, K(\mathbb{f}, \mathbb{g}, \Theta) = 0$ and $Y(\mathbb{f}, \mathbb{g}, \Theta) = 0$ for every value $\Theta > 0$. Therefore, by (f),(g),(r) of definition (3.2) we obtain that the biconvergence $(\mathbb{f}_n, \mathbb{g}_n)$ is biconvergent . $\square$

Proposition 3.7. In a $NnCBMS$, every biconvergent bisequence is a Cauchy bisequence.

Proof
Let $(Q, L, Z, K, Y, \circledast, \circledcirc)$ be a $NnCBMS$ and a biconvergence $(\mathbb{f}_n, \mathbb{g}_n) \in Q \times L$ converges to a point $\mathbb{f}_0 \in Q \cap L$ $\forall\, n, \theta \in \mathbb{N}$ and $\Theta, e > 0$. By (f),(g),(r) of definition (3.2), we have biconvergence

$$\begin{aligned}
Z(\mathbb{f}_n, \mathbb{g}_\theta, \Theta) \geq{}& Z\left(\mathbb{f}_\eta, \mathbb{f}_0, \frac{\frac{\Theta}{e}}{\Upsilon_1(\mathbb{f}_n, \mathbb{f}_0)}\right) \circledast Z\left(\mathbb{f}_0, \mathbb{f}_0, \frac{\frac{\Theta}{e}}{\Upsilon_2(\mathbb{f}_0, \mathbb{f}_0)}\right) \circledast Z\left(\mathbb{f}_0, \mathbb{f}_0, \frac{\frac{\Theta}{e}}{\Upsilon_3(\mathbb{f}_0, \mathbb{f}_0)}\right) \\
&\circledast Z\left(\mathbb{f}_0, \mathbb{f}_0, \frac{\frac{\Theta}{e}}{\Upsilon_4(\mathbb{f}_0, \mathbb{f}_0)}\right) \cdots \circledast Z\left(\mathbb{f}_0, \mathbb{g}_\omega, \frac{\frac{\Theta}{e}}{\Upsilon_n(\vartheta_0, \vartheta_\theta)}\right), \\
K(\mathbb{f}_n, \mathbb{g}_\theta, \Theta) \leq{}& K\left(\mathbb{f}_\eta, \mathbb{f}_0, \frac{\frac{\Theta}{e}}{\Upsilon_1(\mathbb{f}_n, \mathbb{f}_0)}\right) \circledcirc K\left(\mathbb{f}_0, \mathbb{f}_0, \frac{\frac{\Theta}{e}}{\Upsilon_2(\mathbb{f}_0, \mathbb{f}_0)}\right) \circledcirc K\left(\mathbb{f}_0, \mathbb{f}_0, \frac{\frac{\Theta}{e}}{\Upsilon_3(\mathbb{f}_0, \mathbb{f}_0)}\right) \\
&\circledcirc K\left(\mathbb{f}_0, \mathbb{f}_0, \frac{\frac{\Theta}{e}}{\Upsilon_4(\mathbb{f}_0, \mathbb{f}_0)}\right) \cdots \circledcirc K\left(\mathbb{f}_0, \mathbb{g}_\omega, \frac{\frac{\Theta}{e}}{\Upsilon_n(\vartheta_0, \vartheta_\theta)}\right), \\
Y(\mathbb{f}_n, \mathbb{g}_\theta, \Theta) \leq{}& Y\left(\mathbb{f}_\eta, \mathbb{f}_0, \frac{\frac{\Theta}{e}}{\Upsilon_1(\mathbb{f}_n, \mathbb{f}_0)}\right) \circledcirc Y\left(\mathbb{f}_0, \mathbb{f}_0, \frac{\frac{\Theta}{e}}{\Upsilon_2(\mathbb{f}_0, \mathbb{f}_0)}\right) \circledcirc Y\left(\mathbb{f}_0, \mathbb{f}_0, \frac{\frac{\Theta}{e}}{\Upsilon_3(\mathbb{f}_0, \mathbb{f}_0)}\right) \\
&\circledcirc Y\left(\mathbb{f}_0, \mathbb{f}_0, \frac{\frac{\Theta}{e}}{\Upsilon_4(\mathbb{f}_0, \mathbb{f}_0)}\right) \cdots \circledcirc Y\left(\mathbb{f}_0, \mathbb{g}_\omega, \frac{\frac{\Theta}{e}}{\Upsilon_n(\vartheta_0, \vartheta_\theta)}\right).
\end{aligned}$$

