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Data Envelopment Analysis in Three-Stage Network Structure with Slack Based Measure (SBM)

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Abstract Data envelopment analysis (DEA) is a powerful non-parametric technique for measuring and analyzing the relative efficiency of a Decision-Making Unit (DMU) with multiple homogeneous types of inputs and outputs. Traditional DEA treats a DMU as a black box and calculates its efficiency by considering its initial inputs and final outputs. But the overall efficiency of a DMU is directly dependent upon the performances of intermediate operations of a production process. In many situations, DMUs have a three-stage network structure. Against this backdrop, this paper introduces a non-oriented and non-radial slacks-based measure (SBM) of an efficiency framework for the three-stage network production process. In the proposed model, we have considered both series and parallel relationships between the inputs and outputs of different independent and dependent stages. Additionally, this research is aimed at measuring banking efficiency by using a three-stage network DEA model.

Keywords Network Data envelopment analysis, Decision-Making Units, Three-stage structure, Efficiency, Slacks-based measure.

AMS 2010 subject classifications 90C15, 68C05

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1. Introduction

Data envelopment analysis (DEA) is a non-parametric optimization technique and analytical methodology used for efficiency analysis. It is an assumption-free technique regarding the shape of the production function. The advantages of DEA are two-fold: (I) it evaluates the efficiency of each Decision-Making Unit (DMU) and (II) it provides proper knowledge about benchmarking information. However, the drawback of the traditional DEA methodology is that it estimates the overall efficiency of the system based on initial inputs and final outputs and bypasses the role of the internal structure of the production processes. However, the role of the internal structure in the efficiency of a production unit is also essential because every production process cannot produce final outputs in one stage. Some production processes produce intermediate products that are used as inputs for the very next stage to produce the final output. The input passes through several stages connected in series, parallel and mixed of both in a production process. Such types of production operations are mostly identified in the finance and banking sector. We considered Indian banking data for the design of empirical analysis by employing the proposed

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methodology. In the Indian context, commercial banks are considered not merely as money dealers, but also the leaders in the economic development of the country. The economic development of any country largely depends on a well-developed banking system. The role of the Indian banking system has changed significantly with the introduction of liberalization. The Indian banking sector is sufficiently capitalized and well-regulated with the efforts of the Reserve Bank of India (RBI). Credit, market and liquidity risk studies suggest that Indian banks are generally resilient and have withstood the global meltdown well. However, increased competition has forced the banks to reduce their costs, resulting in the closure of unprofitable branches and reduction of staff, and eventually increasing the profitability of the banking system, as discussed by[27].

Studies that have attempted to measure the efficiency of Indian banks with DEA include [4, 36, 38, 37, 43]. An investigation of the impact of the behavioural factor on the supply network by using DEA and network DEA in banking to determine inefficiencies was performed by [6]. The methodology was initially proposed by Charnes et al.[10] by applying LPP in the case of constant returns to scale and later extended by Banker et al.[3] in the case of variable returns to scale. The DEA approach comprehends both technical and scale inefficiencies via the optimal value of the ratio form [7, 46]. Mozambican banking efficiency was analyzed by using integrated techniques, fuzzy DEA and bootstrapping given by [9]. A comparison between stochastic DEA and the fuzzy DEA approach in Angolan bank efficiency was conducted by [15]. DEA & SFA as a powerful tool in decision support system applying in the Indian banking sector by [30]. Mixed orientation in DEA with least distance measure in the Islamic banking sector was given by [31] and sensitive and super-efficiency in the Indian banking sector with DEA given by [32]. According to Charnes et al.[11], the existence of slacks between the efficient and inefficient DMUs developed the additive model where we are dealing directly with such type of input excesses and output shortfalls. SBM efficiency in DEA deals with the input excesses and output shortfalls of the concerned DMU given by Tone [44].

In many situations where DMUs have a two-stage structure, the outputs of the first stage are used as inputs for the next stage (see [28, 14, 1, 26]. The decomposition of overall efficiency in a network DEA is a powerful technique for estimating and identifying the inefficiency in a structure and was initially applied [13, 25, 17, 39]. More than ten thousand papers have been published in the application as well as theory of different aspects of DEA from 1978-2016 given in [18]. The efficiency of the DMU concerned in DEA depends on three types of structures, namely, series, parallel and mixed structures.

Series structure: The three-stage series structure system is a particular case of a multi-stage series structure and is shown in Figure 3. In the three-stage network structure, the overall efficiency of the system is decomposed into three stages. Kao [21] modified the standard DEA model by considering the series relationship of the multi-stages within the overall process. The overall efficiency and congestion of the production process can be decomposed into parts and was studied by [41]. The extension of DEA in two-stage production processes was studied by [12]. The efficiency of energy consumption in the cotton industry was studied by [22], where a dynamic approach was applied to the multi-stage DEA model.

Parallel structure: A graphic representation of a parallel system structure, which is a particular case of a two-stage parallel system, is shown in Figure 4. The [19] is one of the earliest works on parallel systems, and the model is aimed at maximizing the output distance function in the system. The inefficiency of the system can be decomposed into its component units. Estimation of the efficiency of the system can be calculated by taking the individual components into account and making use of the parallel system in the DEA model. Kao [20] observed that decomposition enables the decision-maker to identify the component operating inefficiently and the need to improve the efficiency of that production unit. Bi et al.[5] classified the production activities of a business into core and non-core ones, operating in a parallel manner. The inputs are shared, and the outputs are the contribution of both processes and a conventional input system distance function was used to assess 20 convenience stores by applying a parametric bootstrap efficiency method.

