STATISTICS, OPTIMIZATION AND INFORMATION COMPUTING
*Stat., Optim. Inf. Comput.*, Vol. **x**, Month 202x, pp 0–10.
Published online in International Academic Press (www.IAPress.org)

# Intuitionistic L-Fuzzy Soft Ideal Over Semirings

R Sakthivel [1], S Naganathan [2,*], M Anitha [1]

[1] *Department of Mathematics, Syed Ammal Arts and Science College, Ramanathapuram, India*
[2] *Department of Mathematics, Sethupathy Government Arts College, Ramanathapuram, India*

**Abstract** An intuitionistic L- fuzzy soft ideal is introduced over the intuitionistic L-fuzzy soft semirings. Describe the property of the homomorphic fuction if an intuitionistic L-fuzzy soft semiring and an intuitionistic L- fuzzy soft ideal defined. Also, its application demonstrates a medical diagnosis using fuzzy structures.

**Keywords** Intuitionistic fuzzy set, Intuitionistic L-fuzzy set, Intuitionistic L-fuzzy soft semi-ideal, Intuitionistic L-fuzzy soft ideal.

**AMS 2010 subject classifications** 03F55, 06D27, 08A72, 16Y60

**DOI:** 10.19139/soic-2310-5070-2977

## 1. Introduction

The theory of intuitionistic fuzzy sets plays an important role in modern mathematics. A semiring is a well-known universal algebraic structure within a ring. One major problem shared by these theories is their incompatibility with parameterization tools. Soft set theory, introduced by Molodtsov in (1999) [8]. Provides a way to address this issue. Researchers worldwide are working with soft sets, including fuzzy soft sets and L- fuzzy soft sets. The concept of intuitionistic fuzzy sets has significant applications in modern mathematics. The idea of Intuitionistic L-fuzzy sets (ILFS) was introduced by Atanassov in (1986) [1] as a generalization of Zadeh's fuzzy sets (1965) [15]. Later, Maji P. K., Biswas R., and Roy A. R. (2001) [5, 6, 7] applied the concept of intuitionistic fuzzy soft sets to groups. The concept of fuzzy soft rings was introduced by Pazar Varol B. Intuitionistic fuzzy soft semigroups, based on the notion of intuitionistic fuzzy soft sets, were also investigated. Furthermore, Zhou et al. introduced the notion of intuitionistic fuzzy soft ideals over semigroups and studied their algebraic properties. Motivated by these studies, our work is the first to formulate and analyze intuitionistic L-fuzzy soft ideals in the context of semirings, thus providing a new algebraic structure and examining its basic properties within this framework.

## 2. Preliminaries

In this section, for the sake of completeness, we first give some useful definitions.

**Definition 2.1.** [1] An *intuitionistic fuzzy set* $A$ in a nonempty set $X$ can be defined as follows: $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle \mid x \in X\}$, where $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ satisfy the property $0 \leq \mu_A(x) +$

*Correspondence to: S Naganathan (E-mail: nathanaga@gmail.com). Assistant Professor, Department of Mathematics, Sethupathy Government Arts College, Ramanathapuram-623 502.

ISSN 2310-5070 (online) ISSN 2311-004X (print)

$\gamma_A(x) \leq 1, \quad \forall x \in X$. The values $\mu_A(x)$ and $\gamma_A(x)$ denote the degree of membership and non-membership of $x$ to $A$, respectively.

In general, for all $\epsilon \in A \subseteq E$, $\widetilde{F}(\epsilon)$ is an intuitionistic fuzzy set on $U$, which is called the *intuitionistic fuzzy set of parameter* $\epsilon$. The intuitionistic value $\langle \mu_{\widetilde{F}(\epsilon)}(x), \gamma_{\widetilde{F}(\epsilon)}(x) \rangle$ denotes the degree of $x \in U$ corresponding to the parameter $\epsilon$.

$\bar{F}(\epsilon)$ can be written as:

$$\bar{F}(\epsilon) = \{\langle x, \mu_{\widetilde{F}(\epsilon)}(x), \gamma_{\widetilde{F}(\epsilon)}(x) \rangle \mid x \in U\}.$$

If $\forall x \in U$, $\mu_{\widetilde{F}(\epsilon)}(x) + \gamma_{\widetilde{F}(\epsilon)}(x) = 1$, then $\bar{F}(\epsilon)$ degenerates into a fuzzy set. If $\forall x \in U$ and $\forall \epsilon \in A \subseteq E$, $\mu_{\widetilde{F}(\epsilon)}(x) + \gamma_{\widetilde{F}(\epsilon)}(x) = 1$, then the intuitionistic fuzzy soft set $(\widetilde{F}, A)$ degenerates into a fuzzy soft set.

**Definition 2.2.** [6] Let $U$ be an initial universe, $E$ be a set of parameters, $\text{IFS}(U)$ denote the intuitionistic fuzzy power set of $U$ and $A \subset E$. A pair $(\bar{F}, A)$ is called an *intuitionistic fuzzy soft set* over $U$, where $F$ is a mapping given by: $F : A \to \text{IFS}(U)$. An intuitionistic fuzzy soft set is a parameterized family of intuitionistic fuzzy subsets of $U$. A fuzzy soft set is a special case of an intuitionistic fuzzy soft set, because when all the intuitionistic fuzzy subsets of $U$ degenerate into fuzzy subsets, the corresponding intuitionistic fuzzy soft set degenerates into a fuzzy soft set.

**Definition 2.3.** [6] Let $(\widetilde{F}, A)$ and $(\bar{G}, B)$ be two intuitionistic fuzzy soft sets over a universe $U$. "$(\widetilde{F}, A)\ AND\ (\bar{G}, B)''$ denoted by $(\widetilde{F}, A) \wedge (\bar{G}, B)$, is defined by:

$$(\widetilde{F}, A) \wedge (\widetilde{G}, B) = (\widetilde{H}, A \times B),$$

where

$$\widetilde{H}(\alpha, \beta) = \widetilde{F}(\alpha) \cap \widetilde{G}(\beta), \quad \forall (\alpha, \beta) \in A \times B.$$

**Definition 2.4.** [6] Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft sets over a universe $U$. "$(\widetilde{F}, A)\ OR\ (\bar{G}, B)''$, denoted by $(\widetilde{F}, A) \vee (\widetilde{G}, B)$, is defined by:

$$(\widetilde{F}, A) \vee (\widetilde{G}, B) = (\widetilde{H}, A \times B),$$

where

$$\widetilde{H}(\alpha, \beta) = \widetilde{F}(\alpha) \cup \widetilde{G}(\beta), \quad \forall (\alpha, \beta) \in A \times B.$$

