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A Study on Neutrosophic Cordial Labeling Graphs with Algorithm

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Abstract We present here the neutrosophic cordial labeling graphs, which integrate neutrosophic and cordial labeling graphs. This work explores three types of graph labeling such as fuzzy cordial labeling graphs, intuitionistic fuzzy cordial labeling graphs and neutrosophic cordial labeling graphs. We provide some functions for vertex and edge labeling under specific conditions, such that if the edge labeling is less than 0.5, the cordial labeling is 0 and 1; otherwise, indicating the integral part of that edge labeling. Furthermore, it meets the cordial labeling requirement, which states that the number of edges labeled with 0 and 1 differ at most by 1.

Keywords Fuzzy Cordial Labeling Graphs (FCLG), Intuitionistic Cordial Labeling Graphs (ICLG), Neutrosophic Cordial Labeling Graphs (NCLG)

AMS 2010 subject classifications 03E72,05C72,05C78

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1. Introduction

Graph labeling is one of the fundamental branchs of mathematics; it plays a role in analysing the structure of networks in the real-world applications [1]. Even though, it some times attains the conflicts of imprecision, uncertainty. It is deals with these aspects via the fuzzy logic method. The concept of fuzzy set theory was introduced by Zadeh [1965] [2], and the fuzzy graph was defined by Kaufmann [1986] [3] . In 1975 [4] , Rosenfeld developed a more elaborate definition for fuzzy graphs and several fuzzy analogues. At the same time Yeh and Bang [5] and Bhattacharya [6] introduced various connectedness concepts in fuzzy graphs. Bhuttani [7] and Nagoorgani [8] discussed on structural representation of fuzzy graphs and labeling.

In 1983 [9, 10], Krassimir Atanassov introduced the concept of intuitionistic fuzzy set theory due to occur the inaccuracy in fuzzy. The fuzzy set has only membership grade, the but intuitionistic fuzzy set uses independent membership and non-membership grade. In this paper we propose the membership are [Appearance(A) and Non-Appearance(NA)]. Shannon and Atanassov [11] proposed the idea of the intuitionistic fuzzy relation and intuitionistic fuzzy graph. R.Parvathi [12] presented an extension of fuzzy graph to the intuitionistic fuzzy graph. M.Akaram [13, 14, 15] and Nagoorgani build on various properties of intuitionistic fuzzy graph which help us to the relationship between the real- world components with an intuitionistic fuzzy graph structure.

Smarandache [16] introduced the neutrosophic set, which is an extended concept of (Atanassov) in intuitionistic fuzzy set. Neutrosophic sets can be used to solve the uncertainty, indeterminacy, vagueness of real-life problems. It is a modified version of the fuzzy set and intuitionistic fuzzy set. It is characterised by the membership of Appearance (A), Indeterminate (I) and Non-Appearance (NA) with their sum lying between 0 and 3. These three

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membership degrees are independent. The concept of the netrosophic graph was introduced by Kandasamy et al [17]. It reveals the new dimension of graph theory. Nowadays, many researchers have been actively working on neutrosophic graph theory. Ye et al,Yang [18], Broumi [19] and Arkam. Neutrosophic fuzzy graphs are more effective, flexible and compatible for uncertain real-world problems compared to fuzzy graphs. The application of neutrosophic fuzzy graphs encompasses medical diagnosis, cybersecurity, education, environmental monitoring, financial decision-making, engineering systems, sentiment analysis, social networks, communication networks, and road transportation networks, among others.