Mixed structure: A mixed structure, as shown in Figure 1, is a particular case of the three-stage network structure system. Prieto and Zofio [29] undertook network efficiency analysis within an input and output model that allowed potential technical efficiency gains by comparing technologies corresponding to different economies to be assessed. Avkiran [2] applied an SBM model to measure the performance of 15 UAE banks, where three profit centres where are involved. Wang et al. [45] studied the profitability and marketability efficiencies of 65 high-tech firms in Taiwan. Different from conventional two-stage studies, the first stage in this study was composed of two processes, essential production and R&D effects operating parallel in the system. The weighted average of the input distance parameters of the first stage and the output distance parameter of the second stage in the negative form was minimized. A network DEA model was introduced by [33, 34, 35] and they investigated the so-called black box for the first time. Several authors extended these models. The network DEA model proposed by [24], has a multi-stage structure that is an extension of the two-stage DEA model proposed by [40]. Prieto and Zofio [29] applied network efficiency analysis within an input/output model initiated by [23]. The SBM of efficiency in DEA with a network system was studied by [8].

In this paper, we are decomposing the overall efficiency into three stages connected in series, whereas the intermediate stage has two sub-stages in stage 2, connected in parallel and the overall structure is shown in Figure 1. With the background information from the literature, this study proposed a three-stage network structure DEA model.

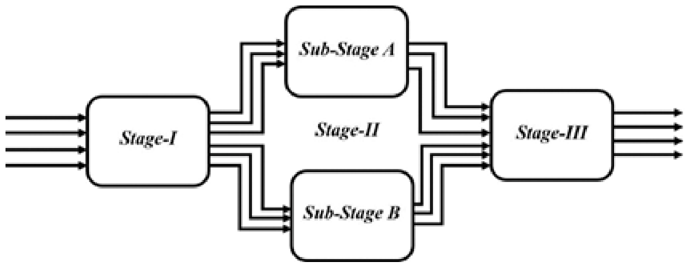

Figure 1. Three-stage networking structures.

The first section of this paper is an overview of two-, three- and multi-stage network DEA models using SBM efficiency. The second section explains the SBM technique as a linear programming problem-based technique and its interpretation. In the third section, the use of SBM in two- and three-stage DEA models, which are connected to the series system, is highlighted. The fourth section discusses SBM in single-stage DEA with two sub-stages connected parallel in a system. The fifth section summarizes the concept of a mixed structures system for preparing the three-stage network DEA-model with SBM efficiency. The critical importance of the study is discussed with empirical illustration, results, and conclusions in the final section.

2. Slack-Based Measure of Efficiency

Let $x_{ij}\ ; i = 1, 2, 3, ...m$ and $y_{rj}\ ; r = 1, 2, 3, ...s$ be the $i^{th} - input$ and $r^{th} - output$ of the $j^{th} - DMU\ ; j = 1, 2, 3, ..., n$ respectively. It is assumed that the data set is know and strictly positive. The production possibility set P for DMU_0 is defined as given below.

$$P = \{(x_{i0}\ ,\ y_{r0})\ s.t.\ x_{i0} \geq \lambda_j x_{ij}\ and\ y_{r0} \leq \lambda_j y_{rj}\ \forall \lambda_j \geq 0\ ; j = 1, 2, 3, ...n\}.$$

P is closed and convex set with boundary points as the efficient production frontier. The relative reduction rate of $i^{th} - input$ and relative increment rate of $r^{th} - output$ for the DMU_0 can expressed as following two equations.

$$(x_{i0} - s_i^-)/x_{io} \Rightarrow\ Relative\ reduction\ rate\ of\ i^{th}\ input\ in\ DMU_0.$$
$$(y_{r0} + s_r^+)/y_{ro} \Rightarrow\ Relative\ increment\ rate\ of\ r^{th}\ output\ in\ DMU_0.$$

Where s_i^- and s_r^+ is the input and output slacks of DMU_O respectively.
Let ρ be the inefficiency rate of DMU_0 assessing the m-inputs and s-outputs is defined as:

$$\rho = \left[\frac{1}{m}\sum_{i=1}^{m}(\frac{x_{i0}-s_i^-}{x_{i0}})\right] * \left[\frac{1}{s}\sum_{r=1}^{s}(\frac{y_{r0}+s_r^+}{y_{r0}})\right]^{-1} \qquad (I)$$

The interpretation of non-oriented and non-radial DEA technique SBM is Minimizing the above inefficiency rate directly on the base of slacks, subject to production possibility set P given by [44]

$$Min\rho = \frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{i0}}}{1+\frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{r0}}}$$

$$Subject\ to$$

$$\sum_{j=1}^{n}\lambda_j x_{ij} + s_i^- = x_{i0}\ ;\ i = 1,2,3,\ldots m. \qquad (2.1.1)$$

$$\sum_{j=1}^{n}\lambda_j y_{rj} - s_r^+ = y_{r0}\ ;\ r = 1,2,3,\ldots s.$$

$$s_i^- \geq 0,\ s_r^+ \geq 0\ and\ \lambda_j \geq 0\ ; j = 1,2,3,\ldots n.$$

Where x_{i0} and y_{r0} are the i^{th}-input and r^{th}-output of the DMU_0 which is under evaluation. s_i^-; $i = 1,2,3\ldots m$ and s_r^+; $r = 1,2,3\ldots s$ are the input excess and output shortfalls and otherwise referred as slacks in DEA. In order to avoid fractional form, we are using Charnes and Cooper transformation with setting given as:

$$t = \frac{1}{1+\frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{ro}}}\ ;\ \forall t \geq 0\ and\ r = 1,2,3,\ldots s. \qquad (I)$$

