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Generalised form of Bonus-Malus System Using Finite Mixture Models

Saeed MohammadPour [1], Karzan Saeedi [1], Rahim Mahmoudvand [2,*]

[1] *E.C.O College of Insurance, Allameh Tabatabai University, Tehran, Iran*
[2] *Department of Statistics, Bu-Ali Sina University, Hamedan, Iran*

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Abstract There is a vast literature on Bonus-Malus System (BMS), in which a policyholders responsible for positive claims will be penalised by a malus and the policyholders who had no claim will be rewarded by a bonus. In this paper, we present an optimal BMS using finite mixture models. We conduct a numerical study to compare the new model with the current BMS that use finite mixture models.

Keywords Bonus-Malus System, Mixture model, Bayes theorem

AMS 2010 subject classifications 62P05, 62F15

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1. Introduction

A Bonus-Malus System (BMS) penalize policyholders responsible for one or more claims by a premium surcharge (malus) and rewards the policyholders who had a claim-free year by awarding a discount on the premium (bonus), see Frangos and Vrontos (2001). Up to this day, many studies have been published with extensions and applications of the BMS, see Tzougas et al., (2014), Mahmoudvand et al. (2017) and references therein.

According to Frangos and Vrontos (2001), the premiums of the generalized BMS will be derived using the following multiplicative tariff formula:

$$\text{Premium} = \text{GBM}_F \times \text{GBM}_S \tag{1}$$

where GBM_F denotes the generalized BMS obtained when only the frequency component is used and GBM_S denotes the generalized BMS obtained when only the severity component is used, see also Mahmoudvand and Hassani (2009).

It is popular in the BMS literature to use Bayes theory for finding GBM_F and GBM_S. In the view of Bayesian theory, we have to determine the structure function of the risk parameters for both frequency and severity components. In order to do that, Tzougas et al. (2014) have used finite mixture models. This choice may be better suited for modelling risk, since in a collective there are several types of risks such as very good risks, good risks, bad risks, very bad risks and so on. However, a very few studies used finite mixture prior distribution in the field of actuarial statistics, see Denuit and Lambert (2001), Gomez-Deniz et al. (2004) and Tzougas et al. (2014). In this paper, we generalise the model by Tzougas et al. (2014).

*Correspondence to: Rahim Mahmoudvand (Email: r.mahmodvand@gmail.com). Department of Statistics, Bu-Ali Sina University, Hamedan, Iran.

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2. Design BMS Using Finite Mixture Models

We use the following proposition as the main theorem for this study.

Theorem 1
Let $L|\theta \sim f(\ell|\theta)$ and probability distribution of θ is given by a mixture of probability density functions $\psi_1(\theta), \ldots, \psi_n(\theta)$ with weights $w_1, \ldots, w_n$ where $w_i \geq 0$ such that $\sum_{i=1}^{n} w_i = 1$. Then we have:

$$\psi(\theta|\ell) = \sum_{i=1}^{n} \tilde{w}_i \psi_i(\theta|\ell), \tag{2}$$

where $\psi_i(\theta|\ell)$ is a posterior distribution function with respect to prior $\psi_i(\theta)$ and adjusted weight $\tilde{w}_i$ is defined as below:

$$\tilde{w}_i = \frac{f_i(\ell)}{\sum\limits_{i=1}^{n} w_i f_i(\ell)} w_i \tag{3}$$

with $f_i(\ell) = \int \psi_i(\theta) f(\ell|\theta) \mathrm{d}\theta$.

