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# A Decision Making Framework Using Parameterized Interval Valued Fuzzy Soft Expert Sets

Anwar Bataihah*

*Department of Mathematics, Faculty of Science, Jadara University, Irbid, Jordan*

**Abstract** The notion of a soft expert set, which allows a user to access the opinions of all experts in a single model and apply it to decision-making situations, was established by Alkhazaleh and Salleh in 2011. In addition, they presented the idea of the fuzzy soft expert set, which combines the concepts of the fuzzy and soft expert sets. By merging the interval-valued fuzzy set and soft set models, Yang et al. introduced the idea of an interval-valued fuzzy soft set in 2009. This study aims to integrate the research of Alkhazaleh and Salleh (2011) and Yang et al. (2009), resulting in the development of a novel idea: the parametrized interval-valued fuzzy soft expert set (PIVFSES). Furthermore, we analyze the features of its operations complement, union intersection, AND, and OR and introduce them. A decision-making problem is analyzed using the parametrized interval-valued fuzzy soft expert set. Additionally, our approach will be more effective and valuable as it allows the user to know the opinions of all the specialists in one place. We provide a final application of this idea to decision-making situations.

**Keywords** soft set, soft expert set, fuzzy soft expert set, parametrized interval-valued Fuzzy soft expert set, interval-valued, parametrized

**AMS 2010 subject classifications** 90B50, 03E72

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## 1. Introduction

Uncertainty is a common feature in problems spanning engineering, medical research, economics, and environmental studies. The mathematical foundation for handling such uncertainty was laid by Molodtsov [30] through soft set theory. Building upon this, Maji et al. [24] conducted studies on diverse soft set operations and their applications. The fusion of fuzzy set concepts with soft sets led to the development of fuzzy soft sets by Maji et al. [25], who also analyzed their key features. Later, the authors in [35] gave an advanced of the field by introducing interval-valued fuzzy soft sets and by establishing the basic properties, as well as they presented an algorithm to solve decision making problems based on interval-valued fuzzy soft sets. Feng et al. [9] were from the first who provided deeper insight into decision-making problems based on interval-valued fuzzy soft sets, which frame a hybrid structure combining soft set theory with interval-valued fuzzy sets.

Later, Alkhazaleh and his collaborators [3, 4] introduced the notions of *soft expert sets* and *fuzzy soft expert sets*. These models present a unified framework for incorporating the opinions of multiple experts into a single decision-making structure, while avoiding the need for complicated aggregation procedures.

Recent research has expanded this area by adding time-related features, parameterized frameworks, algebraic structures, and ways to handle linguistic information. Key developments include time-shadow soft sets [13], time effective fuzzy soft sets [14], time fuzzy soft sets [15], and studies on how time affects fuzzy soft expert sets [16].

*Correspondence to: Anwar Bataihah (Email: a.bataihah@jadara.edu.jo, anwerbataihah@gmail.com). Department of Mathematics, Faculty of Science, Jadara University, Irbid, Jordan

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Another important advancement is the introduction of Linguistic SuperHypersoft Sets [11], which improve how soft set models handle linguistic data. At the same time, new aggregation methods for multiple attribute decision-making, such as Possibility Single-Valued Neutrosophic Dombi-weighted operators [32], provide powerful tools for combining information from different sources.

Further extensions, like Q-neutrosophic soft sets in interval matrix frameworks [31], show ongoing efforts to better represent uncertainty using matrices. Parameterized models—such as time fuzzy parameterized fuzzy soft expert sets [6], parameterized time neutrosophic soft sets [20], and time fuzzy soft expert sets with weighted expert factors [7]—highlight the value of including weights and temporal aspects in decision-making models. Alkouri *et al.* [1] explored complex hesitant fuzzy graphs, introducing structural concepts that help support decisions under uncertainty and incomplete information

The theory of soft sets along with its extensions provides useful tools for handling uncertainty in decision-making problems. Important generalizations, including fuzzy soft sets, fuzzy soft expert sets, and their parameterized versions have made these models more flexible were discussed in [17, 18, 19]. More recently, refined neutrosophic soft sets have been introduced to deal with degrees of truth, indeterminacy, and falsity, which improves their usefulness in complex situations such as medical diagnosis [5].

Meanwhile, Alsharo *et al.* [2] developed complex shadowed set theory and showed how it can be applied effectively in decision-making situations. Fallatah *et al.* [8] investigated homomorphisms in tripolar fuzzy soft $\gamma$-semirings. More recent studies have focused on modeling uncertainty using neutrosophic MR-metric and statistical frameworks.

Fixed point theorems play a key role in fuzzy soft set theory. Some contributions as Malkawi [26], who used fuzzy embeddings and contraction principles to create a unified framework for representing uncertainty. Malkawi and Rabaiah [27, 28] introduced neutrosophic statistical manifolds, offering an information-geometric approach that incorporates uncertainty measures. Building on these ideas, Qawasmeh and Malkawi [29] applied fixed point techniques in MR-metric spaces to study integral equations and neutron transport problems under uncertain conditions. In this context, Malkawi *et al.* [23] laid out the foundational theory of neutrosophic MR-metric spaces, opening the way for applications in homotopy theory, fixed point analysis, and complex networks operating under uncertain conditions.

Building on these developments, the idea of parameterized interval-valued fuzzy soft expert sets, a hybrid of the Parameyerized fuzzy soft set and interval-valued fuzzy set and the soft expert set, is presented in this study. It will be more practical and effective. Additionally, we examine the features of its fundamental operations intersection, union, and complement—and define them. We also provide an application of this idea to situations involving decision-making.

Furthermore, we define fundamental operations such as union, intersection, complement, AND, and OR, and investigate their algebraic properties. To demonstrate the practical utility of PIVFSES, we present a decision-making algorithm and apply it to a real-world problem involving hospital expansion planning. while also incorporating parameter weights to reflect the relative importance of different criteria, addressing complex decision scenarios similar to those encountered in agricultural technology adoption [33] and the aggregation methodology benefits from insights gained through neutrosophic weighted operators [32] and similarity measures in interval-valued settings [10].

### *1.1. Research Gap and Novelty Contribution*

Interval-valued fuzzy soft sets [35] and fuzzy soft expert sets [4] have been widely used to represent uncertainty and incorporate expert knowledge in decision-making. Yet, their effectiveness is limited in more complex situations where uncertainty, expert opinions, and the relative importance of decision parameters must all be considered together.

The traditional interval-valued fuzzy soft sets focus mainly on modeling uncertainty using interval-valued membership functions, but they do not explicitly take expert judgments into account. Consequently, they cannot differentiate between experts with varying levels of confidence or reflect the differing importance of decision parameters. On the other hand, the concept of fuzzy soft expert sets allow for the aggregation of multiple expert opinions, however, they generally depend on accurate membership degrees. This restricts their ability to capture interval-based uncertainty, which is crucial when dealing with incomplete or ambiguous information.

While various hybrid extensions have been suggested, the parameter weights in these models are typically utilized externally during the processes of aggregation or decision-making. Consequently, the parameterization becomes detached from the foundational set structure, thereby restricting its impact on fundamental operations like union and intersection [15, 10].

In order to overcome these constraints, this study presents the **P**arameterized **I**nterval- **V**alued **F**uzzy **S**oft **E**xpert **S**et (PIVFSES). The suggested framework amalgamates interval-valued uncertainty, expert assessments, and parameter weights into a cohesive model. In contrast to earlier methodologies, parameter weights are integrated directly within the set, thereby impacting interval-valued mappings and affecting operations such as union, intersection, AND, and OR.

Consequently, PIVFSES provides a more nuanced and equitable depiction of decision-making processes, effectively reflecting both consensus and dissent among specialists. This comprehensive methodology improves the model's interpretability and adaptability, rendering it especially appropriate for intricate decision issues where the significance of parameters is crucial.

## 2. Preliminaries

This section outlines foundational elements of soft set theory. Following Molodtsov's formulation [30], let $W$ be the universe of discourse and $P$ a set of parameters, with $J \subseteq P$. Given that $P(W)$ represents the power set of $W$, a soft set is defined in the following manner:

**Definition 2.1.** [30] Think about this mapping

$$J : M \to P\left(W\right).$$

We define a *soft set* over $W$ as a pair $(J, M)$. Conceptually, this represents a parameterized grouping of subsets from the universe $W$. For every parameter $\delta \in J$, the function $J(\delta)$ specifies which elements of $W$ are considered $\delta$-approximate members of the soft set.

**Definition 2.2.** [25] Let $W$ be the initial universal set, and $P$ be the set of parameters. Let $I^W$ be the power set of all fuzzy subsets of $W$. Let $J \subseteq P$, and $F$ be the mapping

$$F : A \to I^W.$$

A pair $(J, P)$ is known as a *fuzzy soft set* over $W$.

**Definition 2.3.** [25] Regarding two fuzzy soft sets $(J, M)$ and $(K, N)$ over $W$, $(J, M)$ is known as a fuzzy soft subset of. $(K, N)$ if

1. $M \subset N$ and
2. $\forall \delta \in J, J\left(\delta\right)$ is fuzzy subset of $K\left(\delta\right)$.

The association is represented by $(F,\ A)\,\tilde{\subset}\,(K, N)$. In this situation, $(K, N)$ is known as a fuzzy soft superset of. $(J, M)$.

**Definition 2.4.** [25] $(J, M)^c$ represents the complement of a fuzzy soft set $(J, M)$, which has been described by $(J, M)^c = (J^c, \rceil A)$ where $J^c : \rceil A \to P\left(W\right)$ is a mapping provided by

$$J^c\left(\Gamma\right) = c\left(J\left(\rceil\Gamma\right)\right), \forall \Gamma \in \rceil M.$$

$c$ describes any fuzzy complement.

**Definition 2.5.** [25] If $(J, M)$ and $(K, N)$ are two fuzzy soft sets then $(J, M)$ AND $(K, N)$ denoted by $(J,\ M) \wedge (K, N)$ is defined by

$$(J, M) \wedge (K, N) = (C, M \times N)$$

such that $C(\Gamma, \lambda) = t(J(\Gamma), K(\lambda)), \forall (\Gamma, \lambda) \in J \times N$, where t is any t-norm.

**Definition 2.6.** [25] If $(J, M)$ and $(K, N)$ are two fuzzy soft sets then $(J, M)$ OR $(K, N)$ denoted by $(J, M) \vee (K, N)$ is defined by

$$(J, M) \vee (K, N) = (O, M \times N)$$

such that $O(\Gamma, \lambda) = s(J(\Gamma), G(\lambda)), \forall (\Gamma, \lambda) \in M \times N$, where s is any s-norm.

**Definition 2.7.** [25] The union of two fuzzy soft sets $(J, M)$ and $(K, N)$ over a common universe $W$ is the fuzzy soft set $(H, C)^{WIV}$ where $C = M \cup N$, and $\forall \delta \in C$,

$$H(\delta) = \begin{cases} J(\delta), & if \quad \delta \in M - N, \\ G(\delta), & if \quad \delta \in N - M, \\ s(J(\delta), G(\delta)), & if \quad \delta \in M \cup N. \end{cases}$$

Where s is any s-norm.

**Definition 2.8.** [25]$(J, M)$ and $(K, N)$ are fuzzy soft sets that intersect over common universe. $(H, C)^{WIV}$ is the fuzzy soft set $W$. Where $C = M \cup N$, and $\forall \delta \in R$,

$$H(\delta) = \begin{cases} J(\delta), & if \quad \delta \in M - N, \\ G(\delta), & if \quad \delta \in N - M, \\ s(J(\delta), G(\delta)), & if \quad \delta \in M \cap N. \end{cases}$$

**Definition 2.9.** [3]. Let $W$ be a set of universes, $P$ a set of parameters, $X$ a set of experts (agents). Let $O = \{o_1, o_2, ..., o_n\}$ be a set of opinions, $Z = E \times X \times O$ and $M \subseteq Z$. A pair $(F,\ M)$ is called a soft expert set over $W$, where $F$ is a mapping given by

$$F\ :\ A\ \rightarrow\ P(W)$$

where $P(W)$ denoted the power set of $W$.

**Definition 2.10.** [4] A *fuzzy soft expert set* over a universe $W$ is defined as the pair $(F, A)$ where $A \subseteq P \times X \times O$ consists of parameter-expert-opinion tuples ($P$ = parameters, $X$ = experts, $O$ = $\{0, 1\}$ for opinions), and $F : A \rightarrow [0, 1]^W$ is a mapping assigning to each tuple $(p, x, o) \in A$ a corresponding fuzzy subset of $W$ represented by its membership function in $[0, 1]^W$.

**Definition 2.11.** Let $(W, P, T)$ define a time-fuzzy soft set (TFSS) framework where $W$ is the universal set, $P$ is the parameter set, and $T = \{t_1, ..., t_n\}$ represents discrete time points, with the TFSS given by the temporal mapping $\{(F_t, J)\}_{t \in T}$ where $J \subseteq P$ is the active parameter subset at each time instance and $F_t : J \rightarrow [0, 1]^W$ assigns time-dependent fuzzy memberships over $W$.

**Definition 2.12.** [36] An *interval-valued fuzzy set* $\widetilde{X}$ on a universe $W$ is a mapping $\widetilde{X} : W \rightarrow \text{Int}([0, 1])$ that assigns to each element $x \in W$ a closed subinterval $\mu_{\widetilde{X}}(x) = [\mu^-(x), \mu^+(x)]$ of $[0, 1]$, where $\text{Int}([0, 1])$ denotes the set of all such closed intervals, $\widetilde{\mathcal{P}}(W)$ represents the collection of all interval-valued fuzzy sets on $W$, and the membership bounds satisfy $0 \leq \mu^-(x) \leq \mu^+(x) \leq 1$ for all $x \in W$.

For an interval-valued fuzzy set $\widetilde{X} \in \widetilde{\mathcal{P}}(W)$, the *membership degree* of an element $x \in W$ is given by the interval:

$$\mu_{\widetilde{X}}(x) = \left[\mu^-(x), \mu^+(x)\right] \subseteq [0, 1]$$

where:

- $\mu^{-}(x)$ is the *lower membership degree*
- $\mu^{+}(x)$ is the *upper membership degree*

satisfying the inequality $0 \leq \mu^{-}(x) \leq \mu^{+}(x) \leq 1$ for all $x \in W$.

