Decision Making: Rational Choice or Hyper-Rational Choice

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Abstract   In this paper, we provide an interpretation of the rationality in game theory in which player consider the profit or loss of the opponent in addition to personal profit at the game. The goal of a game analysis with two hyper-rationality players is to provide insight into real-world situations that are often more complex than a game with two rational players where the choices of strategy are only based on individual preferences. The hyper-rationality does not mean perfect rationality but an insight toward how human decision-makers behave in interactive decisions. The findings of this research can help to enlarge our understanding of the psychological aspects of strategy choices in games and also provide an analysis of the decision-making process with cognitive economics approach at the same time.

Keywords   Decision making, Game theory, Rationality, Hyper-rationality, Outcomes.

1. Introduction

To date, the game theory has shown that it can model many interactive situations in which humans are self-interested rational maximizer [4, 19]. Game theory is applied in various fields, such as sociology, psychology, political science, economics, biology, computer science and players can be genes, people, groups, companies or countries [5, 11, 16]. Game theory is a mathematical language for describing strategic interactions in which players think other players what will do[7, 18]. The hyper-rational choice theory suggests that hyper-rational players have considered three classes of hyper-preferences that help determine how to behave in interactive decisions [8]. The first class, set of individual preferences, includes three main behavioral options, individual profit, individual loss, indifferent between profit or loss, relates to the process of decision making when player faced with a problematic and significant choice situation, and choose an action based on self-interested. The second class, set of preferences for others, includes three main behavioral options, profit for others, loss for others and indifferent between profit or loss for others, relates to the process of decision making when player faced with a problematic and significant choice situation, and choose an action based on preferences for others. The third class, both classes above simultaneously, relates to the process of decision making when player faced with a problematic and significant choice situation, and choose an action based on individual preferences and preferences for others, at the same time [2, 9].

Now consider a rational individual. The set of possible choices of rational individual $i \in \{1, 2, ..., n\}$ is shown with $A_i = \{a_1, a_2, ..., a_n\}$. Given hyper-preferences, how will a hyper-rational individual behave? We assume that given a set of choices $B \subseteq \mathcal{A} = A_1 \times A_2 \times ... \times A_n$. We define the weak hyper-preferences of player i' over the

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set B as follows:

$$(a_1, a_2, ..., a_n)_i \succeq' (b_1, b_2, ..., b_n)_i \Leftrightarrow either \ a_1 \succeq b_1 \ or \ b_1 \succeq a_1 \ based \ on \ player \ i'$$
$$preferences \ for \ player \ 1 \ and \ either \ a_2 \succeq b_2 \ or \ b_2 \succeq a_2 \ based \ on \ player \ i'$$
$$preferences \ for \ player \ 2 \ and \ either \ a_i \succeq b_i \ or \ b_i \succeq a_i \ based \ on \ player \ i'$$
$$preferences \ and \ either \ a_n \succeq b_n \ or \ b_n \succeq a_n \ based \ on \ player \ i' \ preferences$$
$$for \ player \ n,$$

where relation $\succeq$ is complete and transitive [8]. In the following, we defined hyper-rationality as follows:

Definition 1
An individual will be called *hyper-rational* under certainty if is a rational and their hyper-preferences for preferences (individual or for others) satisfy at least one of the following conditions:

1. The player chooses from the set of available alternatives (actions) based on individual preferences;
2. The player chooses from the set of available alternatives (actions) based on preferences for other players.

It can be concluded each hyper-rational player is a rational player, but each rational player is not a hyper-rational player [8]. In order to describe a game based on concept of hyper-rational choice, the payoff functions for each player i is given by:

$$_iU_j : A_1 \times A_2 \times ... \times A_n \to \mathbb{R}$$

$$_iU_j(a_1, a_2, ..., a_n) = \begin{cases} _iU_i(a_1, a_2, ..., a_n) & if \ j = i \\ _iU_j(a_1, a_2, ..., a_n) & if \ j \neq i, \end{cases}$$

where $_iU_j$ shows player i' preferences for player j, if player i considers profit (loss) for player j, he will choose an action from a set of available actions which will benefit (lose) player j, for every $i, j \in \{1, 2, ..., n\}$. For more information, see the Missile crisis game in Table 1 and the Chicken game in Table 2.

In this theory, hyper-rational player will renormalize her opinion based on the common knowledge that each player is hyper-rational. The player's best response function is divided into three classes: (1) the best response function of player i based on individual benefit is shown with B_i; (2) the best response function of player i based on profit for other players, is shown with P_i; (3) the best response function of player i based on the loss of other players is shown with L_i. In hyper-rationality choice thoery, based on the concept of hyper-preferences, the players' actions are divided into three classes: (1) strictly dominant action and weakly dominant action based on individual profit; (2) strictly dominant action and weakly dominant action based on profit for other players; (3) strictly dominant action and weakly dominant action based on the loss for others [1]. The following theorem shows a method for finding equilibrium in the game.