**Definition 2.5.** [6] The *intersection* of two intuitionistic fuzzy soft sets $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ over a universe $U$ is an intuitionistic fuzzy soft set denoted by $(\widetilde{H}, C)$, where: $C = A \cap B$, and

$$\widetilde{H}(\epsilon) = \begin{cases} \widetilde{F}(\epsilon), & if \quad \epsilon \in A - B, \\ \widetilde{G}(\epsilon), & if \quad \epsilon \in B - A \\ \widetilde{F}(\epsilon) \wedge \widetilde{G}(\epsilon), & if \quad \epsilon \in A \cap B, \end{cases}$$

for all $\epsilon \in C$. This is denoted by:

$$(\widetilde{H}, C) = (\widetilde{F}, A) \cap (\widetilde{G}, B).$$

**Definition 2.6.** [6] The *union* of two intuitionistic fuzzy soft sets $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ over a universe $U$ is an intuitionistic fuzzy soft set denoted by $(\widetilde{H}, C)$, where: $C = A \cup B$, and

$$\widetilde{H}(\epsilon) = \begin{cases} \widetilde{F}(\epsilon), & if \quad \epsilon \in A - B, \\ \widetilde{G}(\epsilon), & if \quad \epsilon \in B - A \\ \widetilde{F}(\epsilon) \vee \widetilde{G}(\epsilon), & if \quad \epsilon \in A \cup B, \end{cases}$$

for all $\epsilon \in C$. This is denoted by:

$$(\widetilde{H}, C) = (\widetilde{F}, A) \cup (\widetilde{G}, B).$$

In contrast with the above definitions of intuitionistic fuzzy soft set union and intersection, we may sometimes adopt different definitions of union and intersection given as follows.

**Definition 2.7.** [6] Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft sets over a universe $U$ such that $A \cap B \neq \varnothing$. The *bi-union* of $(\widetilde{F}, A)$ and $\widetilde{G}, B)$ is defined to be the intuitionistic fuzzy soft set $(\widetilde{H}, C)$, where: $C = A \cap B$, and

$$\widetilde{H}(\alpha) = \widetilde{F}(\alpha) \cup \widetilde{G}(\alpha), \quad \forall \alpha \in C.$$

This is denoted by:

$$(\widetilde{H}, C) = (\widetilde{F}, A)\widetilde{\sqcup}(\widetilde{G}, B).$$

**Definition 2.8.** [6] Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft sets over a universe $U$ such that $A \cap B \neq \varnothing$. The *bi-intersection* of $(\bar{F}, A)$ and $(\bar{G}, B)$ is defined to be the intuitionistic fuzzy soft set $(H, C)$, where: $C = A \cap B$, and

$$\bar{H}(\alpha) = \widetilde{F}(\alpha) \cap \widetilde{G}(\alpha), \quad \forall \alpha \in C.$$

This is denoted by:

$$(\widetilde{H}, C) = (\widetilde{F}, A)\widetilde{\sqcap}(\widetilde{G}, B).$$

**Definition 2.9.** [6] Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic soft sets over a universe $U$. The *product* of $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ is defined to be the intuitionistic soft set $(\widetilde{F} \circ \widetilde{G}, C)$, where:$C = A \cup B$, and for all $\epsilon \in C$ and $x \in U$:

$$\mu_{(\widetilde{F} \circ \widetilde{G})(\epsilon)}(x) = \begin{cases} \mu_{(\widetilde{F})(\epsilon)}(x), & if \quad \epsilon \in A - B, \\ \mu_{(\widetilde{G})(\epsilon)}(x), & if \quad \epsilon \in B - A, \\ \bigvee\limits_{x = x_1 x_2} \big(\mu_{(\widetilde{F})(\epsilon)}(x_1) \wedge \mu_{(\widetilde{G})(\epsilon)}(x_2)\big), & if \quad \epsilon \in A \cap B, \end{cases}$$

and

$$\gamma_{(\widetilde{F} \circ \widetilde{G})(\epsilon)}(x) = \begin{cases} \gamma_{(\widetilde{F})(\epsilon)}(x), & if \quad \epsilon \in A - B, \\ \gamma_{(\widetilde{G})(\epsilon)}(x), & if \quad \epsilon \in B - A, \\ \bigwedge\limits_{x = x_1 x_2} \big(\gamma_{(\widetilde{F})(\epsilon)}(x_1) \vee \gamma_{(\widetilde{G})(\epsilon)}(x_2)\big), & if \quad \epsilon \in A \cap B. \end{cases}$$

This is denoted by:

$$(\widetilde{F} \circ \widetilde{G}, C) = (\widetilde{F}, A) \circ (\widetilde{G}, B).$$

**Definition 2.10.** [11] Let $\widehat{\omega} = (\widetilde{F}, A)$ be an *intuitionistic L-fuzzy soft set* over $U$ and $A \subseteq E$. An $(s,t)$-level intuitionistic $L$-fuzzy soft set $L(\widehat{\omega}; (s,t))$ of $\widehat{\omega}$ is a crisp soft set, where:

$$L(\widehat{\omega}; (s,t)) = \big(\widetilde{F}_{(s,t)}, A\big) = \{x \in U : \mu_{\widetilde{(F)}(a)}(x) \geq s,\ \gamma_{(\widetilde{F})(a)}(x) \leq t,\ \forall a \in A\},$$

with $s, t \in [0,1]$. In this definition $s$,$t \in [0,1]$ are also called threshold values in which $s \in [0,1]$ can be regarded as a given least threshold on membership values and $t \in [0,1]$ can be regarded as a given greatest threshold on non-membership values.

**Definition 2.11.** [10] Let $f : R \to S$ and $g : A \to B$ be two functions, where $A$ and $B$ are parameter sets for intuitionistic $L$-fuzzy soft sets $R$ and $S$, respectively. Then the pair $(f, g)$ is called an *intuitionistic L-fuzzy soft function* from $R$ to $S$.

**Definition 2.12.** [10] Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic $L$-fuzzy soft rings over $R$ and $S$, respectively. Let $f : R \to S$ be a homomorphism of rings, and $g : A \to B$ be a mapping of sets. We say that $(f, g) : (\widetilde{F}, A) \to (\widetilde{G}, B)$ is an *intuitionistic L-fuzzy soft homomorphism* of intuitionistic $L$-fuzzy soft rings and define by $f(\widetilde{F}, A) = (\widetilde{G}, B)g$, if the following conditions are satisfied

$$f(\mu_{\widetilde{F}(\epsilon)}(x)) = (\mu_{\widetilde{G}(\epsilon)}(x))g, f(\gamma_{\widetilde{F}(\epsilon)}(x)) = (\gamma_{\widetilde{G}(\epsilon)}(x))g$$

**Definition 2.13.** [10] Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic $L$-fuzzy soft rings over $R$ and $S$. Let $(f, g)$ be an intuitionistic $L$-fuzzy soft function from $R$ to $S$.

