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A Multi-device Randomized Response Model for Efficient Estimation of Sensitive Attributes

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Abstract The Randomized Response Technique (RRT), first introduced by Warner, has become a fundamental approach for estimating sensitive characteristics while ensuring respondent anonymity. Over time, enhancements such as the two-device design by Mangat and Singh have improved both the protection of privacy and the accuracy of estimators. Building on this groundwork, the present research proposes a new RRT model that incorporates multiple randomization devices, providing greater flexibility in balancing efficiency with privacy preservation. Theoretical properties of the model are developed, and criteria for efficiency comparison are established. Numerical analyses are conducted, with special emphasis on scenarios involving three randomization devices. In addition to efficiency improvements, the use of layered randomization fosters greater respondent confidence, thereby increasing the likelihood of truthful responses. Overall, the proposed model offers a practical and reliable advancement in sensitive data collection methodologies, with promising applications in social, health, and behavioral research.

Keywords Sensitive Behaviors, Non-sampling Errors, Sample Surveys, Randomized Response Technique

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1. Introduction

Survey research is a principal tool for assessing attitudes and behaviors across diverse fields. However, inquiries into sensitive topics (e.g., substance abuse, illegal activities, or stigmatized conditions) present significant methodological challenges. Researchers must protect respondent privacy while mitigating substantial non-sampling errors, such as refusal to participate or deliberate misreporting, which introduce critical bias. Prior to the development of the Randomized Response Technique (RRT), effective solutions to these problems were limited.

The RRT is a survey method designed to eliminate response bias on sensitive issues by collecting information indirectly. Using a randomizing device (e.g., cards, spinners, dice, or coins), a respondent selects one of two or more questions—at least one of which is sensitive—based on a known probability. The respondent answers truthfully without revealing the selected question, ensuring the interviewer cannot infer their status with certainty. This design protects individual anonymity while providing sufficient aggregate data. By leveraging the known

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randomization probabilities, researchers can derive unbiased statistical estimates, such as the population proportion possessing the sensitive attribute, for the group as a whole. By ensuring respondent confidentiality, the RRT fosters truthful participation, enabling the study of topics, including illegal and socially deviant behaviors, which are often inaccessible through direct questioning.

The Randomized Response Technique (RRT), originally introduced by Warner [19], has inspired extensive research and development. Building on this foundational work, many scholars have proposed alternative estimation approaches, refined parameter selection strategies, and modifications to the original framework aimed at improving efficiency and broadening applicability [1, 2, 3, 4, 5, 7, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20]. Collectively, these contributions have established RRT as an indispensable tool for investigating sensitive behaviors that are otherwise difficult to measure.

Mangat and Singh [16] enhanced Warner's model by introducing two randomization devices, adding an extra layer of protection that increased respondent trust, promoted truthful reporting, and improved estimator efficiency. Building on their contribution, we propose a further refinement of Warner's [19] framework through a more efficient randomized response model that utilizes multi-device. Each additional device reduces the direct link between a respondent's true status and their reported response, thereby reinforcing their perception of privacy and encouraging greater honesty. In practice, using multi-device (e.g., cards, spinners, dice, or coins) instead of a single device significantly enhances both the reliability and effectiveness of the technique.

We begin by reviewing the original Warner design [19] and the Mangat and Singh model [16], with adjustments in notation where necessary. We then introduce the proposed randomized response model employing multi-device, derive its theoretical properties, and establish criteria for efficiency comparison. Finally, we present a detailed description and numerical evaluation for the specific case of three randomizing devices.

2. The original Warner's design

Warner's model [19] is designed to estimate the proportion π_a of a population that possesses a sensitive attribute A. A simple random sample of size n is drawn with replacement. Each respondent uses a randomizing device that selects one of two statements with probabilities p_1 and $1 - p_1$, respectively:

(a) "I belong to sensitive group A."

(b) "I do not belong to sensitive group A."

The respondent answers "yes" or "no" truthfully to the statement selected, without revealing which statement was chosen. The probability of a "yes" response is:

$$\Pr(\text{yes}) = p_1\pi_a + (1 - p_1)(1 - \pi_a). \tag{1}$$

Let $\hat{\lambda}_1$ be the observed proportion of "yes" answers in the sample. An unbiased estimator for π_a is derived from (1):

$$(\hat{\pi}_a)_W = \frac{\hat{\lambda}_1 - (1 - p_1)}{1 - 2(1 - p_1)} \qquad \text{for } p_1 \neq 0.5. \tag{2}$$

The variance of this estimator is:

$$\text{var}(\hat{\pi}_a)_W = \frac{\pi_a(1 - \pi_a)}{n} + \frac{(1 - p_1)\big[1 - (1 - p_1)\big]}{n\,[1 - 2(1 - p_1)]^2}. \tag{3}$$

3. Mangat & Singh's design

Mangat and Singh [16] proposed a model with two random devices, R_1 and R_2.