Theorem 1
The action profile a^* is a equilibrium point of strategic game if and only if hold true in at least one of the following conditions:

- Each action of the player is the best response to actions of other players based on personal benefit:

$$a^* \ is \ in \ B_i(a^*_{-i}) \ for \ every \ player \ i,$$

- Each action of the player is the best response to actions of other players based on the benefit of other players:

$$a^* \ is \ in \ P_i(a^*_{-i}) \ for \ every \ player \ i.$$

- Each action of the player is the best response to actions of other players based on loss of other players:

$$a^* \ is \ in \ L_i(a^*_{-i}) \ for \ every \ player \ i.$$

We consider equilibrium based on concept of Nash [13, 14]. In the next section, we introduce a new game and analyze it using the concept of rationality and the concept of hyper-rationality.

A central tenet of game theory is that individual choose between various actions according to his preferences for increasing their interests [10, 12, 15]. This study, in contrast, shows that individual preferences and preferences for others (hyper-preferences) on outcomes determine action. In this view, the taxonomy of players hyper-preferences which is dependent on environmental condition, type of interacting person, self-evaluation system and evaluation system of other interacting persons, specify the patterns of action selection. Taxonomy of hyper-preference means that if we face a player with two choices of hyper-preferences, she will necessarily have an opinion on which she likes more.

The concept of hyper-rationality extends the advantages of game theory by allowing a player to thinks about profit or loss of other players in addition to his personal profit or loss and choosing an action which is desirable to him. The ability to consider profit or loss for an opponent in the concept of hyper-rational produces different views (collective benefit or collective loss) in the model. The hyper-rational choice analysis shows why tragedy results were obtained in some of games when both sides behave based on collective loss thinking in the conflict. The hyper-rational choice theory more accurately provides solutions for complex theoretic modeling of conflicts than those modeled by rational choice theory.

In this paper, we present a concept of hyper-rational choice, and we apply this concept to the development and generalization of game theory. In section 2, we model the behavior of the United States and the Soviet Union in the Cuban Missile Crisis based on the hyper-rationality. In this game, according to the concept of rationality, players haven't the dominant action, but based on this concept of hyper-rationality, United States has the dominant action of profit for the Soviet Union and dominant action of loss for the Soviet Union, and the Soviet Union hasn't the dominant action. Furthermore, According to the concept of rationality, the Missile crisis game hasn't Nash equilibrium but based on this concept of hyper-rationality, this game has two Nash equilibrium. In section 3, we examine the Chicken game based on concept of hyper-rational choice. In this game, according to the concept of rationality, players haven't the dominant action but based on this concept of hyper-rationality, but based on this concept of hyper-rationality, players have the dominant action of profit and dominant action of loss for other players. Moreover, according to the concept of rationality, the Chicken game has two Nash equilibrium but based on this concept of hyper-rationality, this game has four Nash equilibrium. Section 4 concludes.

2. The Missile crisis game

In *Theory of Moves (TOM),* Steven Brams developed a general dynamic modeling framework and used it to evaluate Cuban missile crisis [3]. In this book, he explains that the goal of the United States was immediate removal of the Soviet Union missiles, and United States (Row player) policymakers seriously considered two strategies to achieve this end: (1) A naval blockade (B), or quarantine as it was euphemistically called, to prevent shipment of further missiles, possibly followed by stronger action to induce the Soviet Union (Colum player) to withdraw those missiles already installed. (2) A surgical air strike (A) to wipe out the missiles already installed, insofar as possible, perhaps followed by an invasion of the island. The alternatives open to Soviet Union policy makers were: (1) Withdrawal (W) of their missiles. (2) Maintenance (M) of their missiles. Instead, he begins by considering the payoff matrix given in Table 1. In Cuban missile crisis, Brams concludes that the two countries reached a compromise. In other words, in this crisis the compromise outcome or $(3, 3)$ is a equilibrium based on TOM. In the following, we analyze the game of missile crisis with the help of the concept of hyper-rationality.

Table 1. Missile crisis game g_1

g_1		
	W	M
B	3, 3	1, 4
A	2, 2	4, 1

1. Each player is thinking of making a profit to another player.

In game g_1, for United States we have: based on concept of hyper-rationality, given fixed W for the Soviet Union, we can see that if United States, seeks to incur profit to his opponent, he will choose B (Soviet, earns a reward 3), it can conclude that pair of action (B, W) is chosen. By choosing M for Soviet Union, we can see that if United States, seeks to incur profit to his opponent, he will choose B (Soviet Union, earns a reward 4), it can conclude that pair of action (B, M) is chosen. Therefore, for the United States, based on the profit of another player, B is a strictly dominant action.