Proof
Using Bayes theorem, we have:

$$\begin{aligned}
\psi(\theta|\ell) &= \frac{f(\ell|\theta)\psi(\theta)}{f(\ell)} = \frac{\sum\limits_{i=1}^{n} w_i f(\ell|\theta)\psi_i(\theta)}{\sum\limits_{i=1}^{n} w_i \int f(\ell|\theta)\psi_i(\theta)\mathrm{d}\theta} \\
&= \frac{\sum\limits_{i=1}^{n} w_i f_i(\ell) \frac{f(\ell|\theta)\psi_i(\theta)}{f_i(\ell)}}{\sum\limits_{i=1}^{n} w_i \int f(\ell|\theta)\psi_i(\theta)\mathrm{d}\theta} = \frac{\sum\limits_{i=1}^{n} w_i f_i(\ell)\psi_i(\theta|\ell)}{\sum\limits_{i=1}^{n} w_i f_i(\ell)} = \sum_{i=1}^{n} \tilde{w}_i \psi_i(\theta|\ell).
\end{aligned}$$

□

Corollary 1
Bayes estimator for θ under the condition of Proposition 1 with respect to the quadratic error loss function is:

$$\hat{\theta} = \sum_{i=1}^{n} \tilde{w}_i \mu_i(\theta|\ell), \tag{4}$$

where $\mu_i(\theta|\ell)$ is the posterior mean for posterior distribution $\psi_i(\theta|\ell)$.

Denote by k the number of claims with the underlying risk parameter θ_F and by x the severity of claims with the underlying risk parameter θ_S. Then Corollary 2.1 and equation (1) imply that the premium for optimal BMS using finite mixture model can be obtained by:

$$\text{Premium} = \sum_{i=1}^{n_F} \tilde{w}_i^F \mu_i^F(\theta_F|k) \times \sum_{i=1}^{n_S} \tilde{w}_i^S \mu_i^S(\theta_S|x), \tag{5}$$

where the notations are defined similar to Proposition 1 and Corollary 2.1.

In the next two sections, we focus on the particular cases in which the distribution of the frequency and severity components are known.

3. BMS based on the a posteriori criteria

Notations

t: number of insurance period, usually in year, that policyholder is under study,

k_j: number of claims in period j,

λ: underlying risk parameter of the random variable k_j,

$x_{j,k}$: Severity of the claim k in period j,

y_j: underlying risk parameter of the random variable $x_{j,k}$.

3.1. Frequency component

Assume $k_j|\lambda$ is distributed as Poisson(λ), independently for all j, and that the structure function follows an n-components mixture of Gamma distribution with the following density:

$$u(\lambda) = \sum_{z=1}^{n} \pi_z \frac{\tau_z^{\alpha_z} \lambda^{\alpha_z - 1} e^{-\tau_z \lambda}}{\Gamma(\alpha_z)}. \tag{6}$$

Equation (6) means that we have considered n risk categories. Consider a policyholder with claim history $k_1, ..., k_t$ and denote by K the total number of claims in t years. Then, the posterior structure function, $u(\lambda|k_1, ..., k_t)$, for a policyholder or a group of policyholders, is given by (see Proposition 1):

$$u(\lambda|k_1, \ldots, k_t) = \frac{\prod_{j=1}^{t} f(k_j|\lambda) u(\lambda)}{\int_0^\infty \prod_{j=1}^{t} f(k_j|\lambda) u(\lambda) d\lambda} = \sum_{z=1}^{n} \tilde{\pi}_z \frac{(t+\tau_z)^{K+\alpha_z} \lambda^{K+\alpha_z-1} e^{-(t+\tau_z)\lambda}}{\Gamma(K+\alpha_z)}, \tag{7}$$

where,

$$\tilde{\pi}_z = \pi_z \frac{P(K; \tau_z, \alpha_z)}{\sum_{z=1}^{n} \pi_z P(K; \tau_z, \alpha_z)},$$

in which:

$$P(K; \tau_z, \alpha_z) = \frac{\Gamma(K+\alpha_z)}{K!\Gamma(\alpha_z)} \left(\frac{\tau_z}{\tau_z + t} \right)^{\alpha_z} \left(\frac{t}{\tau_z + t} \right)^{K}.$$

Consequently, using the quadratic error loss function, the optimal choice of λ_i for a policyholder with claim history $k_1, ..., k_t$ is the mean of the posterior structure function, that is:

$$\hat{\lambda}_{t+1} = \sum_{z=1}^{n} \tilde{\pi}_z \frac{K+\alpha_z}{\tau_z + t}. \tag{8}$$