By using the transformation (I) in the mathematical model (2.1.1) the modified model become non-fractional SBM model are as given:

$$Min\rho = t - \frac{1}{m}\sum_{i=1}^{m}\frac{ts_i^-}{x_{i0}}$$

$$Subject\ to$$

$$t + \frac{1}{s}\sum_{r=1}^{s}\frac{ts_r^+}{y_{r0}} = 1$$

$$\sum_{j=1}^{n}\lambda_j x_{ij} + s_i^- = tx_{i0}\ ;\ i = 1,2,3,\ldots m. \qquad (2.1.2)$$

$$\sum_{j=1}^{n}\lambda_j y_{rj} - s_r^+ = ty_{r0}\ ;\ r = 1,2,3\ldots s$$

$$s_i^- \geq 0,\ s_r^+ \geq 0, t \geq 0\ and \lambda_j \geq 0\ ; j = 1,2,3,\ldots n.$$

The mathematical model (2.1.2) is in non-linear programming program contains the non-linear terms ts_i^- ; $i = 1,2,3,\ldots m$ and ts_r^+ ; $r = 1,2,3,\ldots s$, in order to convert it into linear programming. We are substituting:

$$S_i^- = ts_i^-\ ,\ S_r^+ = ts_r^+\ and\ \wedge_j = t\lambda_j\ ;\ j = 1,2,3,\ldots n \qquad (II)$$

By substituting the transformation (I) and (II) in the model (2.1.1) and converted linear from of SMB-model are given as below:

$$\begin{aligned} &Min\tau = \ t - \frac{1}{m}\sum_{i=1}^{m}\frac{S_i^-}{x_{i0}} \\ &Subject\ to \\ &\qquad t + \frac{1}{s}\sum_{r=1}^{s}\frac{S_r^+}{y_{r0}} = 1 \\ &\qquad \sum_{j=1}^{n}\Lambda_j x_{ij} + S_i^- = x_{i0}\ ;\ i = 1\ ,2\ ,3\ ,\ldots\ m. \qquad (2.1.3) \\ &\qquad \sum_{j=1}^{n}\Lambda_j y_{rj} - S_r^+ = y_{r0}\ ;\ r = 1\ ,2\ ,3\ ,\ldots\ s. \\ &\qquad S_i^- \geq 0\ ,\ S_r^+ \geq 0\ ,\ t \geq 0\ and\ \Lambda_j \geq 0\ ;\ j = 1\ ,2\ ,3\ ,\ldots\ n. \end{aligned}$$

Let an optimal solution of the mathematical model (2.1.3) be $(\tau^*,\ t^*,\ \Lambda^*,\ S^{-*},\ S^{+*})$, then we have an optimal solution of SBM- model is defined as:

$$\rho^* = \tau^*,\ \lambda^* = \frac{\Lambda^*}{t^*},\ s^{-*} = \frac{S^{-*}}{t^*}\ \ and\ \ s^{+*} = \frac{S^{+*}}{t^*} \qquad (III)$$

On the bases of optimal solution (III), we make the decision whether DMU under evaluation is efficient or inefficient depends on the value of $\rho^* = 1$. When $\rho^* = 1$ and all input and output slacks are equal to zero, i.e., $s^{-*}\ =\ s^{+*}\ =\ 0$. This means that there are no input excesses and output shortfalls in the production processes. If the SBM-model are assumed in variable returns to scale (VRS). Then we can express by adding the convexity constraint as $\sum_{j=1}^{n}\lambda_j = 1\ j = 1, 2, 3, \ldots n$, in the model (2.1.1).

3. SBM models for measuring efficiency in case of two-stage series structure

Let us suppose that DMU_O are operating the production process that has two stage series structures as shown in Figure 2. Suppose, we have $n - DMUs$, where each DMU_j $(j = 1\ ,2\ ,3, \ldots n)$ are assessing

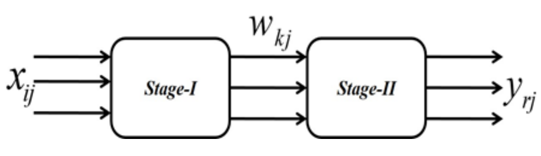

Figure 2. Two-stage series structure production process.

$m - input\ x_{ij}$ $(i = 1\ ,2\ ,3\ ,\ldots\ m)$ in the very first stage in order to produce the $p - outputs\ w_{kj}$ $(k = 1\ ,2\ ,3\ ,\ldots\ p)$. These outputs becomes inputs for the second or intermediate stage for generate the final output y_{rj} $(r = 1\ ,2\ ,3\ ,\ldots\ s)$ of DMU_0 that has two stage production operations.

The overall efficiency score of structure under the VRS assumption can be estimated by applying SBM model for the two individual stages separately and then system as whole. The efficiency scores of the stage-I can be calculated by using the following SBM model.

$$\rho_1^* = \ Min \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^-}{x_{i0}}}{1 + \frac{1}{p}\sum_{k=1}^{p} \frac{s_k^+}{w_{k0}}}$$

Subject to

$$\begin{aligned} &\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{i0} \ ; \ i = 1\ ,2\ ,3\ ,\ldots m. \\ &\sum_{j=1}^{n} \lambda_j w_{kj} - s_k^+ = w_{k0} \ ; \ k = 1\ ,2\ ,3\ ,\ldots p. \\ &\sum_{j=1}^{n} \lambda_j = 1 \ ; \ j = 1, 2, 3, \ldots n. \\ &\lambda_j \geq 0\ , s_i^- \geq 0 \ and \ s_r^+ \geq 0\ . \end{aligned} \tag{3.1.1}$$