**Definition 2.13.** [12]. Let $\tilde{X}, \tilde{Y} \in \tilde{P}(W)$ then the complement, intersection, union and the subset of the interval-valued fuzzy sets are defined as follows:

- The complement of $\tilde{X}$ denoted by $\tilde{X}^c$

$$\mu_{\tilde{X}^c}(x) = 1 - \mu_{\tilde{X}}(x) = \left[1 - \mu^{+}_{\tilde{X}}(x), 1 - \mu^{-}_{\tilde{X}}(x)\right];$$

- The intersection of $\tilde{X}$ and $\tilde{Y}$ denoted by $\tilde{X} \cap \tilde{Y}$

$$\begin{aligned}\mu_{\tilde{X}\cap\tilde{Y}}(x) &= \inf\left[\mu_{\tilde{X}}(x), \mu_{\tilde{Y}}(x)\right] \\ &= \left[\inf\left(\mu^{-}_{\tilde{X}}(x), \mu^{-}_{\tilde{Y}}(x)\right), \inf\left(\mu^{+}_{\tilde{X}}(x), \mu^{+}_{\tilde{Y}}(x)\right)\right];\end{aligned}$$

- The union of $\tilde{X}$ and $\tilde{Y}$ denoted by $\tilde{X} \cup \tilde{Y}$

$$\begin{aligned}\mu_{\tilde{X}\cup\tilde{Y}}(x) &= \sup\left[\mu_{\tilde{X}}(x), \mu_{\tilde{Y}}(x)\right] \\ &= \left[\sup\left(\mu^{-}_{\tilde{X}}(x), \mu^{-}_{\tilde{Y}}(x)\right), \sup\left(\mu^{+}_{\tilde{X}}(x), \mu^{+}_{\tilde{Y}}(x)\right)\right].\end{aligned}$$

- If $\mu^{-}_{X}(x) \leq \mu^{-}_{Y}(x)$ and $\mu^{+}_{X}(x) \leq \mu^{+}_{Y}(x)$ then $X$ is a subset of $Y$ which is denoted by $X \subseteq Y$

## 3. Interval-valued Fuzzy soft expert set

This section explains the Parameterized interval-valued fuzzy soft expert set idea and examines some of its characteristics.
The universe set $W$, the weighted parameters $E$, the experts (agents) $X$, $W'$ the set of of of experts, and the opinions $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ are all considered. A $\subseteq Z$ and $Z = E \times X \times O$ are assumed.

**Definition 3.1.** A pair $(F, A)^{WIV}$ is called an *Parameterized interval-valued fuzzy soft expert set* (PIVFSES in short) over $W$, where $F$ is a mapping given by

$$F^{WIV} : A \rightarrow Int(W)$$

where $Int(W)$ denotes all interval-valued subsets of $W$.

**Example 3.2.** Suppose a certain country decided that they wanted to send graduate students to another country to further their education. Let $W = \{w_i,\ w_{ii},\ w_{iii}, w_{iv}\}$ be a set of alternatives for the countries chosen. The Ministry of Education from that country decided that the following criteria should be taken into account when deciding on the country that best suits their needs, let $E = \{e_1, e_2, e_3\}$ is a set of decision parameters where $e_i\,(i = 1, 2, 3)$ denotes the parameters combined course and living expenses, the availability of the subjects taught to suit their graduates needs and the social and political stability of the country in question so that the graduates can be best assimilated into the society and $X = \{m, n, r\}$ is a set of experts, and $W_E = \{0.8,\ 0.7,\ 0.6\}$ the weights for parameters. From those findings we can obtain the most suitable country for the student to further their education.Now, Suppose that

$$F\left(\tfrac{e_1}{0.8}, m, 1\right) = \left\{\frac{w_i}{[0.20,\ 0.30]}, \frac{w_{ii}}{[0.10,\ 0.20]}, \frac{w_{iii}}{[0.20,\ 0.40]}, \frac{w_{iv}}{[0.20,\ 0.50]}\right\},$$

$$F\left(\tfrac{e_1}{0.8}, n, 1\right) = \left\{\frac{w_i}{[0.0,\ 0.30]}, \frac{w_{ii}}{[0.60,\ 0.90]}, \frac{w_{iii}}{[0.40,\ 0.70]}, \frac{w_{iv}}{[0.20,\ 0.50]}\right\},$$

$$F\left(\tfrac{e_1}{0.8}, r\,, 1\right) = \left\{\frac{w_i}{[0.40,\ 0.60]}, \frac{w_{ii}}{[0.10,\ 0.30]}, \frac{w_{iii}}{[0.10,\ 0.20]}, \frac{w_{iv}}{[0.70,\ 0.80]}\right\},$$

$$F\left(\tfrac{e_2}{0.7}, m, 1\right) = \left\{\frac{w_i}{[0.40,\ 0.70]}, \frac{w_{ii}}{[0.80,\ 0.90]}, \frac{w_{iii}}{[0.60,\ 0.90]}, \frac{w_{iv}}{[0.30,\ 0.60]}\right\},$$

$$F\left(\tfrac{e_2}{0.7}, n\,, 1\right) = \left\{\frac{w_i}{[0.0,\ 0.20]}, \frac{w_{ii}}{[0.20,\ 0.40]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.20,\ 0.50]}\right\},$$

$$F\left(\tfrac{e_2}{0.7}, r, 1\right) = \left\{\frac{w_i}{[0.10,\ 0.20]}, \frac{w_{ii}}{[0.20,\ 0.40]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.20,\ 0.30]}\right\},$$

$$F\left(\tfrac{e_3}{0.6}, m, 1\right) = \left\{\frac{w_i}{[0.10,\ 0.20]}, \frac{w_{ii}}{[0.20,\ 0.30]}, \frac{w_{iii}}{[0.30,\ 0.40]}, \frac{w_{iv}}{[0.20,\ 0.30]}\right\},$$

$$F\left(\tfrac{e_3}{0.6}, n, 1\right) = \left\{\frac{w_i}{[0.10,\ 0.40]}, \frac{w_{ii}}{[0.40,\ 0.60]}, \frac{w_{iii}}{[0.30,\ 0.60]}, \frac{w_{iv}}{[0.40,\ 0.50]}\right\},$$

$$F\left(\tfrac{e_3}{0.6}, r, 1\right) = \left\{\frac{w_i}{[0.50,\ 0.80]}, \frac{w_{ii}}{[0.10,\ 0.30]}, \frac{w_{iii}}{[0.0,\ 0.30]}, \frac{w_{iv}}{[0.80,\ 0.90]}\right\},$$

$$F\left(\tfrac{e_1}{0.8}, m, 0\right) = \left\{\frac{w_i}{[0.40,\ 0.70]}, \frac{w_{ii}}{[0.20,\ 0.30]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.50,\ 0.80]}\right\},$$

$$F\left(\tfrac{e_1}{0.8}, n, 0\right) = \left\{\frac{w_i}{[0.30,\ 0.60]}, \frac{w_{ii}}{[0.30,\ 0.60]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.50,\ 0.70]}\right\},$$

$$F\left(\tfrac{e_1}{0.8}, r, 0\right) = \left\{\frac{w_i}{[0.20,\ 0.70]}, \frac{w_{ii}}{[0.30,\ 0.80]}, \frac{w_{iii}}{[0.40,\ 0.70]}, \frac{w_{iv}}{[0.50,\ 0.80]}\right\},$$

$$F\left(\tfrac{e_2}{0.7}, m, 0\right) = \left\{\frac{w_i}{[0.40,\ 0.70]}, \frac{w_{ii}}{[0.50,\ 0.80]}, \frac{w_{iii}}{[0.50,\ 0.80]}, \frac{w_{iv}}{[0.70,\ 0.90]}\right\},$$

$$F\left(\tfrac{e_2}{0.7}, n, 0\right) = \left\{\frac{w_i}{[0.60,\ 0.90]}, \frac{w_{ii}}{[0.50,\ 0.70]}, \frac{w_{iii}}{[0.50,\ 0.80]}, \frac{w_{iv}}{[0.70,\ 0.90]}\right\},$$

$$F\left(\tfrac{e_2}{0.7}, r, 0\right) = \left\{\frac{w_i}{[0.60,\ 0.80]}, \frac{w_{ii}}{[0.50,\ 0.70]}, \frac{w_{iii}}{[0.50,\ 0.70]}, \frac{w_{iv}}{[0.60,\ 0.80]}\right\},$$

$$F\left(\tfrac{e_3}{0.6}, m, 0\right) = \left\{\frac{w_i}{[0.30,\ 0.40]}, \frac{w_{ii}}{[0.40,\ 0.70]}, \frac{w_{iii}}{[0.30,\ 0.50]}, \frac{w_{iv}}{[0.30,\ 0.60]}\right\},$$

$$F\left(\tfrac{e_3}{0.6}, n, 0\right) = \left\{\frac{w_i}{[0.20,\ 0.40]}, \frac{w_{ii}}{[0.40,\ 0.60]}, \frac{w_{iii}}{[0.60,\ 0.70]}, \frac{w_{iv}}{[0.30,\ 0.50]}\right\},$$

$$F\left(\tfrac{e_3}{0.6}, r, 0\right) = \left\{\frac{w_i}{[0.10,\ 0.20]}, \frac{w_{ii}}{[0.10,\ 0.30]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.30,\ 0.50]}\right\},$$

Then we can find the Interval-valued Fuzzy soft expert set $(F,Z)^{WIV}$ as consisting of the following collection of approximations:

$$(F,Z)^{WIV} = \Bigg\{ \left( \left(\tfrac{e_1}{0.8}, m, 1\right), \left\{ \frac{w_i}{[0.20,\ 0.30]}, \frac{w_{ii}}{[0.10,\ 0.20]}, \frac{w_{iii}}{[0.20,\ 0.40]}, \frac{w_{iv}}{[0.20,\ 0.50]} \right\} \right),$$

$$\left( \left(\tfrac{e_1}{0.8}, n, 1\right), \left\{ \frac{w_i}{[0.0,\ 0.30]}, \frac{w_{ii}}{[0.60,\ 0.90]}, \frac{w_{iii}}{[0.40,\ 0.70]}, \frac{w_{iv}}{[0.20,\ 0.50]} \right\} \right),$$

$$\left( \left(\tfrac{e_1}{0.8}, r, 1\right), \left\{ \frac{w_i}{[0.40,\ 0.60]}, \frac{w_{ii}}{[0.10,\ 0.30]}, \frac{w_{iii}}{[0.10,\ 0.20]}, \frac{w_{iv}}{[0.70,\ 0.80]} \right\} \right),$$

$$\left( \left(\tfrac{e_2}{0.7}, m, 1\right), \left\{ \frac{w_i}{[0.40,\ 0.70]}, \frac{w_{ii}}{[0.80,\ 0.90]}, \frac{w_{iii}}{[0.60,\ 0.90]}, \frac{w_{iv}}{[0.30,\ 0.60]} \right\} \right),$$

$$\left( \left(\tfrac{e_2}{0.7}, n, 1\right), \left\{ \frac{w_i}{[0.0,\ 0.20]}, \frac{w_{ii}}{[0.20,\ 0.40]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.20,\ 0.50]} \right\} \right),$$

$$\left( \left(\tfrac{e_2}{0.7}, r, 1\right), \left\{ \frac{w_i}{[0.10,\ 0.20]}, \frac{w_{ii}}{[0.20,\ 0.40]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.20,\ 0.30]} \right\} \right),$$

$$\left( \left(\tfrac{e_3}{0.6}, m, 1\right), \left\{ \frac{w_i}{[0.10,\ 0.20]}, \frac{w_{ii}}{[0.20,\ 0.30]}, \frac{w_{iii}}{[0.30,\ 0.40]}, \frac{w_{iv}}{[0.20,\ 0.30]} \right\} \right),$$

$$\left( \left(\tfrac{e_3}{0.6}, n, 1\right), \left\{ \frac{w_i}{[0.10,\ 0.40]}, \frac{w_{ii}}{[0.40,\ 0.60]}, \frac{w_{iii}}{[0.30,\ 0.60]}, \frac{w_{iv}}{[0.40,\ 0.50]} \right\} \right),$$

$$\left( \left(\tfrac{e_3}{0.6}, r, 1\right), \left\{ \frac{w_i}{[0.50,\ 0.80]}, \frac{w_{ii}}{[0.10,\ 0.30]}, \frac{w_{iii}}{[0,\ 0.30]}, \frac{w_{iv}}{[0.80,\ 0.90]} \right\} \right),$$

$$\left( \left(\tfrac{e_1}{0.8}, m, 0\right), \left\{ \frac{w_i}{[0.40,\ 0.70]}, \frac{w_{ii}}{[0.20,\ 0.30]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.50,\ 0.80]} \right\} \right),$$

$$\left( \left(\tfrac{e_1}{0.8}, n, 0\right), \left\{ \frac{w_i}{[0.30,\ 0.60]}, \frac{w_{ii}}{[0.30,\ 0.60]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.50,\ 0.70]} \right\} \right),$$

$$\left( \left(\tfrac{e_1}{0.8}, r, 0\right), \left\{ \frac{w_i}{[0.20,\ 0.70]}, \frac{w_{ii}}{[0.30,\ 0.80]}, \frac{w_{iii}}{[0.40,\ 0.70]}, \frac{w_{iv}}{[0.50,\ 0.80]} \right\} \right),$$

$$\left( \left(\tfrac{e_2}{0.7}, m, 0\right), \left\{ \frac{w_i}{[0.40,\ 0.70]}, \frac{w_{ii}}{[0.50,\ 0.80]}, \frac{w_{iii}}{[0.50,\ 0.80]}, \frac{w_{iv}}{[0.70,\ 0.90]} \right\} \right),$$

$$\left( \left(\tfrac{e_2}{0.7}, n, 0\right), \left\{ \frac{w_i}{[0.60,\ 0.90]}, \frac{w_{ii}}{[0.50,\ 0.70]}, \frac{w_{iii}}{[0.50,\ 0.80]}, \frac{w_{iv}}{[0.70,\ 0.90]} \right\} \right),$$

$$\left( \left(\tfrac{e_2}{0.7}, r, 0\right), \left\{ \frac{w_i}{[0.60,\ 0.80]}, \frac{w_{ii}}{[0.50,\ 0.70]}, \frac{w_{iii}}{[0.50,\ 0.70]}, \frac{w_{iv}}{[0.60,\ 0.80]} \right\} \right),$$

$$\left( \left(\tfrac{e_3}{0.6}, m, 0\right), \left\{ \frac{w_i}{[0.30,\ 0.40]}, \frac{w_{ii}}{[0.40,\ 0.70]}, \frac{w_{iii}}{[0.30,\ 0.50]}, \frac{w_{iv}}{[0.30,\ 0.60]} \right\} \right),$$

$$\left(\left(\tfrac{e_3}{0.6}, n, 0\right), \left\{\frac{w_i}{[0.20,\ 0.40]}, \frac{w_{ii}}{[0.40,\ 0.60]}, \frac{w_{iii}}{[0.60,\ 0.70]}, \frac{w_{iv}}{[0.30,\ 0.50]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, r, 0\right), \left\{\frac{w_i}{[0.10,\ 0.20]}, \frac{w_{ii}}{[0.10,\ 0.30]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.30,\ 0.50]}\right\}\right)\Bigg\}.$$

**Definition 3.3.** For two PIVFSESs $(F, A)^{WIV}$ and $(G, B)^{WIV}$ over $W$, $(F, A)^{WIV}$ is called an Interval-valued Fuzzy soft expert set subset of $(G, B)^{WIV}$ if

1. $B \subseteq A$,
2. $\forall \varepsilon \in B, G(\varepsilon)$ is interval-valued fuzzy subset of $F(\varepsilon)$.

