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A net with serial access and the reduction of total work for identical service

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Abstract This paper looks at the open net in conditions of identical service. The strong reduction of total work and time of expectation on the inlying nodes of net is marked. By using the transform of Legendre, the previous results are generalized on net with the relatively general structure.

Keywords Identical Service, Serial Access, Heavy Traffic, Total Time of Work, Waiting Times

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1. Introduction

We consider the nets with serial access in conditions of identical service. Main restriction on the net are the conditions: the customer from any node can arrive in only one next node, and the total speed of service in the inlying note is equal to the sum of speeds of service in the nodes of sources for the inlying node (the sum of speeds of the j-sources for the j-th node, the C1,C2,C3 conditions). A similar condition is necessary, for example, at research of network in the conditions of diffusive approximation in heavy traffic, [1, 2, 3, 4, 5]. A subject of the article is of interest as for applications so in the classic tasks of theory of nets with serial access, [1, 2, 3, 4, 5, 6, 7, 8].

In theorem 1 the previous results are generalized on net with the relatively general structure. In the situation we can use the equality of Legendre ([1, 2, 3]) :

$$f(Y(t) - f(Y(t))) \equiv 0, t \in [0,1],$$

$$f(Y(t)) = Y(t) - t - \inf_{u\in[0,t]}(Y(u) - u) = \sup_{0\leq u\leq t}(Y(t) - Y(u) - (t-u)),$$

for different the $Y(t)$ functions ($Y(t) = S_j(t)$-in the main theorem 1).

Each node contains the infinite set of places for wait . The total number of customers (the examples in the second part for the determined length of service of one customer), the time of wait,the total time of work in the inlying nodes remains limited (for all trajectories and all $t \in [0,\infty)$, t is the time), if the time of service of one customer by one device of all nodes is limited by a constant (the

$$W_j^t \leq \sum_{n=1}^{K_j} \theta_n^{U_1} \leq K_j \max_{i\leq A(t)} \xi_i^1,$$

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expression in the theorem 1). It is the basic result of article. The basic examples of the third part ensue from the result.

The time of expectation and total work on the inlying nodes was considered in works ([1, 2, 3, 4, 5, 6, 7, 8, 9]) (for identical service for devices in series).

For the determined time of service of one customer total number of customers on the inlying node always less than number of sources of input streams for the node (the theorem 1). The fact results in the examples of the third part.

In all the nodes the customers are served in order of arrival. If a customers arrives on a node in group (for non-ordinary process),the customers are disposed in the group in the random order - all results d't rely on the order in the group.

The j-th node consists of 1 service units (device) with infinite set of waiting places, $j = 1, \ldots N$.

In the article we explore the net characteristics :

W_j^t - the total time of service of all the customers being on j-th node at t moment , $j = 1, \ldots, N$,(the virtual time of wait for one channel) ;

ν_j^t - the total number of the customers on j-th node at t moment , $j = 1, \ldots, N$;

V_j^t - the full time of service on the j-th node of the customer, which arrives on the j-th node at t moment $j = 1, \ldots, N$ (the time of wait plus the "length of customer").

By definition, $A_j(t)$ is a number of final customer among all customers arriving on j-th node (the node with the n number) during $[0, t]$. (If the customers arrives on a node in group ,the customers are disposed in the group in random order). For the ordinary process

$$A_j(t) = \max k : t_k \leq t,$$

$A_j(t) = 0$, if $t_1 > t$, where t_i is the moment of arriving on the j-th node of i-th customer, $i = 1, 2, \ldots; j = 1, \ldots, N$, N is the common quantity of nodes in the net.

By definition, the node with the n number (the n-th node) is the j-source for the j-th node, if all customers from the n-th node after service arrive on the j-th node, (the output process for the n-th node is a part of the input process for the j-th node).

The arrival process ($A_j(t)$) for the j-th node is equal to the sum of output processes ($A_n(t)$) on the j-sources for the j-th node:

$$A_j(t) = \sum_{n=1}^{K_j} A_n(t), K_j \in 1, \ldots,$$

where K_j **is the common quantity of the j-sources** .Other limitations on the arrival process are not present.

Let j be the number of the customer arriving on the first node j-th on the account. By definition, ξ_i^j is the time of service of the ith customer on the jth node by one device (unit of service) ; **all the different devices on all nodes are identical:**

$$\xi_i^1 = \xi_i^2 = \ldots = \xi_i^N, i = 1, 2, \ldots,$$

N is the common quantity of nodes in net. The $\{\xi_1^1\}$ values can be dependency upon each other and from the input streams), [3].