The SBM-model (3.1.1) for stage first, where DMU_0 assessing the m-inputs x_{ij} $(i = 1, 2, 3, \ldots m)$ in-order to produce k-outputs w_{kj} $(k = 1, 2, 3, \ldots p)$ With having convexity constraint. The efficiency scores of the stage-II can be estimate by using the following SBM models.

$$\rho_2^* = \ Min \frac{1 - \frac{1}{p}\sum_{k=1}^{p} \frac{s_k^-}{w_{k0}}}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^+}{y_{r0}}}$$

Subject to

$$\begin{aligned} &\sum_{j=1}^{n} \mu_j w_{kj} + s_k^- = w_{k0} \ ; \ k = 1\ ,2\ ,3\ ,\ldots p. \\ &\sum_{j=1}^{n} \mu_j y_{rj} - s_r^+ = y_{r0} \ ; \ r = 1\ ,2\ ,3\ ,\ldots s. \\ &\sum_{j=1}^{n} \mu_j = 1 \ ; \ j = 1\ ,2\ ,3\ ,\ldots n. \\ &\mu_j \geq 0\ , s_i^- \geq 0 \ and \ s_r^+ \geq 0\ . \end{aligned} \tag{3.1.2}$$

The SMB-models in two stage series structure for the estimation an overall efficiency of DMU_0 . Since w_{kj} $(k = 1, 2, 3, \ldots p)$ are outputs of stage-I and same time these becomes inputs for the stage-II and generate the continuity constraints for two stages.

$$\sum_{j=1}^{n} \lambda_j w_{kj} = \sum_{j=1}^{n} \mu_j w_{kj} \ ; \ k = 1, 2, 3, \ldots p. \qquad \& \qquad \sum_{j=1}^{n} \lambda_j = \sum_{j=1}^{n} \mu_j \ ; \ j = 1, 2, 3, \ldots n.$$

Using above continuity constraints, we can propose the following SBM-model for measuring the overall efficiency of DMU_0.

$$\rho^*_{overall} = Min \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^-}{x_{i0}}}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^+}{y_{r0}}}$$

Subject to

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{i0} \ ; \ i = 1 \ ,2 \ ,3 \ ,\ldots m. \quad (3.1.3)$$

$$\sum_{j=1}^{n} \mu_j y_{rj} - s_r^+ = y_{r0} \ ; \ r = 1 \ ,2 \ ,3 \ ,\ldots s.$$

$$\sum_{j=1}^{n} \lambda_j w_{kj} = \sum_{j=1}^{n} \mu_j w_{kj} \ ; \ k = 1 \ ,2 \ ,3 \ ,\ldots p.$$

$$\sum_{j=1}^{n} \lambda j = \sum_{j=1}^{n} \mu_j = 1 \ ; \ j = 1 \ ,2 \ ,3 \ ,\ldots n.$$

$$\lambda_j \geq 0 \ ,\mu_j \geq 0 \ ,s_i^- \geq 0 \ and \ s_r^+ \geq 0.$$

4. SBM models for measuring efficiency in case of three-stage series structure

Let us suppose that DMU_0 are operating the production process that has three stages series structure as shown in Figure 3: Suppose we have $n - DMUs$ where each DMU_j $(j = 1, 2, 3, \ldots n)$ are assessing m-

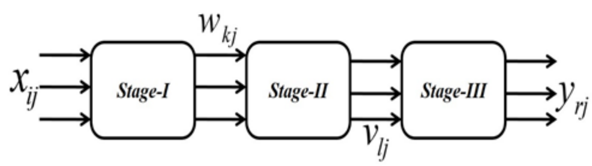

Figure 3. Three-stage series structure production process.

inputs x_{ij} $(i = 1, 2, 3, \ldots m)$ in the very first stage in order to produce the p-outputs w_{kj} $(k = 1, 2, 3, \ldots p)$ and overall the outputs of very first stage are used as inputs for the second/intermediate stage for producing the outputs of that stage as $v_{lj}(l = 1, 2, 3, \ldots q)$. Finally, all the outputs of second/intermediate stage are utilize as inputs for $stage - III$, in order to generate the final output as y_{rj} $(r = 1, 2, 3, \ldots s)$ of DMU_0 that has three stage series structure production process.

The SMB-models for three stages series structure and estimate an overall efficiency of DMU_0. Since all the output of stage-I w_{kj} $(k = 1, 2, 3 \ldots p)$ are used as inputs for stage-II. Then all the outputs $v_{lj}(l = 1, 2, 3 \ldots q)$ of stage-II are further use as inputs for the stage-III in order to produce the final Outputs. In the three stages production processes system the stage-II is called in intermediate stage through which all outputs of stage-I to stage-III. All the output of stage-I w_{kj} $(k = 1, 2, 3, \ldots p)$ and inputs of stage-III $v_{lj}(l = 1, 2, 3, \ldots q)$ are generate the continuity constraints for the stage-(I, II) and Stage- (II,

III) respectively.

$$\begin{aligned}
&\sum_{j=1}^{n}\lambda_j w_{kj} = \sum_{j=1}^{n}\mu_j w_{kj}\ ;\ k = 1\,,2\,,3\,,\ldots p.\\
&\sum_{j=1}^{n}\mu_j w_{kj} = \sum_{j=1}^{n}\mu_j v_{kj} = \sum_{j=1}^{n}\eta_j v_{lj}\ ;\ l = 1\,,2\,,3\,,\ldots q.\\
&\sum_{j=1}^{n}\lambda_j = \sum_{j=1}^{n}\mu_j = \sum_{j=1}^{n}\eta_j = 1\ ;\ j = 1\,,2\,,3\,,\ldots n.
\end{aligned} \tag{C1}$$