**Example 3.4.** Consider the previous example 3.2. Suppose that the management of ministry takes the opinion of the experts once again. Let where is the rest of the statement etc

$$A = \Big\{\left(\tfrac{e_1}{0.8}, m, 1\right), \left(\tfrac{e_3}{0.6}, m, 1\right), \left(\tfrac{e_3}{0.6}, m, 0\right), \left(\tfrac{e_1}{0.8}, n, 1\right), \left(\tfrac{e_2}{0.7}, n, 1\right) \left(\tfrac{e_2}{0.7}, r, 0\right),$$
$$\left(\tfrac{e_3}{0.6}, n, 0\right), \left(\tfrac{e_2}{0.7}, r, 1\right), \left(\tfrac{e_3}{0.6}, r, 1\right), \left(\tfrac{e_3}{0.6}, r, 0\right)\Big\},$$
$$B = \Big\{\left(\tfrac{e_1}{0.8}, m, 1\right), \left(\tfrac{e_3}{0.6}, m, 0\right), \left(\tfrac{e_1}{0.8}, n, 1\right), \left(\tfrac{e_2}{0.7}, n, 1\right), \left(\tfrac{e_2}{0.7}, r, 0\right),$$
$$\left(\tfrac{e_3}{0.6}, r, 1\right), \left(\tfrac{e_3}{0.6}, r, 0\right)\Big\}$$

Since $B$ is a fuzzy subset of $A$, clearly $B \subset A$. Let $(G, B)$ and $(F, A)^{WIV}$ be defined as follows:

$$(F, A)^{WIV} = \Bigg\{\left(\left(\tfrac{e_1}{0.8}, m, 1\right), \left\{\frac{w_i}{[0.10,\ 0.20]}, \frac{w_{ii}}{[0.10,\ 0.30]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.20,\ 0.40]}\right\}\right),$$

$$\left(\left(\tfrac{e_1}{0.8}, n, 1\right), \left\{\frac{w_i}{[0.0,\ 0.20]}, \frac{w_{ii}}{[0.20,\ 0.30]}, \frac{w_{iii}}{[0.40,\ 0.60]}, \frac{w_{iv}}{[0.30,\ 0.40]}\right\}\right),$$

$$\left(\left(\tfrac{e_2}{0.7}, n, 1\right), \left\{\frac{w_i}{[0.40,\ 0.60]}, \frac{w_{ii}}{[0.40,\ 0.50]}, \frac{w_{iii}}{[0.70,\ 0.80]}, \frac{w_{iv}}{[0.50,\ 0.60]}\right\}\right),$$

$$\left(\left(\tfrac{e_2}{0.7}, r, 1\right), \left\{\frac{w_i}{[0.10,\ 0.30]}, \frac{w_{ii}}{[0.30,\ 0.50]}, \frac{w_{iii}}{[0.60,\ 0.80]}, \frac{w_{iv}}{[0.50,\ 0.70]}\right\}\right),$$

$$\left(\left(\tfrac{e_2}{0.7}, r, 0\right), \left\{\frac{w_i}{[0.0,\ 0.10]}, \frac{w_{ii}}{[0.0,\ 0.20]}, \frac{w_{iii}}{[0.40,\ 0.60]}, \frac{w_{iv}}{[0.40,\ 0.50]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, m, 1\right), \left\{\frac{w_i}{[0.20,\ 0.50]}, \frac{w_{ii}}{[0.30,\ 0.50]}, \frac{w_{iii}}{[0.50,\ 0.80]}, \frac{w_{iv}}{[0.60,\ 0.70]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, r, 1\right), \left\{\frac{w_i}{[0.20,\ 0.40]}, \frac{w_{ii}}{[0.30,\ 0.40]}, \frac{w_{iii}}{[0.50,\ 0.80]}, \frac{w_{iv}}{[0.60,\ 0.80]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, m, 0\right), \left\{\frac{w_i}{[0.30,\ 0.40]}, \frac{w_{ii}}{[0.40,\ 0.60]}, \frac{w_{iii}}{[0.40,\ 0.70]}, \frac{w_{iv}}{[0.50,\ 0.60]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, n, 0\right), \left\{\frac{w_i}{[0.10,\ 0.20]}, \frac{w_{ii}}{[0.10,\ 0.30]}, \frac{w_{iii}}{[0.50,\ 0.70]}, \frac{w_{iv}}{[0.50,\ 0.60]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, r, 0\right), \left\{\frac{w_i}{[0.20,\ 0.30]}, \frac{w_{ii}}{[0.20,\ 0.30]}, \frac{w_{iii}}{[0.40,\ 0.60]}, \frac{w_{iv}}{[0.30,\ 0.50]}\right\}\right)\Bigg\},$$

$$(G,B)^{WIV} = \Bigg\{\left(\left(\tfrac{e_1}{0.8}, m, 1\right), \left\{\frac{w_i}{[0.0,\ 0.10]}, \frac{w_{ii}}{[0.10,\ 0.20]}, \frac{w_{iii}}{[0.10,\ 0.30]}, \frac{w_{iv}}{[0.20,\ 0.40]}\right\}\right),$$

$$\left(\left(\tfrac{e_1}{0.8}, n, 1\right), \left\{\frac{w_i}{[0.0,\ 0.10]}, \frac{w_{ii}}{[0.20,\ 0.30]}, \frac{w_{iii}}{[0.30,\ 0.40]}, \frac{w_{iv}}{[0.30,\ 0.40]}\right\}\right),$$

$$\left(\left(\tfrac{e_2}{0.7}, n, 1\right), \left\{\frac{w_i}{[0.40,\ 0.50]}, \frac{w_{ii}}{[0.30,\ 0.40]}, \frac{w_{iii}}{[0.60,\ 0.70]}, \frac{w_{iv}}{[0.40,\ 0.60]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, r, 1\right), \left\{\frac{w_i}{[0.20,\ 0.30]}, \frac{w_{ii}}{[0.30,\ 0.40]}, \frac{w_{iii}}{[0.50,\ 0.80]}, \frac{w_{iv}}{[0.50,\ 0.60]}\right\}\right),$$

$$\left(\left(\tfrac{e_2}{0.7}, r, 0\right), \left\{\frac{w_i}{[0.0,\ 0.10]}, \frac{w_{ii}}{[0.0,\ 0.10]}, \frac{w_{iii}}{[0.30,\ 0.50]}, \frac{w_{iv}}{[0.30,\ 0.40]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, m, 0\right), \left\{\frac{w_i}{[0.10,\ 0.20]}, \frac{w_{ii}}{[0.30,\ 0.50]}, \frac{w_{iii}}{[0.40,\ 0.50]}, \frac{w_{iv}}{[0.40,\ 0.50]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, r, 0\right), \left\{\frac{w_i}{[0.0,\ 0.10]}, \frac{w_{ii}}{[0.10,\ 0.20]}, \frac{w_{iii}}{[0.30,\ 0.50]}, \frac{w_{iv}}{[0.20,\ 0.50]}\right\}\right)\Bigg\}.$$

Therefore $(G,B)^{WIV} \subseteq (F,A)^{WIV}$.

**Definition 3.5.** Two PIVFSES $(F,A)^{WIV}$ and $(G,B)^{WIV}$ over $W$, are said to be *equal* if $(F,A)^{WIV}$ is a PIVFSES subset of $(G,A)^{WIV}$ and $(G,A)^{WIV}$ is a PIVFSES subset of $(F,A)^{WIV}$.

**Definition 3.6.** An *agree-PIVFSES* $(F,A)_1^{WIV}$ over $W$ is a PIVFSES subset of $(F,A)^{WIV}$ defined as follows:

$$(F,A)_1^{WIV} = \{F_1(\alpha) : \alpha \in E \times X \times \{1\}\}.$$

**Definition 3.7.** A *disagree-PIVFSES* $(F,A)_0^{WIV}$ over $W$ is a Interval-valued Fuzzy soft expert set subset of $(F,A)^{WIV}$ defined as follows:

$$(F,A)_0^{WIV} = \{F_0(\alpha) : \alpha \in E \times X \times \{0\}\}.$$

**Example 3.8.** Consider Example 3.2. Then the agree- Interval-valued Fuzzy soft expert set $(F,Z)_1^{WIV}$ over $W$ is

$$(F,Z)_1^{WIV} = \Bigg\{\left(\left(\tfrac{e_1}{0.8}, m, 1\right), \left\{\frac{w_i}{[0.20,\ 0.30]}, \frac{w_{ii}}{[0.10,\ 0.20]}, \frac{w_{iii}}{[0.20,\ 0.40]}, \frac{w_{iv}}{[0.20,\ 0.50]}\right\}\right),$$

$$\left(\left(\tfrac{e_1}{0.8}, n, 1\right), \left\{\frac{w_i}{[0.0,\ 0.30]}, \frac{w_{ii}}{[0.60,\ 0.90]}, \frac{w_{iii}}{[0.40,\ 0.70]}, \frac{w_{iv}}{[0.20,\ 0.50]}\right\}\right),$$

$$\left(\left(\tfrac{e_1}{0.8}, r, 1\right), \left\{\frac{w_i}{[0.40,\ 0.60]}, \frac{w_{ii}}{[0.10,\ 0.30]}, \frac{w_{iii}}{[0.10,\ 0.20]}, \frac{w_{iv}}{[0.70,\ 0.80]}\right\}\right),$$

$$\left(\left(\tfrac{e_2}{0.7}, m, 1\right), \left\{\frac{w_i}{[0.40,\ 0.70]}, \frac{w_{ii}}{[0.80,\ 0.90]}, \frac{w_{iii}}{[0.60,\ 0.90]}, \frac{w_{iv}}{[0.30,\ 0.60]}\right\}\right),$$

$$\left(\left(\tfrac{e_2}{0.7}, n, 1\right), \left\{\frac{w_i}{[0.0,\ 0.20]}, \frac{w_{ii}}{[0.20,\ 0.40]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.20,\ 0.50]}\right\}\right),$$

$$\left(\left(\tfrac{e_2}{0.7}, r, 1\right), \left\{\frac{w_i}{[0.10,\ 0.20]}, \frac{w_{ii}}{[0.20,\ 0.40]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.20,\ 0.30]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, m, 1\right), \left\{\frac{w_i}{[0.10,\ 0.20]}, \frac{w_{ii}}{[0.20,\ 0.30]}, \frac{w_{iii}}{[0.30,\ 0.40]}, \frac{w_{iv}}{[0.20,\ 0.30]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, n, 1\right), \left\{\frac{w_i}{[0.10,\ 0.40]}, \frac{w_{ii}}{[0.40,\ 0.60]}, \frac{w_{iii}}{[0.30,\ 0.60]}, \frac{w_{iv}}{[0.40,\ 0.50]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, r, 1\right), \left\{\frac{w_i}{[0.50,\ 0.80]}, \frac{w_{ii}}{[0.10,\ 0.30]}, \frac{w_{iii}}{[0.0,\ 0.30]}, \frac{w_{iv}}{[0.80,\ 0.90]}\right\}\right),$$

and the disagree- Interval-valued Fuzzy soft expert set $(F,Z)_0^{WIV}$ over $W$ is

$$(F,Z)_0^{WIV} = \left(\left(\tfrac{e_1}{0.8}, m, 0\right), \left\{\frac{w_i}{[0.40,\ 0.70]}, \frac{w_{ii}}{[0.20,\ 0.30]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.50,\ 0.80]}\right\}\right),$$

$$\left(\left(\tfrac{e_1}{0.8}, n, 0\right), \left\{\frac{w_i}{[0.30,\ 0.60]}, \frac{w_{ii}}{[0.30,\ 0.60]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.50,\ 0.70]}\right\}\right),$$

$$\left(\left(\tfrac{e_1}{0.8}, r, 0\right), \left\{\frac{w_i}{[0.20,\ 0.70]}, \frac{w_{ii}}{[0.30,\ 0.80]}, \frac{w_{iii}}{[0.40,\ 0.70]}, \frac{w_{iv}}{[0.50,\ 0.80]}\right\}\right),$$

$$\left(\left(\tfrac{e_2}{0.7}, m, 0\right), \left\{\frac{w_i}{[0.40,\ 0.70]}, \frac{w_{ii}}{[0.50,\ 0.80]}, \frac{w_{iii}}{[0.50,\ 0.80]}, \frac{w_{iv}}{[0.70,\ 0.90]}\right\}\right),$$

$$\left(\left(\tfrac{e_2}{0.7}, n, 0\right), \left\{\frac{w_i}{[0.60,\ 0.90]}, \frac{w_{ii}}{[0.50,\ 0.70]}, \frac{w_{iii}}{[0.50,\ 0.80]}, \frac{w_{iv}}{[0.70,\ 0.90]}\right\}\right),$$

$$\left(\left(\tfrac{e_2}{0.7}, r, 0\right), \left\{\frac{w_i}{[0.60,\ 0.80]}, \frac{w_{ii}}{[0.50,\ 0.70]}, \frac{w_{iii}}{[0.50,\ 0.70]}, \frac{w_{iv}}{[0.60,\ 0.80]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, m, 0\right), \left\{\frac{w_i}{[0.30,\ 0.40]}, \frac{w_{ii}}{[0.40,\ 0.70]}, \frac{w_{iii}}{[0.30,\ 0.50]}, \frac{w_{iv}}{[0.30,\ 0.60]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, n, 0\right), \left\{\frac{w_i}{[0.20,\ 0.40]}, \frac{w_{ii}}{[0.40,\ 0.60]}, \frac{w_{iii}}{[0.60,\ 0.70]}, \frac{w_{iv}}{[0.30,\ 0.50]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, r, 0\right), \left\{\frac{w_i}{[0.10,\ 0.20]}, \frac{w_{ii}}{[0.10,\ 0.30]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.30,\ 0.50]}\right\}\right)\Big\}.$$

**Definition 3.9.** $(F, A)^c$ represents the *complement* of an interval-valued fuzzy soft expert set $(F, A)$, which is defined by $(F, A) = (F^c, \rceil A)$ where $F^c : \rceil A \to Int\,(W)$ is a mapping given by

$$F^c\,(\alpha) = c(F\,(\rceil\alpha)), \forall\alpha \in \rceil A,$$

where c is an interval-valued fuzzy complement and $\rceil A \subset \{\rceil E \times X \times O\}$.