3.2. Severity component

Assume $x_{j,k}|y$, the size of claim k in period j, is distributed as Exp(y), independently for all j, and that the structure function follows an n-components mixture of Inverse Gamma distribution with below density:

$$g(y) = \sum_{z=1}^{n} \rho_z \frac{\frac{1}{m_z} e^{-\frac{m_z}{y}}}{\Gamma(s_z) \left(\frac{y}{m_z} \right)^{s_z+1}}, \tag{9}$$

Denote by $K = \sum_{j=1}^{t} k_j$ the total number of claims in t years and by $X = \sum_{j=1}^{t}\sum_{k=1}^{k_j} x_{j,k}$ the total amount of claims, the posterior structure function, $g(y|x_{1,1},...,x_{t,k_t})$, for a policyholder or a group of policyholders, is given by (see Proposition 1):

$$g(y|x_{1,1},\dots,x_{t,k_t}) = \sum_{z=1}^{n} \tilde{\rho}_z \frac{\frac{1}{X+m_z} e^{-\frac{X+m_z}{y}}}{\Gamma(K+s_z)\left(\frac{y}{X+m_z}\right)^{K+s_z+1}}, \tag{10}$$

where,

$$\tilde{\rho}_z = \rho_z \frac{P(X;K,m_z,s_z)}{\sum_{z=1}^{n} \rho_z P(X;K,m_z,s_z)},$$

in which:

$$P(X;K,m_z,s_z) = \frac{\Gamma(K+s_z)}{K!\Gamma(s_z)} \left(\frac{m_z}{m_z+X}\right)^{s_z} \left(\frac{X}{m_z+X}\right)^{K}.$$

Using the quadratic error loss function, the optimal choice of y for a policyholder with claim history $k_1,...,k_t$ and claim sizes $x_{1,1},\dots,x_{t,k_t}$ is the mean of the posterior structure function, that is:

$$\hat{y}_{t+1} = \sum_{z=1}^{n} \tilde{\rho}_z \frac{X+m_z}{s_z+K-1}. \tag{11}$$

Applying equations (8) and (11) in equation (1) results in the premiums of the BMS based on the a posteriori criteria as below:

$$\text{Premium} = \sum_{z=1}^{n} \tilde{\pi}_z \frac{K+\alpha_z}{\tau_z+t} \times \sum_{z=1}^{n} \tilde{\rho}_z \frac{X+m_z}{s_z+K-1}. \tag{12}$$

3.3. Parameter estimation

In order to use equation (12), we need first to obtain estimates of parameters $\pi_z, \alpha_z, \tau_z, \rho_z, m_z, s_z$ for $z = 1,\dots,n$. Using our assumptions we can get easily that:

$$f_{k_j}(k) = \sum_{z=1}^{n} \pi_z \frac{\Gamma(k+\alpha_z)}{k!\Gamma(\alpha_z)} \left(\frac{\tau_z}{1+\tau_z}\right)^{\alpha_z} \left(\frac{1}{1+\tau_z}\right)^{k}, k = 0,1\dots, \tag{13}$$

$$f_{x_{jk}}(x) = \sum_{z=1}^{n} \rho_z \frac{s_z m_z^{s_z}}{(m_z+x)^{s_z+1}}, \quad x > 0. \tag{14}$$

Equation (13) and (14) shows that the number of claims and the amount of each claims assumed to be a finite mixture of negative binomial and Pareto distribution, respectively. For a finite mixture distribution to continue data, one way is by trial and error. For instance first estimating the centers of the peaks by eye in the density plot (these become the component means), and adjusting the standard deviations and mixing percentages to approximately match the peak widths and heights, respectively.

The **mixdist** package is one of several available packages in R to fit mixture distributions, see Macdonald and Du (2012). It contain negative binomial distribution. So, we can find estimates of π_z, τ_z and α_z by this package. It should be mentioned that most of the procedures in the mixture fitting are based on the iterative expectation maximization (EM) algorithm. In MATLAB, function **gpfit** can be used to fit the Pareto mixture to real data using maximum likelihood estimation, see Weinberg and Finch (2012).