1. The *image* of $(\widetilde{F}, A)$ under $(f, g)$, denoted by $(f, g)(\widetilde{F}, A)$, is the intuitionistic $L$-fuzzy soft ring over $S$ defined by:
$$(f, g)(\widetilde{F}, A) = \big(f(\widetilde{F}), g(A)\big),$$
where for all $k \in g(A)$ and $s \in S$:
$$f(\widetilde{F})_k(s) = \begin{cases} \bigvee\limits_{f(r)=s} \bigvee\limits_{g(a)=k} \mu_{\widetilde{F}(\epsilon)}(r), & \text{if } r \in f^{-1}(s), \\ 0, & \text{otherwise}, \end{cases}$$
$$f(\widetilde{F})_k(s) = \begin{cases} \bigwedge\limits_{f(r)=s} \bigwedge\limits_{g(a)=k} \gamma_{\widetilde{F}(a)}(r), & \text{if } r \in f^{-1}(s), \\ 1, & \text{otherwise}. \end{cases}$$
2. The *preimage* of $(\widetilde{G}, B)$ under $(f, g)$, denoted by $(f, g)^{-1}(G, B)$, is the intuitionistic $L$-fuzzy soft ring over $R$ defined by:
$$(f, g)^{-1}(\widetilde{G}, B) = \big(f^{-1}(\widetilde{G}), g^{-1}(B)\big),$$
where:
$$f^{-1}(\widetilde{G})_{(a)}(r) = (\widetilde{G})_{(g(a)}(f(r)), \quad \forall a \in g^{-1}(B),\ \forall r \in R.$$

If $f$ and $g$ are injective (respectively, surjective), then $(f, g)$ is said to be injective (respectively, surjective).

**Definition 2.14.** [13] Let $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$. An *intuitionistic fuzzy point*, written as $x_{(\alpha,\beta)}$, is defined to be an intuitionistic fuzzy subset of $R$, given by:
$$x_{(\alpha,\beta)}(y) = \begin{cases} (\alpha, \beta), & \text{if } x = y, \\ (0, 1), & \text{if } x \neq y. \end{cases}$$

An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to belong to IFS$(\mu, \gamma)$, denoted by:
$$x_{(\alpha,\beta)} \in (\mu, \gamma),$$
if $\mu(x) \geq \alpha \quad$ and $\quad \gamma(x) \leq \beta$.. Moreover, for $x, y \in R$, we have:
$$x_{(t,s)} + y_{(\alpha,\beta)} = (x + y)_{(t\wedge\alpha,\, s\vee\beta)},$$
$$x_{(t,s)} \cdot y_{(\alpha,\beta)} = (xy)_{(t\wedge\alpha,\, s\vee\beta)}.$$

## 3. Intuitionistic L-fuzzy soft semi-ideals

Now, we are ready to give the definition of the intuitionistic $L$-fuzzy soft semi-ideal and examine it.

**Definition 3.1.** An intuitionistic $L$-fuzzy soft set $(F, A)$ over a semiring $S$ is called an *intuitionistic $L$-fuzzy soft semi-ideal* over $S$ if the following conditions are satisfied:

1. $\mu_{\widetilde{F}(\epsilon)}(x - y) \geq \mu_{\widetilde{F}(\epsilon)}(x) \wedge \mu_{\widetilde{F}(\epsilon)}(y) \quad$ and $\quad \gamma_{\widetilde{F}(\epsilon)}(x - y) \leq \gamma_{\widetilde{F}(\epsilon)}(x) \vee \gamma_{\widetilde{F}(\epsilon)}(y)$,
2. $\mu_{\widetilde{F}(\epsilon)}(xy) \geq \mu_{\widetilde{F}(\epsilon)}(x) \vee \mu_{\widetilde{F}(\epsilon)}(y) \quad$ and $\quad \gamma_{\widetilde{F}(\epsilon)}(xy) \leq \gamma_{\widetilde{F}(\epsilon)}(x) \wedge \gamma_{\widetilde{F}(\epsilon)}(y)$,

for all $x, y \in S$ and $\epsilon \in A$.

**Definition 3.2.** For any intuitionistic $L$-fuzzy soft set $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle \mid x \in X\}$, the sets $\Box A$ and $\Diamond A$ are defined as follows:

$$\Box A = \{\langle x, \mu_A(x), \mu_A^c(x)\rangle \mid x \in X\},$$
$$\Diamond A = \{\langle x, \gamma_A^c(x), \gamma_A(x)\rangle \mid x \in X\}.$$

*Theorem 3.3*
Let $(\widetilde{F}, A)$ be an intuitionistic $L$-fuzzy soft semi-ideal over a semiring $S$. Then $(\Box\widetilde{F}, A)$ and $(\Diamond\widetilde{F}, A)$ are intuitionistic $L$-fuzzy soft semi-ideals over the semiring $S$.

*Proof*
Since $(\widetilde{F}, A)$ is an intuitionistic $L$-fuzzy soft semi-ideal over $S$, for all $x, y \in S$ and $\epsilon \in A$, we have:

$$\mu_{\widetilde{F}(\epsilon)}(x-y) \geq \mu_{\widetilde{F}(\epsilon)}(x) \wedge \mu_{\widetilde{F}(\epsilon)}(y)$$

,

$$\mu_{\widetilde{F}(\epsilon)}(xy) \geq \mu_{\widetilde{F}(\epsilon)}(x) \vee \mu_{\widetilde{F}(\epsilon)}(y)$$

. Then we obtain

$$\begin{aligned} \mu^c_{\widetilde{F})(\epsilon)}(x-y) &\geq \mu^c_{\widetilde{F}(\epsilon)}(x) \wedge \mu^c_{\widetilde{F}(\epsilon)}(y), \\ &\leq (1-\mu^c_{\widetilde{F}(\epsilon)}(x)) \vee (1-\mu^c_{\widetilde{F}(\epsilon)}(y)), \\ \mu^c_{\widetilde{F}(\epsilon)}(x-y) &\leq \mu^c_{\widetilde{F}(\epsilon)}(x) \vee \mu^c_{\widetilde{F}(\epsilon)}(y). \end{aligned}$$

and

$$\begin{aligned} \mu^c_{\widetilde{F}(\epsilon)}(xy) &\geq \mu^c_{\widetilde{F}(\epsilon)}(x) \vee \mu^c_{\widetilde{F}(\epsilon)}(y), \\ &\leq (1-\mu^c_{\widetilde{F}(\epsilon)}(x)) \wedge (1-\mu^c_{\widetilde{F}(\epsilon)}(y)), \\ \mu^c_{\widetilde{F}(\epsilon)}(xy) &\leq \mu^c_{\widetilde{F}(\epsilon)}(x) \wedge \mu^c_{\widetilde{F}(\epsilon)}(y). \end{aligned}$$