As $n, \theta \to \infty$, we obtain $Z(\mathbb{f}_n, \mathbb{g}_\theta, \Theta) \geq 1, K(\mathbb{f}_n, \mathbb{g}_\theta, \Theta) \leq 0$ and $Y(\mathbb{f}_n, \mathbb{g}_\theta, \Theta) \leq 0$ for every value $\Theta, e > 0$. This implies that $Z(\mathbb{f}_n, \mathbb{g}_\theta, \Theta) \to 1, K(\mathbb{f}_n, \mathbb{g}_\theta, \Theta) \to 0$ and $Y(\mathbb{f}_n, \mathbb{g}_\theta, \Theta) \to 0$ for every value $\Theta, e > 0$. Hence $(\mathbb{f}_n, \mathbb{g}_n)$ is a Cauchy bisequence . $\square$

Lemma 3.8. Let $(Q, L, Z, K, Y, \circledast, \circledcirc)$ be a $NnCBMS$ and $\Omega \in Q \cap L$ is a sequence's limit, then it is the sequence's unique limit.

Proof
Get $\{\mathbb{f}_n\} \in Q$ be a sequence. Consider that $\{\mathbb{f}_n\} \to \mathbb{g} \in L$ and also $\{\mathbb{f}_n\} \to \Omega \in Q \cap L$, then for $\Theta_i > 0$, we have

$$\begin{aligned}
&Z(\Omega, \mathbb{g}, \Theta_1 + \Theta_2 + \Theta_3 + \Theta_4 \cdots + \Theta_n) \\
&\geq Z(\Omega, \Omega, \Theta_1) \circledast Z(\Omega, \Omega, \Theta_2) \circledast Z(\Omega, \Omega, \Theta_3) \circledast \cdots \circledast Z(\mathbb{f}_n, \Omega, \Theta_{n-1}) \circledast Z(\mathbb{f}_n, \mathbb{g}, \Theta_n), \\
&K(\Omega, \mathbb{g}, \Theta_1 + \Theta_2 + \Theta_3 + \Theta_4 \cdots + \Theta_n) \\
&\leq K(\omega, \omega, \Theta_1) \circledcirc K(\omega, \omega, \Theta_2) \circledast K(\omega, \omega, \Theta_3) \circledcirc \cdots \circledcirc K(\mathbb{f}_n, \omega, \Theta_{n-1}) \circledcirc K(\mathbb{f}_n, \mathbb{g}, \Theta_n), \\
&Y(\Omega, \mathbb{g}, \Theta_1 + \Theta_2 + \Theta_3 + \Theta_4 \cdots + \Theta_n) \\
&\leq Y(\Omega, \Omega, \Theta_1) \circledcirc Y(\Omega, \Omega, \Theta_2) \circledast Y(\Omega, \Omega, \Theta_3) \circledcirc \cdots \circledcirc Y(\mathbb{f}_n, \Omega, \Theta_{n-1}) \circledcirc Y(\mathbb{f}_n, \mathbb{g}, \Theta_n).
\end{aligned}$$

As $n \to \infty$, we obtain $Z(\Omega, \mathbb{g}, \Theta_1 + \Theta_2 + \Theta_3 + \Theta_4 \cdots + \Theta_n) \geq 1, K(\Omega, \mathbb{g}, \Theta_1 + \Theta_2 + \Theta_3 + \Theta_4 \cdots + \Theta_n) \leq 0$, $Y(\Omega, \mathbb{g}, \Theta_1 + \Theta_2 + \Theta_3 + \Theta_4 \cdots + \Theta_n) \leq 0$ which suggest that $\Omega = g$, i.e., sequence $\{\mathbb{f}_n\}$, has a unique limit. $\square$

Theorem 3.9. Let $(Q, L, Z, K, Y, \circledast, \circledcirc)$ be a complete $NnCBMS$ with n non-comparable functions $\Upsilon_i : Q \times L \to [1, \infty)$ given by

$$\lim_{\Theta\to\infty} Z(\mathbb{f}, \mathbb{g}, \Theta) = 1, \lim_{\Theta\to\infty} K(\mathbb{f}, \mathbb{g}, \Theta) = 0, \lim_{\Theta\to\infty} Y(\mathbb{f}, \mathbb{g}, \Theta) = 0, \text{ for every value } \mathbb{f} \in Q, \mathbb{g} \in L. \tag{3.9.1}$$