- Device R_1 has two options:
  (a) "I belong to group A." (with probability p_2)
  (b) "Use device R_2." (with probability $1 - p_2$)
- Device R_2 is identical to Warner's device, with statements chosen with probability (p_1) and ($1 - p_1$).

The respondent uses R_1 first and only proceeds to R_2 if instructed. The probability of a "yes" answer is

$$\Pr(\text{yes}) = \pi_a p_2 + (1 - p_2)p_1\pi_a - (1 - p_1)(1 - p_2)\pi_a + (1 - p_1)(1 - p_2). \tag{4}$$

The unbiased estimator for π_a and its variance are given by:

$$(\hat{\pi}_a)_{\text{MS}} = \frac{\hat{\lambda}_2 - (1 - p_1)(1 - p_2)}{1 - 2(1 - p_1)(1 - p_2)}, \tag{5}$$

and

$$\text{var}(\hat{\pi}_a)_{\text{MS}} = \frac{\pi_a(1 - \pi_a)}{n} + \frac{(1 - p_1)(1 - p_2)\left[1 - (1 - p_1)(1 - p_2)\right]}{n\left[1 - 2(1 - p_1)(1 - p_2)\right]^2}. \tag{6}$$

where $\hat{\lambda}_2$ is the observed proportion of "yes" answers.

4. The proposed model with multi-device

In the proposed model with multi-device (e.g., cards, spinners, dice, or coins), a sample of size n is selected by simple random sampling with replacement from the population. Each respondent selected in the sample is provided with m random devices $R_i(i = 1, 2, ..., m)$.

- Device $R_i(i = 1, 2, ..., m - 1)$ has two options:
  (a) "I belong to group A." (with probability p_{m-i+1})
  (b) "Use device R_{i+1}." (with probability $1 - p_{m-i+1}$)
- Device R_m is identical to Warner's device, with statements chosen with probability (p_1) and ($1 - p_1$).

All respondents start with R_1 and use only the other random devices $R_i(i = 1, 2, ..., m)$ if they are directed by the random device R_{i-1} (for $i = 1, 2, ..., m$). The respondent answers "yes" or "no" truthfully based on the selected statement and their actual status. All previous procedures are done unseen by the interviewer, since the respondent stands behind a barrier and tells the interviewer "yes" or "no" only unobserved by the interviewer and after complete the whole procedure. The probability of "yes" answers will be:

$$\begin{aligned}\Pr(\text{yes}) = p_m\pi_a + (1 - p_m)\Big[p_{m-1}\pi_a + (1 - p_{m-1})\Big[p_{m-2}\pi_a + (1 - p_{m-2})\Big[\cdots \\ \Big[p_2\pi_a + (1 - p_2)\big[p_1\pi_a + (1 - p_1)(1 - \pi_a)\big]\Big]\cdots\Big].\end{aligned} \tag{7}$$

4.1. Estimation of parameters

Let $(\hat{\pi}_a)_m$ denotes to the estimate of sensitive proportion with m random devices. The estimate of $(\hat{\pi}_a)_m$ with m random devices is unbiased and it will be as:

$$(\hat{\pi}_a)_m = \frac{\hat{\lambda}_m - \prod_{i=1}^{m}(1-p_i)}{1-2\prod_{i=1}^{m}(1-p_i)}, \tag{8}$$

where $\hat{\lambda}_m$ is the observed proportion of ”yes” answers.
The properties of the estimator $(\hat{\pi}_a)_m$ could be reached by the following theorems.

Theorem 1. The estimator $(\hat{\pi}_a)_m$ is unbiased for $(\pi_a)_m$.
Proof. Taking the expectation of (8)

$$E\big[(\hat{\pi}_a)_m\big] = \frac{E(\hat{\lambda}_m)}{1-2\prod_{i=1}^{m}(1-p_i)} - \frac{\prod_{i=1}^{m}(1-p_i)}{1-2\prod_{i=1}^{m}(1-p_i)}. \tag{9}$$

Since $n\hat{\lambda}_m$ follows a binomial distribution with parameters n and λ_m, and using formula (7) it is possible to get:

$$E\big[(\hat{\pi}_a)_m\big] = \pi_a. \tag{10}$$

Thus, $(\hat{\pi}_a)_m$ is unbiased estimator of $(\pi_a)_m$.