Hence, $(\Box\widetilde{F}, A)$ is a intuitionistics L-fuzzy soft semi-ideals over the semiring. Now

$$\gamma_{\widetilde{F}(\epsilon)}(x-y) \leq \gamma_{\widetilde{F}(\epsilon)}(x) \vee \gamma_{\widetilde{F}(\epsilon)}(y), \quad \gamma_{\widetilde{F}(\epsilon)}(xy) \leq \gamma_{\widetilde{F}(\epsilon)}(x) \wedge \gamma_{\widetilde{F}(\epsilon)}(y).$$

Then we obtain

$$\begin{aligned} \gamma^c_{\widetilde{F}(\epsilon)}(x-y) &\leq \gamma^c_{\widetilde{F}(\epsilon)}(x) \vee \gamma^c_{\widetilde{F}(\epsilon)}(y), \\ &\geq (1-\gamma^c_{\widetilde{F}(\epsilon)}(x)) \wedge (1-\gamma^c_{\widetilde{F}(\epsilon)}(y)), \\ \gamma^c_{\widetilde{F}(\epsilon)}(x-y) &\geq \gamma^c_{\widetilde{F}(\epsilon)}(x) \wedge \gamma^c_{\widetilde{F}(\epsilon)}(y). \end{aligned}$$

and

$$\begin{aligned} \gamma^c_{\widetilde{F}(\epsilon)}(xy) &\leq \gamma^c_{\widetilde{F}(\epsilon)}(x) \wedge \gamma^c_{\widetilde{F}(\epsilon)}(y), \\ &\geq (1-\gamma^c_{\widetilde{F}(\epsilon)}(x)) \vee (1-\gamma^c_{\widetilde{F}(\epsilon)}(y)), \\ \gamma^c_{\widetilde{F}(\epsilon)}(x-y) &\geq \gamma^c_{\widetilde{F}(\epsilon)}(x) \vee \gamma^c_{\widetilde{F}(\epsilon)}(y). \end{aligned}$$

Thus, $(\Diamond\widetilde{F}, A)$ is a intuitionistics L-fuzzy soft semi-ideals over the semiring. □

*Theorem 3.4*
$(\widetilde{F}, A)$ is an intuitionistic $L$-fuzzy soft semi-ideal over a semiring $S$ if and only if $(\widetilde{F}(\epsilon)^{r,t}, A)$ is a soft semi-ideal of $S$ for all $r \in (0, 1]$, $t \in [0, 1)$, and $\epsilon \in A$.

*Proof*
Let $(\widetilde{F}, A)$ be an intuitionistic $L$-fuzzy soft semi-ideal over $S$ and let $x, y \in \widetilde{F}(\epsilon)^{r,t}$, $s \in S$. Then:

$$\mu_{\widetilde{F}(\epsilon)}(x) \geq r, \quad \mu_{\widetilde{F}(\epsilon)}(y) \geq r, \quad \gamma_{\widetilde{F}(\epsilon)}(x) \leq t, \quad \gamma_{\widetilde{F}(\epsilon)}(y) \leq t.$$

So, we have

$$\mu_{\widetilde{F}(\epsilon)}(x-y) \geq \mu_{\widetilde{F}(\epsilon)}(x) \wedge \mu_{\widetilde{F}(\epsilon)}(y) \geq r$$

and

$$\mu_{\widetilde{F}(\epsilon)}(xy) \geq \mu_{\widetilde{F}(\epsilon)}(x) \vee \mu_{\widetilde{F}(\epsilon)}(y) \leq t$$

by the assumption. Hence, $x - y \in \widetilde{F}(\epsilon)^{r,t}$. Also, Similarly, for multiplication:

$$\mu_{\widetilde{F}(\epsilon)}(sx) \geq \mu_{\widetilde{F}(\epsilon)}(s) \wedge \mu_{\widetilde{F}(\epsilon)}(x) \geq r$$

and

$$\gamma_{\widetilde{F}(\epsilon)}(sx) \leq \mu_{\widetilde{F}(\epsilon)}(x) \wedge \gamma_{\widetilde{F}(\epsilon)}(x) \leq t$$

Thus, $sx \in \widetilde{F}(\epsilon)^{r,t}$, and $\widetilde{F}(\epsilon)^{r,t}$ is a semi-ideal of $S$.

Conversely, let $\widetilde{F}(\epsilon)^{r,t}$ be a semi-ideal of $S$. Then $x - y \in \widetilde{F}(\epsilon)^{r,t}$ and $sx \in \widetilde{F}(\epsilon)^{r,t}$ for all $x, y \in \widetilde{F}(\epsilon)^{r,t}$ and $s \in S$. Suppose that $\mu_{\widetilde{F}(\epsilon)}(x) = r_1$, $\mu_{\widetilde{F}(\epsilon)}(y) = r_2$ with $r_1 \wedge r_2 = r_2$, and $\gamma_{\widetilde{F}(\epsilon)}(x) = t_1$, $\gamma_{\widetilde{F}(\epsilon)}(y) = t_2$ with $t_1 \vee t_2 = t_2$. So, we can write $x - y \in \widetilde{F}(\epsilon)^{r_2,t_2}$. Thus:

$$\mu_{\widetilde{F}(\epsilon)}(x-y) \geq r_2 = \mu_{\widetilde{F}(\epsilon)}(x) \wedge \mu_{\widetilde{F}(\epsilon)}(y),$$

and

$$\gamma_{\widetilde{F}(\epsilon)}(x-y) \wedge \mu_{\widetilde{F}(\epsilon)}(y), \leq t_2 = \gamma_{\widetilde{F}(\epsilon)}(x) \vee \gamma_{\widetilde{F}(\epsilon)}(y),$$

Similarly, for multiplication:

$$\mu_{\widetilde{F}(\epsilon)}(sx) \geq \mu_{\widetilde{F}(\epsilon)}(s) \vee \mu_{\widetilde{F}(\epsilon)}(x)$$

and

$$\gamma_{\widetilde{F}(\epsilon)}(sx) \leq \gamma_{\widetilde{F}(\epsilon)}(s) \wedge \gamma_{\widetilde{F}(\epsilon)}(x).$$

Hence $(\widetilde{F}, A)$ satisfies the semi-ideal conditions. □

*Remark 3.5.* This proposed structure generalizes: The Classical intuitionistic fuzzy ideals over semigroups, which deal only with a single operation (addition) and do not involve parameters or lattice values. The $L$-fuzzy soft structures by incorporating both membership and non-membership functions along with hesitation degrees. Soft sets by including parameterization via $A$. It allows modeling of multi-criteria decision systems, uncertain environments, and approximate algebraic structures, making it suitable for applications in medical diagnosis, engineering, optimization, pattern recognition, and information processing systems.