Get $\Xi : Q \cup L \to Q \cup L$ be a mapping satisfying
(i) $\Xi(Q) \subseteq Q$ and $\Xi(L) \subseteq L$;
(ii) $\prod_{\varrho}(\Xi(\mathbb{f}), \Xi(\mathbb{g}), \delta\Theta) \geq Z(\mathbb{f}, \mathbb{g}, \Theta)$ for every value $\mathbb{f} \in Q, \mathbb{g} \in L$, and $\Theta > 0$, where $\delta \in (0, 1)$. Assume moreover that $\lim_{\eta\to\infty} \Upsilon_i(\mathbb{f}_n, \mathbb{g})$ and $\lim_{\eta\to\infty} \Upsilon_i(\mathbb{g}, \mathbb{f}_n)$, exist and are finite for each $\mathbb{f} \in Q$. Then Ξ has a FP that is unique.

Proof
Fix $\mathbb{f}_0 \in Q$ and $\mathbb{g}_0 \in L$ and consider that $\Xi(\mathbb{f}_n) = \mathbb{f}_{n+1}$ and $\Xi(\mathbb{f}_n) = \mathbb{g}_{n+1} \,\forall\, n \in \mathbb{N} \cup \{0\}$. Next, we get $(\mathbb{f}_n, \mathbb{g}_n)$ as a biconvergence on $NnCBMS$ $(Q, L, Z, K, Y, \circledast, \circledcirc)$. Now, we have

$$\begin{aligned}
Z(\mathbb{f}_1, \mathbb{g}_1, \Theta) &= Z(\Xi(\mathbb{f}_0), \Xi(\mathbb{g}_0), \Theta) \geq Z\left(\mathbb{f}_0, \mathbb{g}_0, \frac{\Theta}{\delta}\right), \\
K(\mathbb{f}_1, \mathbb{g}_1, \Theta) &= K(\Xi(\mathbb{f}_0), \Xi(\mathbb{g}_0), \Theta) \leq K\left(\mathbb{f}_0, \mathbb{g}_0, \frac{\Theta}{\delta}\right), \\
Y(\mathbb{f}_1, \mathbb{g}_1, \Theta) &= Y(\Xi(\mathbb{f}_0), \Xi(\mathbb{g}_0), \Theta) \leq Y(\mathbb{f}_0, \mathbb{g}_0, \frac{\Theta}{\delta}) \,\forall\, \Theta > 0 \text{ and } n \in \mathbb{N}.
\end{aligned}$$

By simple induction, we get,

$$\begin{aligned}
Z(\mathbb{f}_n, \mathbb{g}_n, \Theta) &= Z(\Xi(\mathbb{f}_{n-1}), \Xi(\mathbb{g}_{n-1}), \Theta) \geq Z\left(\mathbb{f}_0, \mathbb{g}_0, \frac{\Theta}{\delta}^n\right), \\
K(\mathbb{f}_n, \mathbb{g}_n, \Theta) &= K(\Xi(\mathbb{f}_{n-1}), \Xi(\mathbb{g}_{n-1}), \Theta) \leq K\left(\mathbb{f}_0, \mathbb{g}_0, \frac{\Theta}{\delta}^n\right), \\
Y(\mathbb{f}_n, \mathbb{g}_n, \Theta) &= Y(\Xi(\mathbb{f}_{n-1}), \Xi(\mathbb{g}_{n-1}), \Theta) \leq Y\left(\mathbb{f}_0, \mathbb{g}_0, \frac{\Theta}{\delta}^n\right),
\end{aligned} \tag{3.9.2}$$

and

$$\begin{aligned}
Z(\mathbb{f}_{n+1}, \mathbb{g}_n, \Theta) &= Z(\Xi(\mathbb{f}_n), \Xi(\mathbb{g}_{n-1}), \Theta) \geq Z\left(\mathbb{f}_1, \mathbb{g}_0, \frac{\Theta}{\delta}^n\right), \\
K(\mathbb{f}_{n+1}, \mathbb{g}_n, \Theta) &= K(\Xi(\mathbb{f}_n), \Xi(\mathbb{g}_{n-1}), \Theta) \leq K\left(\mathbb{f}_1, \mathbb{g}_0, \frac{\Theta}{\delta}^n\right), \\
Y(\mathbb{f}_{n+1}, \mathbb{g}_n, \Theta) &= Y(\Xi(\mathbb{f}_n), \Xi(\mathbb{g}_{n-1}), \Theta) \leq Y\left(\mathbb{f}_1, \mathbb{g}_0, \frac{\Theta}{\delta}^n\right) \,\forall\, \Theta > 0 \text{ and } n \in \mathbb{N}.
\end{aligned} \tag{3.9.3}$$