Theorem 2. The variance of the unbiased estimator $(\hat{\pi}_a)_m$ is

$$\text{var}(\hat{\pi}_a)_m = \frac{\pi_a(1-\pi_a)}{n} + \frac{\prod_{i=1}^{m}(1-p_i)\left[1-\prod_{i=1}^{m}(1-p_i)\right]}{n\big[1-2\prod_{i=1}^{m}(1-p_i)\big]^2}. \tag{11}$$

Proof. Using formula (8), the variance of $(\hat{\pi}_a)_m$, will be

$$\text{var}(\hat{\pi}_a)_m = \frac{1}{\big[1-2\prod_{i=1}^{m}(1-p_i)\big]^2}\ \text{var}(\hat{\lambda}_m). \tag{12}$$

Since $n\hat{\lambda}_m$ follows a binomial distribution with parameters n and λ_m, then

$$\text{var}(\hat{\lambda}_m) = \frac{\lambda_m(1-\lambda_m)}{n}. \tag{13}$$

Substituting in (12) by (13) we can get,

$$\text{var}(\hat{\pi}_a)_m = \frac{\lambda_m(1-\lambda_m)}{n\big[1-2\prod_{i=1}^{m}(1-p_i)\big]^2}. \tag{14}$$

We can use (7) to calculate $\lambda_m(1-\lambda_m)$ as follow,

$$\begin{aligned}\lambda_m(1-\lambda_m) &= \Big\{\pi_a\big[1-2\prod_{i=1}^{m}(1-p_i)\big]+\prod_{i=1}^{m}(1-p_i)\Big\}\Big\{-\pi_a\big[1-2\prod_{i=1}^{m}(1-p_i)\big]+\big[1-\prod_{i=1}^{m}(1-p_i)\big]\Big\}\\
&= -\pi_a^2\big[1-2\prod_{i=1}^{m}(1-p_i)\big]^2+\pi_a\big[1-2\prod_{i=1}^{m}(1-p_i)\big]\big[1-\prod_{i=1}^{m}(1-p_i)\big]\\
&\quad -\pi_a\big[1-2\prod_{i=1}^{m}(1-p_i)\big]\prod_{i=1}^{m}(1-p_i)+\prod_{i=1}^{m}(1-p_i)\big[1-\prod_{i=1}^{m}(1-p_i)\big]\\
&= \pi_a(1-\pi_a)\big[1-2\prod_{i=1}^{m}(1-p_i)\big]^2+\prod_{i=1}^{m}(1-p_i)\big[1-\prod_{i=1}^{m}(1-p_i)\big]. \qquad (15)\end{aligned}$$

Then, it is easy to get (11) by inserting (15) in (14).

4.2. *Efficiency comparison*

In the present section, we will investigate the conditions under which the proposed model with m random devices will be more efficient than the model with random devices less than m (say $m-r$).

Theorem 3. The model with m random devices is more efficient than the model with $(m-r)$ random devices $(r=1,2,...,m)$ if and only if:

$$\prod_{i=m-r+1}^{m}(1-p_i) < \frac{1-\prod_{i=1}^{m-r}(1-p_i)}{1-\prod_{i=1}^{m}(1-p_i)}. \qquad (16)$$

Proof. Clearly, the proposed estimate with m random devices $(\hat{\pi}_a)_m$ becomes more efficient than the estimate with $m-r$ random devices $(\hat{\pi}_a)_{m-r}$, $r<m$ if

$$\text{var}(\hat{\pi}_a)_m < \text{var}(\hat{\pi}_a)_{m-r}, \quad r<m. \qquad (17)$$

The above inequality can be expressed as:

$$\frac{\pi_a(1-\pi_a)}{n}+\frac{\prod_{i=1}^{m}(1-p_i)\,[1-\prod_{i=1}^{m}(1-p_i)]}{n\,[1-2\prod_{i=1}^{m}(1-p_i)]^2} < \frac{\pi_a(1-\pi_a)}{n}+\frac{\prod_{i=1}^{m-r}(1-p_i)\,[1-\prod_{i=1}^{m-r}(1-p_i)]}{n\,[1-2\prod_{i=1}^{m-r}(1-p_i)]^2}. \qquad (18)$$