Rosa [20] introduced the graph labeling, which is mapping to graph elements and a set of numbers. A. Gallian [21] described the numerous types of labeling in graphs, such as graceful labeling, cordial labeling, and mean labeling etc. These graphs label significant applications in many fields. I. cahit [22, 23] introduced cordial graph labeling, which he regarded as "a weaker version of both graceful and harmonious labeling" and also examined cordiality for various graphs such as trees, cycles, and complete graphs, for example: The cycle c_n with n vertices is cordial. if and only if $n \not\equiv 2(mod4)$. To address these theorems, we introduce fuzzy cordial labeling in cycle graphs. Fuzzy labeling graphs are a significant tool in graph theory. Sundaram and Ponraj [24], [25] presented the prime cordial labeling concept, whereas P. sumathi and Suresh Kumar [26] introduced fuzzy quotient 3 cordial labeling for star-related graphs. We draw inspiration from these ideas and present fuzzy and intuitionistic fuzzy cordial labeling graphs. But this graph labeling affects the real-life situations. For example, automatic channel allocation for transmitters to frequencies. The channel station sends the frequencies to all other transmitters in the same way. Sometimes the difficulties arise from the atmosphere at the time the transmitter receives the moderate frequency or medium signal. This type of signal is an inconsistent or indeterminate state. So, we decide to introduce the neutrosophic cordial labeling.

In this study we focused on simple finite connected undirected graphs. The vertex function and edge condition of NCLGs defined in this paper satisfies the cordial condition. Also, an algorithm is defined for labeling purposes. Section 2 of this paper comprises the fundamental definitions of NCLGs. In section 3, we define fuzzy cordial labeling and some related theorems. Sections 4 and 5 include definitions, theorems, examples and algorithms regarding intuitionistic fuzzy cordial labeling and neutrosophic fuzzy cordial labeling respectively. Section 6 demonstrate the applications of NCLGs.

2. Preliminaries

Let R be the universal set. A fuzzy set in R is represented by S = { ($\mathfrak{x}$, $\mu_S(\mathfrak{x})|\mu_S(\mathfrak{x})$>0, $\mathfrak{x}$∈R}, where the function μ_S: R→[0,1] is the appearance of element x in the fuzzy set S. A fuzzy graph G = (μ, γ) is a pair of functions, where μ is a fuzzy subset of $\mathfrak{V}$, and γ is a symmetric fuzzy relation on μ.

Definition 1
A graph G: (μ, γ) is said to be fuzzy labeling graph, if $\mu : \mathfrak{V} \to [0, 1]$ and $\gamma : \mathfrak{V} \times \mathfrak{V} \to [0, 1]$,$\mu$, γ are bijective and $\gamma_A(uv) \leq min(\mu_A(u), \mu_A(v))$ for all $u, v \in \mathfrak{V}$.

Definition 2
An intuitionistic fuzzy graph G is said to be an intuitionistic fuzzy labeling graph. if μ_A: $\mathfrak{V} \to$ [0,1], μ_{NA}: $\mathfrak{V} \to$[0,1] and γ_A: $\mathfrak{V}\times\mathfrak{V}\to$ [0,1], γ_{NA}: $\mathfrak{V}\times\mathfrak{V}\to$ [0,1] are bijective,

$$\begin{aligned} \gamma_A(u, v) &\leq min(\mu_A(u), \mu_A(v)) \\ \gamma_{NA}(u, v) &\leq max(\mu_{NA}(u), \mu_{NA}(v)) \end{aligned}$$

$0 \leq \gamma_A(u, v) + \gamma_{NA}(u, v) \leq 1$ for every edges $(u, v) \in \mathfrak{V}\times\mathfrak{V}$.

Definition 3
A neutrosophic fuzzy graph is the form G where $\mathfrak{V}$ = $(v_1, v_2, v_3, ...v_n)$ such that μ_A: $\mathfrak{V} \to$[0,1], μ_I:$\mathfrak{V} \to$[0,1],

μ_{NA}:$\mathfrak{V}$ →[0,1] denote the degree of appearance, indeterminacy and non-appearance of the vertices and γ_A : $\mathfrak{V}\times\mathfrak{V}\to$ [0,1] and γ_I : $\mathfrak{V}\times\mathfrak{V}\to$ [0,1], γ_{NA}: $\mathfrak{V}\times\mathfrak{V}\to[0,1]$ edges are bijective such that,

$$\begin{aligned}\gamma_A((v_i),(v_j)) &\leq min(\mu_A(v_i)),\mu_A(v_j))\\ \gamma_I((v_i),(v_j)) &\leq min(\mu_I(v_i),\mu_I(v_j))\\ \gamma_{NA}((v_i),(v_j)) &\leq max(\mu_{NA}(v_i),\mu_{NA}(v_j))\end{aligned}$$

$0\leq \gamma_A((v_i),(v_j))+\gamma_I((v_i),(v_j))+\gamma_{NA}((v_i),(v_j))\leq 3$ for every edge in $\mathfrak{E}$.