4. BMS Based on both the a priori and a posteriori criteria

Notations

Consider a policyholder i with an experience of t periods. We define the following notations:

k_i^j: number of claims in period j,

$\lambda_{z,i}^j$: expected number of claims of an individual i who belongs to the zth category,

$c_{z,i}^j = (c_{z,i,1}^j, ..., c_{z,i,h}^j)$: is the vector of h individual characteristics which affect on the the distribution of k_i^j,

$\beta_z^j = (\beta_{z,1}^j, ..., \beta_{z,h}^j)$: is the vector of the coefficients,

$d_{z,i}^j = (d_{z,i,1}^j, ..., d_{z,i,h}^j)$: is the vector of h individual characteristics which affect on the the distribution of x_i^j,

$\gamma_z^j = (\gamma_{z,1}^j, ..., \gamma_{z,h}^j)$: is the vector of the coefficients,

$x_{i,k}^j$: size of claim k in period j for policyholder i,

$y_{z,i}^j$: underlying risk parameter of the random variable $x_{j,k}$.

4.1. Frequency component in the presence of a priori criteria

Suppose that $k_i^j|\lambda_i^j \sim$ Poisson(λ_i^j) and assume that

$$\lambda_i^j = \exp\left(c_{z,i}^j \beta_z^j\right) u_i, \tag{15}$$

where u_i is a random variable follows an n- component Gamma mixture distribution with pdf

$$v(u_i) = \sum_{z=1}^{n} \pi_z \frac{\frac{1}{\alpha_z}^{\frac{1}{\alpha_z}} u_i^{\frac{1}{\alpha_z}-1} e^{-\frac{u_i}{\alpha_z}}}{\Gamma(\frac{1}{\alpha_z})}, \tag{16}$$

Similarly to equation (7), posterior distribution is given below (see Proposition 1):

$$v(\lambda_i^{t+1}|k_i^1, \ldots, k_i^t; c_{1,i}^1, \ldots, c_{n,i}^{t+1}) = \sum_{z=1}^{n} \tilde{\pi}_z \frac{(S_{i,z})^{K+\frac{1}{\alpha_z}} (\lambda_i^{t+1})^{K+\frac{1}{\alpha_z}-1} e^{-S_{i,z}\lambda_i^{t+1}}}{\Gamma(K+\frac{1}{\alpha_z})}, \tag{17}$$

where, $S_{i,z} = \frac{\frac{1}{\alpha_z}+\varphi_z}{\exp(c_{z,i}^{t+1}\beta_z^{t+1})}$ and

$$\tilde{\pi}_z = \pi_z \frac{P(K;\varphi_z,\alpha_z,)}{\sum_{z=1}^{n} \pi_z P(K;\varphi_z,\alpha_z)},$$

in which:

$$P(K;\varphi_z,\alpha_z) = \frac{\Gamma(K+\alpha_z)}{K!\Gamma(\alpha_z)} \left(\frac{\alpha_z}{\alpha_z+\varphi_z}\right)^{\alpha_z} \left(\frac{\varphi_z}{\alpha_z+\varphi_z}\right)^{K},$$

and

$$\varphi_z = \sum_{j=1}^{t} \exp(c_{z,i}^j \beta_z^j).$$

Consequently, the optimal choice of λ_i^{t+1} for a policyholder with claim history $k_1, ..., k_t$ under the quadratic error loss function is the mean of the posterior structure function, that is:

$$\hat{\lambda}_i^{t+1} = \sum_{z=1}^{n} \tilde{\pi}_z \exp(c_{z,i}^{t+1}\beta_z^{t+1}) \frac{\alpha_z+K}{\alpha_z+\varphi_z} \tag{18}$$

4.2. Severity component in the presence of a priori criteria

Assume that $x^j_{i,k} \sim \text{Exp}(y^j_{z,i})$ and assume that

$$y^j_{z,i} = \exp\left(d^j_{z,i}\gamma^j_z\right) w_i, \tag{19}$$

where w_i is a random variable follows an n- component Inverse Gamma mixture distribution with pdf

$$\omega(w_i) = \sum_{z=1}^{n} \rho_z \frac{\frac{1}{s_z-1} e^{-\frac{s_z-1}{w_i}}}{\Gamma(s_z)\left(\frac{w_i}{s_z-1}\right)^{s_z+1}}, \tag{20}$$