The above inequality can be reduced to

$$\prod_{i=1}^{m}(1-p_i)-\Big[\prod_{i=1}^{m}(1-p_i)\Big]^2 < \prod_{i=1}^{m-r}(1-p_i)-\Big[\prod_{i=1}^{m-r}(1-p_i)\Big]^2. \qquad (19)$$

Thus,

$$\prod_{i=m-r+1}^{m}(1-p_i) < \frac{1-\prod_{i=1}^{m-r}(1-p_i)}{1-\prod_{i=1}^{m}(1-p_i)}. \qquad (20)$$

On the other hand, it is possible to use a converse procedure to prove the necessary condition.

5. The suggested model with three random devices

To illustrate the concept of the proposed design using multiple randomization devices, a specific case involving three distinct random devices is presented. We are concerned with the estimate of the proportion of respondents with sensitive characteristic, π_a in case of three different random devices. The size of the random sample (selected with replacement) is n. Each respondent in the sample is provided with three different random devices (e.g. cards, a spinner, or a coin).

- Device R_1 has two options:
  (a) "I belong to group A." (with probability p_3)
  (b) "Use device R_2." (with probability $1 - p_3$)
- Device R_2 has two options:
  (a) "I belong to group A." (with probability p_2)
  (b) "Use device R_3." (with probability $1 - p_2$)
- Device R_3 has two options:
  (a) "I belong to group A." (with probability p_1)
  (b) "I do not belong to sensitive group A." (with probability $1 - p_1$)

All respondents must use the random device R_1. And use only the other random devices $R_i, i = 2, 3$ if they are directed by the random device $R_{i-1}, i = 2, 3$. In the random devices $R_i, i = 1, 2, 3$, and in statement (a) or R_3 in statement (b), the respondent answers "yes" or "no" according to the statement selected and the actual status he possesses. The respondent unobserved by the interviewer completes the whole procedure (i.e. stands behind a barrier and say "yes" or "no").
In this case, the probability of "yes" answer (λ_3) will be as:

$$\lambda_3 = p_3\pi_a + (1 - p_3)\Big[p_2\pi_a + (1 - p_2)\big[p_1\pi_a + (1 - p_1)(1 - \pi_a)\big]\Big]. \tag{21}$$

5.1. Estimation of parameters

The estimate of $(\hat{\pi}_a)_3$ and its variance and referring to their general form [(8), (11)] will be given by the following corollaries.

Corollary 1. The unbiased estimator of $(\hat{\pi}_a)_3$ in case of random devices is

$$(\hat{\pi}_a)_3 = \frac{\hat{\lambda}_3 - (1 - p_1)(1 - p_2)(1 - p_3)}{1 - 2(1 - p_1)(1 - p_2)(1 - p_3)}, \tag{22}$$

where $\hat{\lambda}_3$ is the observed proportion of "yes" answer in the sample of size n.

Corollary 2. The variance of $(\hat{\pi}_a)_3$ is given by

$$\text{var}(\hat{\pi}_a)_3 = \frac{\pi_a(1 - \pi_a)}{n} + \frac{(1 - p_1)(1 - p_2)(1 - p_3)\left[1 - (1 - p_1)(1 - p_2)(1 - p_3)\right]}{n\left[1 - 2(1 - p_1)(1 - p_2)(1 - p_3)\right]^2}. \tag{23}$$

5.2. Numerical investigation

In the present section, the precision of the suggested model with three random devices, $(\hat{\pi}_a)_3$, will be numerically investigated for different values of the parameters π_a, p_1, p_2, and p_3. The following values were used to calculate the variance of $(\hat{\pi}_a)_3$: $n = 100$, $\pi_a = 0.01, 0.05, 0.1, 0.2$, and $0.55 \le p_1,\ p_2,\ p_3 \le 0.95$. The results of investigation are presented in Figure 1.