Definition 4
A binary vertex labeling of a graph G is called a cordial labeling if $|\mathfrak{v}_\mu(0)-\mathfrak{v}_\mu(1)|\leq 1$ and $|\mathfrak{e}_{\gamma(0)}-\mathfrak{e}_{\gamma(1)}|\leq 1$.
A graph G is cordial if it admits the cordial labeling.

3. Fuzzy Cordial Labeling Graph (FCL)

Definition 5
Let the vertex one-to-one function be defined by $\mu_A=\begin{cases}\frac{3(i+x)+2}{(x-1)\times 10} & if\ i\ is\ odd\\ 1-\frac{3(i+x)+2}{x\times 10} & if\ i\ is\ even\end{cases}$ (where x is the number of vertices, and i is the naming of vertices). For each edge γ_A:$\mathfrak{V}\times\mathfrak{V}\to$ [0,1], $\gamma_A(uv)\leq min(\mu_A(u),\mu_A(v))$, $\mu_A(v)$ is defined. Using the above condition in a fuzzy labeling graph is said to be a fuzzy cordial labeling graph. If the number of edges labeled with 1 and the number of edges labeled with 0 differ by at most 1 $(ie)|e_{\gamma_A}(0)-e_{\gamma_A}(1)|\leq 1$ such that $[\gamma_A(uv)]$ is an integral part <0.5. which is 0; otherwise, it is 1.

3.1. Algorithm for Fuzzy Cordial Labeling of Cycle and Path

Let $\mathfrak{V}=\{v_i|i=123...x\}$ be the vertex set, i.e., $|\mathfrak{V}|=x$.

Step 1: Label the vertex with the function $\mu_A=\begin{cases}\frac{3(i+x)+2}{(x-1)\times 10} & if\ i\ is\ odd\\ 1-\frac{3(i+x)+2}{x\times 10} & if\ i\ is\ even.\end{cases}$

Step 2: Label the edge under the condition $\gamma_A(v_iv_j)\leq min(\mu_A(v_i),\mu_A(v_j))$

Step 3: Get $[\gamma_A]$, i.e $\gamma_A=\begin{cases}1 & if\ \gamma_A(v_iv_j)\geq 0.5\\ 0 & if\ otherwise\end{cases}$

Step 4: Check whether $|e_{\gamma_A}(0)-e_{\gamma_A}(1)|\leq 1$ where $e_{\gamma_A}(0)$ and $e_{\gamma_A}(1)$ are the number of edges with integral 0 and 1 respectively.
If not go to step 2.

Theorem 1
For n $\geq$ 3, cycle C_n is a fuzzy cordial labeling graph.

Proof
Let G = C_n be a cycle graph with length n≥3. Define a fuzzy labeling cycle graph G = $(\mathfrak{V},\mu,\gamma)$.
let μ_A:$\mathfrak{V}\to$ [0,1] be a one-to-one function defined by $\mu_A=\begin{cases}\frac{3(i+x)+2}{(x-1)\times 10} & if\ i\ is\ odd\\ 1-\frac{3(i+x)+2}{x\times 10} & if\ i\ is\ even\end{cases}$ and
γ_A:$\mathfrak{V}\times\mathfrak{V}\to$ [0,1] is bijective, such that the appearance values of vertices are distinct and label the edge $\gamma_A(uv)\leq min(\mu_A(u),\mu_A(v))$. The vertices can be adjusted according to the algorithm (3.1). $|e_{\gamma_A}(0)-e_{\gamma_A}(1)|\leq 1$. Hence, cycle C_n, $n\geq 3$ admits a fuzzy cordial labeling graph. □