The above set of constraints (C1) showing the internal operation of production process, where the same quantity of outputs of very first stage is used as inputs for stage-II. Finally the outputs of second stage are used as the inputs for the final stage with different weights. Using above constraints (C1), we can propose the following SBM-model for measuring the overall efficiency of DMU_0 having three stage series structure.

$$\rho^*_{overall} = Min\frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{i0}}}{1+\frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{r0}}}$$

$$\begin{aligned}
&Subject\ to\\
&\sum_{j=1}^{n}\lambda_j x_{ij} + s_i^- = x_{i0}\ ;\ i = 1\,,2\,,3\,,\ldots m.\\
&\sum_{j=1}^{n}\lambda_j y_{rj} - s_r^+ = y_{r0}\ ;\ r = 1\,,2\,,3\,,\ldots s.\\
&\sum_{j=1}^{n}\lambda_j w_{kj} = \sum_{j=1}^{n}\mu_j w_{kj}\ ;\ k = 1\,,2\,,3\,,\ldots p.\\
&\sum_{j=1}^{n}\mu_j v_{lj} = \sum_{j=1}^{n}\eta_j v_{lj}\ ;\ l = 1\,,2\,,3\,,\ldots q.\\
&\sum_{j=1}^{n}\lambda j = \sum_{j=1}^{n}\mu_j = \sum_{j=1}^{n}\eta_j = 1\ ;\ j = 1\,,2\,,3\,,\ldots n.\\
&\lambda_j \geq 0\,,\mu_j \geq 0\,,\eta_j \geq 0\,,s_i^- \geq 0\ and\ s_r^+ \geq 0\,.
\end{aligned} \tag{4.1.1}$$

The mathematical from (4.1.1) is nonlinear from of SBM-model as a closed system †. where three stages are connected to the series. The nonlinearity can be converted into linearity by using the *Charnes Cooper transformation* and state that every "*linear fractional programming problem*(LFPP) can be transformed into linear programming problem (LPP) under the assumption that the Feasible Region is non-empty and bounded" given in [16].The conventional DEA-model is estimating the efficiency score of DMU and provides information about whether the specific DMU is efficient or not. Where the decomposition of efficiency into stages very much helps us to identify the location of inefficiency ‡ present in the DMU. In such type of decomposition, we are comparing the efficiency as a whole with efficiencies of stages. This

† Closed-system DEA models: the intermediate outputs remain unchanged from one stage to another. Where in the open-system DEA models, the intermediate outputs in one stage are partial inputs in a subsequent stages

‡Location of inefficiency: in the decomposition DEA-model into three stages identifying where or in which stage the inefficiency in present

means that in three-stage DEA-models a specific DMU is said to be overall efficient if and only if it is efficient for every three stages. Otherwise, the specific DMU is treated as inefficient if it is inefficient in anyone stage out of the three stages[8].

5. SBM of efficiency in case of two stages parallel structure

Let us suppose that DMU_0 are operating the production process that has two stages A and B connected parallel in the system as shown in Figure 4. Suppose we have $n-DMUs$, where each

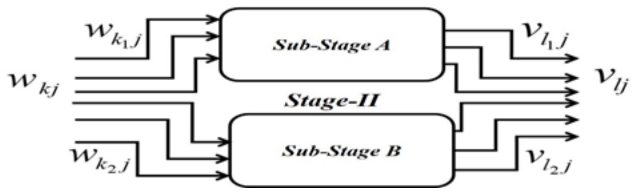

Figure 4. Two-stage parallel structure production process.

DMU_j $(j = 1,2,3,\ldots n)$ are assessing $p-inputs$ in the form of w_{kj} $(k = 1,2,3,\ldots p)$ in order to produce $q-outputs$ in the form of v_{lj} $(l = 1,2,3,\ldots q)$ from the overall stage that has two sub-stages A and B connected parallel in the system. Where in the *sub stage-A* we are using only $p_1-inputs$ w_{k_1j} $(k_1 = 1,2,3,\ldots p_1\ (<\ p))$ in order to producing $q_1-outputs$ v_{l_1j} $(l_1 = 1,2,3,\ldots q_1\ (<\ q))$. Same time in the *sub stage-B* we are using $p_2-inputs$ in the form w_{k_2j} $(k_2 = 1,2,3,\ldots p_2\ (<\ p))$ in order to produce the $q_2-outputs$ v_{l_2j} $(l_2 = 1,2,3,\ldots q_2\ (<\ q))$. Such that $k = k_1 + k_2$ and $l = l_1 + l_2$. This stage is called intermediate stage in our study as estimating the efficiency of DMU_0 that has such type of structure.