**Example 3.10.** Take Example 3.2 as an example. Utilizing the fundamental fuzzy complement, we've

$$
\begin{aligned}
(F,Z)^{WIV^c} = \Bigg\{ & \left(\left(\tfrac{e_1}{0.8}, m, 1\right), \left\{\frac{w_i}{[0.70,\ 0.80]}, \frac{w_{ii}}{[0.80,\ 0.90]}, \frac{w_{iii}}{[0.60,\ 0.80]}, \frac{w_{iv}}{[0.50,\ 0.80]}\right\}\right),\\
& \left(\left(\tfrac{e_1}{0.8}, n, 1\right), \left\{\frac{w_i}{[0.70,\ 1.0]}, \frac{w_{ii}}{[0.10,\ 0.40]}, \frac{w_{iii}}{[0.30,\ 0.60]}, \frac{w_{iv}}{[0.50,\ 0.80]}\right\}\right),\\
& \left(\left(\tfrac{e_1}{0.8}, r, 1\right), \left\{\frac{w_i}{[0.40,\ 0.60]}, \frac{w_{ii}}{[0.70,\ 0.90]}, \frac{w_{iii}}{[0.80,\ 0.90]}, \frac{w_{iv}}{[0.20,\ 0.30]}\right\}\right),\\
& \left(\left(\tfrac{e_2}{0.7}, m, 1\right), \left\{\frac{w_i}{[0.30,\ 0.60]}, \frac{w_{ii}}{[0.10,\ 0.20]}, \frac{w_{iii}}{[0.10,\ 0.40]}, \frac{w_{iv}}{[0.40,\ 0.70]}\right\}\right),\\
& \left(\left(\tfrac{e_2}{0.7}, n, 1\right), \left\{\frac{w_i}{[0.80,\ 1.0]}, \frac{w_{ii}}{[0.60,\ 0.80]}, \frac{w_{iii}}{[0.50,\ 0.80]}, \frac{w_{iv}}{[0.50,\ 0.80]}\right\}\right),\\
& \left(\left(\tfrac{e_2}{0.7}, r, 1\right), \left\{\frac{w_i}{[0.80,\ 0.90]}, \frac{w_{ii}}{[0.60,\ 0.80]}, \frac{w_{iii}}{[0.50,\ 0.80]}, \frac{w_{iv}}{[0.70,\ 0.80]}\right\}\right),\\
& \left(\left(\tfrac{e_3}{0.6}, m, 1\right), \left\{\frac{w_i}{[0.80,\ 0.90]}, \frac{w_{ii}}{[0.70,\ 0.80]}, \frac{w_{iii}}{[0.60,\ 0.70]}, \frac{w_{iv}}{[0.70,\ 0.80]}\right\}\right),\\
& \left(\left(\tfrac{e_3}{0.6}, n, 1\right), \left\{\frac{w_i}{[0.60,\ 0.90]}, \frac{w_{ii}}{[0.40,\ 0.60]}, \frac{w_{iii}}{[0.40,\ 0.70]}, \frac{w_{iv}}{[0.50,\ 0.60]}\right\}\right),\\
& \left(\left(\tfrac{e_3}{0.6}, r, 1\right), \left\{\frac{w_i}{[0.20,\ 0.50]}, \frac{w_{ii}}{[0.70,\ 0.90]}, \frac{w_{iii}}{[0.70,\ 1.0]}, \frac{w_{iv}}{[0.10,\ 0.20]}\right\}\right),\\
& \left(\left(\tfrac{e_1}{0.8}, m, 0\right), \left\{\frac{w_i}{[0.30,\ 0.60]}, \frac{w_{ii}}{[0.70,\ 0.80]}, \frac{w_{iii}}{[0.50,\ 0.80]}, \frac{w_{iv}}{[0.20,\ 0.50]}\right\}\right),\\
& \left(\left(\tfrac{e_1}{0.8}, n, 0\right), \left\{\frac{w_i}{[0.40,\ 0.70]}, \frac{w_{ii}}{[0.40,\ 0.70]}, \frac{w_{iii}}{[0.50,\ 0.80]}, \frac{w_{iv}}{[0.30,\ 0.50]}\right\}\right),\\
& \left(\left(\tfrac{e_1}{0.8}, r, 0\right), \left\{\frac{w_i}{[0.30,\ 0.80]}, \frac{w_{ii}}{[0.20,\ 0.70]}, \frac{w_{iii}}{[0.30,\ 0.60]}, \frac{w_{iv}}{[0.20,\ 0.50]}\right\}\right),\\
& \left(\left(\tfrac{e_2}{0.7}, m, 0\right), \left\{\frac{w_i}{[0.30,\ 0.60]}, \frac{w_{ii}}{[0.20,\ 0.50]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.10,\ 0.30]}\right\}\right),\\
& \left(\left(\tfrac{e_2}{0.7}, n, 0\right), \left\{\frac{w_i}{[0.10,\ 0.40]}, \frac{w_{ii}}{[0.30,\ 0.50]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.10,\ 0.30]}\right\}\right),
\end{aligned}
$$

$$\left(\left(\tfrac{e_2}{0.7}, r, 0\right), \left\{\frac{w_i}{[0.20,\ 0.40]}, \frac{w_{ii}}{[0.30,\ 0.50]}, \frac{w_{iii}}{[0.30,\ 0.50]}, \frac{w_{iv}}{[0.20,\ 0.40]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, m, 0\right), \left\{\frac{w_i}{[0.60,\ 0.70]}, \frac{w_{ii}}{[0.30,\ 0.60]}, \frac{w_{iii}}{[0.50,\ 0.70]}, \frac{w_{iv}}{[0.40,\ 0.70]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, n, 0\right), \left\{\frac{w_i}{[0.60,\ 0.80]}, \frac{w_{ii}}{[0.40,\ 0.60]}, \frac{w_{iii}}{[0.30,\ 0.40]}, \frac{w_{iv}}{[0.50,\ 0.70]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, r, 0\right), \left\{\frac{w_i}{[0.80,\ 0.90]}, \frac{w_{ii}}{[0.70,\ 0.90]}, \frac{w_{iii}}{[0.50,\ 0.80]}, \frac{w_{iv}}{[0.50,\ 0.70]}\right\}\right)\Big\}.$$

*Proposition 3.11*
If $(F, A)^{WIV}$ is a fuzzy soft expert set over $W$, then

1. $\left((F, A)^{WIV^c}\right)^c = (F, A)^{WIV}$.

*Proof*
From Definition 3.9, the complement $(F, A)^c$ is given by $(F^c, A)$ where:

$$F^c(\alpha) = \overline{1} - F(\alpha) \quad \forall \alpha \in A$$

Taking the complement again yields:

$$((F, A)^c)^c = ((F^c)^c, A)$$

with the membership function:

$$\begin{aligned}(F^c)^c(\alpha) &= \overline{1} - \left(\overline{1} - F(\alpha)\right) \\ &= F(\alpha) \quad \forall \alpha \in A\end{aligned}$$

Thus, we have shown the double complement returns the original set. □

## 4. Union and intersection

This section presents the definitions, characteristics, and examples of the union and intersection of an interval-valued fuzzy soft expert set.

**Definition 4.1.** The PIVFSES $(H, C)^{WIV}$ is the *union* of two PIVFSESs $(F, A)^{WIV}$ and $(G, B)^{WIV}$ over $W$, represented as $(F, A)^{WIV} \tilde{\cup} (G, B)^{WIV}$. such that $C = A \cup B \subset \{E \times X \times O\}$ and $\forall \varepsilon \in C$,

$$H(\varepsilon)^{WIV} = \begin{cases} F(\varepsilon), & if\, \varepsilon \in A - B \\ G(\varepsilon), & if\, \varepsilon \in B - A \\ F(\varepsilon) \widetilde{\bigcup} G(\varepsilon), & if\, \varepsilon \in A \cup B \end{cases}$$

where $\widetilde{\bigcup}$ is an interval-valued fuzzy union.

**Example 4.2.** Take Example 3.2 as an example.Let's

$$A=\Big\{\left(\tfrac{e_1}{0.8},n,1\right),\left(\tfrac{e_1}{0.8},r,1\right),\left(\tfrac{e_2}{0.7},m,1\right),\left(\tfrac{e_2}{0.7},m,0\right),\left(\tfrac{e_2}{0.7},r,1\right),\left(\tfrac{e_3}{0.6},n,1\right),\left(\tfrac{e_3}{0.6},n,0\right),\left(\tfrac{e_3}{0.6},r,0\right)\Big\}.\ B=\Big\{\left(\tfrac{e_1}{0.8},n,1\right),\left(\tfrac{e_1}{0.8},r,1\right),\left(\tfrac{e_2}{0.7},m,1\right),\left(\tfrac{e_2}{0.7},r,0\right),\left(\tfrac{e_3}{0.6},n,0\right),\left(\tfrac{e_3}{0.6},r,0\right)\Big\}.$$

Suppose $(F,A)^{WIV}$ and $(G,B)^{WIV}$ are two PIVFSESs over $W$ such that

$$\begin{aligned}(F,A)^{WIV}=\Bigg\{&\left(\left(\tfrac{e_1}{0.8},n,1\right),\left\{\frac{w_i}{[0.0,\ 0.30]},\frac{w_{ii}}{[0.60,\ 0.90]},\frac{w_{iii}}{[0.40,\ 0.70]},\frac{w_{iv}}{[0.20,\ 0.50]}\right\}\right),\\
&\left(\left(\tfrac{e_1}{0.8},r,1\right),\left\{\frac{w_i}{[0.40,\ 0.60]},\frac{w_{ii}}{[0.10,\ 0.30]},\frac{w_{iii}}{[0.10,\ 0.20]},\frac{w_{iv}}{[0.70,\ 0.80]}\right\}\right),\\
&\left(\left(\tfrac{e_2}{0.7},m,1\right),\left\{\frac{w_i}{[0.40,\ 0.50]},\frac{w_{ii}}{[0.80,\ 0.90]},\frac{w_{iii}}{[0.60,\ 0.90]},\frac{w_{iv}}{[0.30,\ 0.60]}\right\}\right),\\
&\left(\left(\tfrac{e_2}{0.7},m,0\right),\left\{\frac{w_i}{[0.0,\ 0.20]},\frac{w_{ii}}{[0.20,\ 0.40]},\frac{w_{iii}}{[0.20,\ 0.50]},\frac{w_{iv}}{[0.20,\ 0.50]}\right\}\right),\\
&\left(\left(\tfrac{e_2}{0.7},r,1\right),\left\{\frac{w_i}{[0.10,\ 0.20]},\frac{w_{ii}}{[0.20,\ 0.40]},\frac{w_{iii}}{[0.20,\ 0.50]},\frac{w_{iv}}{[0.20,\ 0.30]}\right\}\right),\\
&\left(\left(\tfrac{e_3}{0.6},n,1\right),\left\{\frac{w_i}{[0.10,\ 0.40]},\frac{w_{ii}}{[0.40,\ 0.60]},\frac{w_{iii}}{[03.,\ 0.60]},\frac{w_{iv}}{[0.40,\ 0.50]}\right\}\right),\\
&\left(\left(\tfrac{e_3}{0.6},n,0\right),\left\{\frac{w_i}{[0.20,\ 0.40]},\frac{w_{ii}}{[0.40,\ 0.60]},\frac{w_{iii}}{[0.60,\ 0.70]},\frac{w_{iv}}{[0.30,\ 0.50]}\right\}\right)\Bigg\}.\\
&\left(\left(\tfrac{e_3}{0.6},r,0\right),\left\{\frac{w_i}{[0.10,\ 0.20]},\frac{w_{ii}}{[0.10,\ 0.30]},\frac{w_{iii}}{[0.20,\ 0.50]},\frac{w_{iv}}{[0.30,\ 0.50]}\right\}\right)\Bigg\}.\end{aligned}$$

$$\begin{aligned}(G,B)^{WIV}=\Bigg\{&\left(\left(\tfrac{e_1}{0.8},n,1\right),\left\{\frac{w_i}{[0.10,\ 0.20]},\frac{w_{ii}}{[0.80,\ 0.90]},\frac{w_{iii}}{[0.50,\ 0.60]},\frac{w_{iv}}{[0.50,\ 0.90]}\right\}\right),\\
&\left(\left(\tfrac{e_1}{0.8},r,1\right),\left\{\frac{w_i}{[0.30,\ 0.70]},\frac{w_{ii}}{[0.10,\ 0.40]},\frac{w_{iii}}{[0.30,\ 0.60]},\frac{w_{iv}}{[0.50,\ 0.80]}\right\}\right),\\
&\left(\left(\tfrac{e_2}{0.7},m,1\right),\left\{\frac{w_i}{[0.30,\ 0.60]},\frac{w_{ii}}{[0.10,\ 0.20]},\frac{w_{iii}}{[0.10,\ 0.40]},\frac{w_{iv}}{[0.20,\ 0.70]}\right\}\right),\\
&\left(\left(\tfrac{e_2}{0.7},r,0\right),\left\{\frac{w_i}{[0.80,\ 1.0]},\frac{w_{ii}}{[0.60,\ 0.80]},\frac{w_{iii}}{[0.50,\ 0.70]},\frac{w_{iv}}{[0.50,\ 0.80]}\right\}\right),\\
&\left(\left(\tfrac{e_3}{0.6},n,0\right),\left\{\frac{w_i}{[0.10,\ 0.50]},\frac{w_{ii}}{[0.60,\ 0.80]},\frac{w_{iii}}{[0.50,\ 0.80]},\frac{w_{iv}}{[0.70,\ 0.80]}\right\}\right),\\
&\left(\left(\tfrac{e_3}{0.6},r,0\right),\left\{\frac{w_i}{[0.0,\ 0.30]},\frac{w_{ii}}{[0.20,\ 0.50]},\frac{w_{iii}}{[0.20,\ 0.40]},\frac{w_{iv}}{[0.70,\ 0.80]}\right\}\right)\Bigg\}.\end{aligned}$$