**Example 3.6.** Consider the semiring $S = (\mathbb{N} \cup \{0\}, +, \cdot)$ and let $A = \{\epsilon\}$. Define $(\widetilde{F}, A)$ by:

$$\mu_{\widetilde{F}(\epsilon)}(s) = e^{-s}, \quad \gamma_{\widetilde{F}(\epsilon)}(s) = 1 - \frac{e^{-s}}{2}.$$

Then, for any $r \in (0, 1]$ and $t \in [0, 1)$, we have:

$$\widetilde{F}(\epsilon)^{r,t} = \{s \in S \mid e^{-s} \geq r,\ 1 - \frac{e^{-s}}{2} \leq t\},$$

that is:

$$\widetilde{F}(\epsilon)^{r,t} = \{s \in S \mid s \leq -\ln r,\ s \geq -2\ln(1-t)\}.$$

These level sets are semi-ideals, confirming that $(\widetilde{F}, A)$ is an intuitionistic $L$-fuzzy soft semi-ideal.

*Theorem 3.7*
Let $(\widetilde{F},A)$ and $(\widetilde{G},B)$ be two intuitionistic $L$-fuzzy soft semi-ideals over a semiring $S$. Then so are $(\widetilde{F},A)\widetilde{\wedge}\,(\widetilde{G},B)$ and $(\widetilde{F},A)\widetilde{\sqcap}(\widetilde{G},B)$.

*Proof*
we have $(\widetilde{H},C)=(\widetilde{F},A)\ \widetilde{\wedge}(\widetilde{G},B)$, where $C=A\times B$ and: $\widetilde{H}(\alpha,\beta)=\widetilde{F}(\alpha)\cap\widetilde{G}(\beta)$, for all $(\alpha,\beta)\in C$ from Definition 2.3. Since $(\widetilde{F},A)$ and $(\widetilde{G},B)$ are intuitionistic $L$-fuzzy soft semi-ideals, for all $x,y\in S$ and $(\alpha,\beta)\in C$, we have:

$$\begin{aligned}\mu_{\widetilde{H}(\alpha,\beta)}(x-y) &= \{\mu_{\widetilde{F}(\alpha)}(x-y)\wedge\mu_{\widetilde{G}(\beta)}(x-y)\}\\ &\geq \{\big(\mu_{\widetilde{F}(\alpha)}(x)\wedge\mu_{\widetilde{F}(\alpha)}(y)\big)\wedge\big(\mu_{\widetilde{G}(\beta)}(x)\wedge\mu_{\widetilde{G}(\beta)}(y)\big)\}\\ \mu_{\widetilde{H}(\alpha,\beta)}(x-y) &= \{\mu_{\widetilde{H}(\alpha,\beta)}(x)\wedge\mu_{\widetilde{H}(\alpha,\beta)}(y)\},\end{aligned}$$

and

$$\begin{aligned}\gamma_{\widetilde{H}(\alpha,\beta)}(x-y) &= \{\gamma_{\widetilde{F}(\alpha)}(x-y)\vee\gamma_{\widetilde{G}(\beta)}(x-y)\}\\ &\leq \{\big(\gamma_{\widetilde{F}(\alpha)}(x)\vee\gamma_{\widetilde{F}(\alpha)}(y)\big)\vee\big(\gamma_{\widetilde{G}(\beta)}(x)\vee\gamma_{\widetilde{G}(\beta)}(y)\big)\}\\ \gamma_{\widetilde{H}(\alpha,\beta)}(x-y) &= \{\gamma_{\widetilde{H}(\alpha,\beta)}(x)\vee\gamma_{\widetilde{H}(\alpha,\beta)}(y)\},\end{aligned}$$

$$\begin{aligned}\mu_{\widetilde{H}(\alpha,\beta)}(xy) &= \{\mu_{\widetilde{F}(\alpha)}(xy)\wedge\mu_{\widetilde{G}(\beta)}(xy)\}\\ &\geq \{\big(\mu_{\widetilde{F}(\alpha)}(x)\vee\mu_{\widetilde{F}(\alpha)}(y)\big)\wedge\big(\mu_{\widetilde{G}(\beta)}(x)\vee\mu_{\widetilde{G}(\beta)}(y)\big)\}\\ &\geq \{\big(\mu_{\widetilde{F}(\alpha)}(x)\wedge\mu_{\widetilde{F}(\alpha)}(y)\big)\vee\big(\mu_{\widetilde{G}(\beta)}(x)\wedge\mu_{\widetilde{G}(\beta)}(y)\big)\}\\ \mu_{\widetilde{H}(\alpha,\beta)}(xy) &= \{\mu_{\widetilde{H}(\alpha,\beta)}(x)\wedge\mu_{\widetilde{H}(\alpha,\beta)}(y)\},\end{aligned}$$

and

$$\begin{aligned}\gamma_{\widetilde{H}(\alpha,\beta)}(xy) &= \{\gamma_{\widetilde{F}(\alpha)}(xy)\vee\gamma_{\widetilde{G}(\beta)}(xy)\}\\ &\leq \{\big(\gamma_{\widetilde{F}(\alpha)}(x)\wedge\gamma_{\widetilde{F}(\alpha)}(y)\big)\vee\big(\gamma_{\widetilde{G}(\beta)}(x)\wedge\gamma_{\widetilde{G}(\beta)}(y)\big)\}\\ &\leq \{\big(\gamma_{\widetilde{F}(\alpha)}(x)\vee\gamma_{\widetilde{F}(\alpha)}(y)\big)\wedge\big(\gamma_{\widetilde{G}(\beta)}(x)\vee\gamma_{\widetilde{G}(\beta)}(y)\big)\}\\ \gamma_{\widetilde{H}(\alpha,\beta)}(xy) &= \{\gamma_{\widetilde{H}(\alpha,\beta)}(x)\wedge\gamma_{\widetilde{H}(\alpha,\beta)}(y)\}.\end{aligned}$$

Thus, $(\widetilde{F},A)\widetilde{\wedge}(\widetilde{G},B)$ is an intuitionistic $L$-fuzzy soft semi-ideal over $S$. The proof for $(\widetilde{F},A)\widetilde{\sqcap}(\widetilde{G},B)$ is similar. □

*Theorem 3.8*
Let $(\widetilde{F},A)$ and $(\widetilde{G},B)$ be two intuitionistic $L$-fuzzy soft semi-ideals over a semiring $S$. Then so are $(\widetilde{F},A)\cap(\widetilde{G},B)$ and $(\widetilde{F},A)\cup(\widetilde{G},B)$.