The overall efficiency score of DMU_0 for such type of structure under the assumption of VRS to identify the inefficiency in the DMU_0, we are applying the SBM model for the two *sub stages A and B* individually and then as a whole. The efficiency scores of the *sub stage-A* can be calculated by the following SBM model.

$$\rho_A^* = Min \frac{1 - \frac{1}{p_1}\sum_{k_1=1}^{p_1} \frac{s_{k_1}^-}{w_{k_10}}}{1 + \frac{1}{q_1}\sum_{l_1=1}^{q_1} \frac{s_{l_1}^+}{v_{l_10}}}$$

Subject to

$$\sum_{j=1}^{n} \mu_j^1 w_{k_1j} + s_{k_1}^- = w_{k_10}\ ;\ k_1 = 1\ ,2\ ,3\ ,\ldots\ p_1. \qquad (5.1.1)$$

$$\sum_{j=1}^{n} \mu_j^1 v_{l_1j} - s_{l_1}^+ = v_{l_10}\ ;\ l_1 = 1\ ,2\ ,3\ ,\ldots\ q_1.$$

$$\sum_{j=1}^{n} \mu_j^1 = 1\ ;\ j = 1\ ,2\ ,3\ ,\ldots\ n.$$

$$s_{k_1}^- \geq 0\ ,\ s_{l_1}^+ \geq 0\ and\ \mu_j^1 \geq 0\ .$$

The efficiency scores of the *sub stage-B* can be measure by using the following SBM model.

$$\rho_B^* = Min \frac{1 - \frac{1}{p_2}\sum_{k_2=1}^{p_2} \frac{s_{k_2}^-}{w_{k_20}}}{1 + \frac{1}{q_2}\sum_{l_2=1}^{q_2} \frac{s_{l_2}^+}{v_{l_20}}}$$

Subject to

$$\sum_{j=1}^{n} \mu_j^2 w_{k_2 j} + s_{k_2}^- = w_{k_20} \; ; \; k_2 = 1, 2, 3, \dots p_2. \tag{5.1.2}$$

$$\sum_{j=1}^{n} \mu_j^2 v_{l_2 j} - s_{l_2}^+ = v_{l_20} \; ; \; l_2 = 1, 2, 3, \dots q_2.$$

$$\sum_{j=1}^{n} \mu_j^2 = 1 \; ; \; j = 1, 2, 3, \dots n.$$

$$s_{k_2}^- \geq 0 \, , \; s_{l_2}^+ \geq 0 \; and \; \mu_j^2 \geq 0 \, .$$

The SMB-models for the stage that has two sub-stages connected to parallel. Where we are focusing on the overall input and output slacks in the system. These inputs and outputs are passing through two parallel *sub-stages A and B* in a specific DMU_0. The overall efficiency can be calculated by using the following SBM model.

$$\rho_{overall}^* = Min \frac{1 - \frac{1}{p}\sum_{k=1}^{p} \frac{s_k^-}{w_{k0}}}{1 + \frac{1}{q}\sum_{l=1}^{q} \frac{s_l^+}{v_{l0}}}$$

Subject to

$$\sum_{j=1}^{n} \mu_j^1 w_{k_1 j} + \sum_{j=1}^{n} \mu_j^2 w_{k_2 j} + s_k^- = w_{k0} \; ; \; k_1, k_2 \subseteq k. \tag{5.1.3}$$

$$\sum_{j=1}^{n} \mu_j^1 v_{l_1 j} + \sum_{j=1}^{n} \mu_j^2 v_{l_2 j} - s_l^+ = v_{l0} \; ; \; l_1, l_2 \subseteq l.$$

$$\sum_{j=1}^{n} \mu_j^1 = \sum_{j=1}^{n} \mu_j^2 = 1 \; ; \; j = 1, 2, 3, \dots n.$$

$$\mu_j^1 \geq 0 \, , s_k^- \geq 0 \; ; \; k = 1, 2, 3, \dots p.$$

$$\mu_j^2 \geq 0 \, , s_l^+ \geq 0 \; ; \; l = 1, 2, 3, \dots q.$$

6. SBM models for measuring efficiency in case of three-stage network structure

Let us suppose the DMU_0 are operating the production process that has a three-stage network structure shown in Figure 5. Suppose we have $n - DMUs$, where each DMU_j $(j = 1, 2, 3, \dots n)$ are using $m - inputs$ in the form of x_{ij} $(i = 1, 2, 3, \dots m)$ in order to produce the final $s - outputs$ in the form y_{rj} $(r = 1, 2, 3, \dots s)$. Where the production operations of $DMUs$ are passing through in three stages namely as input, intermediate and output stage connected in series. While as in the intermediate the process are passing through two parallel [§] sub stages *sub-stages A and B* as shown in in Figure 5.

[§] Parallel structure is such type structure where all the processes operate independently in the efficiency analysis.

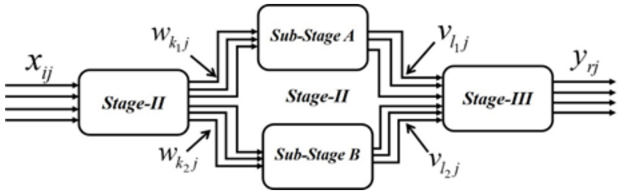

Figure 5. Three-stage network structure production process.

For better understanding of distribution of inputs for each stage and sub-stage. let x_{ij} be the input for very first stage to produce the output w_{k_1j} *and* w_{k_2j} which in turn represent inputs for *sub-stages A and B* under second/intermediate stage. Additionally, the output v_{l_1j} *and* v_{l_2j} of *sub-stage A and B* represent as inputs for stage-III to generate the final output y_{rj}.