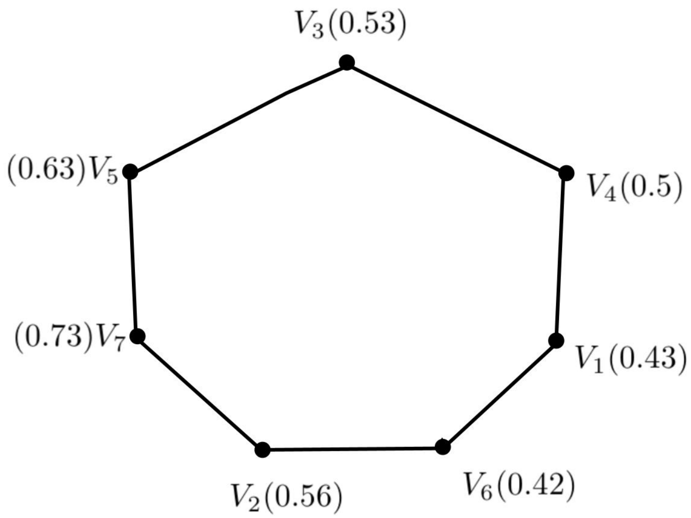

Figure 1. fuzzy cycle graph C_7

$$\begin{array}{cc} v_1 = 0.43 & (v_1v_4) = e_1 = 0.4 \\ v_2 = 0.56 & (v_1v_6) = e_2 = 0.41 \\ v_3 = 0.53 & (v_2v_6) = e_3 = 0.43 \\ v_4 = 0.5 & (v_2v_7) = e_4 = 0.55 \\ v_5 = 0.63 & (v_3v_4) = e_5 = 0.5 \\ v_6 = 0.42 & (v_3v_5) = e_6 = 0.51 \\ v_7 = 0.73 & (v_5v_7) = e_7 = 0.62 \end{array}$$

Cordial labeling for edges $(v_i, v_j) = ([\gamma_A])$

$$(v_1v_4) = e_1 = 0; (v_1v_6) = e_2 = 0;$$
$$(v_2v_6) = e_3 = 0; (v_2v_7) = e_4 = 1;$$
$$(v_3v_4) = e_5 = 1; (v_3v_5) = e_6 = 1;$$
$$(v_5v_7) = e_7 = 1.$$

$$|e_{\gamma_A}(0) - e_{\gamma_A}(1)| \leq 1$$
$$|3 - 4| \leq 1$$

4. Intuitionistic Fuzzy Cordial Labeling Graph (IFCL))

Definition 6

A graph G is said to be an intuitionistic fuzzy labeling graph.

If $\mu_A = \begin{cases} \frac{3(i+x)}{(x-1)\times 10} & if\ i\ is\ odd \\ 1 - \frac{3(i+x)+2}{x\times 10} & if\ i\ is\ even \end{cases}$ and $\mu_{NA} = \begin{cases} \frac{3(i+x)}{x\times 10^2} & if\ i\ is\ odd \\ \frac{3(i+x)+2}{x\times 10} & if\ i\ is\ even \end{cases}$ are one-to-one vertex functions, where x is the number of vertices and i is the naming of vertices. For each edge bijective such that $\gamma_A(uv) \leq min(\mu_A(u), \mu_A(v))$ and $\gamma_{NA}(uv) \leq max(\mu_A(u), \mu_A(v))$. And an intuitionistic fuzzy labeling graph is said to be an intuitionistic fuzzy cordial labeling graph. If the number of edges labeled with 1 and number of edges labeled with 0 differ by at most 1. $(ie)|e_{\gamma_A}(0) - e_{\gamma_A}(1)| \leq 1$ and $|e_{\gamma_{NA}}(0) - e_{\gamma_{NA}}(1)| \leq 1$ such that $[\gamma_A(uv)]$ and $[\gamma_{NA}(uv)]$is integral part < 0.5. which is 0; otherwise, it is 1.