From Figure 1, the following conclusions could be easily reached:

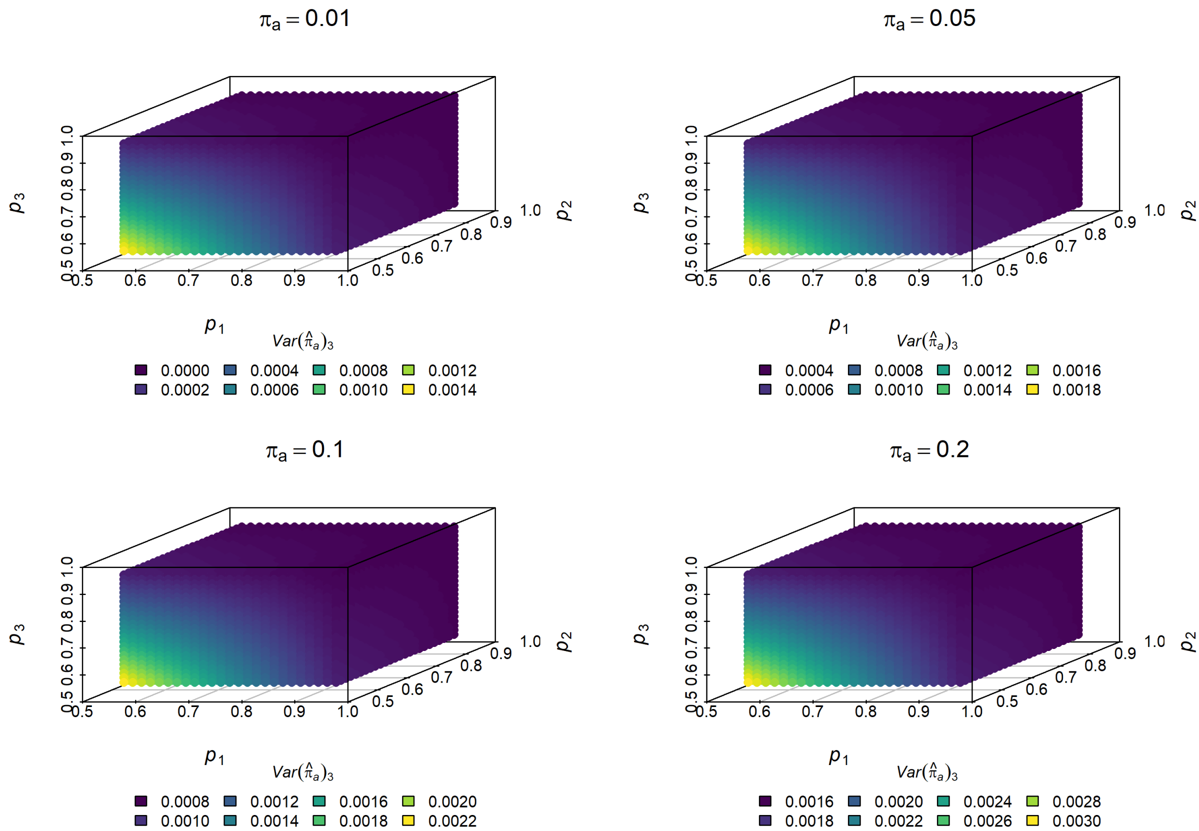

Figure 1. Variance of the suggested model with $m = 3$ for some selected values of π_a, p_1, p_2, and p_3 $(n = 100)$.

- For the fixed values of p_1, p_2, and p_3, $\text{var}(\hat{\pi}_a)_3$ decreases as π_a decreases from 0.2 to 0.01.
- $\text{var}(\hat{\pi}_a)_3$ reaches its minimum values when $\pi_a = 0.01$ whatever values of p_1, p_2, and p_3 are.
- For the fixed values of π_a, p_2, and p_3, $\text{var}(\hat{\pi}_a)_3$ decreases as p_1 increases from 0.55 to 0.95.
- For the fixed values of π_a, p_1, and p_3, $\text{var}(\hat{\pi}_a)_3$ decreases as p_2 increases from 0.55 to 0.95.
- For the fixed values of π_a, p_1, and p_2, $\text{var}(\hat{\pi}_a)_3$ decreases as p_3 increases from 0.55 to 0.95.

5.3. Efficiency comparison

In this section, we will make theoretical and numerical comparisons to investigate the efficiency of proposed estimate $(\hat{\pi}_a)_3$ relative to both the Warner [19] and the Mangat & Singh [16] estimates. These comparisons are made to determine under which conditions one of them will be superior.

Corollary 3. The proposed estimate with $m = 3$, i.e., $(\hat{\pi}_a)_3$ will be more efficient than Warner's estimate iff

$$(1 - p_2)(1 - p_3) < \frac{1 - (1 - p_1)}{1 - (1 - p_1)(1 - p_2)(1 - p_3)}. \tag{24}$$

The numerical comparison between $(\hat{\pi}_a)_3$ and $(\hat{\pi}_a)_W$, based on their variances is investigated in Figure 2, then the relative efficiency of the proposed estimate with $m = 3$ to Warner's estimate $\text{var}(\hat{\pi}_a)_W / \text{var}(\hat{\pi}_a)_3$ is presented. In that comparisons, it is assumed that: $n = 100$, $\pi_a = 0.01, 0.05, 0.1, 0.2$, $p_1 = 0.6, 0.7, 0.8, 0.9$ and $0.6 \leq p_2,\ p_3 \leq 0.9$.