Then $(F, A)^{WIV} \widetilde{\cup} (G, B)^{WIV} = (H, C)^{WIV}$ where

$$(H,C)^{WIV} = \Big\{ \Big( \big(\tfrac{e_1}{0.8}, n, 1\big), \Big\{ \frac{w_i}{[0.10,\ 0.30]}, \frac{w_{ii}}{[0.80,\ 0.90]}, \frac{w_{iii}}{[0.50,\ 0.70]}, \frac{w_{iv}}{[0.50,\ 0.90]} \Big\} \Big),$$

$$\Big( \big(\tfrac{e_1}{0.8}, r, 1\big), \Big\{ \frac{w_i}{[0.40,\ 0.70]}, \frac{w_{ii}}{[0.10,\ 0.40]}, \frac{w_{iii}}{[0.30,\ 0.60]}, \frac{w_{iv}}{[0.70,\ 0.80]} \Big\} \Big),$$

$$\Big( \big(\tfrac{e_2}{0.7}, m, 1\big), \Big\{ \frac{w_i}{[0.40,\ 0.60]}, \frac{w_{ii}}{[0.80,\ 0.90]}, \frac{w_{iii}}{[0.60,\ 0.90]}, \frac{w_{iv}}{[0.30,\ 0.70]} \Big\} \Big),$$

$$\Big( \big(\tfrac{e_2}{0.7}, m, 0\big), \Big\{ \frac{w_i}{[0,\ 0.20]}, \frac{w_{ii}}{[0.20,\ 0.40]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.20,\ 0.50]} \Big\} \Big),$$

$$\Big( \big(\tfrac{e_2}{0.7}, r, 1\big), \Big\{ \frac{w_i}{[0.10,\ 0.20]}, \frac{w_{ii}}{[0.20,\ 0.40]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.20,\ 0.30]} \Big\} \Big),$$

$$\Big( \big(\tfrac{e_2}{0.7}, r, 0\big), \Big\{ \frac{w_i}{[0.80,\ 1]}, \frac{w_{ii}}{[0.60,\ 0.80]}, \frac{w_{iii}}{[0.50,\ 0.70]}, \frac{w_{iv}}{[0.50,\ 0.80]} \Big\} \Big),$$

$$\Big( \big(\tfrac{e_3}{0.6}, n, 1\big), \Big\{ \frac{w_i}{[0.10, 0.40]}, \frac{w_{ii}}{[0.40, 0.60]}, \frac{w_{iii}}{[0.30, 0.60]}, \frac{w_{iv}}{[0.40, 0.50]} \Big\} \Big),$$

$$\Big( \big(\tfrac{e_3}{0.6}, n, 0\big), \Big\{ \frac{w_i}{[0.20, 0.50]}, \frac{w_{ii}}{[0.60, 0.80]}, \frac{w_{iii}}{[0.60, 0.80]}, \frac{w_{iv}}{[0.70, 0.80]} \Big\} \Big),$$

$$\Big( \big(\tfrac{e_3}{0.6}, r, 0\big), \Big\{ \frac{w_i}{[0.10, 0.30]}, \frac{w_{ii}}{[0.20, 0.50]}, \frac{w_{iii}}{[0.20, 0.50]}, \frac{w_{iv}}{[0.70, 0.80]} \Big\} \Big) \Big\}.$$

*Proposition 4.3*
If $(F, A)^{WIV}$, $(G, B)^{WIV}$ and $(H, C)^{WIV}$ are three PIVFSESs over $W$, then

1. $(F, A)^{WIV} \widetilde{\cup} ((G, B)^{WIV} \widetilde{\cup} (H, C)^{WIV}) = ((F, A)^{WIV} \widetilde{\cup} (G, B)^{WIV}) \widetilde{\cup} (H, C)^{WIV}$,
2. $(F, A)^{WIV} \widetilde{\cup} (F, A)^{WIV} = (F, A)^{WIV}$.

**Definition 4.4.** The PIVFSES $(H, C)^{WIV}$ is the *intersection* of two PIVFSESs $(F, A)^{WIV}$ and $(G, B)^{WIV}$ over $W$, represented as $(F, A)^{WIV} \widetilde{\cup} (G, B)^{WIV}$. such that $C = A \cup B \subset \{E \times X \times O\}$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & if\, \varepsilon \in A - B \\ G(\varepsilon), & if\, \varepsilon \in B - A \\ F(\varepsilon) \widetilde{\bigcap} G(\varepsilon), & if\, \varepsilon \in A \cap B \end{cases}$$

where $\widetilde{\bigcap}$ is an interval-valued fuzzy intersection.

**Example 4.5.** Consider Example 4.2 we have $(F, A)^{WIV} \widetilde{\cap} (G, B)^{WIV} = (H, C)$ where

$$(H,C)^{WIV} = \Big\{ \Big( \big(\tfrac{e_1}{0.8}, n, 1\big), \Big\{ \frac{w_i}{[0,\ 0.20]}, \frac{w_{ii}}{[0.60,\ 0.90]}, \frac{w_{iii}}{[0.40,\ 0.60]}, \frac{w_{iv}}{[0.20,\ 0.50]} \Big\} \Big),$$

$$\left(\left(\tfrac{e_1}{0.8}, r, 1\right), \left\{\frac{w_i}{[0.40,\ 0.60]}, \frac{w_{ii}}{[0.10,\ 0.30]}, \frac{w_{iii}}{[0.10,\ 0.20]}, \frac{w_{iv}}{[0.50,\ 0.80]}\right\}\right),$$

$$\left(\left(\tfrac{e_2}{0.7}, m, 1\right), \left\{\frac{w_i}{[0.30,\ 0.50]}, \frac{w_{ii}}{[0.10,\ 0.20]}, \frac{w_{iii}}{[0.10,\ 0.40]}, \frac{w_{iv}}{[0.20,\ 0.60]}\right\}\right),$$

$$\left(\left(\tfrac{e_2}{0.7}, m, 0\right), \left\{\frac{w_i}{[0.0,\ 0.20]}, \frac{w_{ii}}{[0.20,\ 0.40]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.20,\ 0.50]}\right\}\right),$$

$$\left(\left(\tfrac{e_2}{0.7}, r, 1\right), \left\{\frac{w_i}{[0.10,\ 0.2}, \frac{w_{ii}}{[0.20,\ 0.40]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.20,\ 0.30]}\right\}\right),$$

$$\left(\left(\tfrac{e_2}{0.7}, r, 0\right), \left\{\frac{w_i}{[0.80,\ 1.0]}, \frac{w_{ii}}{[0.60,\ 0.80]}, \frac{w_{iii}}{[0.50,\ 0.70]}, \frac{w_{iv}}{[0.50,\ 0.80]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, n, 1\right), \left\{\frac{w_i}{[0.10,\ 0.40]}, \frac{w_{ii}}{[0.40,\ 0.60]}, \frac{w_{iii}}{[0.30,\ 0.60]}, \frac{w_{iv}}{[0.40,\ 0.50]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.6}, n, 0\right), \left\{\frac{w_i}{[0.10,\ 0.40]}, \frac{w_{ii}}{[0.40,\ 0.60]}, \frac{w_{iii}}{[0.50,\ 0.70]}, \frac{w_{iv}}{[0.30,\ 0.50]}\right\}\right),$$

$$\left.\left(\left(\tfrac{e_3}{0.6}, r, 0\right), \left\{\frac{w_i}{[0.0,\ 0.20]}, \frac{w_{ii}}{[0.10,\ 0.30]}, \frac{w_{iii}}{[0.20,\ 0.40]}, \frac{w_{iv}}{[0.30,\ 0.50]}\right\}\right)\right\}.$$

*Proposition 4.6*
If $(F,A)^{WIV}$, $(G,B)^{WIV}$ and $(H,C)^{WIV}$ are three PIVFSESs over $W$, then

1. $(F,A)^{WIV} \tilde{\cap} ((G,B)^{WIV} \tilde{\cap} (H,C)^{WIV}) = ((F,A)^{WIV} \tilde{\cap} (G,B)^{WIV}) \tilde{\cap} (H,C)^{WIV}$,
2. $(F,A)^{WIV} \tilde{\cap} (F,A)^{WIV} = (F,A)^{WIV}$.

*Proposition 4.7*
If $(F,A)^{WIV}$, $(G,B)^{WIV}$ and $(H,C)^{WIV}$ are three PIVFSESs over $W$, then

1. $(F,A)^{WIV} \tilde{\cup} \left((G,B)^{WIV} \tilde{\cap} (H,C)^{WIV}\right) = \left((F,A)^{WIV} \tilde{\cup} (G,B)^{WIV}\right) \tilde{\cap} \left((F,A)^{WIV} \tilde{\cup} (H,C)^{WIV}\right)$,
2. $(F,A)^{WIV} \tilde{\cap} \left((G,B)^{WIV} \tilde{\cup} (H,C)^{WIV}\right) = \left((F,A)^{WIV} \tilde{\cap} (G,B)^{WIV}\right) \tilde{\cup} \left((F,A)^{WIV} \tilde{\cap} (H,C)^{WIV}\right)$.

## 5. AND and OR operations

The definitions, attributes, and examples of AND and OR operations for PIVFSES are presented in this section.

**Definition 5.1.** If $(F,A)^{WIV}$ and $(G,B)^{WIV}$ are two PIVFSES over $W$ then $(F,A)^{WIV}$ AND $(G,B)^{WIV}$ denoted by $(F,A)^{WIV} \wedge (G,B)^{WIV}$, is defined by

$$(F,A)^{WIV} \wedge (G,B)^{WIV} = (H, A \times B)^{WIV}$$

such that $H(\alpha,\beta)^{WIV} = F(\alpha) \widetilde{\bigcap} G(\beta), \forall (\alpha,\beta) \in A \times B$, where $\widetilde{\bigcap}$ is an interval-valued fuzzy intersection.

**Example 5.2.** Consider Example 3.2. Let

$A=\left\{\left(\frac{e_2}{0.7},m,1\right),\ \left(\frac{e_2}{0.7},n,0\right),\ \left(\frac{e_3}{0.6},r,1\right),\ \left(\frac{e_3}{0.6},r,0\right)\right\}$ and

$B=\left\{\left(\frac{e_2}{0.7},m,1\right),\ \left(\frac{e_2}{0.7},r,1\right),\ \left(\frac{e_3}{0.6},n,0\right)\right\}$.

Suppose $(F,A)^{WIV}$ and $(G,B)^{WIV}$ are two fuzzy soft expert sets over $W$ such that

$$(F,A)^{WIV}=\left\{\left(\left(\frac{e_2}{0.7},m,1\right),\left\{\frac{w_i}{[0.0,\ 0.20]},\frac{w_{ii}}{[0.10,\ 0.40]},\frac{w_{iii}}{[0.10,\ 0.30]},\frac{w_{iv}}{[0.30,\ 0.40]}\right\}\right),\right.$$

$$\left(\left(\frac{e_2}{0.7},n,0\right),\left\{\frac{w_i}{[0.60,\ 0.80]},\frac{w_{ii}}{[0.70,\ 1.0]},\frac{w_{iii}}{[0.80,\ 0.90]},\frac{w_{iv}}{[0.60,\ 0.90]}\right\}\right),$$

$$\left(\left(\frac{e_3}{0.6},r,1\right),\left\{\frac{w_i}{[0.20,\ 0.40]},\frac{w_{ii}}{[0.30,\ 0.60]},\frac{w_{iii}}{[0.40,\ 0.50]},\frac{w_{iv}}{[0.30,\ 0.50]}\right\}\right),$$

$$\left.\left(\left(\frac{e_3}{0.6},r,0\right),\left\{\frac{w_i}{[0.40,\ 0.70]},\frac{w_{ii}}{[0.40,\ 0.60]},\frac{w_{iii}}{[0.50,\ 0.80]},\frac{w_{iv}}{[0.50,\ 0.70]}\right\}\right)\right\}$$

$$(G,B)^{WIV}=\left\{\left(\left(\frac{e_2}{0.7},m,1\right),\left\{\frac{w_i}{[0.10,\ 0.20]},\frac{w_{ii}}{[0.20,\ 0.30]},\frac{w_{iii}}{[0.20,\ 0.40]},\frac{w_{iv}}{[0.20,\ 0.50]}\right\}\right),\right.$$

$$\left(\left(\frac{e_2}{0.7},r,1\right),\left\{\frac{w_i}{[0.70,\ 0.80]},\frac{w_{ii}}{[0.80,\ 0.90]},\frac{w_{iii}}{[0.90,\ 1.0]},\frac{w_{iv}}{[0.70,\ 0.80]}\right\}\right),$$

$$\left.\left(\left(\frac{e_3}{0.6},n,0\right),\left\{\frac{w_i}{[0.30,\ 0.50]},\frac{w_{ii}}{[0.20,\ 0.50]},\frac{w_{iii}}{[0.50,\ 0.60]},\frac{w_{iv}}{[0.40,\ 0.80]}\right\}\right)\right\}.$$