4.1. Algorithm for Intuitionistic Fuzzy Cordial Labeling of Cycle

Let $\mathfrak{V} = \{v_i | i = 123...x\}$ be the vertex set, i.e., $|\mathfrak{V}| = x$

Step 1: Label the vertex with the function.

$$\mu_A = \begin{cases} \frac{3(i+x)+2}{(x-1)\times 10} & if\ i\ is\ odd \\ 1 - \frac{3(i+x)+2}{x\times 10} & if\ i\ is\ even \end{cases} \text{ and } \mu_{NA} = \begin{cases} \frac{3(i+x)}{x\times 10^2} & if\ i\ is\ odd \\ \frac{3(i+x)+2}{x\times 10} & if\ i\ is\ even. \end{cases}$$

Step 2: Label the edge under the condition $\gamma_A(uv) \leq min(\mu_A(u), \mu_A(v))$ and $\gamma_{NA}(uv) \leq max(\mu_{NA}(u), \mu_{NA}(v))$

Step 3: Get $\gamma_A = \begin{cases} 1 & if\ \gamma_A(uv) \geq 0.5 \\ 0 & if\ otherwise \end{cases}$ and $\gamma_{NA} = \begin{cases} 1 & if\ \gamma_{NA}(uv) \geq 0.5 \\ 0 & if\ otherwise \end{cases}$

Step 4: Check whether $|e_{\gamma_A}(0) - e_{\gamma_A}(1)| \leq 1$ and $|e_{\gamma_{NA}}(0) - e_{\gamma_{NA}}(1)| \leq 1$ where $e_{\gamma_A}(a)$, $e_{\gamma_{NA}}(a)$ and $e_{\gamma_A}(b)$, $e_{\gamma_{NA}}(b)$ is the number of edges in the integral part of the label a,b = 0,1 respectively
If not go to step 2.

Theorem 2
For n $\geq$ 3, cycle C_n is an intuitionistic fuzzy cordial labeling graph.

Proof
Let G = C_n be a cycle graph with length n$\geq$3. Define an intuitionistic fuzzy labeling cycle graph G = $(\mathfrak{V}, \mu, \gamma)$. Let μ_A:$\mathfrak{V} \to$ [0,1], μ_{NA}:$\mathfrak{V} \to$ [0,1] be a one to one vertex function defined by

$$\mu_A = \begin{cases} \frac{3(i+x)+2}{(x-1)\times 10} & if\ i\ is\ odd \\ 1 - \frac{3(i+x)+2}{x\times 10} & if\ i\ is\ even \end{cases} \text{ and } \mu_{NA} = \begin{cases} \frac{3(i+x)}{x\times 10^2} & if\ i\ is\ odd \\ \frac{3(i+x)+2}{x\times 10} & if\ i\ is\ even \end{cases},$$

and Edge functions defined by γ_A:$\mathfrak{V}\times\mathfrak{V}\to$ [0,1] and γ_{NA}:$\mathfrak{V}\times\mathfrak{V}\to$ [0,1] bijective such that $\gamma_A(uv) \leq min(\mu_A(u), \mu_A(v))$ and $\gamma_{NA}(uv) \leq max(\mu_A(u), \mu_A(v))$. The vertices can be adjusted according to the algorithm (4.1). $|e_{\gamma_A}(0) - e_{\gamma_A}(1)| \leq 1$, $|e_{\gamma_{NA}}(0) - e_{\gamma_{NA}}(1)| \leq 1$. Hence, cycle graph n $\geq$ 3 admits an intuitionistic fuzzy cordial labeling graph. □