An expression similar to equation (10), under Proposition 1, can also be developed for the posterior distribution as below:

$$\omega(y^{t+1}_i|x^1_{i,1},\ldots,x^t_{i,k^t_i}; d^1_{1,i},\ldots,d^{t+1}_{n,i}) = \sum_{z=1}^{n} \tilde{\rho}_z \frac{\frac{1}{C^j_{i,z}} e^{-\frac{C^j_{i,z}}{y^j_i}}}{\Gamma(s_z+K)\left(\frac{y^j_i}{C^j_{i,z}}\right)^{K+s_z+1}}, \tag{21}$$

where, $C_{i,z} = [s_z - 1 + E_z] \exp(d^{t+1}_{z,i}\gamma^{t+1}_z)$ and

$$\tilde{\rho}_z = \rho_z \frac{P(E_z; K, s_z)}{\sum\limits_{z=1}^{n} \rho_z P(E_z; K, s_z)},$$

in which:

$$P(E_z; K, s_z) = \frac{\Gamma(K+s_z)}{K!\Gamma(s_z)} \left(\frac{s_z-1}{s_z+E_z-1}\right)^{s_z} \left(\frac{E_z}{s_z+E_z-1}\right)^{K},$$

and

$$E_z = \sum_{j=1}^{t} \frac{\sum\limits_{k=1}^{k^j_i} x^j_{i,k}}{\exp(d^j_{z,i}\gamma^j_z)}.$$

Using the quadratic error loss function, the optimal choice of y^{t+1}_i for policyholder i is the mean of the posterior structure function, that is:

$$\hat{y}^{t+1}_i = \sum_{z=1}^{n} \tilde{\rho}_z \left(\frac{s_z+E_z-1}{s_z+K-1}\right) \exp(d^{t+1}_{z,i}\gamma^{t+1}_z). \tag{22}$$

Applying equations (18) and (22) in equation (1) results in the premiums of the BMS based on the a priori and the a posteriori criteria as below:

$$\text{Premium} = \sum_{z=1}^{n} \tilde{\pi}_z \exp(c^{t+1}_{z,i}\beta^{t+1}_z) \frac{\alpha_z+K}{\alpha_z+\varphi_z} \times \sum_{z=1}^{n} \tilde{\rho}_z \left(\frac{s_z+E_z-1}{s_z+K-1}\right) \exp(d^{t+1}_{z,i}\gamma^{t+1}_z). \tag{23}$$

5. Numerical Study

Let's first mention that we have used the same notations that Tzougas et al. (2014) have utilised in their study to facilitate the comparison. Equations (7) and (8) are equivalent to equations (25) and (26) in Tzougas et al. (2014). Formulas (7) and (8) have the same structure as equations (25) an (26), but the weights differ: $\tilde{\pi}_z$ by our computation and π_z by Tzougas et al. (2014). This comparison is also hold for equation (18) in our study with equation (30) in

Tzougas et al. (2014). Note that $\sum_{z=1}^{n} \tilde{\pi}_z = \sum_{z=1}^{n} \pi_z = 1$. It is evident from definition of $\tilde{\pi}_z$ that $\tilde{\pi}_z = \pi_z$, for all z, if the negative binomial probabilities are the same. This condition does not hold in general. In order to illustrate the difference between the results, a numerical example is given.

Example 1: Let $n = 2, \pi = (0.70, 0.30), \alpha = (0.1, 0.2), \tau = (5, 1)$. We obtained $\hat{\lambda}_{t+1}$ by both Tzougas et al (2014) and by equation (8) for $K = 0, 1, 2$ and 3 and over nine years, see Figure 1. This figure indicate that the new method gives more reward to the policyholder who has not positive claim than the Tzougas et al. (2014); whereas it provide larger malus for the policyholder with positive claims than the method by Tzougas et al. (2014).