The SBM model is applied both individually for all three stages and sub stages, then as a whole under the assumption of VRS for the identification of inefficiency in underlying system. The individual efficiency scores of *stage (I, II, and III)* are already discussed in previous sections. Overall efficiency score can be calculated by using the following SBM model¶.

$$\rho^*_{overall} = Min \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^-}{x_{i0}}}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^+}{y_{r0}}}$$

Subject to

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{i0} \; ; \; i = 1,2,3,\ldots m. \tag{6.1.1}$$

$$\sum_{j=1}^{n} \lambda j y_{rj} - s_r^+ = y_{r0} \; ; \; r = 1,2,3,\ldots s.$$

$$\sum_{j=1}^{n} \lambda j w_{kj} = \sum_{j=1}^{n} \mu_j^1 w_{k_1j} + \sum_{j=1}^{n} \mu_j^2 w_{k_2j} \; ; \; k_1, k_2 \subseteq k.$$

$$\sum_{j=1}^{n} \mu_j^1 v_{l_1j} + \sum_{j=1}^{n} \mu_j^2 v_{l_2j} = \sum_{j=1}^{n} \eta_j v_{lj} \; ; \; l_1, l_2 \subseteq l.$$

$$\sum_{j=1}^{n} \lambda_j = \sum_{j=1}^{n} \mu_j^1 = \sum_{j=1}^{n} \mu_j^2 = \sum_{j=1}^{n} \eta j = 1 \; ; \; j = 1,2,3,\ldots n.$$

$$\lambda_j \geq 0, \mu_j^1 \geq 0, \mu_j^2 \geq 0, \eta_j \geq 0, s_i^- \geq 0 \text{ and } s_r^+ \geq 0.$$

The mathematical model (6.1.1) calculates the overall efficiency of three stages of network structure. By definition of efficiency *DMU* is considered efficient if and only, it is efficient in all the three-stage [8]. If k_1 is proper subset of k then l_1 is also proper subset of l . Which results that all the three stages are connected in series and the intermediate stage is consist of only one sub-stage that is *sub-Stage A*. where if and k_2 and l_2 are the proper subset of k and l. Then *sub-stage B* will be taken under action. For the three stages network structure we are supposed that bout k_1 , k_2 and l_1 , l_2 are improper subsets of k *and* l respectively.

¶ In the SBM- model we are directly dealing with slacks (input excesses and output shortfalls) of the concerned DMU. Were as in the two or more than two stage SMB-model. we are minimizing the initial inputs excess and final outputs shortfall as shown in the objective function of SBM model (6.1.1).

7. Empirical illustration

In this section, the new approach of network Data envelopment analysis (NDEA) is applied to the 25 banking companies of the Indian banking sector for the financial year 2015. The data structure has been collected and compiled from CMIE(Centre for Monitoring Indian Economy Pvt. Ltd. (CMIE) https://www.cmie.com/) and Bloomberg. At first, the overall production has been decomposed into

Table 1. **Summary Statistics of all Input & Output Variables**

Input/ Output	Range	Min	Max	Mean	Variance
No. of employees	210959	2279	213238	37880.04	1752710140.87
No. of branches	16340	184	16524	3443.60	10488278.33
Total fund	2587418	7814	2595232	417156.41	289128162101.67
Expenses from (FS)	963971	9848	973818	195908.08	36303664419.51
Demand deposits(FS)	179298	15	179313	17817.78	1375070458.78
Term deposits (FS)	1106823	6	1106829	150344.79	64836756248.85
Expenses from (NFS)	649547	6800	656346	108238.26	16674322465.32
Demand deposits(NFS)	1176181	10127	1186308	228382.96	71868451349.90
Term deposits (NFS)	9068970	88113	9157083	1821338.51	3330830792953.30
Income (FS)	1710252	13659	1723911	323537.86	115570645912.21
Income (NFS)	17014	110	17125	4951.60	21047549.31
Interest Income	1511135	12836	1523971	288002.93	88877098266.72
Returns on assists	2	0	2	0.81	0.41
User fee income	131654	74	131728	20096.63	941835999.53

the three-stage series structure, where the intermediate stage has two sub-stages which are connected parallel in the systems. The overall system follows the mixed-structure called network structure where the selection of variables is shown in figure 6 in Appendix-I. The variables are No.of employees (x_1), No. of branches (x_2)and total funds used (x_2) are the inputs of the very first stage and the output of stage-I are divided into two inputs for intermediate stage connected parallel in the system. The sub-stage A is operating the financial services and using the inputs Financial expenses (W_{11}), Demand and Term deposits of financial service (W_{12}) and (W_{13}) to produce the income from the financial services (v_{11}). On the other side, the sub-stage B is operating the non-financial services and using the inputs Non-financial expenses (W_{21}), Demand and Term deposits of non-financial service (W_{22}) and (W_{23}) to produce the income from the non-financial services (v_{21}). Finally, the outputs of the second stage are used as the inputs in the final stage and produce the final outputs: Interest income (y_1), Return on assets (y_2) and User fee income (y_3) of $j^{th} DMU/Bank; j = 1, 2, 3, \ldots, n$.. The summary statistics of all inputs and outputs are shown in table 1.

This type of production process does not have final outputs for the first stage nor in the second and exogenous inputs are used for second and third stages respectively. By applying a simplified version SBM models (3.1.1),(5.1.3), (3.1.2), (5.1.1) and (5.1.2) for single system and corresponding efficiency values are shown in table 2 in column (2), (3), (4), (7) and (8). For the two-stage series structure, we the SBM model (3.1.3) and the corresponding efficiency values are shown in column (5)and (6) in table 2. While the estimation the efficiency in case of two-stage parallel structure system can be calculated by the SBM model (5.1.3). The efficiency value of the three-stage series structure is calculating by applying the mathematical form of the SBM model(4.1.1). Finally, the overall efficiency of the network can be calculated by applying the mathematical form of the SBM model(6.1.1) and the corresponding efficiency value are shown in column (9) in table 2. The stage efficiencies were calculated and the results are shown in Table 1. It is observed that only DMU7 appeared to be efficient across all the stages. DMU7 and

Table 2. Results of Network Data Envelopment Analysis (NDEA).