*Proof*
For any $x,y\in S$ and $\epsilon\in C$, We consider three cases:
**Case 1:** $\epsilon\in A\setminus B$. Then we have

$$\mu_{\widetilde{H}(\epsilon)}(x-y)=\mu_{\widetilde{F}(\epsilon)}(x-y)\geq\mu_{\widetilde{F}(\epsilon)}(x)\wedge\mu_{\widetilde{F}(\epsilon)}(y)=\mu_{\widetilde{H}(\epsilon)}(x)\wedge\mu_{\widetilde{H}(\epsilon)}(y),$$

$$\gamma_{\widetilde{H}(\epsilon)}(x-y)=\gamma_{\widetilde{F}(\epsilon)}(x-y)\leq\gamma_{\widetilde{F}(\epsilon)}(x)\vee\gamma_{\widetilde{F}(\epsilon)}(y)=\gamma_{\widetilde{H}(\epsilon)}(x)\vee\gamma_{\widetilde{H}(\epsilon)}(y),$$

and

$$\mu_{\widetilde{H}(\epsilon)}(xy)=\mu_{\widetilde{F}(\epsilon)}(xy)\geq\mu_{\widetilde{F}(\epsilon)}(x)\vee\mu_{\widetilde{F}(\epsilon)}(y)=\mu_{\widetilde{H}(\epsilon)}(x)\vee\mu_{\widetilde{H}(\epsilon)}(y),$$

$$\gamma_{\widetilde{H}(\epsilon)}(xy) = \gamma_{\widetilde{F}}(xy) \leq \gamma_{\widetilde{F}(\epsilon)}(x) \wedge \gamma_{\widetilde{F}(\epsilon)}(y) = \gamma_{\widetilde{H}(\epsilon)}(x) \wedge \gamma_{\widetilde{H}(\epsilon)}(y).$$

**Case 2:** $\epsilon \in B \setminus A$. Analogous to Case (i).
**Case 3:** $\epsilon \in A \cap B$. Here $\widetilde{H}(\epsilon) = \widetilde{F}(\epsilon) \cap \widetilde{G}(\epsilon)$. It follows that

$$\begin{aligned}
\mu_{\widetilde{H}(\epsilon)}(x-y) &= \{\mu_{\widetilde{F}(\epsilon)}(x-y) \wedge \mu_{\widetilde{G}(\epsilon)}(x-y)\} \\
&\geq \{\big(\mu_{\widetilde{F}(\epsilon)}(x) \wedge \mu_{\widetilde{F}(\epsilon)}(y)\big) \wedge \big(\mu_{\widetilde{G}(\epsilon)}(x) \wedge \mu_{\widetilde{G}(\epsilon)}(y)\big)\} \\
&= \{\mu_{\widetilde{F}(\epsilon)}(x) \wedge \mu_{\widetilde{G}(\epsilon)}(x)\} \wedge \{\mu_{\widetilde{F}(\epsilon)}(y) \wedge \mu_{\widetilde{G}(\epsilon)}(y)\} \\
\mu_{\widetilde{H}(\epsilon)}(x-y) &= \{\mu_{\widetilde{H}(\epsilon)}(x) \wedge \mu_{\widetilde{H}(\epsilon)}(y)\}
\end{aligned}$$

and

$$\begin{aligned}
\gamma_{\widetilde{H}(\epsilon)}(x-y) &= \{\gamma_{\widetilde{F}(\epsilon)}(x-y) \vee \gamma_{\widetilde{G}(\epsilon)}(x-y)\} \\
&\leq \{\big(\gamma_{\widetilde{F}(\epsilon)}(x) \vee \gamma_{\widetilde{F}(\epsilon)}(y)\big) \vee \big(\gamma_{\widetilde{G}(\epsilon)}(x) \vee \gamma_{\widetilde{G}(\epsilon)}(y)\big)\} \\
&= \{\big(\gamma_{\widetilde{F}(\epsilon)}(x) \vee \gamma_{\widetilde{G}(\epsilon)}(x)\big) \vee \big(\gamma_{\widetilde{F}(\epsilon)}(y) \vee \gamma_{\widetilde{G}(\epsilon)}(y)\big)\} \\
\gamma_{\widetilde{H}(\epsilon)}(x-y) &= \{\gamma_{\widetilde{H}(\epsilon)}(x) \vee \gamma_{\widetilde{H}(\epsilon)}(y)\},
\end{aligned}$$

$\mu_{\widetilde{H}(\epsilon)}(xy) = \{\mu_{\widetilde{H}(\epsilon)}(x) \vee \mu_{\widetilde{H}(\epsilon)}(y)\}$ and $\gamma_{\widetilde{H}(\epsilon)}(xy) = \{\gamma_{\widetilde{H}(\epsilon)}(x) \wedge \gamma_{\widetilde{H}(\epsilon)}(y)\}$, are similarly proved.

Consequently, $(\widetilde{F}, A) \cap (\widetilde{G}, B)$ is an intuitionistic $L$-fuzzy soft semi-ideal over a semiring S. $(\widetilde{F}, A) \cup (\widetilde{G}, B)$ can be similarly proved as well. □

*Theorem 3.9*
Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic $L$-fuzzy soft semi-ideals over a semiring $S$. Then so is $(\widetilde{F}, A) \circ (\widetilde{G}, B)$.

*Proof*
Let $x, y \in S$ and let $\epsilon \in A \cup B$, we consider the following cases defending on the membership of $\epsilon$.
**Case 1:** $\epsilon \in A \setminus B$. Then we get:

$$\mu_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(x-y) = \mu_{\widetilde{F}(\epsilon)}(x-y) \geq \mu_{\widetilde{F}(\epsilon)}(x) \wedge \mu_{\widetilde{F}(\epsilon)}(y) = \mu_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(x) \wedge \mu_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(y),$$

$$\gamma_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(x-y) = \gamma_{\widetilde{F}(\epsilon)}(x-y) \leq \gamma_{\widetilde{F}(\epsilon)}(x) \vee \gamma_{\widetilde{F}(\epsilon)}(y) = \gamma_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(x) \vee \gamma_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(y),$$

and

$$\mu_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(xy) = \mu_{\widetilde{F}(\epsilon)}(xy) \geq \mu_{\widetilde{F}(\epsilon)}(x) \vee \mu_{\widetilde{F}(\epsilon)}(y) = \mu_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(x) \vee \mu_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(y),$$

$$\gamma_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(xy) = \gamma_{\widetilde{F}(\epsilon)}(xy) \leq \gamma_{\widetilde{F}(\epsilon)}(x) \wedge \gamma_{\widetilde{F}(\epsilon)}(y) = \gamma_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(x) \wedge \gamma_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(y).$$

Hence the ideal properties hold
**Case 2:** $\epsilon \in B \setminus A$. Analogous to Case (i).
**Case 3:** $\epsilon \in A \cap B$. Then we obtain

$$\begin{aligned}
\mu_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(x) &= \bigvee_{x=x_1x_2} \{\mu_{\widetilde{F}(\epsilon)}(x_1) \wedge \mu_{\widetilde{G}(\epsilon)}(x_2)\} \\
&\leq \bigvee_{xy=x_1x_2y} \{\big(\mu_{\widetilde{F}(\epsilon)}(x_1y) \wedge \mu_{\widetilde{G}(\epsilon)}(x_2y)\} \\
&\leq \bigvee_{xy=zt} \{\mu_{\widetilde{F}(\epsilon)}(z) \wedge \mu_{\widetilde{G}(\epsilon)}(t)\} \\
\mu_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(x) &= \{\mu_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(xy)\}
\end{aligned}$$