In game g_1, for the Soviet Union we have: based on concept of hyper-rationality, given fixed B for United States, we can see that if Soviet Union, seeks to incur profit to his opponent, he will choose W (United States, earns a reward 3), it can conclude that pair of action (B, W) is chosen. By choosing A for United States, we can see that if Soviet Union, seeks to incur profit to his opponent, he will choose M (United States, earns a reward 4), it can conclude that pair of action (A, M) is chosen. So, for the Soviet Union, based on the profit of another player, there is no a strictly dominant action.

2. United States is looking to profit for the Soviet Union, and the Soviet Union seeks to lose of United States.

In game g_1, for United States we have: based on concept of hyper-rationality, given fixed W for the Soviet Union, we can see that if United States, seeks to incur profit to his opponent, he will choose B (Soviet Union, earns a reward 3), it can conclude that pair of action (B, W) is chosen. By choosing M for Soviet Union, we can see that if United States, seeks to incur profit to his opponent, he will choose B (Soviet Union, earns a reward 4), it can conclude that pair of action (B, M) is chosen. Therefore, for the United States, based on the profit of another player, B is a strictly dominant action.

In game g_1, for Soviet Union we have: based on concept of hyper-rationality, given fixed B for United States, we can see that if Soviet Union, seeks to incur loss to his opponent, he will choose M (United States, earns a reward 1), it can conclude that pair of action (B, M) is chosen. By choosing A for United States, we can see that if Soviet Union, seeks to incur loss to his opponent, he will choose W (United States, earns a reward 2), it can conclude that pair of action (A, W) is chosen. So, for the Soviet Union, based on the loss of another player, there is no strictly dominant action.

3. Each player is thinking of making a loss to another player.

In game g_1, for United States we have: based on concept of hyper-rationality, given fixed W for Soviet Union, we can see that if United States, seeks to incur loss to his opponent, he will choose A (Soviet Union, earns a reward 2), it can conclude that pair of action (A, W) is chosen. By choosing M for Soviet Union, we can see that if United States, seeks to incur loss to his opponent, he will choose A (Soviet Union, earns a reward 1), it can conclude that pair of action (A, M) is chosen. Therefore, for United States, based on the loss of another player, A is a strictly dominant action.

In game g_1, for Soviet Union we have: based on concept of hyper-rationality, given fixed B for United States, we can see that if Soviet Union, seeks to incur loss to his opponent, he will choose M (United States, earns a reward 1), it can conclude that pair of action (B, M) is chosen. By choosing A for United States, we can see that if Soviet Union, seeks to incur loss to his opponent, he will choose W (United States, earns a reward 2), it can conclude that pair of action (A, W) is chosen. So, for the Soviet Union, based on the loss of another player, there is no strictly dominant action.

4. United States is looking to loss to the Soviet Union, and the Soviet Union seeks to profit for United States.

In game g_1, for United States we have: based on concept of hyper-rationality, given fixed W for Soviet Union, we can see that if United States, seeks to incur loss to his opponent, he will choose A (Soviet Union, earns a reward 2), it can conclude that pair of action (A, W) is chosen. By choosing M for Soviet Union, we can see that if United

States, seeks to incur loss to his opponent, he will choose A (Soviet Union, earns a reward 1), it can conclude that pair of action (A, M) is chosen. Therefore, for United States, based on the loss of another player, A is a strictly dominant action.

In game g_1, for the Soviet Union we have: based on concept of hyper-rationality, given fixed B for United States, we can see that if Soviet Union, seeks to incur profit to his opponent, he will choose W (United States, earns a reward 3), it can conclude that pair of action (B, W) is chosen. By choosing A for United States, we can see that if Soviet Union, seeks to incur profit to his opponent, he will choose M (United States, earns a reward 4), it can conclude that pair of action (A, M) is chosen. So, for the Soviet Union, based on the profit of another player, there is no a strictly dominant action.