Further we will use for ξ_i^j the term "length of customer" too.

Main restriction on the net are the C1,C2,C3 conditions.

Condition C1: all customer from j-th node can arrive on only one n-th node or all customer from j-th node leave from net. (The multiway branch outputs from one node is impossible).

Condition C2: the total speed of service ([3]) in the inlying j-th note is equal to the sum of speeds of service in the j-sources.

Condition C3: the customers from only the external nodes arrive on the inlying nodes (the sources for net can not be some j-source for the inlying node); the net has not cycles .

2. Main result

By definition, the j-th node is an inlying-nodes, if for the j-th node there are some j-sources **from other nodes** (not the input processes for the net).

The main result in theorem 1 ([1, 2, 3]) is formulated for all fixed $j = 1, \ldots, N_0$ (N_0 is the common quantity of inlying-nodes in net).

Theorem 1.

$$\sup_{0 \le u \le t} \max(W_j^u, V_j^u) \le K_j \Delta(t),$$

$$\Delta(t) = \max_{i \le A(t)} \xi_i^1,$$

$A(t)$ is the total quantity of customers arriving in the net during $[0, t]$, if the j-node is the inlying-node, and the j-th inlying-node has the K_j j-sources, $K_j \in 1, \ldots$.

Proof.

Lets the j-th node is the inlying-node.

Let $A(j, t)$ be the number of the last customer arriving on the j-th node during $[0, t]$,$j = 1, \ldots, N$.

We use (Borovkov,[10], p.41 and [3])

$$W_j^t = S_j(t) - c_j t - \inf_{0 \le u \le t} (S_j(u) - c_j u),$$

$$S_j(t) = \sum_{i=1}^{A(j,t)} \xi_i^1, \; j = 1, \ldots, N,$$

(the equality is for all nodes - not only for the inlying-node), where c_j is the speed of service in the j-th node, $j = 1, \ldots, N$.

At the t moment on the n-th node there are a customer with number $\mu_n(t)$. By definition, θ_n^t is the part of the "length of customer" with number $\mu_n(t)$ is already served at t moment; $\theta_n^t = 0$,if $W_n^t = 0$,$n = 1, \ldots, K_j$.

We will suppose $c_j = 1$. We obtain (as in Pavlov, [1, 2, 3])

$$S_j(t) - t = \sum_{n=1}^{K_j} [S_n(t) - (W_n^t + \theta_n^t)] - t, \theta_n^t \le \xi_{\mu_n(t)}^n,$$

$j = 1, \ldots, N_0$.

With help of

$$\sum_{n=1}^{K_j} c_n = c_j = 1,$$

(the C2 condition) we can write

$$S_j(t) - t = \sum_{n=1}^{K_j} [S_n(t) - [S_n(t) - c_n t - \inf_{0 \le u \le t} (S_n(u) - c_n u)] - \theta_n^t] - t =$$

$$= \sum_{n=1}^{K_j} [c_n t + \inf_{0 \le u \le t} (S_n(u) - c_n u) - \theta_n^t] - t = \sum_{n=1}^{K_j} [\inf_{0 \le u \le t} (S_n(u) - c_n u) - \theta_n^t], \; t \in [0, +\infty),$$

and

$$W_j^t = \sum_{n=1}^{K_j} [\inf_{0 \le u \le t} (S_n(u) - c_n u) - \theta_n^t] - \inf_{0 \le U \le t} \sum_{n=1}^{K_j} [\inf_{0 \le u \le U} (S_n(u) - c_n u) - \theta_n^U];$$

we obtain

$$W_j^t = \sum_{n=1}^{K_j} [\inf_{0 \le u \le t} (S_n(u) - c_n u) - \theta_n^t] -$$

$$- \sum_{n=1}^{K_j} [\inf_{0 \le u \le U_1} (S_n(u) - c_n u) - \theta_n^{U_1}] =$$

$$= \left(\sum_{n=1}^{K_j} [\inf_{0 \le u \le t} (S_n(u) - c_n u) - \inf_{0 \le u \le U_1} (S_n(u) - c_n u)] \right) + \left(\sum_{n=1}^{K_j} [-\theta_n^t + \theta_n^{U_1}] \right) \le$$

$$\le \left(\sum_{n=1}^{K_j} [-\theta_n^t + \theta_n^{U_1}] \right),$$

where U_1 is the moment

$$\inf_{0 \le U \le t} \sum_{n=1}^{K_j} [\inf_{0 \le u \le U} (S_n(u) - c_n u) - \theta_n^U] = \sum_{n=1}^{K_j} [\inf_{0 \le u \le U_1} (S_n(u) - c_n u) - \theta_n^{U_1}], U_1 \le t,$$