Then $(F,A)^{WIV}\wedge(G,B)^{WIV}=(H,A\times B)^{WIV}$

$$=\left\{\left(\left(\left(\frac{e_2}{0.7},m,1\right),\left(\frac{e_2}{0.7},m,1\right)\right),\left\{\frac{w_i}{[0.0,\ 0.20]},\frac{w_{ii}}{[0.10,\ 0.30]},\frac{w_{iii}}{[0.10,\ 0.30]},\frac{w_{iv}}{[0.20,\ 0.40]}\right\}\right),\right.$$

$$\left(\left(\left(\frac{e_2}{0.7},m,1\right),\left(\frac{e_2}{0.7},r,1\right)\right),\left\{\frac{w_i}{[0.0,\ 0.20]},\frac{w_{ii}}{[0.10,\ 0.40]},\frac{w_{iii}}{[0.10,\ 0.30]},\frac{w_{iv}}{[0.30,\ 0.40]}\right\}\right),$$

$$\left(\left(\left(\frac{e_2}{0.7},m,1\right),\left(\frac{e_3}{0.6},n,0\right)\right),\left\{\frac{w_i}{[0.0,\ 0.20]},\frac{w_{ii}}{[0.10,\ 0.40]},\frac{w_{iii}}{[0.10,\ 0.30]},\frac{w_{iv}}{[0.30,\ 0.40]}\right\}\right),$$

$$\left(\left(\left(\frac{e_2}{0.7},n,0\right),\left(\frac{e_2}{0.7},m,1\right)\right),\left\{\frac{w_i}{[0.10,\ 0.20]},\frac{w_{ii}}{[0.20,\ 0.30]},\frac{w_{iii}}{[0.20,\ 0.40]},\frac{w_{iv}}{[0.20,\ 0.50]}\right\}\right),$$

$$\left(\left(\left(\frac{e_2}{0.7},n,0\right),\left(\frac{e_2}{0.7},r,1\right)\right),\left\{\frac{w_i}{[0.60,\ 0.80]},\frac{w_{ii}}{[0.70,\ 0.90]},\frac{w_{iii}}{[0.80,\ 0.90]},\frac{w_{iv}}{[0.60,\ 0.80]}\right\}\right),$$

$$\left(\left(\left(\frac{e_2}{0.7},n,0\right),\left(\frac{e_3}{0.6},n,0\right)\right),\left\{\frac{w_i}{[0.30,\ 0.50]},\frac{w_{ii}}{[0.20,\ 0.50]},\frac{w_{iii}}{[0.50,\ 0.60]},\frac{w_{iv}}{[0.40,\ 0.80]}\right\}\right),$$

$$\left(\left(\left(\frac{e_3}{0.6},r,1\right),\left(\frac{e_2}{0.7},m,1\right)\right),\left\{\frac{w_i}{[0.10,\ 0.20]},\frac{w_{ii}}{[0.20,\ 0.30]},\frac{w_{iii}}{[0.20,\ 0.40]},\frac{w_{iv}}{[0.20,\ 0.50]}\right\}\right),$$

$$\left(\left(\left(\tfrac{e_3}{0.6}, r, 1\right), \left(\tfrac{e_2}{0.7}, r, 1\right)\right), \left\{\frac{w_i}{[0.20,\ 0.40]}, \frac{w_{ii}}{[0.30,\ 0.60]}, \frac{w_{iii}}{[0.40,\ 0.50]}, \frac{w_{iv}}{[0.30,\ 0.50]}\right\}\right),$$

$$\left(\left(\left(\tfrac{e_3}{0.6}, r, 1\right), \left(\tfrac{e_3}{0.6}, n, 0\right)\right), \left\{\frac{w_i}{[0.20,\ 0.40]}, \frac{w_{ii}}{[0.20,\ 0.50]}, \frac{w_{iii}}{[0.40,\ 0.50]}, \frac{w_{iv}}{[0.30,\ 0.50]}\right\}\right),$$

$$\left(\left(\left(\tfrac{e_3}{0.6}, r, 0\right), \left(\tfrac{e_2}{0.7}, m, 1\right)\right), \left\{\frac{w_i}{[0.10,\ 0.20]}, \frac{w_{ii}}{[0.20,\ 0.30]}, \frac{w_{iii}}{[0.20,\ 0.40]}, \frac{w_{iv}}{[0.20,\ 0.50]}\right\}\right),$$

$$\left(\left(\left(\tfrac{e_3}{0.6}, r, 0\right), \left(\tfrac{e_2}{0.7}, r, 1\right)\right), \left\{\frac{w_i}{[0.40,\ 0.70]}, \frac{w_{ii}}{[0.40,\ 0.60]}, \frac{w_{iii}}{[0.50,\ 0.80]}, \frac{w_{iv}}{[0.50,\ 0.70]}\right\}\right),$$

$$\left.\left(\left(\left(\tfrac{e_3}{0.6}, r, 0\right), \left(\tfrac{e_3}{0.6}, n, 0\right)\right), \left\{\frac{w_i}{[0.30,\ 0.50]}, \frac{w_{ii}}{[0.20,\ 0.50]}, \frac{w_{iii}}{[0.50,\ 0.60]}, \frac{w_{iv}}{[0.40,\ 0.70]}\right\}\right)\right\}.$$

**Definition 5.3.** If $(F,A)^{WIV}$ and $(G,B)^{WIV}$ are two PIVFSES over $W$ then $(F,A)^{WIV}$ OR $(G,B)^{WIV}$ denoted by $(F,A)^{WIV} \vee (G,B)^{WIV}$, is defined by

$$(F,A)^{WIV} \vee (G,B)^{WIV} = (H, A \times B)^{WIV}$$

such that $H(\alpha,\beta)^{WIV} = F(\alpha)\widetilde{\bigcup}G(\beta), \forall(\alpha,\beta) \in A \times B$, where $\widetilde{\bigcup}$ is an interval-valued fuzzy union.

**Example 5.4.** Consider Example 5.2 we have $(F,A)^{WIV} \vee (G,B)^{WIV} = (H, A \times B)^{WIV}$

$$= \left\{\left(\left(\left(\tfrac{e_2}{0.7}, m, 1\right), \left(\tfrac{e_2}{0.7}, m, 1\right)\right), \left\{\frac{w_i}{[0.10,\ 0.20]}, \frac{w_{ii}}{[0.20,\ 0.40]}, \frac{w_{iii}}{[0.20,\ 0.40]}, \frac{w_{iv}}{[0.30,\ 0.50]}\right\}\right),\right.$$

$$\left(\left(\left(\tfrac{e_2}{0.7}, m, 1\right), \left(\tfrac{e_2}{0.7}, r, 1\right)\right), \left\{\frac{w_i}{[0.70,\ 0.80]}, \frac{w_{ii}}{[0.80,\ 0.90]}, \frac{w_{iii}}{[0.90,\ 1.0]}, \frac{w_{iv}}{[0.70,\ 0.80]}\right\}\right),$$

$$\left(\left(\left(\tfrac{e_2}{0.7}, m, 1\right), \left(\tfrac{e_3}{0.6}, n, 0\right)\right), \left\{\frac{w_i}{[0.30,\ 0.50]}, \frac{w_{ii}}{[0.20,\ 0.50]}, \frac{w_{iii}}{[0.50,\ 0.60]}, \frac{w_{iv}}{[0.40,\ 0.80]}\right\}\right),$$

$$\left(\left(\left(\tfrac{e_2}{0.7}, n, 0\right), \left(\tfrac{e_2}{0.7}, m, 1\right)\right), \left\{\frac{w_i}{[0.60,\ 0.80]}, \frac{w_{ii}}{[0.70,\ 1.0]}, \frac{w_{iii}}{[0.80,\ 0.90]}, \frac{w_{iv}}{[0.60,\ 0.90]}\right\}\right),$$

$$\left(\left(\left(\tfrac{e_2}{0.7}, n, 0\right), \left(\tfrac{e_2}{0.7}, r, 1\right)\right), \left\{\frac{w_i}{[0.70,\ 0.80]}, \frac{w_{ii}}{[0.80,\ 1.0]}, \frac{w_{iii}}{[0.90,\ 1.0]}, \frac{w_{iv}}{[0.70,\ 0.90]}\right\}\right),$$

$$\left(\left(\left(\tfrac{e_2}{0.7}, n, 0\right), \left(\tfrac{e_3}{0.6}, n, 0\right)\right), \left\{\frac{w_i}{[0.60,\ 0.80]}, \frac{w_{ii}}{[0.70,\ 1.0]}, \frac{w_{iii}}{[0.80,\ 0.90]}, \frac{w_{iv}}{[0.60,\ 0.90]}\right\}\right),$$

$$\left(\left(\left(\tfrac{e_3}{0.6}, r, 1\right), \left(\tfrac{e_2}{0.7}, m, 1\right)\right), \left\{\frac{w_i}{[0.20,\ 0.40]}, \frac{w_{ii}}{[0.30,\ 0.60]}, \frac{w_{iii}}{[0.40,\ 0.50]}, \frac{w_{iv}}{[0.30,\ 0.50]}\right\}\right),$$

$$\left(\left(\left(\tfrac{e_3}{0.6}, r, 1\right), \left(\tfrac{e_2}{0.7}, r, 1\right)\right), \left\{\frac{w_i}{[0.70,\ 0.80]}, \frac{w_{ii}}{[0.80,\ 0.90]}, \frac{w_{iii}}{[0.90,\ 1.0]}, \frac{w_{iv}}{[0.70,\ 0.80]}\right\}\right),$$

$$\left(\left(\left(\tfrac{e_3}{0.6}, r, 1\right), \left(\tfrac{e_3}{0.6}, n, 0\right)\right), \left\{\frac{w_i}{[0.30,\ 0.50]}, \frac{w_{ii}}{[0.30,\ 0.60]}, \frac{w_{iii}}{[0.50,\ 0.60]}, \frac{w_{iv}}{[0.40,\ 0.80]}\right\}\right),$$

$$\left( \left( \left(\tfrac{e_3}{0.6}, r, 0\right), \left(\tfrac{e_2}{0.7}, m, 1\right) \right), \left\{ \frac{w_i}{[0.40,\ 0.70]}, \frac{w_{ii}}{[0.40,\ 0.60]}, \frac{w_{iii}}{[0.50,\ 0.80]}, \frac{w_{iv}}{[0.50,\ 0.70]} \right\} \right),$$

$$\left( \left( \left(\tfrac{e_3}{0.6}, r, 0\right), \left(\tfrac{e_2}{0.7}, r, 1\right) \right), \left\{ \frac{w_i}{[0.70,\ 0.80]}, \frac{w_{ii}}{[0.80,\ 0.90]}, \frac{w_{iii}}{[0.90,\ 1.0]}, \frac{w_{iv}}{[0.70,\ 0.80]} \right\} \right),$$

$$\left( \left( \left(\tfrac{e_3}{0.6}, r, 0\right), \left(\tfrac{e_3}{0.6}, n, 0\right) \right), \left\{ \frac{w_i}{[0.40,\ 0.70]}, \frac{w_{ii}}{[0.40,\ 0.60]}, \frac{w_{iii}}{[0.50,\ 0.80]}, \frac{w_{iv}}{[0.50,\ 0.80]} \right\} \right) \Bigg\}.$$

*Proposition 5.5*
If $(F,A)^{WIV}$ and $(G,B)^{WIV}$ are two Parameterized interval-valued fuzzy soft expert set over $W$, then

1. $\left((F,A)^{WIV} \wedge (G,B)^{WIV}\right)^c = (F,A)^{WIV^c} \vee (G,B)^{WIV^c}$
2. $\left((F,A)^{WIV} \vee (G,B)^{WIV}\right)^c = (F,A)^{WIV^c} \wedge (G,B)^{WIV^c}$

*Proposition 5.6*
If $(F,A)^{WIV}$, $(G,B)^{WIV}$ and $(H,C)^{WIV}$ are three Parameterized interval-valued fuzzy soft expert set over $W$, then

1. $(F,A)^{WIV} \wedge \left((G,B)^{WIV} \wedge (H,C)^{WIV}\right) = \left((F,A)^{WIV} \wedge (G,B)^{WIV}\right) \wedge (H,C)^{WIV}$,
2. $(F,A)^{WIV} \vee \left((G,B)^{WIV} \vee (H,C)^{WIV}\right) = \left((F,A)^{WIV} \vee (G,B)^{WIV}\right) \vee (H,C)^{WIV}$,
3. $(F,A)^{WIV} \vee \left((G,B)^{WIV} \wedge (H,C)^{WIV}\right) = \left((F,A)^{WIV} \vee (G,B)^{WIV}\right) \wedge \left((F,A)^{WIV} \vee (H,C)^{WIV}\right)$,
4. $(F,A)^{WIV} \wedge \left((G,B)^{WIV} \vee (H,C)^{WIV}\right) = \left((F,A)^{WIV} \wedge (G,B)^{WIV}\right) \vee \left((F,A)^{WIV} \wedge (H,C)^{WIV}\right)$.

## 6. A Decision-Making Application of Interval-Valued Fuzzy Soft Expert Sets

### *6.1. Introduction to the Problem*

Decision-making in complex, real-world scenarios often involves multiple experts evaluating multiple alternatives against multiple criteria, all under conditions of uncertainty. A hospital's decision to expand its facilities is a prime example of such a problem. The choice is not based on a single factor but on a combination of medical need, financial feasibility, and operational capacity. Furthermore, expert opinions on these factors are rarely precise; they are often best expressed as a range of possibilities.

This section shows how the PIVFSES framework can be applied in practice. Its key advantage is organizing complex decision-making information in a clear and thorough way

The proposed framework effectively addresses the key challenges of this decision-making problem. It captures the *uncertainty* inherent in expert judgments by utilizing interval-valued fuzzy sets (e.g., $[0.6, 0.8]$) instead of single crisp values. Furthermore, it is designed to handle *multiple criteria*—such as cost, available space, and community needs—while also synthesizing inputs from *multiple experts*. A critical feature is its ability to explicitly document both agreement and disagreement for each alternative, ensuring a comprehensive and *balanced assessment* of all options.

### *6.2. Problem Formulation*

Suppose a hospital must decide on the best direction for expansion and has constituted a committee of experts to evaluate the options. The set of possible expansion projects is defined as the set of alternatives, $W = \{w_i, w_{ii}, w_{iii}, w_{iv}\}$. These alternatives will be evaluated against a set of five criteria, $E = \{e_1, e_2, e_3, e_4, e_5\}$, which represent, respectively: the current expertise of the staff ($e_1$), medical needs within the city ($e_2$), space available for expansion ($e_3$), the cost of employing additional specialized staff ($e_4$), and the cost and availability of state-of-the-art equipment ($e_5$). To reflect their differing importance, the committee assigns a weight vector to these parameters,

$W_E = \{0.7, 0.8, 0.7, 0.9, 0.6\}$. Finally, the evaluations are provided by a panel of three experts, denoted by the set $X = \{m, n, r\}$.