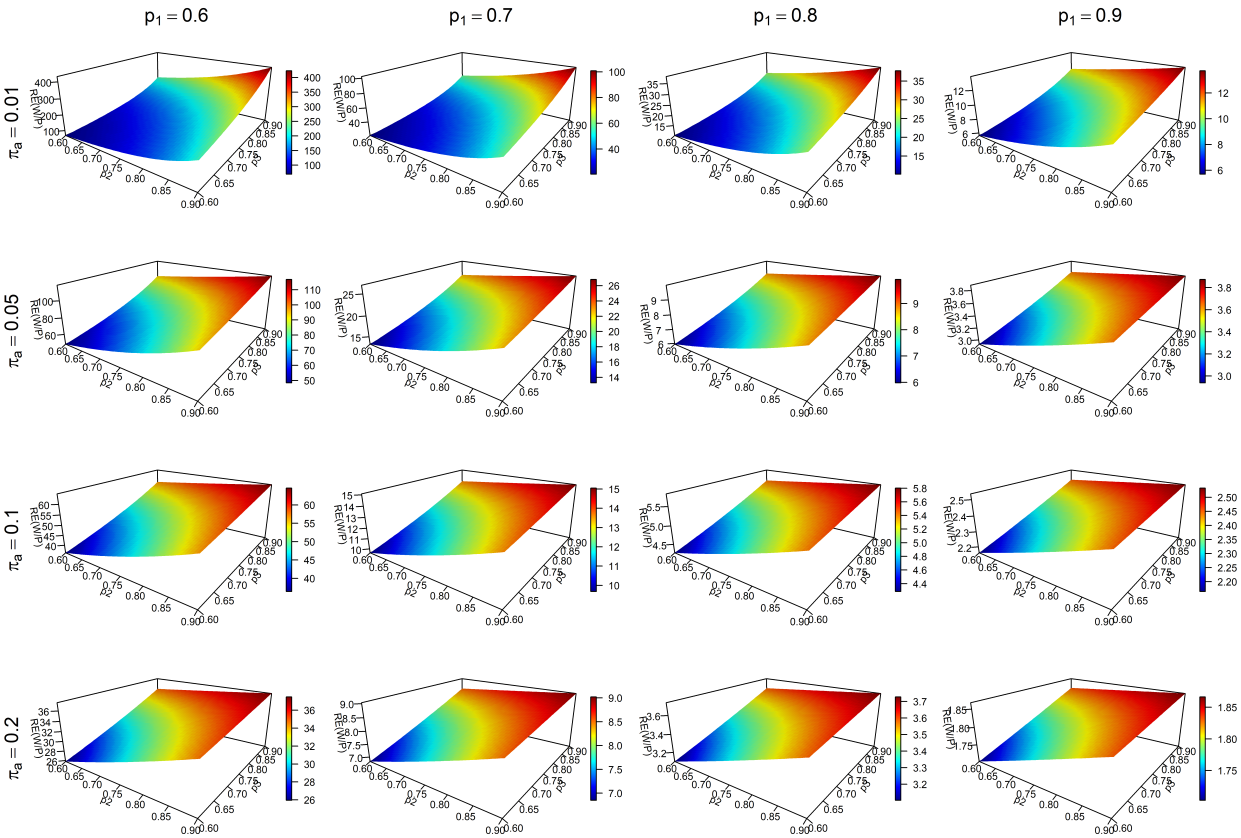

Figure 2. Relative efficiency of the proposed estimate with $m = 3$ to Warner's estimate (RE(W/P) = $\mathrm{var}(\hat{\pi}_a)_W / \mathrm{var}(\hat{\pi}_a)_3$)

From Figure 2, it may be noted that:

- For different values of p_1, p_2, p_3, and π_a, the proposed estimate is more efficient than Warner's estimate.
- For fixed values of p_1, p_2, and p_3 the efficiency of the proposed estimate relative to Warner's estimate, increases as π_a decreases from 0.2 to 0.01.
- For fixed values of p_2, p_3, and π_a the efficiency of the proposed estimate relative to Warner's estimate, increases as p_1 decreases from 0.9 to 0.6.
- For fixed values of p_1, p_3, and π_a the efficiency of the proposed estimate relative to Warner's estimate, increases as p_2 increases from 0.6 to 0.9. (Since, the variance of the proposed estimate with $m = 3$ decreases as p_2 increases from 0.6 to 0.9, but the variance of Warner's estimate is fixed.)
- For fixed values of p_1, p_2, and π_a the efficiency of the proposed estimate relative to Warner's estimate, increases as p_3 increases from 0.6 to 0.9. (since, the variance of the proposed estimate with $m = 3$ decreases as p_3 increases from 0.6 to 0.9, but the variance of Warner's estimate is fixed.)

Corollary 4. The proposed estimate with $m = 3$, i.e., $(\hat{\pi}_a)_3$ will be more efficient than Mangat & Singh's estimate iff

$$1 - p_3 < \frac{1-(1-p_1)(1-p_2)}{1-(1-p_1)(1-p_2)(1-p_3)}. \tag{25}$$

For numerical investigation between $(\hat{\pi}_a)_3$ and $(\hat{\pi}_a)_{MS}$, the efficiency of the proposed estimate with $m = 3$ relative to Mangat & Singh's estimate $\operatorname{var}(\hat{\pi}_a)_{MS}/\operatorname{var}(\hat{\pi}_a)_3$ will be presented in Figure 3. The following combination of parameters values are used: $n = 100$, $\pi_a = 0.01, 0.05, 0.1, 0.2$, $p_1 = 0.6, 0.7, 0.8, 0.9$ and $0.6 \le p_2,\ p_3 \le 0.9$.