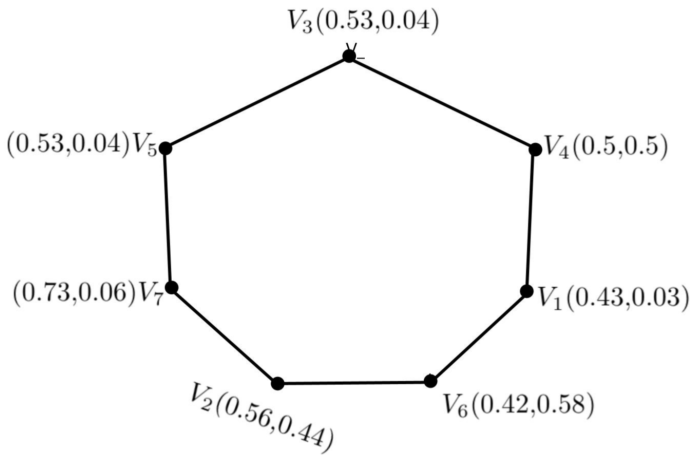

Figure 2. Intuitionistic fuzzy cycle graph C_7

$$\begin{array}{ll} v_1 = (0.43, 0.03) & (v_1v_4) = e_1 = (0.4, 0.45) \\ v_2 = (0.56, 0.44) & (v_1v_6) = e_2 = (0.41, 0.53) \\ v_3 = (0.53, 0.04) & (v_2v_6) = e_3 = (0.43, 0.52) \\ v_4 = (0.5, 0.5) & (v_2v_7) = e_4 = (0.55, 0.42) \\ v_5 = (0.63, 0.05) & (v_3v_4) = e_5 = (0.5, 0.5) \\ v_6 = (0.42, 0.42) & (v_3v_5) = e_6 = (0.51, 0.042) \\ v_7 = (0.73, 0.06) & (v_5v_7) = e_7 = (0.62, 0.055) \end{array}$$

Cordial labeling for edges $(v_i, v_j) = ([\gamma_A], [\gamma_{NA}])$

$$(v_1v_4) = e_1 = (0,0)\ ;\ (v_1v_6) = e_2 = (0,1);$$
$$(v_2v_6) = e_3 = (0,1);\ (v_2v_7) = e_4 = (1,0);$$
$$(v_3v_4) = e_5 = (1,1);\ (v_3v_5) = e_6 = (1,0);$$
$$(v_5v_7) = e_7 = (1,0)$$

$$|e_{\gamma_A}(0) - e_{\gamma_A}(1)| \leq 1 \quad \text{and} \quad |e_{\gamma_{NA}}(0) - e_{\gamma_{NA}}(1)| \leq 1$$
$$|3-4| \leq 1 \quad \text{and} \quad |4-3| \leq 1$$

5. Neutrosophic Cordial Labeling Graph(NCLG)

Definition 7

A neutrosophic graph is of the form G = $(\mathfrak{V}, \mathfrak{E})$. If $\mu_A = \begin{cases} \frac{3(i+x)+2}{(x-1)\times 10} & if\ i\ is\ odd \\ 1 - \frac{3(i+x)+2}{x\times 10} & if\ i\ is\ even \end{cases}$, $\mu_I = \begin{cases} \frac{5i+x+5)}{x\times 10} & if\ i\ is\ odd \\ \frac{5i+x+5}{(x-1)\times 10} & if\ i\ is\ even \end{cases}$, and $\mu_{NA} = \begin{cases} \frac{3(i+x)}{x\times 10^2} & if\ i\ is\ odd \\ \frac{3(i+x)+2}{x\times 10} & if\ i\ is\ even \end{cases}$ one-to-one vertex functions, where x is the number of vertices and i is the naming of vertices. For each edge is bijective such that $\gamma_A(uv) \leq min(\mu_A(u), \mu_A(v)), \gamma_I(uv) \leq min(\mu_I(u), \mu_I(v))$ and $\gamma_{NA}(uv) \leq max(\mu_{NA}(u), \mu_{NA}(v))$. A neutrosophic labeling graph is said to be a neutrosophic cordial labeling graph. If the number of edges labeled with 1 and the number of edges labeled with 0 differ by at most 1 $(ie)|e_{\gamma_A}(0) - e_{\gamma_A}(1)| \leq 1, |e_{\gamma_I}(0) - e_{\gamma_I}(1)| \leq 1$ and $|e_{\gamma_{NA}}(0) - e_{\gamma_{NA}}(1)| \leq 1$ such that $[\gamma_A(uv)]$, $[\gamma_I(uv)]$ and $[\gamma_{NA}(uv)]$is integral part < 0.5, which is 0; otherwise, it is 1