This game has two Nash equilibrium (A, W) and (B, W). In game g_1, for the US, based on damage for Soviet Union, A is a strictly dominant action of loss for Soviet Union, and for the US, based on profit for Soviet Union, B is a strictly dominate action of profit for Soviet Union. Why did not the US an air strike that destroys the missiles? Bram's write: an air strike that destroys the missiles that the Soviet Union were maintaining is an "honorable" U.S. action, (its best state) and thwarts the Soviet Union (their worst state)- $(4, 1)$. An air strike that destroys the missiles that the Soviet Union were withdrawing is a "dishonorable" U.S. action (its next-worst state) and thwarts the Soviet Union (their next-worst state)- $(2, 2)$. According to the concept of hyper-rationality, based on the loss for the US, pair of actions (A, W) is a pair of hyper-rationality. So, a pair of actions (A, W) is a Nash equilibrium. In this game, for the US, based on benefit for Soviet Union, B is a strictly dominant action of profit for Soviet Union and A is a strictly dominated action of profit for Soviet Union. Therefore, pairs of actions (B, W) and (B, M) are rational for Soviet Union. According to the concept of hyper-rationality, based on benefit for the US, a pair of actions (B, W) is a pair of hyper-rationality. So, a pair of actions (B, W) is a Nash equilibrium. Consequently, the United States has two dominant actions: B based on benefit to Soviet Union and A based on the loss to Soviet Union. On the other hand, if interaction between the United States and the Soviet Union is based on collective benefit thinking, both player prefers: either $(B, W) \succeq' (A, W) \succeq' (A, M) \succeq' (B, M)$ or $(B, W) \succeq' (A, W) \succeq' (B, M) \succeq' (A, M)$ $((B, W) \succeq' (A, W) \succeq' (B, M) \sim' (A, M))$. Hence, Based on collective benefit thinking $(3, 3)$ is preferred to $(2, 2)$ by two players. Therefore, in Missile crisis game, it can be said that the two countries considered the collective benefits and reached a compromise.

3. The Chicken game

The game of chicken, also known as the hawkdove game or snowdrift game, is a model of conflict for two players in game theory [6, 17]. Many conflicts that lead to war are because of players regardless of their cost willing to incur the most loss for the opponent. The traits vary from person to person and can produce different actions or behavior from each person. Here we explore the human behavior in the Chicken game g_2 in Table 2. We assume that each player in the game is hyper-rational. So, the Chicken game for both players based on loss of other players D is a strictly dominant action and C is a strictly dominated action of loss. Therefore, in game g_2, equilibrium point (D, D) is an equilibrium in which players choose based on loss of the opponent or collective loss thinking is considered. Since, it can be predicted that if players consider collective loses, the likelihood of a collision is extremely high. For both players, based on the profit for other players, C is a strictly dominant action and D is a strictly dominated action. Consequently, equilibrium point (C, C) is an equilibrium in which players choose based on the profit of other players or collective profit thinking is considered. According to the definition of classic rationality, the equilibrium points (C, C) and (D, D) are not Nash equilibrium. Therefore, it can be predicted that if players consider collective profit, players are likely to reach a compromise. In this game, Nash equilibria are (D, C) and (C, D) which players have considered individual profit and loss of other players at the same time. As a result, based on the concept of hyper-rationality, the chicken game has four equilibrium points. According to the concept of rationality, players haven't the dominant action, but based on this concept of hyper-rationality, players have the dominant action of profit and dominant action of loss for other players.

Table 2. Chicken game g_2

g_2	C	D
C	3, 3	2, 4
D	4, 2	1, 1

We explore the players' behavior with help of the concept of hyper-rationality. These interpretations help to enlarge our understanding of psychological aspects of strategy choices in games and also provide an analysis of the decision-making process with cognitive economics approach at the same time. For example, selecting a pair of actions (D, D) shows that players are spiteful individuals and only thought to harm others. In other words, the hyper-preferences indicate that the type of interaction, environmental conditions, and valuation system are based on hostility and consideration of maximum loss for other at this point. The concept of hyper-rationality explains that, based on the profit of other players, cooperation is a strictly dominant action. Moreover, $(3, 3)$ is preferred to $(1, 1)$ by two players. On the other hand, based on the lose of other players, defecte is a strictly dominant action. in addition, $(1, 1)$ is preferred to $(3, 3)$ by two players.

4. Conclusion

In this article, we compared the concept of rationality and hyper-rationality. In the hyper-rationality concept , the player thinks about profit or loss of other players in addition to his personal profit or loss and then will choose an action which is desirable to him, but in the the concept of rationality, each player only seeks his personal profit maximizer. The hyper-rationality concept will help to model the behavior of people considering environmental conditions, the kind of behavior interactive, valuation system of itself and others and the system of beliefs and internal values of societies. Hyper-rationality helps us understand how human decision makers behave in interactive decisions. The findings of this research can help to enlarge our understanding of the psychological aspects of strategy choices in games and also provide an analysis of the decision-making process with cognitive economics approach at the same time. The hyper-rationality does not mean perfect rationality but an insight toward how human decision-makers behave in interactive decisions.

Acknowledgement

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