Let $n < \omega \,\forall\, n, \omega \in \mathbb{N}$. Then

$$\begin{aligned}
Z(\mathbb{f}_n, \mathbb{g}_\omega, \Theta) \geq{} & Z\left(\mathbb{f}_n, \mathbb{g}_n, \frac{\frac{\Theta}{r}}{\Upsilon_1(\mathbb{f}_n, \mathbb{g}_n)}\right) \circledast Z\left(\mathbb{f}_{n+1}, \mathbb{g}_n, \frac{\frac{\Theta}{r}}{\Upsilon_2(\mathbb{f}_{n+1}, \mathbb{g}_n)}\right) \circledast Z\left(\mathbb{f}_{n+1}, \mathbb{g}_{n+1}, \frac{\frac{\Theta}{r}}{\Upsilon_3(\mathbb{f}_{n+1}, \mathbb{g}_{n+1})}\right) \circledast \\
& Z\left(\mathbb{f}_{n+2}, \mathbb{g}_{n+1}, \frac{\frac{\Theta}{r}}{\Upsilon_4(\mathbb{f}_{n+2}, \mathbb{g}_{n+1})}\right) \circledast \cdots \circledast Z\left(\mathbb{f}_\omega, \mathbb{g}_{\omega-1}, \frac{\frac{\Theta}{r}}{\Upsilon_{n-1}(\mathbb{f}_\omega, \mathbb{g}_{\omega-1})}\right) \circledast Z\left(\mathbb{f}_\omega, \mathbb{g}_\omega, \frac{\frac{\Theta}{r}}{\Upsilon_n(\mathbb{f}_\omega, \mathbb{g}_\omega)}\right), \\
K(\mathbb{f}_n, \mathbb{g}_\omega, \Theta) \leq{} & K\left(\mathbb{f}_n, \mathbb{g}_n, \frac{\frac{\Theta}{r}}{\Upsilon_1(\mathbb{f}_n, \mathbb{g}_n)}\right) \circledcirc K\left(\mathbb{f}_{n+1}, \mathbb{g}_n, \frac{\frac{\Theta}{r}}{\Upsilon_2(\mathbb{f}_{n+1}, \mathbb{g}_n)}\right) \circledcirc K\left(\mathbb{f}_{n+1}, \mathbb{g}_{n+1}, \frac{\frac{\Theta}{r}}{\Upsilon_3(\mathbb{f}_{n+1}, \mathbb{g}_{n+1})}\right) \circledcirc
\end{aligned}$$

$$K\left(\mathbb{f}_{n+2},\mathbb{g}_{n+1},\frac{\frac{\Theta}{r}}{\Upsilon_4(\mathbb{f}_{n+2},\mathbb{g}_{n+1})}\right)\circledcirc\cdots\circledcirc K\left(\mathbb{f}_{\omega},\mathbb{g}_{\omega-1},\frac{\frac{\Theta}{r}}{\Upsilon_{n-1}(\mathbb{f}_{\omega},\mathbb{g}_{\omega-1})}\right)\circledcirc K\left(\mathbb{f}_{\omega},\mathbb{g}_{\omega},\frac{\frac{\Theta}{r}}{\Upsilon_n(\mathbb{f}_{\omega},\mathbb{g}_{\omega})}\right),$$