5.1. *Algorithm for Neutrosophic Cordial Labeling of Star Graph*

Let $\mathfrak{V} = \{v_i | i = 123...x\}$ be the vertex set, i.e., $|\mathfrak{V}| = x$

Step 1: Label the vertex with the function $\mu_A = \begin{cases} \frac{3(i+x)+2}{(x-1)\times 10} & if\ i\ is\ odd \\ 1 - \frac{3(i+x)+2}{x\times 10} & if\ i\ is\ even, \end{cases}$ $\mu_I = \begin{cases} \frac{5i+x+5)}{x\times 10} & if\ i\ is\ odd \\ \frac{5i+x+5}{(x-1)\times 10} & if\ i\ is\ even, \end{cases}$ and $\mu_{NA} = \begin{cases} \frac{3(i+x)}{x\times 10^2} & if\ i\ is\ odd \\ \frac{3(i+x)+2}{x\times 10} & if\ i\ is\ even \end{cases}$

Step 2: Label the edge under the condition $\gamma_A(uv) \leq min(\mu_A(u), \mu_A(v))$, $\gamma_I(uv) \leq min(\mu_I(u), \mu_I(v))$ and $\gamma_{NA}(uv) \leq max(\mu_{NA}(u), \mu_{NA}(v))$

Step 3: Get $[\gamma_A] = \begin{cases} 1 & if\ \gamma_A(uv) \geq 0.5 \\ 0 & if\ otherwise \end{cases}$, $[\gamma_I] = \begin{cases} 1 & if\ \gamma_I(uv) \geq 0.5 \\ 0 & if\ otherwise \end{cases}$ and $[\gamma_{NA}] = \begin{cases} 1 & if\ \gamma_{NA}(uv) \geq 0.5 \\ 0 & if\ otherwise \end{cases}$

Step 4: Check whether $|e_{\gamma_A}(0) - e_{\gamma_A}(1)| \leq 1$, $|e_{\gamma_I}(0) - e_{\gamma_I}(1)| \leq 1$ and $|e_{\gamma_{NA}}(0) - e_{\gamma_{NA}}(1)| \leq 1$ where $e_{\gamma_A}(a), e_{\gamma_I}(a), e_{\gamma_{NA}}(a)$ and $e_{\gamma_A}(b), e_{\gamma_I}(b), e_{\gamma_{NA}}(b)$ are the number of edges in the integral part of the label a,b = 0,1

If not go to step 2.

Theorem 3

For n $\geq$ 3, the Star graph S_n is a neutrosophic cordial labeling graph.

Proof

Let G = S_n be a star graph with length n$\geq$3. Define a neutrosophic labeling Star graph G = $(\mathfrak{V}, \mu, \gamma)$.
let μ_A:$\mathfrak{V} \rightarrow$ [0,1], μ_I:$\mathfrak{V} \rightarrow$ [0,1], μ_{NA}:$\mathfrak{V} \rightarrow$ [0,1] be a one- to-one vertex function defined by

$\mu_A = \begin{cases} \frac{3(i+x)+2}{(x-1)\times 10} & if\ i\ is\ odd \\ 1 - \frac{3(i+x)+2}{x\times 10} & if\ i\ is\ even \end{cases}$, $\mu_I = \begin{cases} \frac{5i+x+5)}{x\times 10} & if\ i\ is\ odd \\ \frac{5i+x+5}{(x-1)\times 10} & if\ i\ is\ even \end{cases}$, and $\mu_{NA} = \begin{cases} \frac{3(i+x)}{x\times 10^2} & if\ i\ is\ odd \\ \frac{3(i+x)+2}{x\times 10} & if\ i\ is\ even \end{cases}$