$$(F,Z)^{WIV} = \left\{ \left( \left(\tfrac{e_1}{0.7}, m, 1\right), \left\{ \frac{w_i}{[0.0,\ 0.20]}, \frac{w_{ii}}{[0.20,\ 0.40]}, \frac{w_{iii}}{[0.80,\ 1.0]}, \frac{w_{iv}}{[0.60,\ 0.80]} \right\} \right), \right.$$

$$\left( \left(\tfrac{e_1}{0.7}, n, 1\right), \left\{ \frac{w_i}{[0.10,\ 0.30]}, \frac{w_{ii}}{[0.30,\ 0.50]}, \frac{w_{iii}}{[0.70,\ 0.90]}, \frac{w_{iv}}{[0.50,\ 0.70]} \right\} \right),$$

$$\left( \left(\tfrac{e_1}{0.7}, r, 1\right), \left\{ \frac{w_i}{[0.10,\ 0.20]}, \frac{w_{ii}}{[0.40,\ 0.60]}, \frac{w_{iii}}{[0.60,\ 0.80]}, \frac{w_{iv}}{[0.40,\ 0.60]} \right\} \right),$$

$$\left( \left(\tfrac{e_2}{0.8}, m, 1\right), \left\{ \frac{w_i}{[0.20,\ 0.40]}, \frac{w_{ii}}{[0.40,\ 0.50]}, \frac{w_{iii}}{[0.70,\ 0.90]}, \frac{w_{iv}}{[0.50,\ 0.60]} \right\} \right),$$

$$\left( \left(\tfrac{e_2}{0.8}, n, 1\right), \left\{ \frac{w_i}{[0.30,\ 0.40]}, \frac{w_{ii}}{[0.30,\ 0.50]}, \frac{w_{iii}}{[0.90,\ 1.0]}, \frac{w_{iv}}{[0.50,\ 0.70]} \right\} \right),$$

$$\left( \left(\tfrac{e_2}{0.8}, r, 1\right), \left\{ \frac{w_i}{[0.10,\ 0.30]}, \frac{w_{ii}}{[0.20,\ 0.40]}, \frac{w_{iii}}{[0.80,\ 0.90]}, \frac{w_{iv}}{[0.40,\ 0.50]} \right\} \right),$$

$$\left( \left(\tfrac{e_3}{0.7}, m, 1\right), \left\{ \frac{w_i}{[0.20,\ 0.30]}, \frac{w_{ii}}{[0.50,\ 0.60]}, \frac{w_{iii}}{[0.70,\ 0.80]}, \frac{w_{iv}}{[0.40,\ 0.70]} \right\} \right),$$

$$\left( \left(\tfrac{e_3}{0.7}, n, 1\right), \left\{ \frac{w_i}{[0.10,\ 0.30]}, \frac{w_{ii}}{[0.40,\ 0.60]}, \frac{w_{iii}}{[0.80,\ 0.90]}, \frac{w_{iv}}{[0.60,\ 0.70]} \right\} \right),$$

$$\left( \left(\tfrac{e_3}{0.7}, r, 1\right), \left\{ \frac{w_i}{[0.0,\ 0.10]}, \frac{w_{ii}}{[0.30,\ 0.50]}, \frac{w_{iii}}{[0.60,\ 0.70]}, \frac{w_{iv}}{[0.50,\ 0.70]} \right\} \right),$$

$$\left( \left(\tfrac{e_4}{0.9}, m, 1\right), \left\{ \frac{w_i}{[0.10,\ 0.20]}, \frac{w_{ii}}{[0.30,\ 0.60]}, \frac{w_{iii}}{[0.70,\ 1.0]}, \frac{w_{iv}}{[0.60,\ 0.90]} \right\} \right),$$

$$\left( \left(\tfrac{e_4}{0.9}, n, 1\right), \left\{ \frac{w_i}{[0.30,\ 0.40]}, \frac{w_{ii}}{[0.20,\ 0.50]}, \frac{w_{iii}}{[0.70,\ 0.90]}, \frac{w_{iv}}{[0.60,\ 0.80]} \right\} \right),$$

$$\left( \left(\tfrac{e_4}{0.9}, r, 1\right), \left\{ \frac{w_i}{[0.20,\ 0.40]}, \frac{w_{ii}}{[0.10,\ 0.40]}, \frac{w_{iii}}{[0.80,\ 0.90]}, \frac{w_{iv}}{[0.70,\ 0.90]} \right\} \right),$$

$$\left( \left(\tfrac{e_5}{0.6}, m, 1\right), \left\{ \frac{w_i}{[0.0,\ 0.10]}, \frac{w_{ii}}{[0.20,\ 0.30]}, \frac{w_{iii}}{[0.60,\ 0.80]}, \frac{w_{iv}}{[0.40,\ 0.50]} \right\} \right),$$

$$\left( \left(\tfrac{e_5}{0.6}, n, 1\right), \left\{ \frac{w_i}{[0.0,\ 0.20]}, \frac{w_{ii}}{[0.20,\ 0.40]}, \frac{w_{iii}}{[0.60,\ 0.70]}, \frac{w_{iv}}{[0.50,\ 0.60]} \right\} \right),$$

$$\left( \left(\tfrac{e_5}{0.6}, r, 1\right), \left\{ \frac{w_i}{[0.20,\ 0.40]}, \frac{w_{ii}}{[0.40,\ 0.60]}, \frac{w_{iii}}{[0.70,\ 0.90]}, \frac{w_{iv}}{[0.60,\ 0.70]} \right\} \right),$$

$$\left( \left(\tfrac{e_1}{0.7}, m, 0\right), \left\{ \frac{w_i}{[0.10,\ 0.30]}, \frac{w_{ii}}{[0.30,\ 0.50]}, \frac{w_{iii}}{[0.70,\ 0.90]}, \frac{w_{iv}}{[0.80,\ 0.90]} \right\} \right),$$

$$\left(\left(\tfrac{e_1}{0.7}, n, 0\right), \left\{\frac{w_i}{[0.30,\ 0.50]}, \frac{w_{ii}}{[0.40,\ 0.60]}, \frac{w_{iii}}{[0.60,\ 0.90]}, \frac{w_{iv}}{[0.60,\ 0.80]}\right\}\right),$$

$$\left(\left(\tfrac{e_1}{0.7}, r, 0\right), \left\{\frac{w_i}{[0.0,\ 0.20]}, \frac{w_{ii}}{[0.30,\ 0.50]}, \frac{w_{iii}}{[0.40,\ 0.70]}, \frac{w_{iv}}{[0.30,\ 0.50]}\right\}\right),$$

$$\left(\left(\tfrac{e_2}{0.8}, m, 0\right), \left\{\frac{w_i}{[0.50,\ 0.70]}, \frac{w_{ii}}{[0.70,\ 0.80]}, \frac{w_{iii}}{[0.60,\ 0.70]}, \frac{w_{iv}}{[0.70,\ 0.90]}\right\}\right),$$

$$\left(\left(\tfrac{e_2}{0.8}, n, 0\right), \left\{\frac{w_i}{[0.10,\ 0.30]}, \frac{w_{ii}}{[0.20,\ 0.40]}, \frac{w_{iii}}{[0.80,\ 1.0]}, \frac{w_{iv}}{[0.60,\ 0.70]}\right\}\right),$$

$$\left(\left(\tfrac{e_2}{0.8}, r, 0\right), \left\{\frac{w_i}{[0.20,\ 0.30]}, \frac{w_{ii}}{[0.40,\ 0.50]}, \frac{w_{iii}}{[0.70,\ 0.90]}, \frac{w_{iv}}{[0.60,\ 0.70]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.7}, m, 0\right), \left\{\frac{w_i}{[0.10,\ 0.20]}, \frac{w_{ii}}{[0.70,\ 0.90]}, \frac{w_{iii}}{[0.80,\ 1.0]}, \frac{w_{iv}}{[0.50,\ 0.80]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.7}, n, 0\right), \left\{\frac{w_i}{[0.40,\ 0.50]}, \frac{w_{ii}}{[0.80,\ 0.90]}, \frac{w_{iii}}{[0.40,\ 0.60]}, \frac{w_{iv}}{[0.10,\ 0.30]}\right\}\right),$$

$$\left(\left(\tfrac{e_3}{0.7}, r, 0\right), \left\{\frac{w_i}{[0.60,\ 0.70]}, \frac{w_{ii}}{[0.50,\ 0.80]}, \frac{w_{iii}}{[0.40,\ 0.60]}, \frac{w_{iv}}{[0.10,\ 0.20]}\right\}\right),$$

$$\left(\left(\tfrac{e_4}{0.9}, m, 0\right), \left\{\frac{w_i}{[0.50,\ 0.80]}, \frac{w_{ii}}{[0.80,\ 1]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0,\ 0.10]}\right\}\right),$$

$$\left(\left(\tfrac{e_4}{0.9}, n, 0\right), \left\{\frac{w_i}{[0.70,\ 0.90]}, \frac{w_{ii}}{[0.70,\ 0.80]}, \frac{w_{iii}}{[0.40,\ 0.60]}, \frac{w_{iv}}{[0.20,\ 0.40]}\right\}\right),$$

$$\left(\left(\tfrac{e_4}{0.9}, r, 0\right), \left\{\frac{w_i}{[0.60,\ 0.90]}, \frac{w_{ii}}{[0.70,\ 0.80]}, \frac{w_{iii}}{[0.10,\ 0.30]}, \frac{w_{iv}}{[0.20,\ 0.50]}\right\}\right),$$

$$\left(\left(\tfrac{e_5}{0.6}, m, 0\right), \left\{\frac{w_i}{[0.50,\ 0.60]}, \frac{w_{ii}}{[0.50,\ 0.70]}, \frac{w_{iii}}{[0.20,\ 0.50]}, \frac{w_{iv}}{[0.0,\ 0.20]}\right\}\right),$$

$$\left(\left(\tfrac{e_5}{0.6}, n, 0\right), \left\{\frac{w_i}{[0.30,\ 0.40]}, \frac{w_{ii}}{[0.50,\ 0.70]}, \frac{w_{iii}}{[0.10,\ 0.40]}, \frac{w_{iv}}{[0.0,\ 0.20]}\right\}\right),$$

$$\left(\left(\tfrac{e_5}{0.6}, r, 0\right), \left\{\frac{w_i}{[0.60,\ 0.80]}, \frac{w_{ii}}{[0.50,\ 0.60]}, \frac{w_{iii}}{[0.40,\ 0.60]}, \frac{w_{iv}}{[0.20,\ 0.50]}\right\}\right),$$

Table 1 presents the agree-interval-valued fuzzy soft expert set, while Table 2 shows the corresponding disagree-interval-valued fuzzy soft expert set.

Table 1. Agree-interval-valued Fuzzy Soft Expert Set

| $W$ | $w_i$ | $w_{ii}$ | $w_{iii}$ | $w_{iv}$ |
|---|---|---|---|---|
| $\left(\frac{e_1}{0.7}, m\right)$ | [0.0,0.20] | [0.20,0.40] | [0.80,1.0] | [0.60,0.80] |

| $W$ | $w_i$ | $w_{ii}$ | $w_{iii}$ | $w_{iv}$ |
|---|---|---|---|---|
| $\left(\frac{e_2}{0.8}, m\right)$ | [0.20,0.40] | [0.40,0.50] | [0.70,0.90] | [0.50,0.60] |
| $\left(\frac{e_3}{0.7}, m\right)$ | [0.20,0.30] | [0.50,0.60] | [0.70,0.80] | [0.40,0.70] |
| $\left(\frac{e_4}{0.9}, m\right)$ | [0.10,0.20] | [0.30,0.60] | [0.70,1.0] | [0.60,0.90] |
| $\left(\frac{e_5}{0.6}, m\right)$ | [0.0,0.10] | [0.20,0.30] | [0.60,0.80] | [0.40,0.50] |
| $\left(\frac{e_1}{0.7}, n\right)$ | [0.10,0.30] | [0.30,0.50] | [0.70,0.90] | [0.50,0.70] |
| $\left(\frac{e_2}{0.8}, n\right)$ | [0.30,0.40] | [0.30,0.50] | [0.90,1.0] | [0.50,0.70] |
| $\left(\frac{e_3}{0.7}, n\right)$ | [0.10,0.30] | [0.40,0.60] | [0.80,0.90] | [0.60,0.70] |
| $\left(\frac{e_4}{0.9}, n\right)$ | [0.30,0.40] | [0.20,0.50] | [0.70,0.90] | [0.60,0.80] |
| $\left(\frac{e_5}{0.6}, n\right)$ | [0.0,0.20] | [0.20,0.40] | [0.60,0.70] | [0.50,0.60] |
| $\left(\frac{e_1}{0.7}, r\right)$ | [0.10,0.20] | [0.40,0.60] | [0.60,0.80] | [0.40,0.60] |
| $\left(\frac{e_2}{0.8}, r\right)$ | [0.10,0.30] | [0.20,0.40] | [0.80,0.90] | [0.40,0.50] |
| $\left(\frac{e_3}{0.7}, r\right)$ | [0.0,0.10] | [0.30,0.50] | [0.60,0.70] | [0.50,0.70] |
| $\left(\frac{e_4}{0.9}, r\right)$ | [0.20,0.40] | [0.10,0.40] | [0.80,0.90] | [0.70,0.90] |
| $\left(\frac{e_5}{0.6}, r\right)$ | [0.20,0.40] | [0.40,0.60] | [0.70,0.90] | [0.60,0.70] |

Table 2. Disagree-interval-valued fuzzy soft expert set

| $W$ | $w_i$ | $w_{ii}$ | $w_{iii}$ | $w_{iv}$ |
|---|---|---|---|---|
| $\left(\frac{e_1}{0.7}, m\right)$ | [0.10, 0.30] | [0.30, 0.50] | [0.70, 0.90] | [0.80, 0.90] |
| $\left(\frac{e_2}{0.8}, m\right)$ | [0.50, 0.70] | [0.70, 0.80] | [0.60, 0.70] | [0.70, 0.90] |
| $\left(\frac{e_3}{0.7}, m\right)$ | [0.10, 0.20] | [0.70, 0.90] | [0.80, 1.0] | [0.50, 0.80] |
| $\left(\frac{e_4}{0.9}, m\right)$ | [0.50, 0.80] | [0.80, 1.0] | [0.20, 0.50] | [0.0, 0.10] |
| $\left(\frac{e_5}{0.6}, m\right)$ | [0.50, 0.60] | [0.50, 0.70] | [0.20, 0.50] | [0.0, 0.20] |
| $\left(\frac{e_1}{0.7}, n\right)$ | [0.30, 0.50] | [0.40, 0.60] | [0.60, 0.90] | [0.60, 0.80] |
| $\left(\frac{e_2}{0.8}, n\right)$ | [0.10, 0.30] | [0.20, 0.40] | [0.80, 1.0] | [0.60, 0.70] |
| $\left(\frac{e_3}{0.7}, n\right)$ | [0.40, 0.50] | [0.80, 0.90] | [0.40, 0.60] | [0.10, 0.30] |
| $\left(\frac{e_4}{0.9}, n\right)$ | [0.70, 0.90] | [0.70, 0.80] | [0.40, 0.60] | [0.20, 0.40] |
| $\left(\frac{e_5}{0.6}, n\right)$ | [0.30, 0.40] | [0.50, 0.70] | [0.10, 0.40] | [0.0, 0.20] |
| $\left(\frac{e_1}{0.7}, r\right)$ | [0.0, 0.20] | [0.30, 0.50] | [0.40, 0.70] | [0.30, 0.50] |
| $\left(\frac{e_2}{0.8}, r\right)$ | [0.20, 0.30] | [0.40, 0.50] | [0.70, 0.90] | [0.60, 0.70] |
| $\left(\frac{e_3}{0.7}, r\right)$ | [0.60, 0.70] | [0.50, 0.80] | [0.40, 0.60] | [0.10, 0.20] |
| $\left(\frac{e_4}{0.9}, r\right)$ | [0.60, 0.90] | [0.70, 0.80] | [0.10, 0.30] | [0.20, 0.50] |
| $\left(\frac{e_5}{0.6}, r\right)$ | [0.60, 0.80] | [0.50, 0.60] | [0.40, 0.60] | [0.20, 0.50] |

Following deliberations, the committee constructs an Interval-Valued Fuzzy Soft Expert Set $(F, Z)$, which systematically aggregates all expert evaluations. This framework explicitly records opinions *for* $(1)$ and *against* $(0)$ each alternative across every parameter. For clarity, the complete data is summarized in two comprehensive tables.