$$Y(\mathbb{f}_n,\mathbb{g}_\omega,\Theta)\le Y\left(\mathbb{f}_n,\mathbb{g}_n,\frac{\frac{\Theta}{r}}{\Upsilon_1(\mathbb{f}_n,\mathbb{g}_n)}\right)\circledcirc Y\left(\mathbb{f}_{n+1},\mathbb{g}_n,\frac{\frac{\Theta}{r}}{\Upsilon_2(\mathbb{f}_{n+1},\mathbb{g}_n)}\right)\circledcirc Y\left(\mathbb{f}_{n+1},\mathbb{g}_{n+1},\frac{\frac{\Theta}{r}}{\Upsilon_3(\mathbb{f}_{n+1},\mathbb{g}_{n+1})}\right)\circledcirc$$
$$Y\left(\mathbb{f}_{n+2},\mathbb{g}_{n+1},\frac{\frac{\Theta}{r}}{\Upsilon_4(\mathbb{f}_{n+2},\mathbb{g}_{n+1})}\right)\circledcirc\cdots\circledcirc Y\left(\mathbb{f}_{\omega},\mathbb{g}_{\omega-1},\frac{\frac{\Theta}{r}}{\Upsilon_{n-1}(\mathbb{f}_{\omega},\mathbb{g}_{\omega-1})}\right)\circledcirc Y\left(\mathbb{f}_{\omega},\mathbb{g}_{\omega},\frac{\frac{\Theta}{r}}{\Upsilon_n(\mathbb{f}_{\omega},\mathbb{g}_{\omega})}\right).$$

With (3.9.2) and (3.9.3) applied to each and every term on the RHS of the inequality above, we now get

$$Z(\mathbb{f}_n,\mathbb{g}_\omega,\Theta)\ge Z\left(\mathbb{f}_0,\mathbb{g}_0,\frac{\frac{\Theta}{r}}{q^n\Upsilon_1(\mathbb{f}_n,\mathbb{g}_n)}\right)\circledast Z\left(\mathbb{f}_1,\mathbb{g}_0,\frac{\frac{\Theta}{r}}{q^{n+1}\Upsilon_2(\mathbb{f}_{n+1},\mathbb{g}_n)}\right)\circledast Z\left(\mathbb{f}_1,\mathbb{g}_1,\frac{\frac{\Theta}{r}}{q^{n+2}\Upsilon_3(\mathbb{f}_{n+1},\mathbb{g}_{n+1})}\right)\circledast$$
$$Z\left(\mathbb{f}_2,\mathbb{g}_1,\frac{\frac{\Theta}{r}}{q^{n+3}\Upsilon_4(\mathbb{f}_{n+2},\mathbb{g}_{n+1})}\right)\circledast\cdots\circledast Z\left(\mathbb{f}_1,\mathbb{g}_0,\frac{\frac{\Theta}{r}}{q^{\omega-1}\Upsilon_{n-1}(\mathbb{f}_{\omega},\mathbb{g}_{\omega-1})}\right)\circledast Z\left(\mathbb{f}_0,\mathbb{g}_0,\frac{\frac{\Theta}{r}}{q^{\omega}\Upsilon_n(\mathbb{f}_{\omega},\mathbb{g}_{\omega})}\right),$$

$$K(\mathbb{f}_n,\mathbb{g}_\omega,\Theta)\le K\left(\mathbb{f}_0,\mathbb{g}_0,\frac{\frac{\Theta}{r}}{q^n\Upsilon_1(\mathbb{f}_n,\mathbb{g}_n)}\right)\circledcirc K\left(\mathbb{f}_1,\mathbb{g}_0,\frac{\frac{\Theta}{r}}{q^{n+1}\Upsilon_2(\mathbb{f}_{n+1},\mathbb{g}_n)}\right)\circledcirc K\left(\mathbb{f}_1,\mathbb{g}_1,\frac{\frac{\Theta}{r}}{q^{n+2}\Upsilon_3(\mathbb{f}_{n+1},\mathbb{g}_{n+1})}\right)\circledcirc$$
$$K\left(\mathbb{f}_2,\mathbb{g}_1,\frac{\frac{\Theta}{r}}{q^{n+3}\Upsilon_4(\mathbb{f}_{n+2},\mathbb{g}_{n+1})}\right)\circledcirc\cdots\circledcirc K\left(\mathbb{f}_1,\mathbb{g}_0,\frac{\frac{\Theta}{r}}{q^{\omega-1}\Upsilon_{n-1}(\mathbb{f}_{\omega},\mathbb{g}_{\omega-1})}\right)\circledcirc K\left(\mathbb{f}_0,\mathbb{g}_0,\frac{\frac{\Theta}{r}}{q^{\omega}\Upsilon_n(\mathbb{f}_{\omega},\mathbb{g}_{\omega})}\right),$$