DMUs	**Single Stages**			**Double Stages**		**Sub-stages**		**Network**	**Efficiency**
	I	II	II	(I - II)	(II-III)	A	B	NDEA	Stage Status
(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**	**(10)**
DMU1	0.8420	0.2950	0.6410	0.3940	0.7030	1.0000	0.3950	0.3770	Only 7
DMU2	0.8730	0.2560	0.6120	0.3970	0.5990	1.0000	0.8390	0.4300	Only 7
DMU3	1.0000	1.0000	0.3820	0.3600	0.9860	0.9450	0.4180	0.5870	2 and 3
DMU4	1.0000	1.0000	0.4250	0.9250	0.9980	1.0000	0.5180	0.3930	2, 3 and 7
DMU5	0.9010	0.8690	0.4910	0.9070	0.8230	0.4100	1.0000	0.6300	Only 8
DMU6	0.7950	0.3940	0.6440	0.3760	1.0000	0.3200	0.2140	0.2840	Only 6
DMU7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	Overall
DMU8	1.0000	0.6010	1.0000	0.8250	0.8780	1.0000	0.3510	0.8470	2, 4 and 7
DMU9	1.0000	0.4650	1.0000	1.0000	1.0000	0.9440	0.0460	1.0000	2, 4, 5, 6 and 9
DMU10	1.0000	0.2750	0.5550	0.7640	0.8320	0.5560	0.5430	0.5870	Only 2
DMU11	1.0000	0.4330	0.5360	0.3900	0.7600	1.0000	0.5360	0.2450	2 and 7
DMU12	1.0000	0.0720	0.8410	0.6460	0.3480	0.1270	1.0000	0.6700	1 and 8
DMU13	1.0000	0.0950	1.0000	0.7160	0.5530	1.0000	0.3200	1.0000	2, 4, 7 and 9
DMU14	1.0000	0.3600	0.5390	0.4130	0.6860	1.0000	0.5220	0.3270	2 and 7
DMU15	1.0000	0.7630	0.5890	0.9450	0.5700	0.9330	0.6580	0.6320	Only 2
DMU16	1.0000	0.0030	1.0000	1.0000	0.7110	1.0000	0.9010	0.3490	2, 4, 5 and 7
DMU17	0.7230	0.1040	0.7740	0.4520	0.7960	0.1030	0.3660	0.7400	None
DMU18	1.0000	1.0000	1.0000	0.4920	0.0970	1.0000	0.1320	1.0000	2, 3,4,7 and 9
DMU19	0.7770	0.4510	0.4690	0.3710	0.7940	0.9270	0.5510	0.5120	None
DMU20	1.0000	0.4180	0.9770	0.3780	0.9720	0.7680	0.0870	0.8700	Only 2
DMU21	1.0000	0.3520	0.5330	0.4060	0.6960	0.4080	0.6820	0.5380	Only 2
DMU22	0.9420	0.5320	0.5850	0.3710	0.7790	0.6320	1.0000	0.5780	Only 8
DMU23	0.8670	0.1330	0.7060	0.4450	0.3810	0.1070	0.6330	0.2820	None
DMU24	0.9710	0.2260	0.5240	0.4460	0.5650	1.0000	1.0000	0.4090	7 and 8
DMU25	1.0000	0.1500	0.8870	0.7230	0.4230	0.0720	0.9310	1.0000	2 and 9
Mean	0.9480	0.4390	0.7090	0.6060	0.7180	0.7300	0.5860	0.6120	

DMU18 are observed to be efficient under all three single stages. In the two-stages structure, DMU7 and DMU9 have been found efficient, while as in the three-stage network structure system DMU [7, 9, 13, 18, and 25] appeared as efficient units. To maintain the property of efficiency the DMUs are treated to be overall efficient if and only if it is efficient in all stages otherwise, it is treated as inefficient.

The main aim of decomposing the overall efficiency is to identify the proper location of inefficiency present in the specific DMU. As mentioned in the above Table 1: the DMU1 is efficient in sub-stage A as shown under column 7 corresponding to the DMU1. The unity value in the above table indicates the efficiency indicator corresponding to specific DMU under the specific stage. As mentioned in Table 1: DMU17 and DMU23 are overall inefficient, because under none of the stages its efficient score is unity. Under the last column in table 2 indicate the efficiency stage status. The last row of table 6 represents mean of efficiencies in respective stages.

8. Conclusions

In the real world, many problems have a structure resembling a three-stage network structure. However, evaluation of the efficiency of this type of network structure is a very complicated task, and it can be solved with network DEA models with SBM approach, which has good discrimination power over the CCR and BCC DEA models. The identification of inefficiency in network production systems is challenging and has been widely discussed in the professional community in the last three decades. Many papers have been published with the main focus on theoretical aspects and applications in this field. The model developed in this paper is focused on the most straightforward system, which is the three-stage network production system. In contrast, the intermediate stage consists of two parallel-connected sub-stages in the system, and problems can be solved more realistically. The decomposition approach of efficiency is a better technique to identify the processes that cause poor performance in the production system. The processes responsible for inefficiency in a production system are identified by the network DEA. Improving the efficiency of these processes will thus be the most effective way to improve the performance of the overall system. Finally, all analyses in this paper were made under the assumption that the variable returns to scale. How to extend the idea to situations of the multi-stage networking system will be a challenging task for future studies. In this paper, we assume a closed production system, which implies that the study is more applicable to only the type of DMU where a production unit produces finished output in the final stage. The study is not applicable where a production unit produces a partially finished output in the final stage or finished output at early stages.

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