Similarly, we can write $\mu_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(y) \leq \{\mu_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(xy)\}$.
Therefore $\{\mu_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(xy)\} \geq \{\mu_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(x)\} \vee \{\mu_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(y)\}$

$$\begin{aligned}
\gamma_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(x) &= \bigwedge_{x=x_1x_2} \{\gamma_{\widetilde{F}(\epsilon)}(x_1) \vee \gamma_{\widetilde{G}(\epsilon)}(x_2)\} \\
&\geq \bigwedge_{xy=x_1x_2y} \{(\gamma_{\widetilde{F}(\epsilon)}(x_1y) \vee \gamma_{\widetilde{G}(\epsilon)}(x_2y)\} \\
&\geq \bigwedge_{xy=zt} \{\gamma_{\widetilde{F}(\epsilon)}(z) \vee \gamma_{\widetilde{G}(\epsilon)}(t)\} \\
\gamma_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(x) &= \{\gamma_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(xy)\}
\end{aligned}$$

Similarly, we can write $\gamma_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(y) \geq \{\gamma_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(xy)\}$.
Therefore, $\{\gamma_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(xy)\} \leq \{\gamma_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(x)\} \wedge \{\gamma_{(\widetilde{F}\circ\widetilde{G})(\epsilon)}(y)\}$. Then the proof is completed. □

## 4. Intuitionistic *L*-Fuzzy Soft Semiring Homomorphism

**Definition 4.1.** Let $A = \{\langle x, \mu_{\widetilde{F}(A)}(x), \gamma_{\widetilde{F}(A)}(x)\rangle \mid x \in R\}$ be an intuitionistic $L$-fuzzy soft semiring. Let $B = \{\langle x, \mu_{\widetilde{F}(B)}(x), \gamma_{\widetilde{F}(B)}(x)\rangle \mid x \in R\}$ be another intuitionistic $L$-fuzzy soft semiring with $A \subseteq B$.
Then $A$ is called an *intuitionistic $L$-fuzzy soft ideal* of $B$ if for all $x, y \in R$:

1. $\mu_{\widetilde{F}(A)}(x-y) \geq \mu_{\widetilde{F}(A)}(x) \wedge \mu_{\widetilde{F}(A)}(y)$, and $\gamma_{\widetilde{F}(A)}(x-y) \leq \gamma_{\widetilde{F}(A)}(x) \vee \gamma_{\widetilde{F}(A)}(y)$.
2. $\mu_{\widetilde{F}(A)}(xy) \geq (\mu_{\widetilde{F}(B)}(x) \wedge \mu_{\widetilde{F}(A)}(y)) \vee (\mu_{\widetilde{F}(A)}(x) \wedge \mu_{\widetilde{F}(B)}(y))$, and $\gamma_{\widetilde{F}(A)}(xy) \leq (\gamma_{\widetilde{F}(B)}(x) \vee \gamma_{\widetilde{F}(A)}(y)) \wedge (\gamma_{\widetilde{F}(A)}(x) \vee \gamma_{\widetilde{F}(B)}(y))$

Since $R$ is commutative, $\gamma_{\widetilde{F}(A)}(xy) \leq (\gamma_{\widetilde{F}(B)}(x) \vee \gamma_{\widetilde{F}(A)}(y)) \wedge (\gamma_{\widetilde{F}(A)}(x) \vee \gamma_{\widetilde{F}(B)}(y))$ for all $x, y \in R$ if and only if $\gamma_{\widetilde{F}(A)}(xy) \leq (\gamma_{\widetilde{F}(B)}(x) \vee \gamma_{\widetilde{F}(A)}(y))$ , for all $x, y \in R$.

*Theorem 4.2*
Let $B = \{\langle x, \mu_{\widetilde{F}(B)}(x), \gamma_{\widetilde{F}(B)}(x)\rangle \mid x \in R\}$ be an intuitionistic $L$-fuzzy soft semiring. Let $A = \{\langle x, \mu_{\widetilde{F}(A)}(x), \gamma_{\widetilde{F}(A)}(x)\rangle \mid x \in R\}$ be an intuitionistic $L$-fuzzy soft ideal of $B$. Let $f : R \to S$ be an onto homomorphism. Then $f(A)$ is an intuitionistic $L$-fuzzy soft ideal of $f(B)$.

*Proof*
Clearly $f(A)$ and $f(B)$ are intuitionistic $L$-fuzzy soft semirings of $S$ with $f(A) \subseteq f(B)$. For all $x, y \in S$:

$$\begin{aligned}
f(\gamma_{\widetilde{F}(A)})(xy) &= \wedge\{\gamma_{\widetilde{F}(A)}(w) : w \in R, f(w) = xy\} \\
&\leq \wedge\{\gamma_{\widetilde{F}(A)}(uv) : u, v \in R, f(u) = x, f(v) = y\} \\
&\leq \wedge\{\gamma_{\widetilde{F}(B)}(u) \vee \gamma_{\widetilde{F}(A)}(v)/f(u) = x, f(v) = y \ : u, v \in R\} \\
&= (\wedge\{\gamma_{\widetilde{F}(B)}(u)/u \in R, f(u) = x\}) \vee (\wedge\{\gamma_{\widetilde{F}(A)}(v)/v \in R, f(v) = y\,\} \\
f(\gamma_{\widetilde{F}(A)})(xy) &= f(\gamma_{\widetilde{F}(B)})(x) \vee f(\gamma_{\widetilde{F}(A)})(y)
\end{aligned}$$

Thus $f(A)$ is an intuitionistic $L$-fuzzy soft ideal of $f(B)$. □

*Theorem 4.3*
Let $f : R \to S$ be an onto homomorphism. Let $B = \{\langle x, \mu_{\widetilde{F}(B)}(x), \gamma_{\widetilde{F}(B)}(x)\rangle \mid x \in S\}$ be an intuitionistic $L$-fuzzy soft semiring of S. Let $A = \{\langle x, \mu_{\widetilde{F}(A)}(x), \gamma_{\widetilde{F}(A)}(x)\rangle \mid x \in S\}$ be an intuitionistic $L$-fuzzy soft ideal of $B$. Then $f^{-1}(A)$ is an intuitionistic $L$-fuzzy soft ideal of $f^{-1}(B)$.