$$Y(\mathbb{f}_n,\mathbb{g}_\omega,\Theta)\le Y\left(\mathbb{f}_0,\mathbb{g}_0,\frac{\frac{\Theta}{r}}{q^n\Upsilon_1(\mathbb{f}_n,\mathbb{g}_n)}\right)\circledcirc Y\left(\mathbb{f}_1,\mathbb{g}_0,\frac{\frac{\Theta}{r}}{q^{n+1}\Upsilon_2(\mathbb{f}_{n+1},\mathbb{g}_n)}\right)\circledcirc Y\left(\mathbb{f}_1,\mathbb{g}_1,\frac{\frac{\Theta}{r}}{q^{n+2}\Upsilon_3(\mathbb{f}_{n+1},\mathbb{g}_{n+1})}\right)\circledcirc$$
$$Y\left(\mathbb{f}_2,\mathbb{g}_1,\frac{\frac{\Theta}{r}}{q^{n+3}\Upsilon_4(\mathbb{f}_{n+2},\mathbb{g}_{n+1})}\right)\circledcirc\cdots\circledcirc Y\left(\mathbb{f}_1,\mathbb{g}_0,\frac{\frac{\Theta}{r}}{q^{\omega-1}\Upsilon_{n-1}(\mathbb{f}_{\omega},\mathbb{g}_{\omega-1})}\right)\circledcirc Y\left(\mathbb{f}_0,\mathbb{g}_0,\frac{\frac{\Theta}{r}}{q^{\omega}\Upsilon_n(\mathbb{f}_{\omega},\mathbb{g}_{\omega})}\right).$$

From **(4)**, as $n,\omega\to\infty$, we get $Z(\mathbb{f}_n,\mathbb{g}_\omega,\Theta)\ge 1, K(\mathbb{f}_n,\mathbb{g}_\omega,\Theta)\le 0, Y(\mathbb{f}_n,\mathbb{g}_\omega,\Theta)\le 0$ for every value $\Theta>0$. $Bisqnce$ $(\mathbb{f}_n,\mathbb{g}_n)$ is hence a Cauchy bisequence . Given the completeness of $(Q,L,Z,K,Y,\circledast,\circledcirc)$, the biconvergence $(\mathbb{f}_n,\mathbb{g}_n)$ is a convergent Cauchy bisequence . Proposition (3.6) states that the biconvergence $(\mathbb{f}_n,\mathbb{g}_n)$ is a biconvergent sequence.

Due to the biconvergence of $(\mathbb{f}_n,\mathbb{g}_n)$, both the $\{\mathbb{f}_n\}$ and $\{\mathbb{g}_n\}$ sequences have a limit at $\Omega\in Q\cap L$. Lemma (3.8) states that each of the sequences $\{\mathbb{f}_n\}$ and $\{\mathbb{g}_n\}$ has a distinct limit. Examine (f), (g), and (r) of definition (3.2).

$$Z(\Xi(\Omega),\Omega,\Theta)\ge Z\left(\Xi(\Omega),\Xi(\mathbb{g}_n),\frac{\frac{\Theta}{r}}{\Upsilon_1(\Omega,\mathbb{g}_n)}\right)\circledast Z\left(\Xi(\mathbb{f}_n),\Xi(\mathbb{g}_n),\frac{\frac{\Theta}{r}}{\Upsilon_1(\mathbb{f}_n,\mathbb{g}_n)}\right)\circledast$$
$$Z\left(\Xi(\mathbb{f}_n),\Xi(\mathbb{g}_{n+1}),\frac{\frac{\Theta}{r}}{\Upsilon_3(\mathbb{f}_{n+1},\mathbb{g}_{n+1})}\right)\circledast Z\left(\Xi(\mathbb{f}_{n+1}),\Xi(\mathbb{g}_{n+1}),\frac{\frac{\Theta}{r}}{\Upsilon_4(\mathbb{f}_{n+1},\mathbb{g}_{n+1})}\right)\circledast\cdots\circledast$$
$$Z\left(\Xi(\mathbb{f}_{\omega-1}),\Xi(\mathbb{g}_{\omega-1}),\frac{\frac{\Theta}{r}}{\Upsilon_{n-1}(\mathbb{f}_{\omega-1},\mathbb{g}_{\omega-1})}\right)\circledast Z\left(\Xi(\mathbb{f}_{\omega}),\Xi(\Omega),\frac{\frac{\Theta}{r}}{\Upsilon_n(\mathbb{f}_{\omega},\Omega)}\right),$$