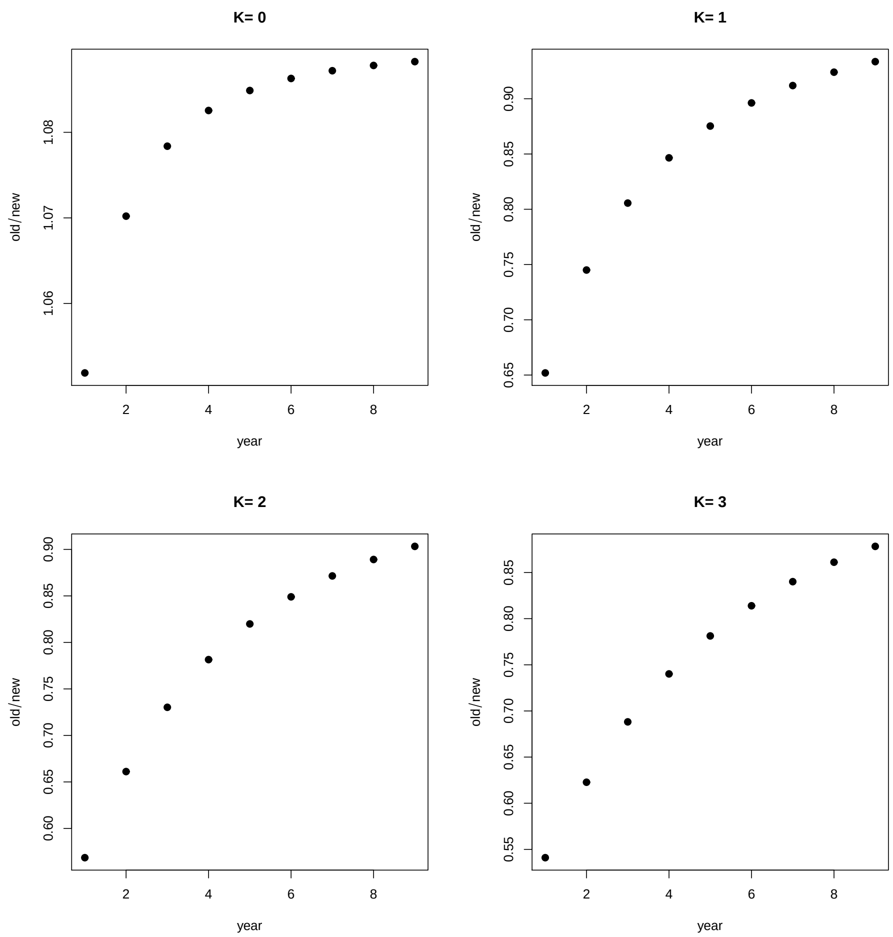

Figure 1. Ratio of posterior mean by equation (26) in Tzougas et al. (2014) over the equation (8) in the current study

Similarly, equations (10)-(11) and equation (22) in this study, which are provided for the severity models, are equivalent to equations (28), (29) and (31) by Tzougas et al (2014). Our formula is based on weights $\tilde{\rho}_z$, whereas Tzougas et al (2014) used ρ_z. We observe again that $\sum_{z=1}^{n} \tilde{\rho}_z = \sum_{z=1}^{n} \rho_z = 1$. In addition, $\tilde{\rho}_z = \rho_z$ provided that the negative binomial probabilities are the same.

Example 2: In this example, we analyse a real data set to illustrate the discrepancy between new method and the method of Tzougas et al. (2014). Table 1 show the frequency distribution of the number of claim along with the mean and variance of it per year for an Iranian insurance company in the period 2009-2011. The data for 2011 shows a very significant increase of over 40% in the number of policy and a very significant decrease of over 50% in the mean claim frequency compared with 2010. The reason was that the insurance company have tie-ups with leading automobile manufacturers in Iran in this year.