*Proof*
Clearly $f^{-1}(A)$ and $f^{-1}(B)$ are intuitionistic $L$-fuzzy soft semirings of $R$ with $f^{-1}(A) \subseteq f^{-1}(B)$. Now:

$$\begin{aligned} f^{-1}(\gamma_{\widetilde{F}(A)})(xy) &= \gamma_{\widetilde{F}(A)}(f(xy)) \\ &= \gamma_{\widetilde{F}(A)}(f(x)f(y)) \\ &\leq \gamma_{\widetilde{F}(B)}(x) \vee \gamma_{\widetilde{F}(A)})(y) \\ f^{-1}(\gamma_{\widetilde{F}(A)})(xy) &= f^{-1}(\gamma_{\widetilde{F}(B)})(x)) \vee f^{-1}(\gamma_{\widetilde{F}(A)})(x) \end{aligned}$$

Therefore $f^{-1}(A)$ is an intuitionistic $L$-fuzzy soft ideal of $f^{-1}(B)$. □

## 5. Application of Intuitionistic L-Fuzzy Soft Ideal Based Medical Diagnosis System over Semiring

This example demonstrates a medical diagnosis system using intuitionistic $L$-fuzzy soft ideals over semirings. The system models uncertainty in medical symptoms and diagnoses using advanced fuzzy structures.

**Example: Medical Diagnois**

**Step 1: Define Parameters.** Let $S = \{s_1, s_2, s_3\}$ be a semiring of medical tests, where $+$ and $\cdot$ represent test combinations. Let $P = \{p_1, p_2, p_3\}$ be the set of patients, $E = \{e_1 \text{ (fever)}, e_2 \text{ (cough)}, e_3 \text{ (fatigue)}\}$ be the set of symptoms, and $D = \{d_1 \text{ (flu)}, d_2 \text{ (COVID-19)}, d_3 \text{ (allergy)}\}$ be the set of possible diagnoses.

**Step 2: Create Intuitionistic $L$-Fuzzy Soft Ideal.** Define $F : E \to L^S$ such that for each symptom $e \in E$, $F(e)$ is an intuitionistic $L$-fuzzy ideal of $S$.

For example, for symptom $e_1$ (fever):

$$F(e_1)(s_1) = (0.8, 0.1), \quad F(e_1)(s_2) = (0.5, 0.4), \quad F(e_1)(s_3) = (0.3, 0.6),$$

representing strong, moderate, and weak indications of fever respectively. Similarly, $F(e_2)$ and $F(e_3)$ are defined for cough and fatigue.

**Step 3: Patient–Symptom Mapping.** For each patient $p \in P$, define an intuitionistic fuzzy soft set $G_p : E \to L^S$ representing their test results.

For patient $p_1$:

$$G_{p_1}(e_1)(s_1) = (0.9, 0.05)((very\ high\ fever),), \quad G_{p_1}(e_2)(s_2) = (0.6, 0.3), (Moderate\ cough)$$

,

$$G_{p_1}(e_3)(s_3) = (0.4, 0.5)(Some\ fatigue).$$

**Step 4: Diagnosis Decision Making.** Using operations on the semiring $S$ and the lattice $L$, compute similarity measures between patient symptoms and known disease patterns.

For diagnosis $d_1$ (flu) with characteristic pattern:

$$F_{\text{flu}}(e_1) = (0.7, 0.2), \quad F_{\text{flu}}(e_2) = (0.6, 0.3), \quad F_{\text{flu}}(e_3) = (0.5, 0.4),$$

the similarity measure is given by:

$$(p_1, \text{flu}) = \sum_{e \in E} \left[\mu_{p_1}(e) \odot \mu_{\text{flu}}(e) + (1 - \gamma p_1(e)) \odot (1 - \gamma \text{flu}(e))\right].$$

**Step 5: Results Interpretation.** After computing similarity for all diagnoses:

$$p_1 : \quad \text{flu } (0.82), \quad \text{COVID}-19 \ (0.65), \quad \text{allergy } (0.45).$$

The system concludes that patient $p_1$ most likely has the flu.

## 6. Conclusion

This paper introduced intuitionistic L-fuzzy soft ideals within the framework of intuitionistic L-fuzzy soft semirings and studied their properties under homomorphisms. Building on these foundations, the proposed structures extend the algebraic foundation of fuzzy soft semirings and provide new insights into their substructures. To illustrate practical utility, a medical diagnosis example demonstrated their effectiveness in handling uncertainty and the similarity measure. Looking forward, future work will focus on broader applications, computational aspects, and potential extensions to other algebraic systems.

REFERENCES


1. K. T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, **20**(1) (1986), 87–96.
2. Baldev Banarjee and Dhiven Kr. Basnet, Intuitionistic fuzzy subrings and ideals, *Journal of Fuzzy Mathematics*, **11**(1) (2003), 139–155.
3. B. A. Ersoy and N. A. Özkirişci, Intuitionistic fuzzy soft semi-ideals, *Azerbaijan Journal of Mathematics*, **6**(2) (2016), 44–54.
4. D.S.Hooda,Kumari Reetu, On application of fuzzy soft sets in dimension reduction and medical diagnosis, *Advances in Research*, **12**(2) (2017), 1- 9.
5. P. K. Maji, A. R. Roy and R. Biswas, Fuzzy Soft Sets, *Journal of Fuzzy Mathematics*, **9** (2001), 589–602.
6. P. K. Maji, A. R. Roy and R. Biswas, Intuitionistic Fuzzy Soft Sets, *Journal of Fuzzy Mathematics*, **9** (2001), 677–692.
7. P. K. Maji, R. Biswas and A. R. Roy, Soft Sets Theory, *Computers and Mathematics with Applications*, **45** (2003), 555–562.
8. D. Molodtsov, Soft Set Theory – First Results, *Computers and Mathematics with Applications*, **37** (1999), 19–31.
9. S. Naganathan, K. Arjunan and N. Palaniappan, A Study on intuitionistic L-fuzzy subgroups, *International Journal of Applied Mathematical Sciences*, **3**(53) (2009), 2619–2624.
10. R. Sakthivel and S. Naganathan, Intuitionistic L-Fuzzy Soft Semirings, *South East Asian Journal of Mathematics and Mathematical Science*, **17**(2) (2021), 195–202.
11. B.K.Tripathy, R.K.Mohanty T.R.Sooraj, On intuitionistic fuzzy soft sets and their application in decision- making in proceedings of the international conference on Signal, Networks, *Computing and systems*, **2016** .
12. Yuncheng Jiang, Yong Tang and Qimai Chen, An adjustable approach to intuitionistic fuzzy soft sets based decision making, *Applied Mathematical Modelling*, **35** (2011), 824–836.
13. Young Bae Jun, J. Neggers and Hee Sik Kim, On L-Fuzzy Ideals in Semirings I, *Czechoslovak Mathematical Journal*, **48**(123) (1998), Praha.
14. Young Bae Jun, Hee Sik Kim and Dall Sun Yoon, Intuitionistic Fuzzy Ideals in Semiring, *NIFS*, **12**(2) (2006), 1–10.
15. L. A. Zadeh, Fuzzy Sets, *Information and Control*, **8** (1965), 338–353.