$$K(\Xi(\Omega),\Omega,\Theta)\le K\left(\Xi(\Omega),\Xi(\mathbb{g}_n),\frac{\frac{\Theta}{r}}{\Upsilon_1(\Omega,\mathbb{g}_n)}\right)\circledcirc K\left(\Xi(\mathbb{f}_n),\Xi(\mathbb{g}_n),\frac{\frac{\Theta}{r}}{\Upsilon_1(\mathbb{f}_n,\mathbb{g}_n)}\right)\circledcirc$$
$$K\left(\Xi(\mathbb{f}_n),\Xi(\mathbb{g}_{n+1}),\frac{\frac{\Theta}{r}}{\Upsilon_3(\mathbb{f}_{n+1},\mathbb{g}_{n+1})}\right)\circledcirc K\left(\Xi(\mathbb{f}_{n+1}),\Xi(\mathbb{g}_{n+1}),\frac{\frac{\Theta}{r}}{\Upsilon_4(\mathbb{f}_{n+1},\mathbb{g}_{n+1})}\right)\circledcirc\cdots\circledcirc$$
$$K\left(\Xi(\mathbb{f}_{\omega-1}),\Xi(\mathbb{g}_{\omega-1}),\frac{\frac{\Theta}{r}}{\Upsilon_{n-1}(\mathbb{f}_{\omega-1},\mathbb{g}_{\omega-1})}\right)\circledcirc K\left(\Xi(\mathbb{f}_{\omega}),\Xi(\Omega),\frac{\frac{\Theta}{r}}{\Upsilon_n(\mathbb{f}_{\omega},\Omega)}\right),$$

$$Y(\Xi(\Omega),\Omega,\Theta)\le Y\left(\Xi(\Omega),\Xi(\mathbb{g}_n),\frac{\frac{\Theta}{r}}{\Upsilon_1(\Omega,\mathbb{g}_n)}\right)\circledcirc Y\left(\Xi(\mathbb{f}_n),\Xi(\mathbb{g}_n),\frac{\frac{\Theta}{r}}{\Upsilon_1(\mathbb{f}_n,\mathbb{g}_n)}\right)\circledcirc$$
$$Y\left(\Xi(\mathbb{f}_n),\Xi(\mathbb{g}_{n+1}),\frac{\frac{\Theta}{r}}{\Upsilon_3(\mathbb{f}_{n+1},\mathbb{g}_{n+1})}\right)\circledcirc Y\left(\Xi(\mathbb{f}_{n+1}),\Xi(\mathbb{g}_{n+1}),\frac{\frac{\Theta}{r}}{\Upsilon_4(\mathbb{f}_{n+1},\mathbb{g}_{n+1})}\right)\circledcirc\cdots\circledcirc$$

$$Y\left(\Xi(\mathbb{f}_{\omega-1}), \Xi(\mathbb{g}_{\omega-1}), \frac{\frac{\Theta}{r}}{\Upsilon_{n-1}(\mathbb{f}_{\omega-1}, \mathbb{g}_{\omega-1})}\right) \circledcirc Y\left(\Xi(\mathbb{f}_{\omega}), \Xi(\Omega), \frac{\frac{\Theta}{r}}{\Upsilon_{n}(\mathbb{f}_{\omega}, \Omega)}\right), \forall\, n, \omega \in \mathbb{N} \text{ and } \Theta > 0.$$

As $n \to \infty$, implies
$Z(\Xi(\Omega), \Omega, \Theta) \to 1 \circledast 1 \circledast 1 \circledast 1 \cdots \circledast 1 \circledast 1 = 1$,
$K(\Xi(\Omega), \Omega, \Theta) \to 0 \circledcirc 0 \circledcirc 0 \circledcirc 0 \cdots \circledcirc 0 \circledcirc 0 = 0$,
$Y(\Xi(\Omega), \Omega, \Theta) \to 0 \circledcirc 0 \circledcirc 0 \circledcirc 0 \cdots \circledcirc 0 \circledcirc 0 = 0$.
From (d) of definition (3.2), we obtain $\Xi(\Omega) = \Omega$. Let $\mathbb{f} \in Q \cap L$ be one more FP of Ξ. Then

$$Z(\Omega, k, \Theta) = Z(\Xi(\mu), \Xi(\mathbb{f}), \Theta) \geq Z\left(\Omega, k, \frac{\Theta}{q}\right),$$

$$K(\Omega, k, \Theta) = K(\Xi(\Omega), \Xi(\mathbb{f}), \Theta) \leq K\left(\Omega, k, \frac{\Theta}{q}\right),$$

$$Y(\Omega, k, \Theta) = Y(\Xi(\Omega)(, \Xi(\mathbb{f}), \Theta) \leq Y\left(\Omega, k, \frac{\Theta}{q}\right).$$