(U_1 is the moment of reach of external minimum). We use, that

$$\left(\sum_{n=1}^{K_j} [\inf_{0 \le u \le t} (S_n(u) - c_n u) - \inf_{0 \le u \le U_1} (S_n(u) - c_n u)] \right) \le 0,$$

with help of

$$\inf_{0 \le u \le t} (S_n(u) - c_n u) \le \inf_{0 \le u \le U_1} (S_n(u) - c_n u),\ n = 1, \ldots, K_j,$$

if $t \ge U_1$.

We get

$$W_j^t \le \left(\sum_{n=1}^{K_j} [-\theta_n^t + \theta_n^{U_1}] \right) \le \sum_{n=1}^{K_j} \theta_n^{U_1},\ \theta_n^{U_1} \le \max_{i \le A(t)} \xi_i^1,$$

(we use $\theta_n^t \ge 0, n = 1, \ldots, K_j$).

The theorem 1 about W_j^t is proved.

For V_j^u we use $V_j^u = W_j^u, u \in [0, t]$.

3. Examples of limited work on the inlying nodes

We consider the net from the introduction with the C1-C3 conditions.

Example 1

We consider the j-th inlying nodes with the K_j j-sources (with the $K_j \ge 1$ input nodes) in the condition of identical service for all time $t \in [0, +\infty)$.

If the time of service of one customer by one device more of m constant, and it is always limited by $2m$ constant:

$$m < \xi_1^1 < 2m, 2m = \max_i \xi_i^1 < \infty,\ K_j \in 1, \ldots,$$

the number of customers on the j-th inlying nodes **will be only in the set** $0, 1, \ldots, 2K_j - 1$ **independently from an intensity of arriving of customers on the first device** : $\nu_j^t \leq 2K_j - 1$, for all $t \in [0, \infty), j = 1, \ldots, N_0$ (in the net are located N_0 the inlying nodes), if $\nu_j^0 = 0, j = 1, \ldots, N_0$ (we use the theorem 1).

The situation (the theorem 1) is executed for all input streams of customers on first device and for all $\{\xi_1^1\}$,(the $\{\xi_1^1\}$ values can be dependency upon each other and from the input streams), [3].

Example 2

As in the first example we consider the j-th inlying nodes with the K_j j-sources (with the $K_j \geq 1$ input nodes) in the condition of identical service for all time $t \in [0, +\infty)$.

From the theorem 1 the number of customers **on the j-th inlying nodes** $\nu_j^t \in 0, 1, \ldots, K_j, K_j \in 1, \ldots, t \in [0, \infty)$ **, if the time of service of every customer by one device of all nodes** ξ_1^1 **is the** M **constant**:

$$\nu_j^t \leq K_j,$$

for all $t \in [0, \infty), j = 1, \ldots, N_0$,if $\nu_j^0 = 0, j = 1, \ldots, N_0$,

$$\xi_j^1 \equiv M = const. < \infty, j = 1, 2, \ldots,$$

[3].

As in the first example the situation (the theorem 1) is executed for all input streams of customers on first device and for all $\{\xi_1^1\}$,(the $\{\xi_1^1\}$ values can be dependency upon each other and from the input streams), [3].

Example 3

If the sources for the net are considered as a control processor (the virtual node or nodes with number 0) for all the inlying nodes in conditions of identical service, we obtain for the inlying nodes the limited time of total work and the time of wait:

$$V_j^t < K_j q = const., \sum_{j=1,\ldots,N_0} W_j^t < Q < \infty, Q = const.$$

for all the inlying nodes with the j-th number $j = 1, \ldots, N_0, t \in [0, \infty)$, if

$$\xi_j^1 < q = \max_i \xi_i^1 < \infty,$$

where $Q = K_j q N_0$. (We use the theorem 1 too), [3] .

At the use of the processor we use no information from the working nodes. The situation (as in the theorem 1) is executed for all input streams of customers on first device and for all $\{\xi_1^1\}$,(the $\{\xi_1^1\}$ values can be dependency upon each other and from the input streams), [3].

Final Remark: In opinion of author the methods of the article are not applicable to the nets of more difficult structure. Probably, the marked reduction of total work on the inlying nodes is impossible in the more general situation.

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