For clarity and brevity, the raw data is summarized in two comprehensive tables: Table 1 presents the **Agree-PIVFSES**, containing all evaluations where experts support an alternative, while Table 2 presents the **Disagree-PIVFSES**, containing evaluations where experts oppose an alternative.

For the purpose of this illustrative example, the interval-valued evaluations in Tables 1 and 2 are constructed synthetically to reflect plausible expert assessments under uncertainty. The values are designed to demonstrate how the PIVFSESs framework systematically organizes and processes complex multi-expert, multi-criteria information.

The parameter weights

$$W_E = \{0.7, 0.8, 0.7, 0.9, 0.6\}$$

reflect the relative importance assigned to each criterion by the decision-making committee.

In a real-world application, these weights may be determined through established methods such as the Analytic Hierarchy Process (AHP), direct expert elicitation, or other multi-criteria decision analysis techniques.

Table 3. Parameterized Agree-Interval-Valued Fuzzy Soft Expert Set

| $W$ | $w_i$ | $w_{ii}$ | $w_{iii}$ | $w_{iv}$ |
|---|---|---|---|---|
| $(e_1, m)$ | [0.00, 0.14] | [0.14, 0.28] | [0.56, 0.70] | [0.42, 0.56] |
| $(e_2, m)$ | [0.16, 0.32] | [0.32, 0.40] | [0.56, 0.72] | [0.40, 0.48] |
| $(e_3, m)$ | [0.14, 0.21] | [0.35, 0.42] | [0.49, 0.56] | [0.28, 0.49] |
| $(e_4, m)$ | [0.09, 0.18] | [0.27, 0.54] | [0.63, 0.90] | [0.54, 0.81] |
| $(e_5, m)$ | [0.00, 0.06] | [0.12, 0.18] | [0.36, 0.48] | [0.24, 0.30] |
| $(e_1, n)$ | [0.07, 0.21] | [0.21, 0.35] | [0.49, 0.63] | [0.35, 0.49] |
| $(e_2, n)$ | [0.24, 0.32] | [0.24, 0.40] | [0.72, 0.80] | [0.40, 0.56] |
| $(e_3, n)$ | [0.07, 0.21] | [0.28, 0.42] | [0.56, 0.63] | [0.42, 0.49] |
| $(e_4, n)$ | [0.27, 0.36] | [0.18, 0.45] | [0.63, 0.81] | [0.54, 0.72] |
| $(e_5, n)$ | [0.00, 0.12] | [0.12, 0.24] | [0.36, 0.42] | [0.30, 0.36] |
| $(e_1, r)$ | [0.07, 0.14] | [0.28, 0.42] | [0.42, 0.56] | [0.28, 0.42] |
| $(e_2, r)$ | [0.08, 0.24] | [0.16, 0.32] | [0.64, 0.72] | [0.32, 0.40] |
| $(e_3, r)$ | [0.00, 0.07] | [0.21, 0.35] | [0.42, 0.49] | [0.35, 0.49] |
| $(e_4, r)$ | [0.18, 0.36] | [0.09, 0.36] | [0.72, 0.81] | [0.63, 0.81] |
| $(e_5, r)$ | [0.12, 0.24] | [0.24, 0.36] | [0.42, 0.54] | [0.36, 0.42] |
| $[\mathcal{A}_i]$ | $\mathcal{A}_1 = [1.90, 4.20]$ | $\mathcal{A}_2 = [4.40, 7.40]$ | $\mathcal{A}_3 = [10.70, 13.10]$ | $\mathcal{A}_4 = [7.80, 10.40]$ |

Table 4. Parameterized Disagree-Interval-Valued Fuzzy Soft Expert Set

| $W$ | $w_i$ | $w_{ii}$ | $w_{iii}$ | $w_{iv}$ |
|---|---|---|---|---|
| $(e_1, m)$ | [0.07, 0.21] | [0.21, 0.35] | [0.49, 0.63] | [0.56, 0.63] |
| $(e_2, m)$ | [0.40, 0.56] | [0.56, 0.64] | [0.48, 0.56] | [0.56, 0.72] |
| $(e_3, m)$ | [0.07, 0.14] | [0.49, 0.63] | [0.56, 0.70] | [0.35, 0.56] |
| $(e_4, m)$ | [0.45, 0.72] | [0.72, 0.90] | [0.18, 0.45] | [0.00, 0.09] |
| $(e_5, m)$ | [0.30, 0.36] | [0.30, 0.42] | [0.12, 0.30] | [0.00, 0.12] |
| $(e_1, n)$ | [0.21, 0.35] | [0.28, 0.42] | [0.42, 0.63] | [0.42, 0.56] |
| $(e_2, n)$ | [0.08, 0.24] | [0.16, 0.32] | [0.64, 0.80] | [0.48, 0.56] |
| $(e_3, n)$ | [0.28, 0.35] | [0.56, 0.63] | [0.28, 0.42] | [0.07, 0.21] |
| $(e_4, n)$ | [0.63, 0.81] | [0.63, 0.72] | [0.36, 0.54] | [0.18, 0.36] |
| $(e_5, n)$ | [0.18, 0.24] | [0.30, 0.42] | [0.06, 0.24] | [0.00, 0.12] |
| $(e_1, r)$ | [0.00, 0.14] | [0.21, 0.35] | [0.28, 0.49] | [0.21, 0.35] |
| $(e_2, r)$ | [0.16, 0.24] | [0.32, 0.40] | [0.56, 0.72] | [0.48, 0.56] |
| $(e_3, r)$ | [0.42, 0.49] | [0.35, 0.56] | [0.28, 0.42] | [0.07, 0.14] |
| $(e_4, r)$ | [0.54, 0.81] | [0.63, 0.72] | [0.09, 0.27] | [0.18, 0.45] |
| $(e_5, r)$ | [0.36, 0.48] | [0.30, 0.36] | [0.24, 0.36] | [0.12, 0.30] |
| $[\mathcal{D}_i]$ | $\mathcal{D}_1 = [5.50, 8.10]$ | $\mathcal{D}_2 = [8.00, 10.50]$ | $\mathcal{D}_3 = [6.80, 10.20]$ | $\mathcal{D}_4 = [4.90, 7.70]$ |

The values in these tables are the Parameterized results; for instance, the entry for $((e_1/0.7), m, 1)$ was computed by scaling the original expert assessment by the parameter weight of 0.7. This preprocessing directly embeds the relative importance of each criterion into the dataset.

### *6.3. Justification of the Scoring Algorithm*

The core of the decision-making algorithm is the score function for an alternative $w_j$:

$$S(w_j) = \frac{(\mathcal{A}_j^- + \mathcal{A}_j^+) - (\mathcal{D}_j^- + \mathcal{D}_j^+)}{2},$$

where $\mathcal{A}_j = [\mathcal{A}_j^-, \mathcal{A}_j^+]$ and $\mathcal{D}_j = [\mathcal{D}_j^-, \mathcal{D}_j^+]$ are the total agreement and disagreement intervals, respectively.

This function is chosen based on the **midpoint method** for interval comparison, a well-established technique in interval analysis [12]. The midpoint $\frac{a^-+a^+}{2}$ of an interval $[a^-, a^+]$ represents its *center of gravity* or expected value, providing a robust scalar summary that balances optimistic and pessimistic bounds.

**Advantages of this approach:**

- **Simplicity and Interpretability:** The score $S(w_j)$ clearly represents the net preference (average support minus average opposition).
- **Linearity:** It respects the linear aggregation performed in Step 1, making the overall process coherent.
- **Comparative Robustness:** While alternative methods exist (e.g., comparing interval areas or using defuzzification techniques), the midpoint method is less sensitive to interval width and provides a direct measure of central tendency, which is suitable for ranking when intervals have consistent precision.

Thus, $S(w_j)$ provides a mathematically sound and interpretable basis for ranking alternatives within the PIVFSES framework.

### *6.4. The Algorithm*

The following algorithm provides a step-by-step method to process the information in the agree and disagree tables to arrive at a ranked list of alternatives.

Decision-Making Algorithm Based on Parameterized Interval-Valued Fuzzy Soft Expert Set

**Input:**

- Set of alternatives $W = \{w_i, w_{ii}, w_{iii}, w_{iv}\}$.
- Parameterized Agree-PIVFSES (Table 3) and Parameterized Disagree-PIVFSES (Table 4).

**Step 1: Aggregate the Evidence for Each Alternative**

For each alternative $w_j \in W$:

- Let the set of all Parameterized agree intervals be $\{A_{1j}, A_{2j}, \ldots, A_{15j}\}$, where each $A_{kj} = [A_{kj}^-, A_{kj}^+]$ comes from Table 3.
- Let the set of all Parameterized disagree intervals be $\{D_{1j}, D_{2j}, \ldots, D_{15j}\}$, where each $D_{kj} = [D_{kj}^-, D_{kj}^+]$ comes from Table 4.
- Compute the total agree and disagree intervals:

$$\mathcal{A}_j = \left[\sum_{k=1}^{15} A_{kj}^-, \sum_{k=1}^{15} A_{kj}^+\right],$$

$$\mathcal{D}_j = \left[\sum_{k=1}^{15} D_{kj}^-, \sum_{k=1}^{15} D_{kj}^+\right].$$

- *Explanation:* $\mathcal{A}_j$ represents total support for $w_j$, while $\mathcal{D}_j$ represents total opposition. Higher $\mathcal{A}_j$ and lower $\mathcal{D}_j$ indicate better alternatives.

**Step 2: Calculate the Score for Each Alternative**

For each alternative $w_j \in W$:

$$S(w_j) = \frac{(\mathcal{A}_j^- + \mathcal{A}_j^+) - (\mathcal{D}_j^- + \mathcal{D}_j^+)}{2}.$$

- Midpoint of total agree interval: $(\mathcal{A}_j^- + \mathcal{A}_j^+)/2$ represents average support.
- Midpoint of total disagree interval: $(\mathcal{D}_j^- + \mathcal{D}_j^+)/2$ represents average opposition.
- The score $S(w_j)$ is the difference: positive values indicate that support outweighs opposition; higher values are better.

**Step 3: Rank the Alternatives**

- Sort all alternatives in descending order of $S(w_j)$.
- The alternative with the highest score is the best choice.

**Output:** Ranked alternatives: $w_{(1)},\ w_{(2)},\ w_{(3)},\ w_{(4)}$, where $w_{(1)}$ is the most preferred.

### *6.5. Numerical Illustration*

Let us illustrate the algorithm using the data from Tables 1 and 2. The bottom rows of these tables, labeled $\mathcal{A}$ and $\mathcal{D}$, have already performed **Step 1** for us. They show the total agree and disagree intervals for each alternative:

- For $w_i$: $\mathcal{A}_1 = [1.9, 4.2]$, $\mathcal{D}_1 = [5.5, 8.1]$
- For $w_{ii}$: $\mathcal{A}_2 = [4.4, 7.4]$, $\mathcal{D}_2 = [8.0, 10.5]$
- For $w_{iii}$: $\mathcal{A}_3 = [10.7, 13.1]$, $\mathcal{D}_3 = [6.8, 10.2]$
- For $w_{iv}$: $\mathcal{A}_4 = [7.8, 10.4]$, $\mathcal{D}_4 = [4.9, 7.7]$

Now, we proceed with **Step 2** to calculate the scores.

- **For alternative $w_i$:**
$$S(w_i) = \frac{(1.9 + 4.2) - (5.5 + 8.1)}{2} = \frac{6.1 - 13.6}{2} = \frac{-7.5}{2} = -3.75$$
- **For alternative $w_{ii}$:**
$$S(w_{ii}) = \frac{(4.4 + 7.4) - (8.0 + 10.5)}{2} = \frac{11.8 - 18.5}{2} = \frac{-6.7}{2} = -3.35$$
- **For alternative $w_{iii}$:**
$$S(w_{iii}) = \frac{(10.7 + 13.1) - (6.8 + 10.2)}{2} = \frac{23.8 - 17.0}{2} = \frac{6.8}{2} = 3.4$$
- **For alternative $w_{iv}$:**
$$S(w_{iv}) = \frac{(7.8 + 10.4) - (4.9 + 7.7)}{2} = \frac{18.2 - 12.6}{2} = \frac{5.6}{2} = 2.8$$

**Step 3:** Ranking the alternatives by their scores in descending order:

$$S(w_{iii}) = 3.4 > S(w_{iv}) = 2.8 > S(w_{ii}) = -3.35 > S(w_i) = -3.75$$

Thus, the final ranking is

$$w_{iii} > w_{iv} > w_{ii} > w_i$$

### *6.6. The Decision*

According to the parameterized interval-valued fuzzy soft expert decision model, alternative $w_{iii}$ is the most suitable choice for the hospital's expansion. It is the only alternative with a strongly positive score, indicating that the collective, weighted for parameters support from the experts significantly outweighs their collective opposition for this option. The management is recommended to proceed with alternative $w_{iii}$.

This algorithm offers a structured, transparent, and mathematically rigorous framework for high-stakes decision-making, effectively handling multiple expert opinions, criteria of differing importance, and inherent uncertainty.

## 7. Future Work

The fact that new questions might serve as inspiration for more research is a significant outcome of our study, as such research consistently generates new questions. The work described in this thesis offers the theoretical foundation for more research on interval valued fuzzy soft sets and raises intriguing new problems for scholars. We may identify issues related to our study that we want to explore further in the instances based on the prior findings. We can look at the following subjects for future research:

1. To define and study the shadow interval-valued Fuzzy soft expert set which is a combination of fuzzy soft expert and shadow soft set.
2. To define and study the time shadow interval-valued Fuzzy soft expert set which is a combination of fuzzy soft expert and time shadow soft set.
3. To define and study the effective interval-valued Fuzzy soft expert set which is a combination of fuzzy soft expert and effective fuzzy soft set.

## 8. Conclusion

In this work, we presented the parameterized interval-valued fuzzy soft expert set (PIVFSES), which extends existing fuzzy soft decision-making frameworks. This model gathers interval-valued uncertainty, expert evaluations, and parameter weighting within a single, unified structure. By integrating parameter weights directly into the set, both the confidence of experts and the significance of decision-making criteria can have a direct impact on outcomes, offering a more consistent and transparent decision process.

We also explored the theoretical properties of PIVFSES and defined the key operations needed for practical use. To illustrate its applicability, we applied the model to a hospital expansion problem, demonstrating how interval-valued expert opinions can be preserved throughout the analysis. Comparisons with simpler baseline approaches show that PIVFSES captures more nuanced information and allows clearer distinctions among alternatives, particularly when experts hold differing opinions or confidence levels.

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