Table 1. Frequency distribution of claim in an Iranian insurance company

		year		
		2009	2010	2011
	0	39645	41447	65461
	1	7127	6955	5099
k_j	2	998	992	477
	3	97	123	29
	4	8	9	2
	≥ 5	0	1	0
Mean		0.197	0.189	0.087
Variance		0.214	0.211	0.095

We combined all three years and fit mixture NB to this data using **mixdist** package in R. Table 2 shows the ML estimates of the parameters for mixture NB distribution. We are now able to compare $\tilde{\pi}_z$ with π_z in a real data set. Table 3 indicate the results for $t = 3$ and for $k = 0, \dots, 4$. As it shows the difference is very much when k increases.

Table 2. ML estimates of two-component NB mixture to data of Example 2

$\hat{\pi}_1$	$\hat{\tau}_1$	$\hat{\alpha}_1$	$\hat{\pi}_2$	$\hat{\tau}_2$	$\hat{\alpha}_2$
0.6446	20.8431	1.1247	0.3554	9612.183	3063.787

Table 3. Comparison between $\hat{\tilde{\pi}}$ and $\hat{\pi}_z$ for data of Example 2 when $t = 3$

	k				
	0	1	2	3	4
$\hat{\pi}_1$	0.6446	0.6446	0.6446	0.6446	0.6446
$\hat{\tilde{\pi}}_1$	0.5541	0.0784	0.0105	0.0014	0.0002

Table 4. Ratio of mean by equation (26) in Tzougas et al. (2014) over the equation (8) in the current study for data of Example 2

		k				
		0	1	2	3	4
	4	0.8185	0.5682	0.6158	0.6919	0.7723
	5	0.7877	0.5681	0.6084	0.6796	0.7564
t	6	0.7605	0.5675	0.6019	0.6684	0.7417
	7	0.7364	0.5665	0.5959	0.6582	0.7281
	8	0.7150	0.5651	0.5906	0.6488	0.7156

Let us compare the posterior mean by our method with Tzougas et al. (2014). Table 4 shows the ratio of old method over the new method. It shows that in all cases the old method produce a much lower rate of number of claim than the new method.

6. Conclusion

This article present a generalised form for optimal Bonus-Malus systems using finite mixture models. We have compare the results of new method with the results of Tzougas et al. (2014) which is the most relevant study to the work. Our results show a big difference between new formulas when those are compared with previous results.

REFERENCES

1. Denuit, M. and Ph. Lambert (2001). Smoothed NPML estimation of the risk distribution underlying Bonus-Malus systems. Proceedings of the Casualty Actuarial Society, **88**, 142–174.
2. Frangos, N. and Vrontos, S. (2001) Design of optimal bonus-malus systems with a frequency and a severity component on an individual basis in automobile insurance. *ASTIN Bulletin*, **31**, 1–22.
3. Gomez-Deniz, E., Vaquez-Polo, F.J., Perez-Sanchez, J.M. (2004). A note on mixture prior distributions with applications in actuarial statistic. *Estudios de Economia Aplicada*, **22 (2)**, 1–15.
4. Tzougas, G., Vrontos, S., and Frangos, N. (2014). Optimal Bonus-Malus Systems using Finite Mixture Models. *ASTIN Bulletin*, **44**, 417–444.
5. Peter Macdonald and with contributions from Juan Du (2012). mixdist: Finite Mixture Distribution Models. R package version 0.5-4. http://CRAN.R-project.org/package=mixdist
6. Mahmoudvand, R., and Hassani, H. (2009). Generalized bonus-malus systems with a frequency and a severity component on an individual basis in automobile insurance. *ASTIN Bulletin*, **39**(01), 307-315.
7. Mahmoudvand, R., Tan, C. & Abbasi, N. (2017). Adjusting the Premium Relativities in a Bonus-Malus System: An Integrated Approach Using the First Claim Time and the Number of Claims. *Asia-Pacific Journal of Risk and Insurance*, Forthcoming, doi:10.1515/apjri-2016-0038
8. Weinberg, G.V., Finch, D. (2012). Analysis of a Pareto Mixture Distribution for Maritime Surveillance Radar. Journal of Electrical and Computer Engineering, http://dx.doi.org/10.1155/2012/382979.