For $q \in (0, 1)$ and $\forall\, \Theta > 0$. By Lemma (3.4), finally we get $\Omega = \mathbb{f}$. □

Example 3.10. Let $Q = \left[0, \frac{1}{2}^n\right], L = \left[\frac{1}{2}^{n+1}, 1\right]$, and $\Upsilon_i : Q \times L \to [1, \infty), (1 \leq i \leq 9)$ be nine non-comparable mappings defined as
$\Upsilon_1(\mathbb{f}_1, \mathbb{g}_1) = \mathbb{f}_1 + \mathbb{g}_1 + 1, \Upsilon_2(\mathbb{f}_2, \mathbb{g}_1) = \mathbb{f}_2 + \mathbb{g}_1 + 1, \Upsilon_3(\mathbb{f}_2, \mathbb{g}_2) = \mathbb{f}_2 + \mathbb{g}_2 + 1,$
$\Upsilon_4(\mathbb{f}_3, \mathbb{g}_2) = \mathbb{f}_3 + \mathbb{g}_2 + 1, \Upsilon_5(\mathbb{f}_3, \mathbb{g}_3) = \mathbb{f}_3 + \mathbb{g}_3 + 1, \Upsilon_6(\mathbb{f}_4, \mathbb{g}_3) = \mathbb{f}_4 + \mathbb{g}_3 + 1,$
$\Upsilon_7(\mathbb{f}_4, \mathbb{g}_4) = \mathbb{f}_4 + \mathbb{g}_4 + 1, \Upsilon_8(\mathbb{f}_5, \mathbb{g}_4) = \mathbb{f}_5 + \mathbb{g}_4 + 1, \Upsilon_9(\mathbb{f}_5, \mathbb{g}_5) = \mathbb{f}_5 + \mathbb{g}_5 + 1.$
Define $Z(\mathbb{f}_1, \mathbb{g}_5, \Theta_i) = \frac{\Theta_i}{\Theta_i + |\mathbb{f}_1 - \mathbb{g}_5|}$, $K(\mathbb{f}_1, \mathbb{g}_5, \Theta_i) = \frac{|\mathbb{f}_1 - \mathbb{g}_5|}{\Theta_i + |\mathbb{f}_1 - \mathbb{g}_5|}$, $Y(\mathbb{f}_1, \mathbb{g}_5, \Theta_i) = \frac{|\mathbb{f}_1 - \mathbb{g}_5|}{\Theta_i}$ for every value $\Theta_i > 0$, $\mathbb{f} \in Q$ and $\mathbb{g} \in L$. Clearly, $(Q, L, Z, K, Y, \circledast, \circledcirc)$ is a $NnCBMS$, where $\circledast$ is a CTC defined as $\Theta_1 \circledast \Theta_2 = \Theta_1\Theta_2$ and $\circledcirc$ is a CTN defined as $\Theta_1 \circledcirc \Theta_2 = \max\{\Theta_1, \Theta_2\}$.
Define $\Xi : Q \cup L \to Q \cup L$ by $\Xi(\Omega) = \begin{cases} \frac{\Omega}{2}, & \text{if} \quad \Omega \in \left[0, \frac{1}{2^n}\right], \\ 0, & \text{if} \quad \Omega \in \left[\frac{1}{2^{n+1}}, 1\right], \end{cases}$ for every value $\Omega \in Q \cup L$. Obviously, the criteria's of Theorem (3.9) are satisfied. Hence Ξ has a unique FP, i.e., $\Omega = 0$. On the same lines, we can prove for higher value of n.

4. Conclusion

We provide the idea of $Nn - CBMS$ in this study. Several FP outcomes are established using n-non-comparable functions. We also extend the results using a variation of the Banach contraction principle and provide a number of non-trivial instances. We then use the key results to solve fractional differential equation issues related to financial modelling. Future study on additional criteria that would ensure the presence of FPs in $Nn - CBMS$ is something we find intriguing.

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