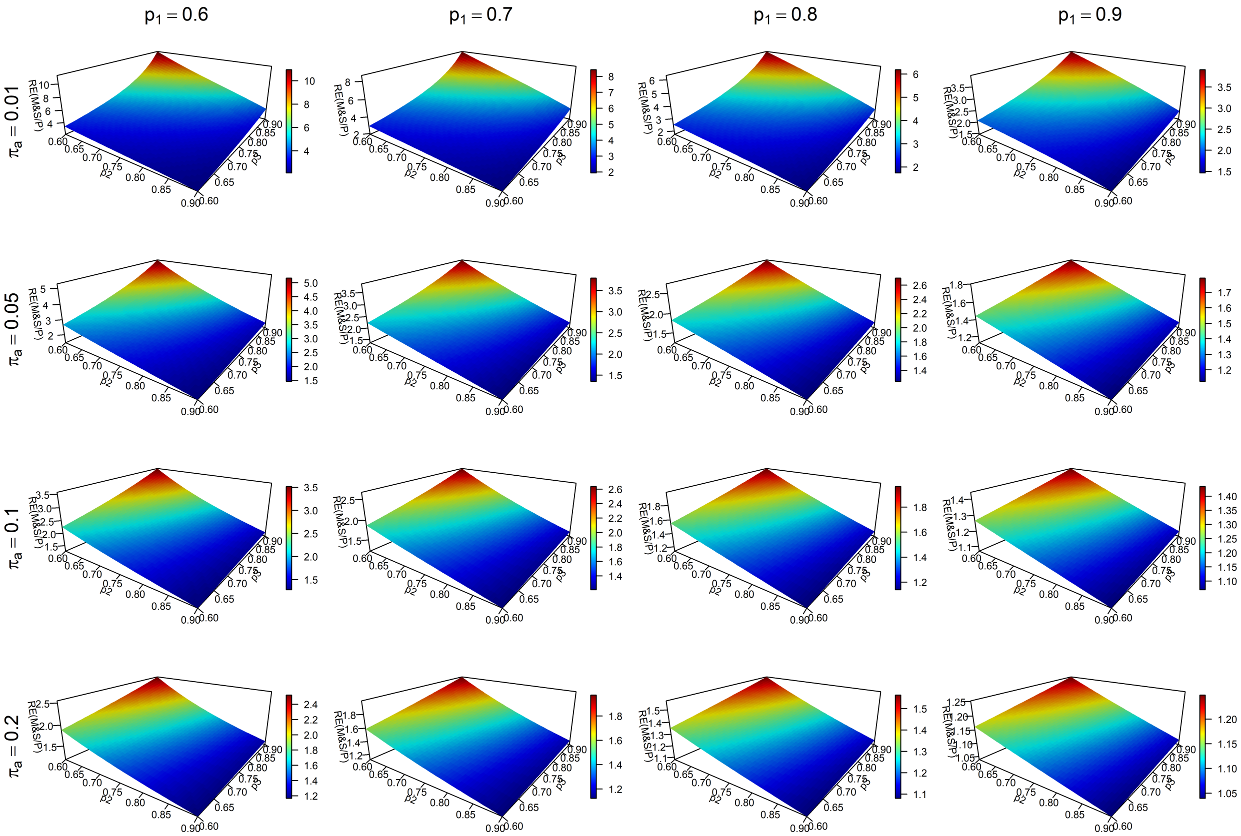

Figure 3. Relative efficiency of the proposed estimate with $m = 3$ to Mangat & Singh's estimate (RE(M&S/P) = $\operatorname{var}(\hat{\pi}_a)_{MS}/\operatorname{var}(\hat{\pi}_a)_3$)

From Figure 3, it may be noted that:

- For different values of p_1, p_2, p_3, and π_a, the proposed estimate is more efficient than Mangat & Singh's estimate.
- For fixed values of p_1, p_2, and p_3 the efficiency of the proposed estimate relative to Mangat & Singh's estimate, increases as π_a decreases from 0.2 to 0.01.
- For fixed values of p_2, p_3, and π_a the efficiency of the proposed estimate relative to Mangat & Singh's estimate, increases as p_1 decreases from 0.9 to 0.6.
- For fixed values of p_1, p_3, and π_a the efficiency of the proposed estimate relative to Mangat & Singh's estimate, increases as p_2 decreases from 0.9 to 0.6.

- For fixed values of p_1, p_2, and π_a the efficiency of the proposed estimate relative to Mangat & Singh's estimate, increases as p_3 increases from 0.6 to 0.9. (Since, the variance of the proposed estimate with $m = 3$ decreases as p_3 increases from 0.6 to 0.9, but the variance of Mangat & Singh's estimate is fixed.)

6. Discussion

The present study introduced and analyzed a generalized randomized response model employing multiple randomization devices. By extending Warner's [19] and Mangat & Singh's [16] classical frameworks, the proposed design strengthens respondent privacy while achieving superior efficiency under a broad range of conditions.

The theoretical results and numerical investigations highlight several important findings. First, the addition of multiple randomization devices systematically reduces the direct connection between a respondent's true status and their observed answer, thereby reinforcing the perception of anonymity. This enhancement is crucial for studies involving highly sensitive attributes, where reluctance to respond truthfully often threatens data reliability. Second, efficiency comparisons demonstrate that the proposed model consistently outperforms both Warner's and Mangat & Singh's models across feasible parameter values. In particular, efficiency gains are most pronounced when the prevalence of the sensitive attribute is low, and when the randomization probabilities are chosen closer to extremes, reflecting realistic survey conditions.

From a practical standpoint, the model offers researchers a flexible yet conceptually simple tool for reducing response bias. The use of multiple devices, while theoretically more complex, can be implemented with straightforward randomization mechanisms (e.g., coins, dice, or cards) that remain accessible to both interviewers and respondents. This balance between design sophistication and operational simplicity is central to the appeal of the proposed framework.

There is a direct trade-off between efficiency and privacy: higher probabilities (e.g., $p_i = 0.9$) favor efficiency, while lower probabilities (e.g., $p_i = 0.1$) favor privacy. Achieving a workable balance between these two objectives is essential.

In sum, the proposed design advances the methodological toolkit for handling sensitive survey questions by combining enhanced privacy safeguards with improved statistical efficiency. These contributions reinforce the role of randomized response techniques as indispensable instruments for studying sensitive behaviors in social, health, and behavioral sciences.

7. Limitations and future research

Several limitations of the current study should be noted. The analysis primarily emphasizes theoretical efficiency and controlled numerical simulations. To fully assess the model's robustness, real-world applications across diverse populations—accounting for cultural and psychological differences in response to sensitive questions—are necessary. While the efficiency gains are evident, the balance between respondent burden and practical feasibility requires further empirical evaluation. Moreover, the model relies on the assumption of complete truthful reporting, which may not hold in contexts involving highly sensitive topics.

Future research should aim to address these limitations by conducting field studies to observe respondent behavior under different configurations of randomization devices. Comparative analyses with other recent RRT methodologies would further contextualize the proposed model within the broader literature. Additionally, investigating adaptive strategies for parameter selection could help optimize both efficiency and privacy in practical applications.

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