Edge function defined by γ_A:$\mathfrak{V}\times\mathfrak{V}\rightarrow$ [0,1] and γ_I:$\mathfrak{V}\times\mathfrak{V}\rightarrow$ [0,1]γ_{NA}:$\mathfrak{V}\times\mathfrak{V}\rightarrow$ [0,1] are bijection such that, $\gamma_A(uv) \leq min(\mu_A(u), \mu_A(v))$ and $\gamma_I(uv) \leq min(\mu_I(u), \mu_I(v))$, $\gamma_{NA}(uv) \leq max(\mu_A(u), \mu_A(v))$. The vertices can be adjusted according to the algorithm (5.1) $|e_{\gamma_A}(0) - e_{\gamma_A}(1)| \leq 1$, $|e_{\gamma_I}(0) - e_{\gamma_I}(1)| \leq 1$, $|e_{\gamma_{NA}}(0) - e_{\gamma_{NA}}(1)| \leq 1$ Hence, a star graph n $\geq$ 3 has a neutrosophic fuzzy cordial labeling graph. ☐

6. Application of neutrosophic cordial labeling graph

Communication Network For Automatic Channel Allocation:

The transmitters are automatically assigned channel numbers and frequencies. The channel station assigns frequencies to all transmitters in the same manner. The transmitter may receive a high, medium, or low frequency signal created by the current atmosphere. Consider the following example.

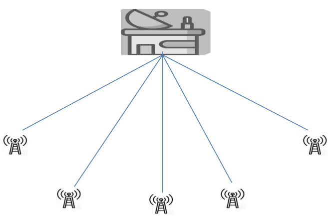

Figure 3. Communication Network For Automatic Channel Allocation

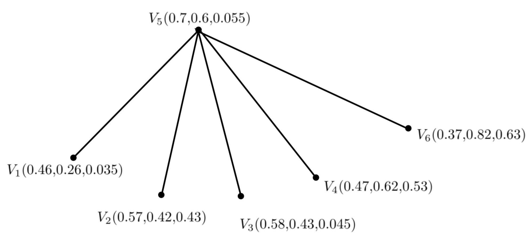

Figure 4. Neutrosophic Cordial labeling Star graph S_6

$$\begin{array}{ll} v_1 = (0.46, 0.26, 0.035) & (v_1v_5) = e_1 = (0.44, 0.11, 0.045) \\ v_2 = (0.57, 0.42, 0.43) & (v_2v_5) = e_2 = (0.51, 0.48, 0.41) \\ v_3 = (0.58, 0.43, 0.045) & (v_3v_5) = e_3 = (0.54, 0.4, 0.047) \\ v_4 = (0.47, 0.62, 0.53) & (v_4v_5) = e_4 = (0.45, 0.56, 0.52) \\ v_5 = (0.7, 0.6, 0.055) & (v_5v_6) = e_5 = (0.26, 0.55, 0.62) \\ v_6 = (0.37, 0.82, 0.63) & \end{array}$$

Cordial labeling for edges $(v_i, v_j) = ([\gamma_A], [\gamma_I], [\gamma_{NA}])$

$$(v_1v_5) = e_1 = (0, 0, 0); (v_2v_5) = e_2 = (1, 0, 0);$$
$$(v_3v_5) = e_3 = (1, 0, 0); (v_4v_5) = e_4 = (0, 1, 1);$$
$$(v_5v_6) = e_5 = (0, 1, 1)$$

$$|e_{\gamma_A}(0) - e_{\gamma_A}(1)| \leq 1, \quad |3 - 2| \leq 1$$
$$|e_{\gamma_I}(0) - e_{\gamma_I}(1)| \leq 1, \quad |3 - 2| \leq 1,$$
$$|e_{\gamma_{NA}}(0) - e_{\gamma_{NA}}(1)| \leq 1, \quad |3 - 2| \leq 1.$$

We consider the channel station and transmitters to be vertices, and the frequency between the transmitters and the channel station to be edged. The vertices and edges are now labeled using our technique. If we label using our algorithm, then neutrosophic cordial labeling exists. Once the neutrosophic cordial labeling is established, the Automatic channel allocation frequency does not overlap.

7. Conclusion

The contribution of this paper is to introduce the neutrosophic cordial labeling graphs. In this paper, we have described the idea of the fuzzy cordial labeling graphs, intuitionistic fuzzy cordial labeling graphs, neutrosophic cordial labeling graphs and their algorithms. Also, an application related to communication network for channel Allocation is given to the NCLGs. In the future, we will focus on the study of various neutrosophic cordial labeling graphs.

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